Hybridization can facilitate species invasions, even without enhancing local adaptation

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The founding population in most new species introductions, or at the leading edge of an ongoing invasion, is likely to be small. Severe Allee effects—reductions in individual fitness at low population density—may then result in a failure of the species to colonize, even if the habitat could support a much larger population. Using a simulation model for plant populations that incorporates demography, mating systems, quantitative genetics, and pollinators, we show that Allee effects can potentially be overcome by transient hybridization with a resident species or an earlier colonizer. This mechanism does not require the invocation of adaptive changes usually attributed to invasions following hybridization. We verify our result in a case study of sequential invasions by two plant species where the outcrosser Cakile maritima has replaced an earlier, inbreeding, colonizer Cakile edentula (Brassicaceae). Observed historical rates of replacement are consistent with model predictions from hybrid-alleviated Allee effects in outcrossers, although other causes cannot be ruled out.

Colonizing species rarely encounter empty environments and may, as a result, come into contact with close relatives. The most intensively discussed case of related colonizer–resident interactions, although by no means the only example, is the entry of modern humans into regions occupied by Neanderthals (1), eventually replacing them. Colonizing and resident species can interact in various ways, with consequences for their population dynamics. Competition, for example, is likely to impede successful establishment and population growth of newcomers; in contrast, resident species may facilitate new colonizers through hybridization or by shared interactions with organisms from other trophic levels. Although some interactions have been identified and are widely considered to predominate, there may be other interactions whose significance is yet to be explored. These interactions may be important in some or many cases, and a better knowledge of them might alter our interpretations of empirical observations. Here, we will investigate one such potential interaction, arising from hybridization between plant species sharing animal pollinators.

Hybridization can introduce adaptive alleles that contribute to expansion into new habitats (2, 3) and may result in new taxa. Local adaptation, resulting from changes in fitness, is the only positive role for species hybridization considered in the invasion literature; the same is true for admixtures of species genotypes from multiple sources. However, is it possible that there could be other benefits from hybridization, purely demographic ones? Furthermore, are these additional benefits capable of altering invasion dynamics over similar timescales and magnitudes? If so, such interactions would need to be excluded before adaptation can merely be assumed.

Many colonizing populations are susceptible to the demographic challenge of Allee dynamics, either because the initial founding population is small (4, 5) or because rare, long distance dispersers, beyond the present range, will initiate low-density satellite populations (6). Allee effects could slow range expansion or even halt it completely (7). In plants, a small founding population can experience Allee effects because of a scarcity of compatible mates, low pollinator visitation, or both (8, 9); the term “pollen limitation” is often used as a generic term when the exact mechanism is unclear. In the extreme case of a single arriving adult, population persistence would normally be impossible unless the species is capable of asexual reproduction or self-fertilization (“Baker’s Law” (10)). Here, we propose a positive role for hybridization in species invasions and range expansion, a purely demographic mechanism without the requirement for any new adaptation to result.

If a mate-limited colonizing species is capable of hybridizing with a species already present, the colonizing species could overcome the otherwise insurmountable limitations imposed by Allee effects. Early-generation interspecific crosses (i.e., hybrids) could enable the colonizer genes to establish. Pure colonizer-type individuals can subsequently arise through crossing among hybrid lineages (11) or repeated backcrossing with the colonizer parents. We further hypothesize that this hybridization-rescue effect is more likely to eventuate if the new species and hybrids are more attractive to pollinators or when the hybrids are more compatible (i.e., more likely to produce viable offspring) with the newcomer than with the established species. That is, after sufficient generations of asymmetric breeding (backcrossing to the new colonizer), plants will increasingly come to resemble the original newcomers. The arriving species will essentially have been reconstituted, at least in its nuclear genome, and its population can increase now that it has escaped the critical effects of Allee dynamics. In effect, resident populations of a cross-compatible

Significance

Species colonizations frequently occur in regions already occupied by other species of the same genus, or by other genotypes of the same species. When hybrid genotypes become more invasive than their parents, improved local adaptation is inevitably assumed. Here, we show that there is another way that hybridization could facilitate invasions, by temporarily overcoming pollen limitation when populations are small. We verify this conclusion using two contemporary sequential invaders, a plant analog of the Neanderthal–modern human system. Our findings indicate that it may be misleading to assume that all cases of invasive hybrids result from enhanced phenotypic suitability. Our hypothesis may be even more relevant to animal species because, unlike plants, animals can rarely escape mate limitation by self-fertilization.

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species, which may otherwise negatively affect the newly colonizing species through competition, become stepping stones through the landscape for a self-incompatible species via hybridization.

We develop a model that confirms that transient hybridization can overcome Allee effects under a wide range of ecological settings in relation to breeding system, pollinator behavior, and life history traits. Moreover, we show in a case study using a simplified version of the model that the parameter values required for the demographic mechanism to be as effective as genetic changes in fitness are fulfilled by the sequential invaders Cakile edentula and Cakile maritima. Our hypothesis therefore provides a possible explanation for the rapid replacement of C. edentula over a large part of its invasive range by C. maritima in the west of North America, New Zealand, and Australia (12–15).

Results

Our hypothesis is clearly supported by the model results. In the absence of hybridization, when we introduce a small number of seeds, the incoming species population declines rapidly to extinction (Fig. 1). The same propagule pressure but with hybridization transforms the population from one dominated by resident species and resident-like genotypes (blue in Fig. 1B) to one dominated by colonizer and colonizer-like genotypes (red in Fig. 1B). Over time, the colonizer genotype is reconstituted and the resident species is eventually replaced by the new colonizer. Allee population thresholds are always smaller with hybridization than without it, proving that introgression is asymmetric in favor of the colonizer [i.e., when $W_0 > 1$ (Fig. 2A) or $\beta > 1$ (Fig. 2B)]. Even with no bias in pollinator behavior ($W_0 = 1$) or compatibility ($\beta = 1$), hybridization still reduces the Allee threshold slightly (Fig. 2) because some of the otherwise wasted pollen results in seed production via the resident species, thus contributing positively to the dynamics of colonizer genes.

Consistent with the established theory that selfing provides reproductive assurance (16), the Allee threshold for establishment of colonizing species decreases with its selfing rate for both

![Figure 1](image1.png)

**Fig. 1.** Dynamics predicted by the generic model when a small number of a colonizing species arrives at a new location currently occupied by a resident species: no hybridization (A) and hybridization (B). Although in both A and B, pollinator preference parameter $W$ is set to favor more backcrossing to the incoming genotype than to the resident species, the incoming genotype fails to establish when it does not hybridize with the resident species (A). However, when there is hybridization (B), the population steadily accumulates a higher proportion of individuals similar to the colonizer, whereas the resident genotypes decline in abundance and are finally replaced by the colonizer. Dark red indicates individuals whose genome is indistinguishable from the colonizer, dark blue indicates genomes indistinguishable from the resident species, and gray represents hybrid genotypes comprising 50% of each parental species’ genome. See Tables S1 and S2 for lists of parameters and their values.

![Figure 2](image2.png)

**Fig. 2.** The critical number of propagules for the establishment of a colonizing species, the predicted Allee threshold, as a function of either pollinator behavior (assuming equal compatibility, $\beta = 1$) (A) or reproductive compatibility of hybrids with the parental genotypes (assuming equal pollinator preference, $W_0 = 1$) (B). Note that both A and B would also indicate the results expected from passive pollination, such as by wind. Compatibility, $\beta$, has no effect on the dynamics of either species when they do not hybridize: the Allee threshold is therefore constant for the no-hybridization scenario (solid line in B). When $W_0 = 1$ and $\beta = 1$, backcrossing is symmetric, but when $\beta$ is larger than 1, F1 and later generation hybrids are more compatible with the colonizing-type individuals. In our simulations, the criterion for the Allee threshold was that the population size (seed number) after 100 generations was the same as the initial population size. See Tables S1 and S2 for lists of parameters and their values.
hybridization and no-hybridization scenarios. However, Allee thresholds were always smaller for a hybridizing colonizer than the nonhybridizing one over a wider range of simulated breeding systems of both species (Fig. 3). Because the resident species adopts a more autogamous breeding system, Allee thresholds become larger in the colonizing species, but with hybridization, the incoming species could still invade the resident population (Fig. 3).

To test our predictions in a contemporary invasion, we parameterized a simplified version of our model with a single hybrid category (F1 hybrids), using empirical data from Cakile maritima [self-compatible (SC) (17)] and C. edentula [self-incompatible (SI) (17, 18)]. Why and how the established invader (C. maritima) in three parts of the world has remained a mystery over the 40 y since the replacement was first reported (13). Could hybridization with the prior SC invading plant have increased the probability of subsequent establishment of an SI species? Hybrids between the two species can be produced easily by hand (17). We have also confirmed, using genetic markers, that individuals with intermediate morphology near the leading edge of the C. maritima invasion in Tasmania are the results of hybridization (19).

We simulated the dynamics of C. maritima with and without hybridization with C. edentula. The results indicate that hybridization could facilitate the establishment of a small (otherwise sub-Allee threshold) population of the primarily self-incompatible C. maritima (Fig. 4), with a strong correspondence between the timescales for species replacement predicted by the model simulation (Fig. 4A) and the frequency of C. maritima relative to C. edentula in herbarium specimens (Fig. 4B) (20). Without hybridization, small initial populations of C. maritima were predicted to fail to establish (Fig. 4C). A note of caution is required for this comparison: our model simulates the population dynamics within a single location (with no account of subsequent spread), whereas herbarium data give a (very crude) representation of invasion dynamics over a wider geographic scale (21). Our simulation results are, however, also in good agreement with the replacement time-scale observed on Lord Howe Island, where C. maritima replaced C. edentula in perhaps 20–30 y (14).

Our simulation model and the Cakile system provide support for our hypothesis of an overlooked, purely demographic, role for hybridization in both establishment and spread. A robust test of the hypothesis would require deliberate introductions into regions lacking one or other species, which would probably be unethical or unwise. However, in the case of SI and SC Cakile species in Australia, there is now clear evidence that interspecific hybridization has occurred during the establishment and expansion of C. maritima (19). Genetic and morphological evidence across the current C. maritima invasion wave-front in Tasmania...
(19) match what would be expected if species replacement by this mechanism had occurred. Specifically, in places where C. edentula has a very high relative frequency, hybrids are common and even plants that are morphologically indistinguishable from one parent often have the chloroplast genotypes of the other (19). In regions where C. edentula has long since disappeared and there is now little evidence of nuclear introgression, C. edentula chloroplasts may still be common in C. maritima, indicating extensive past hybridization and reconstitution of parental phenotypes. Although our artificial intercrossing provided no evidence for bias in cross-compatibility of hybrids with the two parental species (Fig. S3), pollinators clearly preferred to visit C. maritima-like individuals over C. edentula when both were present (SI Text, Simplified Model for C. maritima and C. edentula, Sources of data for parameterization). We note that introgression has been proposed before for these species (22) but based on less convincing data and without any mechanism being proposed (19).

If this demographic phenomenon does occur in nature, is it idiosyncratic—a unique combination of species sharing pollinators and invading the same habitat in the same sequence—and of little relevance to ecology and evolution in general, or is it more common? We believe that the latter may be true. First, both hybridization (2) and Allee effects are common in biological invasions (7). Evidence is mounting for the importance of hybridization as one of the processes driving invasions, although so far, the explanations have relied solely upon genetic consequences of hybridization endowing adaptive benefits (refs. 2 and 3 but see ref. 11). Indeed, the risks of extinction and genetic swamping of native species have been raised as dire consequences of hybridization (2). Allee effects or, more specifically, pollen-limited seed production, are expected to be common, because more than 80% of plants rely on pollen transfer for reproduction (23) and ~50% of plant species are obligate outcrossers (24). Pollen limitation, resulting from low availability of both compatible mates and pollinators, seems to be more common in introduced species than their native counterparts (25, 26). First, our model shows that the presence of another species can alleviate such pollen limitation and reduce Allee effects through hybridization with related species. Second, our model shows that the rescue effect of hybridization will be stronger when there is an asymmetry in the direction of introgression (backcrossing); this phenomenon seems to be common in both natural and artificial hybridizations. Pollinators rarely commit to random foraging bouts but rather discriminate among plant types (27); such behavior can result in assortative mating and produce the required asymmetry. Intercrossing success (i.e., the production of viable seeds) is also known to be affected by the direction of crossing (28, 29). The interplay between these pre- and/or postzygotic processes can result in unequal transfer of parental genetic material into the genome of hybrid progeny (28, 29). Whereas the short-term demographic consequence of this asymmetry is facilitation of establishment, the long-term outcome can be species replacement as we see in Cakile species.

Materials and Methods

We developed a density-dependent, time-discrete, deterministic model based on the life cycle of an annual plant with no persistent seedbank and occurring in a small, isolated patch of suitable habitat. The model incorporates demography, mating systems, pollinators, and quantitative genetics through the following recurrence difference equation:

\[ N_{x(t+1)} = \sum_{y} M_{y} \delta_{y} F_{y}(M) \left[ s_{y}(H(x,y),y) - (1 - s_{y}) \sum_{z} C_{y}(z,2) V_{y}(M;H(x,y,z)) \right], \]

where \( N_{x(t)} \) is the number of seeds from genotype \( x \) in generation \( t + 1 \), and \( M_{y} = N_{0} R_{y} S_{N} / N \) denotes the total number of female adults with genotype \( y \), surviving from \( N = \sum_{N_{x(t)}} \) total seeds according to a Beverton–Holt type recruitment function, \( S(N) \) (Eq. S1); \( \delta_{y} \) is the per capita ovule production of female genotype \( y \) in the absence of neighboring plants, which decreases with total adult population size \( M = \sum_{x} M_{x} \) according to a rectangular hyperbolic function \( F_{y}(M) \) (Eq. S3) because of resource competition and other negative interactions between plants. A fraction \( s_{y} \) of flowers produces seeds through selfing, whereas the \( 1 - s_{y} \) nonselfed proportion relies on pollinators for seed production. Not all pollen from other genotypes will result in viable offspring; hence the compatibility of genotype \( y \) with genotype \( z \) is given by \( C(x,y) \) (Fig. S2). The relative frequency of genotype \( y \), adjusted for pollinator preference (Fig. S3), is \( V_{y} \), whereas the probability of pollination is related to the total adult population size following a Holling type III functional response, \( H(x,y,z) \) (30).

We used the number of chromosomes as a proxy for determining the total number of genotype classes (which is \( 2n + 1 \)) and the chromosome combination of the two parental species to characterize \( x \) (Fig. S4). That is, a pure individual of the incoming species, denoted by \( x = 0 \), has no alien chromosome from the resident species whereas the karyotype of an F1 hybrid embodies an equal number of chromosomes from the two parental species, so it is shown as \( x = 0.5 \). The proportion of ancestry from species decreases as \( x \) approaches 1, whereas that of the resident species increases, with \( x = 1 \) indicating a pure individual of the resident species with no alien chromosome from the incoming species. A diploid individual then undergoes gametogenesis following the principle of independent assortment of chromosomes, assuming no crossing over, where a haploid gamete randomly receives a mixture of chromosomes from the two parental species. By means of the convolution of the genotype types of the two parents, we can provide the probability of two alleles from the two crossing diploid individuals, we obtained the probability \( H(x,y,z) \) of two parents with genotype \( y \) and \( z \) producing an offspring with genotype \( x \). In essence, our quantitative genetic model is similar to the hypergeometric phenotypic model used previously to study sympatric speciation (31, 32) and risk of extinction by hybridization (33). We mapped genotypic class \( x \) to a phenotypic character using a Gaussian model (30) (Fig. S5). We simulated the population dynamics of 19 genotype classes corresponding to a diploid species with \( 2n = 18 \) over 100 generations. This value was chosen to correspond to Cakile spp. (\( 2n = 18 \)) which we consider later. A smaller or larger number of chromosomes will respectively either increase or decrease the speed at which species replacement will occur but without altering the qualitative dynamics. Full details of the model and simulations are provided in Supplementary Text, Methods, and the Dynamics of Hybridizing Annual Species, Generic Model formulation and SI Appendix.

We compared the population dynamics of the newcomer under two scenarios: (i) hybridization with an established species; and (ii) an established relative present but no hybridization. Fecundity and survival parameters were chosen to be constant and equal for the two species and for all hybrids to ensure no fitness advantage or disadvantage from hybridization. These assumptions can be altered according to the question being investigated. As a measure of establishment success, we estimated “Allee thresholds” as the minimum number of colonizing individuals required to ensure positive population growth for each scenario. A facilitation effect for hybridization would be apparent as a smaller Allee threshold than that observed for the no-hybridization scenario. We investigated the sensitivity of the model predictions over a wide range of parameter values related to initial population size, mating systems of the interacting genotypes, intercrossability, pollinator behavior, and pollination-plant density relationship (Table S2).

To verify the findings of the above theoretical model, a reduced version including only three genotypes, the two parental species, and a single hybrid class (34), was parameterized using empirical data from Cakile maritima and C. edentula (see Fig. S6 for example simulation). Parameter values related to fecundity were obtained from a common garden experiment. The setting rates and cross-compatibility of the genotypes were quantified in two artificial crossing experiments (Fig. S1). Pollinator visitation rates were obtained by monitoring a population where the two species had similar relative frequencies. For hybrids, we used the average phenotypic values of the two parental species when no data existed. For all other parameters, we used published data and if no data were available, we ran the model over a wide range of parameter values to ensure consistent outcomes (e.g., Fig. S7). See SI Text, Simplified Model for C. maritima and C. edentula for detailed descriptions of the estimation of parameters.

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Supporting Information

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SI Text

Generic Model to Simulate the Dynamics of Hybridizing Annual Species.

**Generic model formulation.** We first construct a deterministic population model for a single annual species, of a single genotype, and then show how it can be extended to two species, to include more cooccurring genotypes that arise because of hybridization and later-generation backcrossing. Throughout we use population density and population size (or number) interchangeably, assuming that for a fixed area, changes in density are equal to changes in number.

**Single species.** Suppose that a population consists of $N$ diploid seeds at time $t$, $N(t)$. Seeds germinate and are recruited into the population as seedlings and assumed to be spread evenly within the receiving patch of habitat. The proportion surviving to maturity is a function of both density-dependent and density-independent factors, collectively represented by $S(N)$. Each surviving plant has the potential to produce $\theta$ ovules in the absence of competition from other plants, but its per capita ovule production decreases with population density, following a negative density-dependent function $F(M)$, where $M$ indicates number of adult plants: $M = NS(N)$. Each plant can self-fertilize a fraction $s$ of its ovules, whereas the remaining $(1 - s)$ fraction is available for outcrossing. Because of pollinator or mate limitations, however, not all $1 - s$ will be outcrossed and some will be aborted. We assume that the probability of successful pollination is positively related to the population density (i.e., there is an Allee effect) following $\theta(M)$. A positive correlation between population density and pollinator visitation rate has been observed in many studies and has been implemented in population dynamics models (35). The difference equation with both negative (ovule production and survival) and positive (pollination success) density regulation for a single species’ population dynamics is therefore given by

$$N(t + 1) = M(N(t))0F(M(N(t))[s + (1 - s)\theta(M(N(t)))]).$$  \[S1\]

For density-dependent and density-independent recruitment of seeds into the population, we use a Beverton–Holt type model:

$$S(N) = \frac{L}{1 + bN}.$$  \[S2\]

where $L$ denotes the proportion of seeds recruited independent of population density, whereas $b$ is the rate of decrease in survivorship of a plant as density increases. We used a rectangular hyperbola to describe ovule reduction as related to density of mature plants, $M$:

$$F(M) = \theta \left[1 - \frac{dM}{1 + dM}\right].$$  \[S3\]

where $d$ is a coefficient; the larger the value of $d$, the stronger the negative effect of density on reproduction. Given that $M = NS(N)$, Eq. S3 can be written as a function of $N$:

$$F(M(N)) = \theta \left[1 - \frac{LdN}{1 + LdN}\right].$$  \[S4\]

which can be simplified to

$$F(M(N)) = \frac{\theta(bN + 1)}{N(b + Ld) + 1}.$$  \[S5\]

For pollination probability, or “pollinator foraging choice” (30), we used Holling type III functional response to relate the probability of pollination to population size:

$$\phi(M) = \frac{(\rhoM)^\gamma}{1 + (\rhoM)^\gamma}.$$  \[S6\]

and, again,

$$\phi(M(N)) = \frac{\left[\frac{\rho NL}{1 + N\theta} \right]^\gamma}{1 + \left[\frac{\rho NL}{1 + N\theta}\right]^\gamma}.$$  \[S7\]

This type of functional response is expected for generalists that frequently switch between flower types (35) or if pollinators engage in an area-restricted search strategy (36). The reciprocal of $\rho$ gives the population size at which 50% of plants will be visited (and pollinated) by pollinators. The parameter $\gamma$ determines the shape of the curve: when $\gamma$ is large, there is little change in probability of pollination at low densities compared with a small $\gamma$, which gives a higher pollination rate at the same low densities. When $\gamma = 1$, the model reduces to a type II or saturating functional response. Eq. S7 can be simplified to $\phi(M(N)) = \frac{1 - \rho NL}{1 + \rho NL}$, and Eq. S1 becomes

$$N(t + 1) = \frac{N(t)L}{1 + bN(t)} \left(1 - \frac{\theta(bN(t) + 1)}{N(t)(b + Ld) + 1}\right) \times \frac{s - (1 - s)}{s - (1 - s)} \left[\frac{\rho NL}{1 + N\theta} \right]^\gamma \frac{1}{1 + \left[\frac{\rho NL}{1 + N\theta}\right]^\gamma}. \quad \text{[S8]}$$

This model results in component Allee dynamics (37) by negatively relating recruitment and per capita fecundity to population density, while positively scaling pollination probability.

**Two species.** Now suppose that an equally competitive congenic species already occurs in the habitat (we will call this a resident, which could be either a native or a previously established invader) and can interact with the above species in both positive and negative ways. The assumption is made that both species are homogeneously distributed across space. A negative interaction occurs because of competition between the two species for limited abiotic resources and is reflected in their reproduction, $F(M(N))$, and survival, $S(N)$, functions. The intensity of competition, embedded within $F(M(N))$ and $S(N)$, is assumed to depend on the total number of both species regardless of the species identity. This equality in competitiveness of species is required to ensure that any difference between hybridization and nonhybridization scenarios is solely attributable to hybridization and not to an a priori fitness differential. However, the model can accommodate species inequality in any traits, as shown in Eq. S16. A positive interaction can occur when there is a positive relationship.
between the chance of pollination and the total population size, such that \( M \) on the right hand side of Eq. S6 is replaced by the combined number of individuals from the two species. However, the two species may also compete for pollinators and, depending on their relative attractiveness to pollinators, may experience differential outcrossing success, accounted for by the \( V_x \) function (see below). Using subscript \( x \), where \( x = 0 \) denotes the invader and \( x = 1 \) indicates the resident species, the difference equation for two interacting species is given by

\[
N_x(t+1) = M_x(N_x(t))F_t \left( \sum_{k=0}^{1} M_k(N(t)) \right) \times 
\left[ 1 - (1 - s_x)V_x(N_x(t))\phi \left( \sum_{k=0}^{1} M_k(N_x(t)) \right) \right].
\]

[S9]

where \( N \) is the total density summed across all genotypes: \( M_x(N) = \sum_{k=0}^{1} M_k(N) \) with \( k = 0 \) (invader) and \( k = 1 \) (resident). \( V_x(N) \) is the proportion of pollinator visits to species \( x \). If both species are equally preferred by pollinators (i.e., visited randomly) then the likelihood of a visit to any given species is proportional to its frequency in the population, i.e., \( V_x(N) = M_x(N)/\sum_{k=0}^{1} M_k(N) = R_x \), where \( R_x \) is the relative frequency of adult plants. However, the two species may differ in their attractiveness such that pollinators visit \( W_x \) plants of the invasive species for every one plant of the resident species. Then the proportion of visits to the invasive species is

\[
V_o = \frac{WR_0}{WR_0 + R_1},
\]

[S10]

and to the resident is

\[
V_i = \frac{R_1}{WR_0 + R_1} = 1 - V_o,
\]

[S11]

where \( R_0 \) and \( R_1 \) are the relative frequencies of invader and resident plants in the mixed population (\( R_0 + R_1 = 1 \)), whereas \( V_o \) and \( V_i \) are adjusted frequencies after accounting for the bias in pollinator preference (see Fig. S1 for a graphical explanation of the pollination submodel). When the two species hybridize and produce viable offspring, the population will consist of three genotypes in the next generation: two parental pure species plus the \( F_1 \) hybrid, so \( x = (0,0.5,1) \). We use \( x = 0.5 \) to indicate that the hybrid genotype is intermediate relative to its parents. In the later generations, more intermediate genotypes will be produced through backcrossing and segregation (accounted for in Eq. S12), so \( x \) can then take any values between 0 and 1. However, as a workable assumption, we consider variable \( x \) as a discrete variable with \( G \) genotypic classes. We use the number of chromosomes as a proxy for determining \( G \) (the total number of genotype classes) and the chromosome combination of the two parental species to characterize \( x \). That is, suppose that species \( A \) (e.g., invasive) and \( B \) (resident) both have \( 2n = 18 \) chromosomes and can freely cross with each other. Individuals from this mixed population can then be categorized in \( 19 \) genotype classes (\( G = 2n + 1 \)) based on the number of chromosomes that they receive from the two species [i.e., (18,0), (17,1), ... (1,17), (0,18)]. The first element in (\( \bullet, \bullet \)) indicates the number of chromosomes from species \( A, A_o \), whereas the second element is the number of chromosomes from species \( B, B_o \). An individual with an (18,0) chromosome combination is regarded as a pure individual of species \( A \), whereas (0,18) denotes a pure individual of species \( B \). We map chromosome combinations to variable \( x \) using \( x = A_o/2n \), which gives \( x = 1 \) for species \( A \) and \( x = 0 \) for species \( B \) (the use of \( A_o \) in the numerator is arbitrary). In this paper, we assign \( x = 0 \) to the invading species and \( x = 1 \) to the resident species. Crossing-over can result in more intermediate genotypes, where a single chromosome can comprise various proportions of the genome from the two species, but for simplicity, we do not consider this possibility in our model. It should not affect our qualitative conclusions.

A diploid genotype with a total of \( t \) alien chromosomes (\( t \leq n \) and \( t \neq 0 \)) undergoing gametogenesis can produce \( t + 1 \) haploid gametes (an \( F_1 \) hybrid which has \( n \) alien chromosomes can produce the maximum number of gamete types, which is \( n + 1 \)). A gamete can possess \( \nu \) alien chromosomes where \( \nu \) range from 0 up to \( t \) (i.e., \( \nu \in \{0,1, \ldots, t\} \)) with probability given by

\[
P(\nu|t) = \frac{\binom{n}{\nu} \left( \frac{n}{n+1} \right)^t}{\sum_{i=\nu}^{n} \binom{n}{i} \left( \frac{n}{n+1} \right)^{t-i}}.
\]

[S12]

This formula is based on the number of ways that chromosomes can line up at the cell equator during the metaphase of the first meiotic division. As shown in Fig. S2, with \( \ell = 4 \), various arrangements are possible: (i) all alien chromosomes (red) can occur on the same strand, (ii) one alien chromosome on the left strand and three on the right, and (iii) two alien chromosomes on either side. Within each of these arrangements, many different combinations (i.e., positioning of aliens along the native strand) can take place. Indeed, \( \binom{n}{\nu} \) in the numerator of Eq. S12 gives the number of possible ways that \( \nu \) alien chromosomes (out of \( t \)) can be positioned on the left strand, whereas \( \binom{n}{t-\nu} \) gives the number of positions for the remaining alien chromosomes (i.e., \( t-\nu \)) on the right strand. Note that when the number of alien chromosomes on the left and right strands are the same (i.e., \( \nu = t-\nu \)), the number of combinations for that gamete type doubles. The denominator in Eq. S12 simply gives the total number of all possible combinations.

Now, suppose that a female genotype \( y \) with \( t_i \) alien chromosomes is crossed with a male genotype \( z \) with \( t_z \) alien chromosomes. We obtained the probability of producing offspring \( x \) (with \( x \in \{0, \ldots, n\} \), where \( L = t_i + t_z \)) from parents with genotypes \( y \) and \( z \) using a discrete convolution:

\[
H(x:y,z) = \sum_{h} y_h z_{x-h+1}.
\]

[S13]

In this formula, \( y = P(\psi|\ell) \) and \( z = P(\psi|\ell) \) are two vectors containing the probabilities for gamete types that can be produced by parents \( y \) and \( z \), respectively, following Eq. S12, and \( h \) ranges over all legal subscripts for \( y_h \) and \( z_{x-h+1} \), specifically, \( h = \max(1,x+1-L), \ldots, \min(x,L) \). Note that we transform \( t_i \) to \( x \) using \( x = 1 - t_i/2n \), which represents the proportion of alien chromosomes within the genome ranging from 0 (pure invasive species) to 1 (pure resident species). Our genetic model gives rise to similar qualitative outcomes as the hypergeometric phenotypic model (31, 32). The difference equation model that allows for hybridization then takes the form:

\[
N_x(t+1) = \sum_{j=1}^{G} M_x(N_x(j))\theta_{F_t} \left( \sum_{k=1}^{G} M_k(N_x(j)) \right) \times 
\left[ 1 - (1 - s_x)C(y,z)V_x\phi \left( \sum_{k=1}^{G} M_k(N_x(j)) \right) \right] \times H(x:y,z),
\]

[S14]

where \( G = 2n + 1 \) indicates the total number of genotypes in the population. The function \( C(y,z) \) in Eq. S14 indicates the compatibility (i.e., the likelihood of producing viable offspring
once pollinated) of female genotype $y$ with the male genotype $z$ and is calculated using

$$C(y, z) = 1 - (\alpha - 1) |y^2 - 2y^2 z^n + z^n|,$$  \[\text{S15}\]

where parameter $\alpha$ indicates the compatibility between the two parental species, whereas $\beta$ introduces asymmetry in the direction of backcrossing (asymmetric compatibility). The model (Eq. S15) assumes that the compatibility between two genotypes increases as these two genotypes come to share more genome from the same species. For the specific cases of $y = z = 0$ (cross between two pure individuals of the incoming species) and $y = z = 1$ (cross between two pure individuals of the established species), the model gives $C(y, z) = 1$ but $C(y, z) \neq 1$ if $y = z = \beta$ or $y = z = 1$. Note that $C(y, z) = 1$ always holds if $\beta = 0$ or $\alpha = 1$. When $\beta > 1$, $F_1$ and later generation hybrids are more compatible with the incoming-type individuals, whereas for $\beta < 1$ compatibility is biased toward the resident species and for $\beta = 1$ introgression is symmetric (Fig. S3). Studies with both natural and artificial crossing have shown that the intercrossing success rate can be affected by the direction of crossing (38), and our model accounts for such an asymmetry by incorporating the bias parameter $\beta$.

To relate genotype $x$ to a phenotypic character $T$ (any model parameter), we assumed that trait values (e.g., selfing rate) are normally distributed among genotypes, following Hall et al. (39) with minor modifications:

$$T(x) = T_{\text{max}} \exp \left( -\frac{(x - x_{\text{max}})^2}{2\sigma^2} \right),$$  \[\text{S16}\]

where $T_{\text{max}}$ is the maximum value of a given trait, whereas $x_{\text{max}}$ is the genotype which acquires that maximum trait value. The parameter $\sigma^2$ represents the variance, which can be solved to give the minimum value of $T$ for a specific genotype. For example, if disturbance occurs first whereby many seedlings die, then the density-dependent mortality would be minimal.

A list of model parameters is given in Table S1. To summarize, some of the assumptions of our model are as follows:

i) There is no persistent seedbank.

ii) Both density-dependent and density-independent factors act at the same time to determine the survival rate. For example, if disturbance occurs first whereby many seedlings die, then the density-dependent mortality would be minimal.

iii) Self-fertilization occurs first, and the remaining flowers then out-cross.

iv) Pollination success is only determined by population size and attractiveness of genotypes (the frequency of pollen from any genotype in the pollen pool is proportional to the relative abundance of adult plants of that genotype in the mixed population and the per capita pollen production is therefore the same for all genotypes).

v) There is no difference between chromosomes when mapping genotype to phenotype (i.e., it is the number of chromosomes that matters, not their type).

vi) There is no crossing over.

vii) Self-fertilization incurs no inbreeding depression. Inbreeding depression could easily be incorporated into the model as another multiplier (for example, of selfing rate $s$) in Eq. S1 and its multigenotype derivation (Eq. S14), but the results would not change qualitatively.

**Simulations.** We simulated the population dynamics of 19 genotypic classes corresponding to a diploid species with $2n = 18$ over 100 generations. All simulations were run using Matlab (version 2014a); code is given in SI Appendix. Table S2 provides details of parameter values (and range) used for various simulations related to the effect of pollinator behavior, mating system, compatibility between genotypes under hybridization, and no-hybridization scenarios.

**Simplified Model for C. maritima and C. edentula.**

**Model assumptions.** To test the hypothesis that hybridization could have facilitated the invasion of *C. maritima* without enhancing fitness, we used a reduced version of the generic model. This simplified model simulates the invasion dynamics of only three genotype classes (34): *C. maritima*, *C. edentula*, and their F$_1$ hybrid. The variable $x$ thus takes three values: 0 for *C. maritima*, 1 for *C. edentula*, and 0.5 for hybrids. The main reason for reducing the number of genotypes is that there is no information on the demography and biology of intermediate genotypes (later generation hybrids). Furthermore, this simplification makes the model more tractable without affecting the qualitative conclusions. We used intercrossing compatibility rates obtained from artificial hybridization experiments rather than using Eq. S15.

The model was parameterized using both the literature and our own experimental or observational data; for a few parameters, for which there were no estimates, we evaluated a range of plausible parameter values to test whether their precise values affect the qualitative results. If no parameter estimate was available for the hybrid genotype, we used the average of the two parental species, because hybrids tend to be intermediate in most characters relative to their parents (17). See Table S3 for a list of parameter values and their sources: although some parameters are in favor of *C. maritima* (e.g., higher fecundity and survival) compared with *C. edentula*, this inherent fitness advantage alone is insufficient for *C. maritima* to drive a well-established *C. edentula* population to extinction (Fig. 4).

**Initial population sizes.** Almost no data exist on the number of arriving propagules in accidental introductions (4); clearly, the number could be as few as one or it could be several orders of magnitude greater, depending on the dispersal vector and chance. We have no information on the founding population sizes (number of seeds arrived) of *C. maritima* when the species was first introduced to Australia, or even for its dispersal from one beach to the next during subsequent spread. However, we know that *C. edentula* had time to saturate most of its preferred habitats by the time *C. maritima* arrived as the second invader. The number of *Cakile* adult plants that can be found on a beach is highly variable, so it is equally hard to say what a plausible carrying capacity would be. Highly disturbed beaches may function as sink populations and never support more than a few individuals, whereas a sheltered beach may be a source population of many thousands of plants and a net exporter of seeds. In our simulations, we fixed the carrying capacity of a beach at 500 adult plants (regardless of the species: this value is within the range of our field observations for small beaches) and then introduced 1–600 seeds of *C. maritima* (with unit increment).

We simulated the dynamics of the three genotypes over 40 generations. To estimate the Allee threshold for *C. maritima*, we compared the number of *C. maritima* seeds after 40 generations with the initial size of the founding population: the minimum initial seed number that resulted in a positive population growth was taken with the initial size of the founding population: the minimum initial seed number that resulted in a positive population growth was taken compared to the number of seeds arrived of *C. maritima* when the species was first introduced to Australia, or even for its dispersal from one beach to the next during subsequent spread. However, we know that *C. edentula* had time to saturate most of its preferred habitats by the time *C. maritima* arrived as the second invader. The number of *Cakile* adult plants that can be found on a beach is highly variable, so it is equally hard to say what a plausible carrying capacity would be. Highly disturbed beaches may function as sink populations and never support more than a few individuals, whereas a sheltered beach may be a source population of many thousands of plants and a net exporter of seeds. In our simulations, we fixed the carrying capacity of a beach at 500 adult plants (regardless of the species: this value is within the range of our field observations for small beaches) and then introduced 1–600 seeds of *C. maritima* (with unit increment).

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Plants were derived from seeds of *C. maritima* and *C. edentula* and the progeny of putative hybrids (later confirmed as hybrids by simple sequence repeat markers) collected from five, six, and two locations, respectively, across their invaded ranges in Australia. The number of flowers produced by each plant over its lifetime was determined when plants were no longer flowering, by counting the number of scars and pedicels left on the racemes. *Cakile maritima*, *C. edentula*, and putative hybrids produced an average of 1,195 (SE = 72; n = 54), 906 (SE = 59; n = 63), and 1,486 (SE = 128; n = 38) flowers, respectively. These numbers are within the ranges estimated in beach populations in Tasmania.

Data on intercrossing compatibility were determined in two glasshouse experiments, where we first produced F₁ hybrids and then established a series of reciprocal crosses between the F₁ hybrids and their parental species. The probability of producing viable offspring when crossed was calculated from the number of viable (germinable) seeds divided by the total number of crosses made between genotypes. Selfing rates were determined by excluding pollinators. No viable seeds were produced when *C. maritima* was selfed by hand, although a very small number of putative spontaneous selfs occurred on caged plants; it was therefore not possible to measure inbreeding depression. Fig. S6 shows the average compatibility for the various conspecific crossings implemented in these experiments; as can be seen, backcrosses are almost equally compatible in either direction. However, hybrids are more likely to backcross with *C. maritima* than with *C. edentula* because they have larger flowers than *C. edentula*: in our common garden experiment, mean petal dimensions (displayed section of petal only) were 5.42 × 2.01 mm for putative hybrids, 6.19 × 2.45 mm for *C. maritima*, but only 4.35 × 1.19 mm for the inbreeder *C. edentula*, which, on some occasions, may not even produce all four petals (17).

We monitored pollinator visits to 47 *C. maritima* and 37 *C. edentula* plants on a beach in Tasmania where both species were approximately in equal abundance. *C. maritima* has a more conspicuous display, with more open flowers per raceme and larger flowers (see above). Both species were mainly visited by honey bees (*Apis mellifera*); other common visitors were European bumblebees (*Bombus terrestris*) and cabbage white butterflies (*Pieris brassicae*). Although hybrids and their back-crosses may have been present, we chose plants that had all of the characteristics of their assumed parent species. We recorded the time from the start of recording until the first visitor and the number of flowers subsequently visited on that plant. We analyzed these time-to-event data with the Proc Reliability procedure of SAS to fit a lognormal distribution (version 9.3; SAS Institute). The mean times to first visit were 110 s for *C. maritima* and 830 s for *C. edentula*, implying that *C. maritima* was visited by pollinators 7.5 times as frequently as *C. edentula*. Pollinators also visited more flowers per visit on *C. maritima* than *C. edentula*: using a Poisson distribution, the average number of flowers visited per pollinator was 11.7 for *C. maritima* and 6.2 for *C. edentula*.

All details of experiments outlined above can be obtained from R.D.C. For ρ and γ, parameters that describe the relationship between population density and probability of pollinator visit, we investigated the sensitivity of the model predictions over a wide range of parameter values (Table S3), because we had no empirical estimates for these two parameters. As shown in Fig. S7, a model with hybridization always gave smaller Allee thresholds over all tested ranges of these two parameters (Fig. S7).
Fig. S2. Compatibility of F1 hybrid pollen (i.e., $x = 0.5$) with the ovules of various genotypes as predicted by Eq. S15 for three $\beta$ values. When $\beta$ is larger than 1 (here, $\beta = 3$; blue), the hybrid is more compatible (i.e., more likely to produce viable offspring) with the incoming genotype, whereas it has lower compatibility with the resident genotype. When $\beta$ is smaller than 1 (here, $\beta = 0.2$, red), the opposite is true, whereas for $\beta = 1$ (green), all genotypes exhibit the same level of compatibility with the hybrid. In this example, the compatibility of the two pure parental genotypes, $\alpha$, was set at 0.5.

Fig. S3. Fraction of pollinator visits to a given species, as a function of its relative abundance and attractiveness ($W$) in a mixed population of two species which share pollinator services. The likelihood of a visit to the mixed population is initially determined by the total size of the mixed population using Eq. S6, whereas the proportion of visits (of the total probable visits) to a given species is then determined by the attractiveness of that species, using Eqs. S10 and S11. For example, in A, because the total size of population was large (500 individuals), the likelihood of a visit was $\sim 1$, whereas in B, with 50 plants, the maximum likelihood of a visit was 0.5 (these probabilities were calculated using Eq. S6 with $\gamma = 1/50$ and $\gamma = 3.5$). When both species are equally preferred by pollinators ($W_0 = W_1 = 1$), the proportion of visits to any species is equal to their frequency (green line), whereas if the species is either more attractive or less attractive, the visitation probability exhibits a convex ($W = 4$; red line) or concave ($W = 0.25$; blue line) relationship, respectively.
Fig. S4. Some chromosome arrangements for a hypothetical diploid genotype with \(2n = 12\) where eight chromosomes descend from the resident species (blue) and four (\(l =4\)) from the alien species (red). All alien chromosomes may be located on a single strand (A) or occur on the two strands (B and C). The number of ways that alien chromosomes can be positioned within each strand is given by \(\binom{n}{\nu}\) and \(\binom{n}{\ell - \nu}\) for left and right strands, respectively (Eq. S12), and the total number of combinations is the product of these two values. For example, there are 15 different ways to position 4 alien chromosomes on the right strand while having no alien chromosome on the left strand. This arrangement will result in two gamete types (one with no alien chromosomes and one with four) with the equal probability of 0.0208. The inset table provides the probability values for each of the five gamete types (i.e., \(\nu \in f(0, \ldots, 4)g\)) that can be produced by a diploid genotype with \(n = 6\) and \(l =4\).
Fig. S5. An example of mapping genotype $x$ to a phenotypic character (e.g., selfing rate, $s$) using Eq. S16. $x_{\text{max}}$ determines the location (i.e., genotype) at which the maximum value of a trait, $T_{\text{max}}$, occurs (here a selfing rate of 0.9).

Fig. S6. Predicted Allee thresholds for the establishment of *C. maritima*. The critical number of arriving seeds, estimated for two invasion scenarios of *C. maritima* at various initial plant densities of *C. edentula* (relative to the combined carrying capacity). The minimum number of seeds necessary for *C. maritima* establishment is always smaller with (solid line) than without (dashed line) hybridization and is more than halved by hybridization. See Table S3 for parameter values used in above simulations.
Fig. S7. Predicted Allee thresholds for the establishment of C. maritima, as a function of pollination parameters $\rho$ and $\gamma$. The reciprocal of $\rho$ is the population size at which the probability of pollination is 50%, whereas $\gamma$ is a shape parameter. The minimum number of seeds necessary for C. maritima establishment is smaller with (solid line) than without hybridization (dotted line). See Table S3 for other parameter values used in above simulations.
Table S1. List of parameters used in the generic model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>Density-independent survival (i.e., fraction of seeds that germinate and survive to maturity) ( L \in [0,1] )</td>
</tr>
<tr>
<td>( B )</td>
<td>Density-dependent coefficients for seedling survivorship to maturity ( b \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Per capita flower production ( \theta \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( d )</td>
<td>Density-dependent coefficients for flower production reduction ( d \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( s )</td>
<td>Selfing rate (i.e., proportion of flowers that produce seeds through self-fertilization) ( s \in [0,1] )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Pollination switching density parameter: ( 1/\rho ) gives the population size at which the chance of pollination is 50% ( \rho \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Switching exponent parameter in pollination equation (shape parameter): when ( \gamma = 1 ), the Eq. S6 will collapse to type II functional response ( \gamma \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Compatibility of the two parental species when they intercross ( \alpha \in [0,1] )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Compatibility bias parameter ( \beta \in \mathbb{R}^+ \beta \neq 0 )</td>
</tr>
<tr>
<td>( n )</td>
<td>No. of chromosomes in haploid ( n \in \mathbb{N} )</td>
</tr>
<tr>
<td>( i )</td>
<td>Total no. of alien chromosomes in a diploid genotype ( i \leq n )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>No. of alien chromosomes in a haploid gamete ( \nu \in {0, 1, \ldots, n - 2, n - 1, 1} )</td>
</tr>
<tr>
<td>( x )</td>
<td>Index variable indicating the identity of a genotype (e.g., ( x = 0 ) represents the newcomer species, whereas ( x = 1 ) denotes the resident species). Total no. of elements (genotypes) in ( x ) is 2( n + 1 ).</td>
</tr>
<tr>
<td>( R )</td>
<td>Relative frequency of a given genotype in the population ( R \in [0,1] )</td>
</tr>
<tr>
<td>( V )</td>
<td>Relative frequency of a given genotype in the population after adjustment for pollinator preference ( V \in [0,1] )</td>
</tr>
<tr>
<td>( W )</td>
<td>A weight parameter that adjusts the relative frequency of genotype ( x ) according to its degree of attractiveness to pollinators ( W \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( y )</td>
<td>An index variable indicating the identity of a female genotype (i.e., pollen recipient) ( y \in {1, 2, \ldots, n - 2, n - 1, 1} )</td>
</tr>
<tr>
<td>( z )</td>
<td>An index variable indicating the identity of a male genotype (i.e., pollen donor) ( z \in {1, 2, \ldots, n - 2, n - 1, 1} )</td>
</tr>
<tr>
<td>( T_{\text{max}} )</td>
<td>Maximum value of a given trait (model parameter) ( T_{\text{max}} \in \mathbb{R}^+ )</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>Variance of a phenotypic trait across genotypes ( \sigma^2 \in \mathbb{R}^+ )</td>
</tr>
</tbody>
</table>

Table S2. Parameter values (and ranges) used for simulations of the generic model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_x )</td>
<td>0.1</td>
</tr>
<tr>
<td>( b_x )</td>
<td>0.0001</td>
</tr>
<tr>
<td>( \theta_x )</td>
<td>1,000</td>
</tr>
<tr>
<td>( d_x )</td>
<td>0.1</td>
</tr>
<tr>
<td>( s_x )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>0.035</td>
</tr>
<tr>
<td>( \gamma_x )</td>
<td>3.5</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0 and 0.5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1</td>
</tr>
<tr>
<td>( n )</td>
<td>18</td>
</tr>
<tr>
<td>( W_0 )</td>
<td>5</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>50</td>
</tr>
<tr>
<td>( N_1 )</td>
<td>1,950</td>
</tr>
</tbody>
</table>

Parameter values used for sensitivity analysis are shown in boldface.

*Parameter setting used for the invasion scenarios presented in Fig. 1.
†Parameter setting used for estimating Allee thresholds presented in Fig. 2.
‡Parameter setting used for estimating Allee thresholds presented in Fig. 3.
Table S3. Parameters of the simplified model and their values (range) used in the simulations

<table>
<thead>
<tr>
<th>Genotype</th>
<th>L*</th>
<th>b†</th>
<th>θ‡</th>
<th>d§</th>
<th>s{</th>
<th>ρ#</th>
<th>γjj</th>
<th>W**</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. maritima</td>
<td>0.08</td>
<td>0.0001</td>
<td>1,195</td>
<td>0.1</td>
<td>0.01</td>
<td>0.0001</td>
<td>1,195</td>
<td>0.1</td>
</tr>
<tr>
<td>C. edentula</td>
<td>0.03</td>
<td>0.0001</td>
<td>906</td>
<td>0.1</td>
<td>0.8</td>
<td>0.0001</td>
<td>906</td>
<td>0.1</td>
</tr>
<tr>
<td>Hybrid</td>
<td>0.055</td>
<td>0.0001</td>
<td>1,486</td>
<td>0.1</td>
<td>0.2</td>
<td>0.0001</td>
<td>1,486</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*L is the fraction of the seedbank that results in mature plants. The average survival to flowering (across a foredune and open beach habitats) was ∼8 and 3% for C. maritima and C. edentula, respectively, in California (15). For hybrids, we assumed the average of the two parental species.

†b is the density-dependent mortality parameter, assumed to be constant across the three genotypes. A b value of 0.0001 would result in a carrying capacity ∼500 adult plants (regardless of the genotype).

‡θ is the number of flowers produced by a single plant in the absence of competition and was obtained from our common garden experiment of five, six, and two populations of C. maritima, C. edentula, and putative hybrids.

§d indicates the proportional reduction in the number of flowers as the population density increases. This coefficient was estimated from the data of Keddy (40) for C. edentula. There are no similar data for C. maritima or hybrids, so we assumed that the three genotypes exhibit the same response to density.

{s is the fraction of flowers that produce seeds through selfing. Selfing rates for the three genotypes were obtained from our hand-crossing experiment.

*ρ is a coefficient in the pollination foraging submodel (Eq. S6): the coefficient’s reciprocal is the population size at which the probability of pollination is 50%. Because there are no empirical data, a range of values was tested for this parameter. For the simulation presented in Fig. 4 and Fig. S5, we used a ρ value of 0.035 corresponding to a population size of ∼30 plants for 50% pollination. The results of simulations using the given range in the Table S3 is shown in Fig. S7.

|γ is a shape parameter in the pollination foraging model (Eq. S6). Because there are no empirical data, a range of values was tested. For the simulation presented in Fig. 4 and Fig. S5, we used a γ value of 3.5, which results in a sigmoid response [type III functional response (30, 41)]. The results of simulations using the given range in the Table S3 is shown in Fig. S7.

**W is a weighting parameter that adjusts the relative frequency of genotypes according to their attractiveness to pollinators. These values were obtained from our field observations in a mixed population. For hybrid genotypes, we assumed the average of the two parental species.
%% Example: The below code will produce Figure 1B as in the main text of paper.
%% Make sure all four "functions" are in the same folder. These functions are:
%% 1- "hybrid_facil.m" (this is the main function that executes the model: you only need to work this
%% function);
%% 2- "crossing.m" (this function gives the identity of offspring genotypes);
%% 3- "geno2pheno.m" (this function maps genotype to phenotype);
%% 4- "selfingrate.m" (this function uses "geno2pheno.m" function to gives the selfing rate of genotypes)

Mohsen B Mesgaran (mohsenm@unimelb.edu.au): May 2016

%%
clear % clear memory
close all % close all figures;
%% Parameters: see "hybrid_facil" function for a description of all the listed parameters
n_chromosome = 9;
n_colonizer = 50;
n_resident = 1950;
L = 0.1;
b = L/1000;
d = 0.1;
theta = 1000;
rho = 0.035;
gamma = 3.5;
s_0 = 0.01;
s_1 = 0.01;
x_max = 0;
W_max = 5;
alpha = 0.5;
beta = 1;
time = 99;

n = 2*n_chromosome+1;  % number of genotypes

%% Run the model
SeedBank = hybrid_facil(n_chromosome,n_colonizer,n_resident,L,b,d,theta,rho,gamma,s_0,s_1,x_max,W_max,alpha,beta,time);
SeedBank = [n_colonizer; zeros(n-2,1); n_resident]'; SeedBank
RF=bsxfun(@(x,y) x./y,SeedBank,sum(SeedBank,2)); % convert numbers to relative frequency

%% Figure configuration
set(0,'units','centimeter')
scsz = get(0,'screensize');
figWidth = 15;
figHeight = 15;
fc = figure ('Units','centimeters','Position',[(scsz(3)-figWidth)/2 (scsz(4)-figHeight)/2 figWidth figHeight], 'Color', [1 1 1]);

%% Color for bars (color-coding genotypes)
red = [ones(1,fix(n/2)); linspace(0,0.9,fix(n/2)); linspace(0,0.9,fix(n/2))];
blue = [linspace(0.9,0,fix(n/2)); linspace(0.9,0,fix(n/2));ones(1,fix(n/2))];
col2 = [red; [0.97 1 .97]; blue];
col3=col2*.9;

%% plot the 3D bar graph
b1 = bar3(RF,1);
ylim([1 time+2])
xlim([0.25 19.75])
zlim([0 1])
for i=1:n
set(b1(i),'facecolor',col3(i,:),'EdgeColor', col2(i,:),'LineWidth',0.05);
end
set(gca,'FontSize',8,'YTick', [1 20:20:100], 'XTick', 1:n,'XTickLabel',...
    {'100' '' '75' '' '50' '' '25' '' '0'}, 'XDir','normal');
axis square
x2 = xlabel('Genome derived\newlinefrom colonizer (%)', 'HorizontalAlignment', 'center');
set(x2,'rotation',-26,'FontSize', 11);
x2.Position = [-6.5719   29.4396   -0.8152];
y2 = ylabel('Time (generations)');
set(y2,'rotation',34,'FontSize', 11);
y2.Position = [4.5   -10.6476   -1.0063];
z1 = zlabel('Relative frequency','FontSize', 11);
z1.Position = [-16.9728   27.1791   -0.2612];
ax = gca;
grid off
view(42,34);
annotation('textbox','String', 'Population dynamics of 19 genotypic classes over time as result of hybridization between colonizing and resident species','...'
    'LineStyle', 'none','FontWeight','bold','FontSize', 11,'Position',[.1 .85 .75 .05],'HorizontalAlignment','center');

%%  Save figures in three different formats
set(fc,'PaperUnits','centimeters','PaperSize',figWidth figHeight,'PaperPosition',[0 0 figWidth figHeight], 'PaperPositionMode', 'manual');
print(fc,'Fig1_PNAS', '-depsc','-painters');
print(fc,'Fig1_PNAS', '-dpdf','-painters');
print(fc,'Fig1_PNAS', '-dpng','-r300');
function SeedBank = hybrid_facil(n_chromosome,n_colonizer,n_resident,L,b,theta,rho,gamma,s_0,s_1,x_max,W_max,alpha,beta,time)

% n_chromosome = number of haploid chromosomes
% n_colonizer = initial population size of colonizer genotype
% n_resident = initial population size of resident genotype
% s_0 = selfing rate in colonizer [0,...,1]
% s_1 = selfing rate in resident [0,...,1]
% L = the proportion of seeds recruited independent of population density (Eq.2 in Supplementary Information)
% b = mortality rate due to density: b/L gives the max number of adult plants (sort of carrying capacity for adult plant stage not seed stage: see Eq.2 in Supplementary Information)
% theta = the maximum number of ovules/flowers produce by a single plant (Eq.3 in Supplementary Information)
% d = rate of decrease in ovule/flower production due to density (Eq.3 in Supplementary Information)
% rho = location parameter in pollination function: 1/rho gives the plant density at which the probability of pollination is 50% (Eq.6 in Supplementary Information)
% gamma = shape parameter in pollination function (Eq.6 in Supplementary Information)
% alpha = compatibility between parental genotypes i.e. colonizer*resident (Eq.15 in Supplementary Information).
% When alpha = 0 the whole model is reduced to a "competition model" with two species
% beta = bias parameter in backcrossing (Eq.15 in Supplementary Information)
% time = number of generations/years
% x_max = genotype with the highest value of attractiveness (see Eq.16 in Supplementary Information)
% W_max = attractiveness of the most attractive genotype (that of the least will be 1). if W_max = 1, then all "n" genotypes will have the same attractiveness value of 1

% Mesgaran B Mesgaran (mohsenm@unimelb.edu.au): May 2016

%% Genotyping
n = 2*n_chromosome+1; %number of genotypes
N_x = [n_colonizer; zeros(n-2,1); n_resident]'; % number of seeds from genotype x
x = linspace(0,1,n); % Vector of genotype ranging from 0 to 1 with n classes where 0 indicates colonizer and 1 resident
[Sx , Sx_x] = selfingrate(s_0,s_1,n); % this function gives the selfing rate of all "n" genotypes (based on Eq.16 in Supplementary Information) in vector "Sx" and n replicates of them in "Sx_x"
[cross_offs , self_offs] = crossing(n); % this function gives the identity of genotypes resulted from all pairwise crossings between genotypes in the matrix "cross_offs" with n^2 by n dimension while the identity offspring from selfing is given in "self_offs" which has a dimension n*n (this function is based on E.12 and Eq. 13 in Supplementary Information)
xx = repmat(x,1,n); % repeating the vector "x" for pairwise crossings
z = sort(xx(:),1); % vector of male plants
y = xx(:); % vector of female plants
W_x = geno2pheno(x_max,W_max,1,n); % this function maps genotype to phenotypic (here attractiveness) using Eq. 16 in Supplementary Information

% the start of the loop
for j=1:time

%% Survival to maturity (flowering stage)
N = sum(N_x(:)); % total number of seeds summed over all genotypes
S_N = L/(1+b*N); % Survival function(Eq.2 in Supplementary Information)
M_N = S_N*N_x; % number of mature plants
M_N2 = [reshape(repmat(M_N,n,1),n*n,1) repmat(M_N',n,1) ]; % repeating the vector "M_N" for pairwise crossings

%% Adjusting relative frequencies for pollinator preference (attractiveness)
M_total = sum(M_N(:)); % total number of adult plants
R_x = M_N./M_total; % unadjusted (for pollinator preference) relative frequency of each genotype
V_x = R_x.*W_x./sum(R_x.*W_x); % adjusted (for pollinator preference) relative frequency of each genotype (Eq.10 in Supplementary Information)
RV_x = repmat(V_x,1,n); % repeating the vector "V_x" for pairwise crossings
RV_x=reshape(RV_x',[],1); % reshape it to get the proper column vector

%% Pollination
phi_M = (rho*M_total)^gamma/(1+ (rho*M_total)^gamma); % pollination functional response relating probability of pollination to total plant density (Eq.6 in Supplementary Information)
phi_M_x = phi_M*RV_x; % probability of pollination for genotype x while accounting for the for pollinator preference

%% Probability of out-crossing
out_cross_x = phi_M_x.*(1-Sx_x); % while "s" fraction of ovules get fertilised through selfing the remaining "(1-s)" need to be fertilized through pollination

%% Compatibility
C_yz = z.^beta.*(alpha - 1) - y.^beta.*(2*z.^beta.*(alpha - 1) - alpha + 1) + 1; % the compatibility of female genotype y with the male genotype z (Eq.15 in Supplementary Information)
fertile_out_cross_x = out_cross_x.*C_yz ; % the probability of fertile outcrossing between genotypes after accounting for probability pollination (phi_M_x), non-selfing fraction (1-s) and compatibility (C_yz)

%% Flower production
F_M = 1-(d*M_total)/(1+(d*M_total)); % reduction in capacity of ovule production due to density-dependent competition (Eq.3 in Supplementary Information)
theta_M_x = F_M*(M_N2(:,2).*theta); % total number of ovules produced by adult plants of genotype x

%% Outcrossed seed production
out_seed = theta_M_x.*fertile_out_cross_x; % number of seeds produced on genotype x through outcrossing (note that these are not seeds with genotype x, the identity of these seeds need to be determined by function "cross_offs" which is based on Eq.12 and Eq.13 in Supplementary Information

%% Selfed seed production
theta_M_x = F_M*(M_N.*theta'); % total number of ovules produced by adult plants of genotype x, here M_N which is a vector of length n is used not M_N2 which has a length of n^2 (the values are the same but as in the previous section we were dealing with outcrossing we needed to have all pairwise combinations)
self_seed = (theta_M_x.*Sx'); % number of seeds produce on genotype x through outcrossing (note that these are not seeds with genotype x, the identity of these seeds need to be determined by function "self_offs" which is based on Eq.12 and Eq.13 in Supplementary Information

%% Total seed production
out_seed_x = out_seed'*cross_offs; %number of seeds with genotype x that has been produced through outcrossing
self_seed_x = self_seed*self_offs; %number of seeds with genotype x that has been produced through selfing
out_seed_x(isnan(out_seed_x)) = 0; % convert NaNs to zero (if for any unforeseen reason the model gives NaN values)
self_seed_x(isnan(self_seed_x)) = 0; % convert NaNs to zero (if for any unforeseen reason the model gives NaN values)
N_x = self_seed_x+ out_seed_x; % total number of seeds with genotype x after on generation

%% End of the loop
SeedBank (j,:) = N_x;   % we save all these data into the new matrix SeedBank
% This matrix SeedBank has "n" columns (equal to the number of genotypes) and "time" rows (equal to the % number of generations). The first column represents the "colonizer" while the last column is the "resident" species. % If "alpha = 0" it means there is no hybridisation, so all the first and last columns of matrix "SeedBank" will have "non-zero" data.
end
function [cross_offs, self_offs] = crossing(n)
    % This function gives the identity of genotypes resulted from all pairwise crossings between
    % genotypes in the matrix "cross_offs" with n^2 by n dimension while the identity offspring from
    % selfing is given in "self_offs" which has a dimension n*n.
    % This function is based on E.12 and Eq. 13 in Supplementary Information.

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    n = (n-1)/2;
    L = 1:n;
    N=cell(n,1);

    for j=1:length(L)
        for i=1:L(j)+1
            K = 0:L(j);
            if K(i)==L(j)-K(i)
                N{j,1}(i) = 2*(nchoosek(n,K(i))*nchoosek(n,L(j)-K(i)));
            else
                N{j,1}(i) = nchoosek(n,K(i))*nchoosek(n,L(j)-K(i));
            end
        end
    end

    P = cellfun(@(x) x./sum(x),N,'uni',0);
    for i=1:n
        l = length(P{i});
        P{i}(1,l+1:n+1)=0;
    end

    gmt =[[1 zeros(1,n)]; cell2mat(P)];
    gmt2 = fliplr(flip(gmt));
    mios = [gmt; gmt2(2:end,:)];

    % For outcrossed flowers
    cross_offs = cell(2*n+1,1);
    for i=1:2*n+1
        for j=1:2*n+1
            cross_offs{i}(j,:) = conv(mios(i,:),mios(j,:));
        end
    end

    cross_offs = cell2mat(cross_offs);

    % For selfed flowers
    self_offs = zeros(2*n+1);
    for i = 1:2*n+1
        self_offs (i,:) = conv(mios(i,:),mios(i,:));
    end
function F = geno2pheno(u,T,z,n)
% This function maps genotype to phenotype using Eq.16 in Supplementary Information. The output
% vector F is scaled to vary from T to 1 (if T<1) or 1 to T (if T > 1). The function uses the "solve"
% to estimate the value of sigma in Eq. 16.
% u = the location of the genotype with largest or smallest phenotypic value
% T = is the phenotypic value of a trait
% if T > 1 u is the location for the genotype with largest phenotypic value
% if T < 1 u is the location for the genotype with smallest phenotypic value
% if T = 1 all genotypes are equally fit
% z = can take 3 values: 1, 2 and 3
% if z = 1, the output F is of length of n (= number of genotypes);
% if z = 2, the output F is of length of n^2 and includes the phenotypic values of genotypes acting%
% as female in all pairwise combinations of genotypes
% if z = 3, the output F is of length of n^2 and includes the phenotypic values of genotypes acting%
% as male in all pairwise combinations of genotypes
%
% Mesgaran B Mesgaran (mohsenm@unimelb.edu.au): May 2016
%
% Total number of genotypes
G = linspace(0,1,n); % genotypes vector
E = repmat(G,1,n);
Y=[sort(E(:),1) E(:)];
Ym = Y(:,1);
Yf = Y(:,2);

if z == 1;
    G = G;
elseif z == 2;
    G = Yf;
else z == 3;
    G = Ym;
end

if T < 1;
    if u <= .5;
        X = 1;
    else X = 0;
    end
    sym v
    v=solve(T*exp((((X-u)^2)/(2*v^2)))==1,v);
    v = double(v);
    v=v(v>0);
    F = T*exp(((G-u).^2)./(2*v^2));
elseif T > 1;
    if u <= .5;
        X = 1;
    else X = 0;
    end
    sym v;
    v=solve(T*exp(-(((X-u)^2)/(2*v^2)))==1,v);
    v = double(v);
    v=v(v>0);
    F = T*exp(-((G-u).^2)./(2*v^2));
else T == 1;
    F = ones(1,length(G));
end
end
    F = reshape(F,length(G),1);
end
function [Sx, Sx_x] = selfingrate(s_0, s_1, n)
% this function gives the selfing rate of all "n" genotypes (based on Eq.16 in Supplementary Information)
% in vector "Sx" and n replicates of them in "Sx_x"
% n = number of genotypes
% s_0 = selfing rate in colonizer [0,...,1]
% s_1 = selfing rate in resident [0,...,1]

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[m, e] = meshgrid(s_0, s_1);
Sx_x = [m(:) e(:)];

Lm = length(s_0);
Le = length(s_1);
Sx = zeros(n, Lm*Le);

for i=1:Lm*Le
    if Sx_x(i,1) >= Sx_x(i,2)
        Sx(:,i) = tdg(0, Sx_x(i,1)/Sx_x(i,2), 1, n)*Sx_x(i,2);
    else
        Sx(:,i) = tdg(1, Sx_x(i,2)/Sx_x(i,1), 1, n)*Sx_x(i,1);
    end
end
Sx_x = repmat(Sx, n, 1);
end