Positional Analysis for Social Networks & Health
Outline

1. Introduction:
   a. Positions vs. Connections
2. Types of positional models
   a. “Centrality”
   b. Block Models
3. Centrality Models
4. Block Models
5. Conclusions
## Positional Analysis

### Introduction

### Connections & Positions: Network Problems

<table>
<thead>
<tr>
<th>Ego</th>
<th>Complete</th>
<th>Multiple</th>
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</table>
| Connectionist: *Networks as pipes* | • Structural Holes  
• Density  
• Mixing Models  
• Size | • Community detection  
• Reachability  
• Homophily  
• Degree Distribution  
• Social Balance  
• ERGM | • Multi-layer networks  
• Multi-level models of multiple networks |
| Positional: *Networks as roles* | Local Roles  
(Mandel 1983, Mandel & Winship 1984) | • Relational Block Models  
• Motifs | 2 ideas:  
• Patterns in networks  
• Patterns of networks |
Overview

• Social life can be described (at least in part) through social roles.
• To the extent that roles can be characterized by regular interaction patterns, we can summarize roles through common relational patterns.
• Identifying these sets is the goal of block-model analyses.

Nadel: The Coherence of Role Systems
• Background ideas for White, Boorman and Brieger. Social life as interconnected system of roles
• Important feature: thinking of roles as connected in a role system = social structure

White, Boorman and Breiger: Social structure from Multiple Networks I. Blockmodels of Roles and Positions
• The key article describing the theoretical and technical elements of block-modeling
Nadel: The Coherence of Role Systems

Elements of a *Role*:

- *Rights and obligations with respect to other people or classes of people*
- *Roles require a ‘role compliment’ another person who the role-occupant acts with respect to*

*Examples:*
*Parent - child, Teacher - student, Lover - lover, Friend - Friend, Husband - Wife, etc.*

Nadel (Following functional anthropologists and sociologists) defines ‘logical’ types of roles, and then examines how they can be linked together.
Nadel describes how various roles fit together to form a coherent whole. Roles are collected in people through the ‘summation of roles’

**Necessary:**
Some roles fit together necessarily. For example, the expected interaction patterns of “son-in-law” are implied through the joint roles of “Husband” and “Spouse-Parent”

**Coincidental:**
Some roles tend to go together empirically, but they need not (businessman & club member, for example).

Distinguishing the two is a matter of usefulness and judgement, but relates to social substitutability. The distinction reverts to how the system as a whole will be held together in the face of changes in role occupants.
Positional Analysis

Introduction

White et al.: From logical role systems to empirical social structures

Start with some basic ideas of what a role is: An exchange of something (support, ideas, commands, etc) between actors. Thus, we might see an exchange network such as:

- Romantic Love
- Provides food for
- Bickers with
Positional Analysis

Introduction

*White et al*: From logical role systems to empirical social structures

Start with some basic ideas of what a *role* is: An exchange of something (support, ideas, commands, etc) between actors. Thus, we might see an exchange network such as:

![Positional Analysis Diagram]

(and there are, of course, many other relations inside the family)
Positional Analysis
Types of positional models

Informally we tend to think of “position” in two senses, a node-metric sense (“Centrality”) and a collective property/group sense (“blockmodel”).

The node metric sense uses some graph-theoretic property of the node to characterize a social position of interest. Generally treats positional features as scales rather than categories.

Examples:
“Loners” → operationalized as nodes with degree zero
“Leaders” → operationalized as nodes with lower network constraint
“Bridges” → operationalized as nodes with high betweenness
“Social butterfly” → operationalized as nodes that change relations frequently

These are not all strictly centrality scores, but often they are (which is why the connotations of being in the middle of thigs implied by the term “centrality” is really sort of not useful).
Positional Analysis
Types of positional models

Informally we tend to think of “position” in two senses, a node-metric sense (“Centrality”) and a collective property/group sense (“blockmodel”).

The **collective property** sense identifies sets of nodes with equivalent* tie patterns to define a partition of nodes into classes. It makes the assumption that these are substantively discrete – classes rather than scales.

Examples:

- set of all people who can hire faculty $\rightarrow$ “Deans”
- set of all people descendant from the King $\rightarrow$ “Potential heir”
- set of all female siblings of a father $\rightarrow$ “Aunts”
- set of all allies of an enemy $\rightarrow$ “Enemy”

Positions differ from communities in two ways:

a) Adjacency – two members of the same community should have a very high probability of being adjacent; two members of the same position need not.

b) Ability to compound the relation – two-step links through a position is meaningful (i.e. “father's brother” is “uncle”), but is not for communities.

*Equivalence means something special in this context*
Positional Analysis
Types of positional models

Both approaches tend to be used in health as ways to create variables in a GLM.

a) Generally that simplifies things as the final modeling parts are not substantively different from standard health behavior models.

b) Does raise some obvious non-independence issues worth at least thinking through.
Positional Analysis
Centrality models

The general approach is to model a health outcome as a function of a node’s network characteristic.

\[
\text{Self-rated health} = \text{network metric} + \text{<other stuff>}
\]

The logic behind this model is that we expect some particular feature of the node’s position in the network to be associated with health.

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<tr>
<th>Metric Family</th>
<th>Example</th>
<th>Mechanism</th>
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<tbody>
<tr>
<td>Relational Volume</td>
<td>Isolate</td>
<td>Loneliness leads to self-doubt, depression</td>
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<td></td>
<td>Popularity</td>
<td>Approval of others boosts self esteem</td>
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<tr>
<td>Multiplexity</td>
<td>Proportion of coworkers who are friends</td>
<td>Integration of social worlds leads to consistency of expectations and better mental health</td>
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<td>Bridging</td>
<td>Betweenness centrality</td>
<td>Access to different populations promotes diverse information</td>
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<td>Intransitivity of local friendships</td>
<td>Negotiating friends who dislike each other causes stress</td>
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<tr>
<td>Cohesion</td>
<td>Reciprocity ratio</td>
<td>Other’s recognition improves sense of self</td>
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<tr>
<td></td>
<td>Proportion of friends that overlap</td>
<td>Tight-knit social units provide care</td>
</tr>
</tbody>
</table>

(and many others)
Positional Analysis
Centrality models

And many others... is huge!

http://schochastics.net/sna/periodic.html
Positional Analysis
Centrality models

Degree Centrality: Count of the number of adjacent nodes

\[ C_D = d(n_i) = X_{i+} = \sum_j X_{ij} \]

Closeness Centrality: Inverse of average distance to all other nodes in the network

\[ C_c(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1} \]

Betweenness Centrality: Sum of pairs who’s geodesic a node sits on

\[ C_B(n_i) = \sum_{j<k} g_{jk}(n_i) / g_{jk} \]

Eigenvector Centrality: Normalized recursive sum of adjacent nodes’ degrees.
\[ \rightarrow \text{eigenvector of the largest}(1^{st}) \text{ eigenvalue} \]
Positional Analysis
Centrality models

Borgatti & Everett (2020) provide 3 perspectives to characterize centrality:

1) Walk structure participation perspective - organizes centrality by the pattern of walks & paths in the definition

2) Induced Centrality perspective – centralities matter because of what they do to the overall network structure – captures the extent to which a node contributes to a graph level property.

3) Flow outcomes perspective – focuses on the ways in which the node affects a propagation process outcome of the network.

Most* common centralities can be discussed from each of these perspectives; they are ways to provide understanding for the network process of interest.

*Most because it is possible to define silly path or graph properties that’d technically count, but nobody uses these.
Positional Analysis
Centrality models

1) Walk structure participation perspective: 3 dimensions

- Kind of traversal: Geodesic, Path, Trail, Walk

- Position on the traversal: Endpoint (radial) or interior (medial)

- Property of the traversal: Frequency or Length?

For example:

Degree is a frequency, endpoint & geodesic
Closeness is length, endpoint, geodesic
betweenness is frequency, interior, geodesic
eigenvector is length*frequency, endpoint, walks
Positional Analysis
Centrality models

1) Walk structure participation perspective: 3 dimensions

<table>
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<tr>
<th>Table 17.1 Three-Dimensional Typology of Centrality Measures</th>
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<tr>
<td><strong>Radial measures</strong></td>
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<tr>
<td>Geodesics</td>
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<tr>
<td>Length</td>
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<tr>
<td>Length x Freq</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
</tbody>
</table>

This is comprehensive, but often not clearly substantive.

i.e. doesn’t always provide a clear “which should I use” sort of guide.
2) Induced Centrality perspective

A node is as important to the network as its removal would be consequential.

There are induced-interpretations of most standard graph metrics (though not all). Some of these are trivial (degree’s relation to density, say) others highly dependent on the path structure (betweenness and distance).

Particularly useful for thinking about mechanisms on the network. For example, if you calculate the speed with which a bit diffuses through a network then recalculate removing one node at a time, you get each node’s unique contribution to the total diffusion risk in the network.

Note as well you can apply this idea to groups of nodes – all nodes in some class, or all pairs of nodes, etc.
3) Flow outcomes perspective

Metrics matter based on how they govern a particular kind of flow. Borgatti & Everett (2020) give a handful of archetypes:

<table>
<thead>
<tr>
<th>Name</th>
<th>Traversal</th>
<th>Contagion</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>Used Book</td>
<td>Trail</td>
<td>Move / Transfer</td>
<td>Read a book, pass it on. But if it returns to you, pass it to somebody new.</td>
</tr>
<tr>
<td>News/Gossip</td>
<td>Trail</td>
<td>Copy, directed</td>
<td>Pass a story to confidants, who pass it on, but not to same person repeatedly.</td>
</tr>
<tr>
<td>Itinerant</td>
<td>Path</td>
<td>Move/ Transfer</td>
<td>Live with somebody for a while, but outstay welcome so can’t come back</td>
</tr>
<tr>
<td>Virus</td>
<td>Path</td>
<td>Copy, directed</td>
<td>SIR model, highly infective reaches all neighbors quickly</td>
</tr>
<tr>
<td>Coin</td>
<td>Walk</td>
<td>Move/ Transfer</td>
<td>Coin moves through the economy – only in one place at a time.</td>
</tr>
<tr>
<td>Attitude</td>
<td>Walk</td>
<td>Copy, bidirected</td>
<td>All continuously affecting each other</td>
</tr>
<tr>
<td>Travel</td>
<td>Geodesic</td>
<td>Move</td>
<td>Search out fastest route.</td>
</tr>
</tbody>
</table>

Interestingly, most off-the-shelf centrality scores don’t map onto these common processes exactly (mainly because most work w. geodesics rather than paths/walks)
Positional Analysis
Centrality models

3) Flow outcomes perspective

Metrics matter based on how they govern a particular kind of flow. Borgatti (2005) version:

<table>
<thead>
<tr>
<th>Typology of flow processes</th>
<th>Parallel duplication</th>
<th>Serial duplication</th>
<th>Transfer</th>
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</thead>
<tbody>
<tr>
<td>Geodesics</td>
<td>&lt;No process&gt;</td>
<td>Mitotic reproduction</td>
<td>Package delivery</td>
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<tr>
<td>Paths</td>
<td>Internet name-server</td>
<td>Viral infection</td>
<td>Mooch</td>
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<tr>
<td>Trails</td>
<td>E-mail broadcast</td>
<td>Gossip</td>
<td>Used goods</td>
</tr>
<tr>
<td>Walks</td>
<td>Attitude influencing</td>
<td>Emotional support</td>
<td>Money exchange</td>
</tr>
</tbody>
</table>
A key element of the positional approach to networks is to consider nodes as combinations of metrics, not just a single dimension. This creates a linkage between the “metric” and “classes” version of positional models.

<table>
<thead>
<tr>
<th>Centrality models</th>
<th>Low Degree</th>
<th>Low Closeness</th>
<th>Low Betweenness</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Degree</td>
<td></td>
<td>Embedded in cluster that is far from the rest of the network</td>
<td>Ego's connections are redundant - communication bypasses him/her</td>
</tr>
<tr>
<td>High Closeness</td>
<td>Key player tied to important important/active alters</td>
<td></td>
<td>Probably multiple paths in the network, ego is near many people, but so are many others</td>
</tr>
<tr>
<td>High Betweenness</td>
<td>Ego's few ties are crucial for network flow</td>
<td>Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.</td>
<td></td>
</tr>
</tbody>
</table>
Positional Analysis
Block models

*White et al*: From logical role systems to empirical social structures

Start with some basic ideas of what a *role* is: An exchange of something (support, ideas, commands, etc) between actors. Thus, we might represent a family as:

\[(\text{P}) \rightarrow (\text{C}) \rightarrow (\text{P}) \rightarrow (\text{C}) \leftrightarrow (\text{P}) \rightarrow (\text{C}) \rightarrow (\text{P}) \rightarrow (\text{C}) \rightarrow (\text{C}) \rightarrow (\text{C}) \rightarrow (\text{C}) \]

(and there are, of course, many other relations inside a family)

- Red arrows: Romantic Love
- Blue arrows: Provides food for
- Green arrows: Bickers with
Blockmodeling: basic steps

In any positional analysis, there are 4 basic steps:

1) Identify a definition of equivalence
2) Measure the degree to which pairs of actors are equivalent
3) Develop a representation of the equivalencies
4) Assess the adequacy of the representation

At the end of the day, this is community detection on a role-relevant similarity matrix rather than an adjacency matrix.

The “trick” is defining similar-with-respect-to-what
If the model is going to be based on asymmetric or multiple relations, you simply stack the various relations, usually including both “directions” of asymmetric relations:

**Positional Analysis**

**Block models**

```
If


P

C

C

C

P

Provided food for

Romantic Love

Bickers with

Positional Analysis

Block models

If the model is going to be based on asymmetric or multiple relations, you simply stack the various relations, usually including both “directions” of asymmetric relations:

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Positional Analysis
Block models

Traditional equivalence measures

Classic models build on a continuum between structural equivalence regular equivalence:

**Structural equivalence:** Same ties to the exact same people. Nodes are only distinguishable by their label. Example: Two pendants around the same hub.

**Automorphic equivalence:** Same pattern of ties to all others in the network. Nodes are indistinguishable on any summary metric. Example: Sports team positions

**Regular Equivalence:** Same types of ties to similar types of people. Idea is that nodes of one class relate similarly to nodes of another class, though they may differ in volume. Example: Nurses to Doctors; managers to vice-presidents, kids to parents.

*In practice, it tends to be fairly difficult to distinguish the three forms as the operationalization rarely generates pure SE. So a poor operationalization of SE gives you AE, or something like a mix of AE and RE...*
Positional Analysis
Block models

Traditional equivalence approaches

ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

Here I have blocked structurally equivalent actors
### Positional Analysis

#### Block models

**Traditional equivalence approaches**

**ConCor:** Convergence of Iterated Correlations (Boorman, Breiger & White)

Base similarity is correlation across the rows/columns of each pari to all other pairs

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### Positional Analysis

Block models

Traditional equivalence approaches

ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

#### Concor iteration 1:

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Positional Analysis
Block models

Traditional equivalence approaches
ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

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Positional Analysis
Block models

Traditional equivalence approaches
ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

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Positional Analysis
Block models

Traditional equivalence approaches
ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

**Concor iteration 3: Permutated**

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Positional Analysis

Block models

Traditional equivalence approaches
ConCor: Convergence of Iterated Correlations (Boorman, Breiger & White)

Because CONCOR splits every sub-group into two groups, you get a partition tree that looks something like this:

The question is how to stop splitting – one level, each branch, etc. Very hands on and inductive. Original advice was to fit until you had all one/zero blocks, but that rarely works.
CONCOR example:

Consider a simple senate voting network:

Network is dense, since every cell has some score and dynamic the pattern changes over time.

Color by structural equivalence…
CONCOR example:

Consider a simple senate voting network:

Network is dense, since every cell has some score and dynamic the pattern changes over time.

Adjust position to collapse SE positions.
CONCOR example:

Consider a simple senate voting network:

Network is dense, since every cell has some score and dynamic the pattern changes over time.

And then adjust color, line width, etc. for clarity.

While we’ve gone some distance with identifying relevant information from the mass, how do we account for time?
CONCOR example:

Repeat at each wave, linking positions over time
CONCOR example:
Automorphic and Regular equivalence are more difficult to find, and require iteratively searching over possible class assignments for sets that have the same graph theoretic patterns. Usually start with a set of nodes defined as similar on a number of network measures, then look within these classes for automorphic equivalence classes.

The classic reference is REGE (White & Reitz 1985), which recursively defines the degree of equivalence between pairs and then adjusts for as many iterations as you specify. Slow and doesn’t always converge.

A theoretically appealing method for finding structures that are very similar to regular equivalence, role equivalence, uses the triad census. Each node is involved in \((n-1)(n-2)/2\) triads, and occupies a particular position in each of these triads.

Burt (1990) “Detecting Role Equivalence” Social Networks
**Positional Analysis**
Block models

**Triad Census:** *The periodic table of social elements*

Triads also provide a tight coupling between behavior rules and (local) structure

16 directed triads
Positional Analysis

Block models

Triad Census: *The periodic table of social elements*

Triads also provide a tight coupling between behavior rules and (local) structure

16 directed triads
Positional Analysis
Block models

Triad Census: *The periodic table of social elements*

Triads also provide a tight coupling between behavior rules and (local) structure

16 directed triads
An Example of the triad census

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Sum (2 - 16): 63
Limits on the set of triads constrains the global structure

1) All triads are 030T:

A perfect linear hierarchy.
Positional Analysis
Block models

Triads allowed: \{300, 102\}
Eugene Johnsen (1985, 1986) specifies a number of structures that result from various triad configurations.
Positional Analysis
Block models

PRC{300,102, 003, 120D, 120U, 030T, 021D, 021U}   Ranked Cluster:

And many more...
Positional Analysis
Block models

(0)  003
(1)  012
(2)  102
(3)  111D
(4)  201
(5)  210
(6)  300

- Intransitive
- Transitive
- Mixed
Triadic Position Census: 36 Positions within 16 Directed Triads
Indicates the position.
### Triad position vectors for a simple example network with 3 positions:

<table>
<thead>
<tr>
<th>Positional Analysis</th>
<th>Block models</th>
</tr>
</thead>
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- **A**
  - 003

- **B**
  - 102_D

- **C**
  - 102_I

- **D**
  - 201_S

- **E**
  - 201_B

- **F**
  - 300

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Positional Analysis
Block models

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</table>
Positional Analysis
Block models

Triad position vectors for a simple example network with 3 positions:

How do we do it at scale?
Positional Analysis
Block models

One Prosper School
(6th grade)…of 368.
Positional Analysis

Block models

Stage 1: Within settings:
- Build triadic involvement distance matrix
- Ward’s min Variance Clustering
- Calculate modularity score for the partition applied to the similarity matrix at each cut level
- Accept the cut with the highest modularity score
- Units are students
Positional Analysis
Block models

One Prosper School
(6th grade)....each color is a position
Positional Analysis
Block models

Stage 1: Within settings:
- Build triadic involvement distance matrix
- Ward’s min Variance Clustering to build dendrogram
- Calculate modularity score for the partition applied to the similarity matrix
- Accept the cut with the highest modularity score
- \( \rightarrow \) 2912 clusters

Thus far…standard single-network model.

But how do you compare blocks across networks when label values are meaningless?

Example positions identified in a single school network
(role 7 is a “leading crowd” in the simplest sum-of-in-degree sense)
Positional Analysis
Block models

Stage 2: 2nd-order clustering across settings
- Calculate the triad position profile for each within-setting cluster
- Identify similarity across the cluster profiles by clustering a 2nd time
- Units are clusters (of students)
### Positional Analysis

#### Block models

#### 2nd Order Clustering Dendrogram

- **Popular Loners**
  - **High(er) out degree**
  - **Low out-degree**

- **Uninvolved Outsiders**
  - **Lower indegree**

- **Hangers-on**
  - **Asym**
  - **Leading Crowd**

- **Aloofs**
  - **Asym**
  - **Leading Crowd**
  - **T300**

- **Segmented Peers**
  - **Asym**

- **Lieutenants**
  - **Asym**

- **Federated Friends**
  - **Transitivity**

- **Core**
  - 1521

- **Peripheral**
  - 1391

- **2912 within-school clusters**

- **Positional Analysis**
  - **Block models**

- **165**
  - Higher indegree

- **763**
  - Lower indegree

- **928**

- **463**

- **534**

- **740**

- **1149**

- **409**

- **372**

- **341**

- **343**

- **427**

- **492**

- **400**

- **454**

- **468**

- **509**

- **507**

- **420**
Positional Analysis
Block models

Uninvolved outsiders

Popular Loners

Hangers-on

Power Centrality
Closeness Centrality
Total Degree
Ego Density
Ego Transitivity
In-Degree
Information Centrality
Out-Degree
Two-step Reach
Reciprocity
Betweenness

Power Centrality
Closeness Centrality
Total Degree
Ego Density
Ego Transitivity
In-Degree
Information Centrality
Out-Degree
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Power Centrality
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Total Degree
Ego Density
Ego Transitivity
In-Degree
Information Centrality
Out-Degree
Two-step Reach
Reciprocity
Betweenness
Positional Analysis
Block models

Role set characteristics: Core
→ Secondary Core Branch
Positional Analysis
Block models
Role set characteristics: Core → Leading Crowd
Positional Analysis

Block models

Local view of Archetypal Role Positions by the three role activating dimensions

Reciprocity

High

Low

Volume

High

Low

Hierarchy

High

Low

Uninvolved Outsiders

Leading Crowd: Self Recognized & Popular

Federated Friends: Intimate close ties, loosely organized locally.

(8)

(6)

(7)

(5)

(4)

Popular Loners: Expansively low or isolated but still attractive.

Uninvolved Outsiders: No/few ties to anyone else, low popularity.

Aloof Elite: (1) Expansively low but attractive, (2) Attractively low but nominated by other elites, with small number of mutual friends.

(1) Expansively active but unpopular; often (2) “Hangers-on:” ties to elite but not recognized by anyone else.

Hangers-on

Segmented Peers

Leaders

Lieutenants

Federated Friends

Peer Influence: Intimate close ties, loosely organized locally.

Positional Analysis

Block models
Positional Analysis

Block models

Role positions largely cross-cut demographics and behaviors:
Wave 1 to Wave 2 Mobility

(row normalized mobility table; cells shaded by ratio of observed to expected if >2.5)
Positional Analysis
Block models

Wave 2 to Wave 3 Mobility

<table>
<thead>
<tr>
<th></th>
<th>1 LeadingCrd</th>
<th>2 Aloofs</th>
<th>3 FedFriends</th>
<th>4 Segmented</th>
<th>5 Lieutenant</th>
<th>6 Hangerson</th>
<th>7 Pop Loner</th>
<th>8 Outsiders</th>
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<td>31</td>
<td>18</td>
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<td>7</td>
</tr>
<tr>
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<td>9</td>
<td>7</td>
<td>26</td>
<td>18</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
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<td>7</td>
<td>7</td>
<td>30</td>
<td>24</td>
<td>3</td>
<td>14</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>4 Segmented</td>
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<td>19</td>
<td>24</td>
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<td>6 Hangerson</td>
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<td>3</td>
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<td>20</td>
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<tr>
<td>7 Pop Loner</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>19</td>
<td>3</td>
<td>19</td>
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<td>21</td>
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<tr>
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<td>4</td>
<td>9</td>
<td>17</td>
<td>2</td>
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<td>0</td>
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(row normalized mobility table; cells shaded by ratio of observed to expected if >2.5)
Positional Analysis
Block models

Wave 3 to Wave 4 Mobility

(row normalized mobility table; cells shaded by ratio of observed to expected if >2.5)
Positional Analysis
Block models

Wave 4 to Wave 5 Mobility

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<td>11</td>
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</table>

(row normalized mobility table; cells shaded by ratio of observed to expected if >2.5)
Positional Analysis
Block models

Individual mobility models*

Female

Log-odds:

Statistically significant cell values as bold OR

Girls tend to migrate to the “Federated Friends” position (blue column), and are unlikely to leave (red row)

*Setting level random effects multinomial logistic regression models conditional on origin, including wave, sex, race, slun, deviance, church, school attachment, grades, school-level reciprocity, transitivity, degree centralization, structural cohesion & hierarchy score.
### Positional Analysis

**Block models**

#### Individual mobility models*

**Female**

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**Log-odds:**

-0.4  -0.2  0.0  0.2  0.4

Statistically significant cell values as bold OR

---

*Setting level random effects multinomial logistic regression models conditional on origin, including wave, sex, race, slun, deviance, church, school attachment, grades, school-level reciprocity, transitivity, degree centralization, structural cohesion & hierarchy score.

Girls are not generally drawn to the Leading crowd (no column effect), but once there, likely to stay.
Positional Analysis
Block models

Individual mobility models*

School Lunch

Log-odds:

1. LeadingCrd
2. Aloofs
3. Fed.Frnds
4. Segmented
5. Lieutenant
6. Hangers-on
7. Pop.Loner
8. Outsiders

Log-odds: -1.0  -0.5  0.0  0.5  1.0

Popular Loners: Expansively low or isolated but still attractive.

Uninvolved Outsiders: No/few ties to anyone else, low popularity

High-status poor is unstable, likely to become popular loners or outsiders, and outside status is unlikely to move to anything else.

Statistically significant cell values as bold OR

*Setting level random effects multinomial logistic regression models conditional on origin, including wave, sex, race, slun, deviance, church, school attachment, grades, school-level reciprocity, transitivity, degree centralization, structural cohesion & hierarchy score.
**Positional Analysis**

Block models

**Individual mobility models***

**Deviant**

Deviant kids are more likely to move from periphery positions to core positions: *deviance confers status...*

Statistically significant cell values as bold OR

*Setting level random effects multinomial logistic regression models conditional on origin, including wave, sex, race, slun, deviance, church, school attachment, grades, school-level reciprocity, transitivity, degree centralization, structural cohesion & hierarchy score.*
Positional Analysis
Block models

Individual mobility models*

Grades (GPA)

...but good grades also predict being in the most central position, and are protective against moving to peripheral position.

Statistically significant cell values as bold OR

*Setting level random effects multinomial logistic regression models conditional on origin, including wave, sex, race, slun, deviance, church, school attachment, grades, school-level reciprocity, transitivity, degree centralization, structural cohesion & hierarchy score.
Positional Analysis
Block models
Alternative approach: Cluster node metrics.

**Automorphic equivalence:** Same pattern of ties to all others in the network. **Nodes are indistinguishable on any summary metric.** Example: Sports team positions

This implies that if you can characterize a node’s position within a (set of) network(s) as a vector of summary scores, then you can simply cluster that vector to find nodes that are similar across a wide set. This can be very effective for large networks.
It's also possible to use simple-to-calculate scores in unique ways.

So while it's time and space/bandwidth consuming to run a full triad-based structural equivalence model over a giant network; you can calculate a host of local and bridging sorts of scores, then cluster those to get positions quickly.

![Role profile plots](image)
It's also possible to use simple-to-calculate scores in unique ways.

So while it's time and space/bandwidth consuming to run a full triad-based structural equivalence model over a giant network; you can calculate a host of local and bridging sorts of scores, then cluster those to get positions quickly.
Positional Analysis
Block models
Using the results

So once we have positions, what do we do with them?

\[
\text{Self-rated health} = \text{network position} + \text{<other stuff>}
\]

Generally these are metrics on the network that can be mapped back to variables for nodes and used in GLM.

One issue is that your cases are not independent – by definition of being in a network.

On the one hand, measuring the position should capture a lot of what might be driving non-independence, that’s the purpose of “position.” So in that sense, might well be conditionally independent. *(but good luck convincing reviewers of that).*

Simplest thing is to build a model that allows for network autocorrelation in the error structure.
Positional Analysis
Block models
Using the results

Example: Simulate an outcome as a function of node centrality, with and without auto correlated errors. Do 1000 trials, collect estimated coefficients & compare to known truth.
Positional Analysis
Block models
Using the results

Example: Simulate an outcome as a function of node centrality, with and without autocorrelated errors. Do 1000 trials, collect estimated coefficients & compare to known truth.

![Standard Errors for the estimates](image)
Positional Analysis
Block models
Using the results

Redo adding a network autocorrelation of the error (spatial error mode (SEM))
Positional Analysis
Block models
Using the results

Redo adding a network autocorrelation of the error (spatial error mode (SEM))

![Standard Errors for the estimates](image)

No autocorrelation  autocorrelation
Positional Analysis

Block models

Conclusions

Positional approaches are under-used but promising ways to think about networks & health.

Simplest way in is via node metrics or combinations of metrics that capture a unique pattern of ties within the path/flow system of the (perhaps multiple) networks.

Class based approaches are a little more involved, but not terribly so and modern approaches to classification and clustering make it much simpler than in the past.

Still requires a fair amount of investigator judgement.

Models are difficult due to non-independence, but do provide a way to capture some of the otherwise unobserved structure in the network.

Tests for unobserved autocorrelation might be helpful (stay tuned for more on that!).