String Stability Analysis for Vehicle Platooning Under Unreliable Communication Links With Event-Triggered Strategy

Zhicheng Li, Bin Hu, Ming Li, and Gengnan Luo

Abstract—**Vehicle platooning systems are often equipped with vehicle-to-vehicle (V2V) communication technologies to improve both the road efficiency and road safety by exchanging vehicle information over wireless networks to maintain relatively small inter-vehicle distance. The road safety often relies on the quality of service delivered by the V2V communication. The V2V networks are, however, subject to channel fading and often have limited communication bandwidth. The limited bandwidth inevitably compromises the road safety as the vehicular network will easily get congested if the number of vehicles joining the network increases. To address the challenges of ensuring vehicle safety under unreliable wireless network with limited bandwidth, this paper first uses a two-state Markov chain to model the channel fading in vehicular networks. Under the Markov chain model, this paper further develops a novel distributed event triggered strategies under which the vehicle platooning system achieves stochastic $\mathcal{L}_2$ string stability while ensuring an efficient use of the limited communication bandwidth. The simulation results are provided to verify the effectiveness and advantages of the proposed methods.**

Index Terms—**Event-triggered, vehicle platoon, unreliable communication links, $\mathcal{L}_2$ string stability, fading channel model.**

I. Introduction

A. Motivation

In the past few decades, a number of research efforts and new vehicle technologies have been developed to improve system efficiency and road safety for intelligent transportation systems (ITS). As an important part of ITS, vehicle platooning has been extensively recognized as an effective model means to attain an efficient use of limited highway capacity [1] by maintaining relatively small distance between vehicles. The system efficiency achieved by vehicle platooning, however, may conflict with the road safety in many situations due to its inherent string stability issues caused by the chain structure of the vehicle platoons [2]. To address the string stability issues, the vehicle to vehicle (V2V) communication technologies have been recently introduced in the adaptive cruise control system (ACC) to improve the road safety by increasing vehicles' situational awareness of their surrounding environments [1], [3]. As discussed in recent research work [4], [5], the string stability issues can be greatly alleviated if other vehicles' state information (e.g., position and velocities) can be perfectly and reliably accessed through V2V networks and used in the local controller design. However, such assumptions of perfection and reliability are hardly satisfied in practical situations because V2V wireless communications are inherently unreliable and have limited bandwidth [6]–[8]. To address the challenges of unreliable V2V channels with limited bandwidths, this paper first adopts a two-state Markov chain model to characterize the channel fading of the V2V wireless communication. Based on the proposed channel model, this paper further develops a novel distributed event-triggered strategies to ensure stochastic $\mathcal{L}_2$ string stable for vehicle platooning systems under the unreliable wireless fading channels.

The limited bandwidth of V2V communications has become one of the most critical issues that may potentially compromise the road safety in vehicle platooning applications [3], [9]. The traditional protocol for vehicles to establish the mutual communication relies heavily on the periodic transmission scheme where the information exchange occurs at predefined and evenly distributed time instants. Under periodic transmission schemes, the bandwidth issue naturally arises when a large number of vehicles joining and sharing the V2V communication network. In comparison to periodic communication scheme, event-triggered communication transmits vehicle information only when the changes in the vehicle states exceed some predesigned state-dependent thresholds [10], [11]. By only transmitting the information when needed, the event-triggered communication scheme potentially improves the efficient use of the limited bandwidth while still achieving desired system performance comparable to the periodic communication scheme. Motivated by the advantages as well as recent developments of event-triggered communication scheme, one of the main

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contributions of this paper is the development of new distributed event-triggered strategies for vehicle platooning systems.

B. Literature Review and Contributions

Since there is a large body of research work on the topics of event-triggered/self-triggered control and communication, it is beyond the scope of this paper to do a complete literature review, please refer to recent review paper [12] for further details. Instead, this paper will discuss the prior work of event-triggered/self-triggered control and communication for vehicle platooning systems under unreliable wireless communications. By reviewing the relevant literature, this paper will stress the relationship and difference between this paper and prior work. Main contributions of this paper will be summarized at the end of this section.

As promising methods to ensure communication efficiency, event-triggered based communication strategies have been recently developed for vehicle platooning systems. These research work can be classified into two categories. The first category involves the design of event-triggered strategies for vehicle systems assuming that the V2V communication is perfect and reliable. Under the assumption of perfect V2V communication, a nonlinear event-triggered predecessor-following control is developed for vehicle platooning in [13], [14] where a lower bound on the inter-event times is guaranteed.

The second category considers the impact of the network induced imperfections (e.g., packet losses and communication delay) on the event-triggered design [8], [15]–[17]. Specifically, the work in [16] showed that the vehicle platooning systems can achieve $\mathcal{L}_2$ string stability under a dynamic event-triggered control and communication scheme which is designed based on the dynamic event-triggering and periodic event-triggered frameworks developed in [18], [19]. In particular, the results showed that performance of the proposed event-triggered scheme in [16] can be preserved in the presence of time varying but deterministic communication delays. Besides the communication delays [20], other work in [8], [15], [17], [21] considered the unreliability issues from the perspective of packet losses. In particular, event-triggered control strategies were proposed in [17] to show that the $\mathcal{L}_p$ stability can be preserved under both transmission control protocol (TCP) and user diagram protocol (UDP) if the maximum allowable number of successive packet dropouts is bounded. By modeling the events of packet losses as Bernoulli processes, an event-triggered transmission policy was designed in [21] to ensure second-moment stabilization of linear systems under the Bernoulli packet drops.

It is well recognized that the V2V wireless communication in vehicle platooning systems is often subject to channel fading where the communication conditions vary stochastically over time due to vehicles' mobility [6], [7], [22]. The channel models used in the aforementioned work (e.g., Bernoulli processes) cannot capture the time varying nature of V2V fading channels. As a result, the existing event-triggered scheme proposed prior work [15], [17], [21], [23] cannot be directly applied to vehicle platooning systems with V2V fading channels. Motivated by this challenge, this paper proposes the use of a two-state Markov chain [24]–[26] to model the time varying nature of V2V fading channels, which has been experimentally proved to be effective models for wireless mobile fading channels with memories [27]. Under the proposed two-state Markov chain model, the main contributions of this paper are summarized as below,

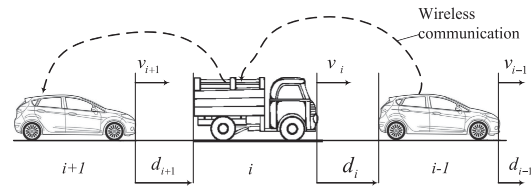

Fig. 1. Vehicle platoon.

- This paper introduces a novel stochastic $\mathcal{L}_2$ string stability concept for the vehicle platooning systems. Such stochastic string stability concept is more suitable for vehicle systems with V2V wireless fading channels than the traditional deterministic $\mathcal{L}_2$ string stability concept in [4], [17] because it characterizes the impact of the stochastic variations of fading channels on the system.
- The second contribution is the design of novel distributed event-triggered strategies for both TCP and UDP transmission protocols under which the vehicle platooning system achieves the stochastic $\mathcal{L}_2$ string stability while ensuring an efficient use of communication bandwidth. To the best of our knowledge, this is the first set of results that consider the distributed event-triggered control and communication design for vehicle platooning systems with V2V fading channels modeled by Markov chain.
- The third contribution lies in the design of a new distributed algorithm that achieves the proposed event-triggered strategies.

The following sections of the paper are organized as follows. The new platoon error dynamics model with fading channel model is formulated in Section II. In Section III, an event-triggered analysis is investigated considering unreliable communication links. In Section IV, the $\mathcal{L}_2$ stabilization analysis is studied, and a new algorithm is presented to stabilize the vehicular networked systems (VNS). An example is given in Section V. Finally the conclusions are given in Section VI.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent the set of real n-vector and $m \times n$ matrices respectively. The superscript $''T''$ stands for matrix transpose. The notation $P > 0\,(\geq 0)$ means that the matrix P is positive (semi)definite. I_n denotes an identity matrix with dimension n and $0_{m,n}$ denotes an $m \times n$ dimension zero matrix. We use $''*''$ to denote the symmetric terms in a block matrix P, $\{P\}_i$ to represent the ith row of its explicitly expressed block structure, and sym(P) to represent $P + P^T$. For a given $x(t)$, the $\mathcal{L}_2$-norm is defined as $\|x\|_{\mathcal{L}_2} = \sqrt[2]{\int_0^\infty x^T(t)x(t)dt}$. A function α is a class $\mathcal{K}$ function if it is strictly increasing and $\alpha(0) = 0$.

II. Vehicle Platooning System Under Wireless Fading Channels

A. Platoon Error Dynamics

Consider a string of m vehicles shown in Fig. 1, d_i denotes the distance between i and its preceding vehicle $i-1$, v_i denotes the velocity of vehicle i and a_i denotes the acceleration of vehicle i. The vehicle platoon is formed in a chain structure with a desired safe distance $d_{r,i}$, which is characterized by a constant time-headway spacing policy [4],

$$d_{r,i}(t) = r_i + hv_i(t),\ 2 \le i \le m, \tag{1}$$

where h is the time headway, and r_i is the standstill distance. The spacing error $e_i(t)$ is thus defined as

$$\begin{aligned} e_i(t) &= d_i(t) - d_{r,i}(t) \\ &= (s_{i-1}(t) - s_i(t) - L_i) - (r_i + hv_i(t)), \end{aligned}$$

with $s_i(t)$ representing the rear-bumper position of vehicle i and L_i its length.

This paper adopts the following vehicle model [5] to characterize the dynamics of each vehicle in the platoon,

$$\begin{pmatrix} \dot{d}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau}a_i + \frac{1}{\tau}u_i \end{pmatrix},\ 2 \le i \le m,$$

where a_i is the acceleration of vehicle i, u_i is an external input representing a desired acceleration, and τ a time constant that characterizes the dynamics of the vehicle engine.

By defining the tracking error as $e_i = d_i - d_{r,i}$, one has

$$\begin{aligned} \dot{e}_i &= v_{i-1} - v_i - ha_i, \\ \ddot{e}_i &= a_{i-1} - a_i - h\left(-\frac{1}{\tau}a_i + \frac{1}{\tau}u_i\right). \end{aligned}$$

The error states can be formulated as

$$\begin{pmatrix} e_{1,i} \\ e_{2,i} \\ e_{3,i} \end{pmatrix} = \begin{pmatrix} e_i \\ \dot{e}_i \\ \ddot{e}_i \end{pmatrix},\ 2 \le i \le m.$$

and

$$\dot{e}_{3,i} = -\frac{1}{\tau}e_{3,i} - \frac{1}{\tau}q_i + \frac{1}{\tau}u_{i-1}, \tag{2}$$

with a new input defined as

$$q_i \triangleq h\dot{u}_i + u_i.$$

Let $x_i = [e_{1,i}; e_{2,i}; e_{3,i}; u_i]$ and $y_i = [u_i]$ denote the state and output of the tracking error subsystem i in the vehicle platoon respectively. The state-space representation of the tracking error subsystem can then be written as

$$\dot{x}_i = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{1}{\tau} & 0 \\ 0 & 0 & 0 & -\frac{1}{h} \end{bmatrix}}_{A_i} x_i + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\tau} \\ \frac{1}{h} \end{bmatrix}}_{B_{1i}} q_i + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \\ 0 \end{bmatrix}}_{B_{2i}} u_{i-1}, \tag{3a}$$

$$y_i = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}}_{C_i} x_i. \tag{3b}$$

It is clear from (4) that the control strategy must consist of both feedback and feedforward parts to ensure asymptotic stability for the output y_i of the tracking subsystem i. Thus, let $K_{1i} \in \mathbb{R}^{1\times 4}$, $K_{2i} \in \mathbb{R}$ denote the gains of feedback and feedforward respectively, and the control input signal q_i can be constructed as

$$q_i = K_{1i}x_i + K_{2i}u_{i-1}.$$

Suppose that the controller vector gains K_{1i} and K_{2i} are selected to satisfy that $\overline{A}_{y,i} \triangleq C_i(A_i + B_{1i}K_{1i})C_i^T$ is a Hurwitz matrix (i.e., every eigenvalue of $\overline{A}_i$ has strictly negative real part) and $C_i(B_{1i}K_{2i} + B_{2i}) = 0$. Note that the asymptotic stability of the output of the tracking system i can only be guaranteed if the states of vehicle i x_i and its preceding vehicle u_{i-1} are both perfectly known to the controller of subsystem i. Such assumptions are generally not possible in practical vehicle platooning system where information of other vehicles, such as u_{i-1}, are difficult to obtain by local on-board sensors and are thus often accessed by wireless communications as shown in Fig. 1. However, the wireless communication used in vehicle platooning is inherently unreliable and subject to deep fades. In the presence of such deep fades, the communication channels often exhibit time varying features caused by fast moving vehicles, which inevitably compromises system stability. To design efficient and stable control and communication strategies, it is important to model such time varying features in wireless communication. In Section II-B, a two-state Markov chain is used to model deep fading channels. Note that from the wireless communication's standpoint, there are many methods, such as channel coding scheme and retransmission protocols [3], to address the unreliability issues caused by channel fading. This paper, however, is more focused on the development of event-triggered strategies as well as control strategies to ensure stochastic string stability of the vehicle platooning systems when the channel fading is present.

Let $\alpha_i(t) \in \{0, 1\}$ denote a binary random variable that models the conditions of wireless link in subsystem i at time instant t. Then, $\alpha_i = 0$ represents the event that the transmitted packet in subsystem i is dropped while $\alpha_i = 1$ stands for the success of packet transmission. The impact of the wireless link conditions on the control system is formulated as follows,

$$q_i = \alpha_i(t)K_{1i}\begin{pmatrix} e_{1,i} \\ e_{2,i} \\ e_{3,i} \\ u_i \end{pmatrix} + \alpha_i(t)K_{2i}u_{i-1},\ 2 \le i \le m, \tag{4}$$

with $K_{1i} = [k_{pi}, k_{di}, k_{ddi}, 0]$. By incorporating the equation (4) into (2), we have

$$\begin{aligned} \dot{u}_i = &-\frac{1}{h}u_i + \frac{1}{h}\alpha_i(t)\left(k_{pi}e_{1,i} + k_{di}e_{2,i} + k_{ddi}e_{3,i}\right) \\ &+ \frac{1}{h}\alpha_i(t)K_{2i}u_{i-1}. \end{aligned}$$

With the controller defined above, the 4th-order closed-loop model cab be reformulated to incorporate the impact of wireless link conditions

$$\dot{x}_i = A_ix_i + \alpha_i(t)B_{1i}K_{1i}x_i + \alpha_i(t)B_{1i}K_{2i}u_{i-1} + B_{2i}u_{i-1} \tag{5}$$

where matrices A_i, B_{1i} and B_{2i} are defined in (3) and $\alpha_i(t)$ is binary random variable that follows a two-state Markov chain, which is further investigated in detail in the following subsection.

B. Wireless Fading Channel Model: Two-State Markov Chain

This section introduces a two-state Markov chain model to characterize the fading characteristics of the wireless communication in vehicle platooning. Note that the two-state Markov chain is well recognized as a fading channel model for a variety of communication environments, particularly suitable for channels with memories [28].

Formally, a two-state Markov chain model for wireless fading channels consists of two states with one state representing the "good" channel condition and the other one representing the "bad" channel conditions. When the wireless communication is in the "good" channel condition, it is more likely that the information transmission succeeds than that under the "bad" channel condition.

In wireless communication, the information is often assembled into a single packet and transmitted at discrete time instants. The transmitted packet may drop with a certain probability due to unreliable communications. Specifically, let $\{t_k\}_{k=0}^{\infty}$ denote an infinite sequence of transmission time instants with $t_{k+1} > t_k, \forall k \in \mathbb{Z}_{\geq 0}$, and the binary sequence $\{\alpha_i(t_k)\}$ denote a random process for the wireless link in subsystem i with $\alpha_i(t_k) \in \{0, 1\}$ being defined as a binary random variable at time instant t_k. In particular, when the random variable takes the value $\{\alpha_i(t_k) = 1\}$, it represents the event that the packet is successfully received and decoded by the controller while $\{\alpha_i(t_k) = 0\}$ represents the event that the packet loses or fails to be decoded correctly. A two-state Markov chain model is then used to characterize the stochastic properties of these two events along the time. Mathematically, the wireless fading channel is said to be a two-state Markov chain if it satisfies the following definition.

Definition 1 (Gilbert-Elliott Channel-Two-State Markov Chain): A two-state Markov chain is a tuple $\mathcal{M} = (S, s_0, P)$ where $S = \{G, B\}$ is a finite set with two states, $s_0 \in S$ is the initial state and $P : S \times S \to [0, 1]$ is the transition matrix that characterizes the transition probability between the states in S. In particular, the state G denotes the "good" channel conditions and B denotes the "bad" channel conditions. The transition matrix Π is defined as below,

$$\Pi = \begin{bmatrix} \pi_{GG} & \pi_{GB} \\ \pi_{BG} & \pi_{BB} \end{bmatrix},$$

where $\pi_{ss'} = \Pr\{s|s'\}$ is a transition probability of channel state transiting from s' to s with $s, s' \in \{G, B\}$. Under the channel condition $s \in S$, the probability of packet loss at time instant t_k can then be defined as $\Pr\{\alpha_i(t_k) = 0|s\} \triangleq \overline{\alpha}_i(s)$ and $\Pr\{\alpha_i(t_k) = 1|s\} = 1 - \overline{\alpha}_i(s)$. The wireless fading channel is a Gilbert-Elliott channel if it satisfies the above properties.

Remark 1: Under the two-state Markov chain channel, the binary random process $\{\alpha_i(t_k)\}_{k=0}^{\infty}$ used for modeling random packet loss is often related to so called Markov modulated Bernoulli processes [29], [30] that are widely studied and used to model queuing networks. It relates the probability distribution of the random packet loss to different channel conditions, thereby providing an effective means to model the time varying feature in vehicular wireless network.

Under the two-state Markov chain model, the closed-loop system can be rewritten as follows:

$$\dot{x}_i = A_i x_i + \alpha_i(t|s) B_i K_{1i} x_i + \alpha_i(t|s) B_i K_{2i} u_{i-1} + B_{2i} u_{i-1}. \tag{6}$$

By defining the sampling time instant T, the continuous model can be discretized as

$$\begin{aligned} x_i(k+1) &= \bar{A}_i x_i(k) + \alpha_i(k|s) \bar{B}_i K_{1i} x_i(k) \\ &\quad + \alpha_i(k|s) \bar{B}_i K_{2i} u_{i-1}(k) + \bar{B}_{2i} u_{i-1}(k) \end{aligned} \tag{7}$$

where $x_i(k) = \begin{bmatrix} e_{1,i}^T & e_{2,i}^T & e_{3,i}^T & u_i^T \end{bmatrix}^T$, $\bar{A}_i = e^{A_i T}$, $\bar{B}_i = \int_0^T e^{A_i t} dt B_i$, $\bar{B}_{2i} = \int_0^T e^{A_i t} dt B_{2i}$.

III. $\mathcal{L}_2$ String Stability and Stability Analysis and Controller Design for Vehicle Platoon

The homogeneous platoon model (7) is a special case of the following cascaded state-space system:

$$\begin{aligned} x_0(k+1) &= f_r\left(x_0(k), u_r\right) \\ x_i(k+1) &= f_i\left(x_i(k), x_{i-1}(k)\right), \ i \in S_m \\ y_i(k) &= h(x_i(k)), \ i \in S_m \end{aligned} \tag{8}$$

This section introduces the notion of *stochastic string stability* defined as follows,

Definition 2 (stochastic $\mathcal{L}_2$ string stability): Consider the interconnected system in (8). Let $x = \begin{pmatrix} x_0^T & x_1^T & \ldots & x_m^T \end{pmatrix}^T$ be the lumped state vector and let $\bar{x} = \begin{pmatrix} \bar{x}_0^T & \bar{x}_1^T & \ldots & \bar{x}_m^T \end{pmatrix}^T$ denote the constant equilibrium solution of (8) for $u_r = 0$. The system (8) is stochastic $\mathcal{L}_2$ string stable if there exist class $\mathcal{K}$ functions $\breve{\alpha}$ and β such that, for any initial state $x(0) \in \mathbb{R}^{(m+1)n}$ and any $u_r \in \mathcal{L}_2$

$$\begin{aligned} E\left\{\|y_i(k) - h\left(\bar{x}_0\right)\|_{\mathcal{L}_2}\right\} &\leq \breve{\alpha}\left(E\left\{\|u_r(k)\|_{\mathcal{L}_2}\right\}\right) \\ &\quad + \beta\left\|x(0) - \bar{x}\right\|, \\ &\quad \forall i \in S_m \text{ and } \forall m \in \mathbb{N}. \end{aligned}$$

If, in addition, with $x(0) = \bar{x}$ it also holds that

$$\begin{aligned} E\left\{\|y_i(k) - h\left(\bar{x}_0\right)\|_{\mathcal{L}_2}\right\} &\leq E\left\{\|y_{i-1}(k) - h\left(\bar{x}_0\right)\|_{\mathcal{L}_2}\right\}, \\ &\quad \forall i \in S_m \backslash \{1\} \text{ and } \forall m \in \mathbb{N} \backslash \{1\}, \end{aligned}$$

Lemma 1 [4]: Consider the interconnected system in (7) and assume that the unstable and marginally stable modes are unobservable, then it is strictly stochastically $\mathcal{L}_2$ string stable, if and only if the following inequality holds

$$\begin{aligned} &E\left\{\|P_1(s)\|_{H_\infty}\right\} < \infty, \\ &E\left\{\|\Gamma_i(s)\|_{H_\infty}\right\} \leq 1, \ \forall i \in \mathbb{N} \backslash \{1\}. \end{aligned}$$

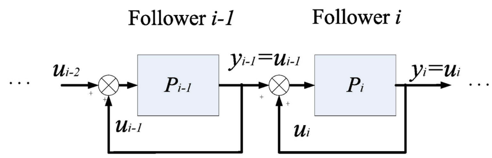

Fig. 2. The interconnection system.

where

$$\|P_1(s)\|_{H_\infty} \triangleq \sup_{u_r \neq 0} \frac{\|y_1(k)\|_{\mathcal{L}_2}}{\|u_r(k)\|_{\mathcal{L}_2}},$$

$$\|\Gamma_i(s)\|_{H_\infty} \triangleq \sup_{y_{i-1} \neq 0} \frac{\|y_i(k)\|_{\mathcal{L}_2}}{\|y_{i-1}(k)\|_{\mathcal{L}_2}}.$$

With the above definitions and lemmas, the following theorem shows the necessary and sufficient conditions under which the cascaded system in (8) is strictly stochastically $\mathcal{L}_2$ string stable.

Theorem 3.1: The interconnected system in (7) is strictly stochastically $\mathcal{L}_2$ string stable, if and only if the following inequality holds

$$E\left\{\sup_{u_r \neq 0} \frac{\|y_1(k)\|_{\mathcal{L}_2}}{\|u_r(k)\|_{\mathcal{L}_2}}\right\} \leq \gamma_1 < \infty,$$

$$E\left\{\sup_{u_{i-1} \neq 0} \frac{\|y_i(k)\|_{\mathcal{L}_2}}{\|u_{i-1}(k)\|_{\mathcal{L}_2}}\right\} \leq \gamma_i \leq 1, \ \forall i \in \mathbb{N}\backslash\{1\},$$

where γ_1 is the supremum of $E\left\{\|P_1(s)\|_{H_\infty}\right\}$, and γ_i is the the supremum of $E\left\{\|\Gamma_i(s)\|_{H_\infty}\right\}$, $\forall i \in \mathbb{N}\backslash\{1\}$.

Proof: Consider the structure of the feedback controller, the first three states $(e_{1,i}, e_{2,i}, e_{3,i})$ of the system is controllable and the fourth state (u_i) is the only state that may be unstable. Thus, one can show that

$$C_i = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, we have

$$u_{i-1}(k) = y_{i-1}(k).$$

The proof is complete.

With the Theorem 1, the structure of the vehicle platoon networked control system can be shown as Fig. 2.

IV. Distributed Event-Triggered Control Scheme Under Unreliable Communication

This section focuses on presenting the information structure of the packet for transmission as well as the event-triggered scheme based on the information structure. From practical standpoints, the information structure of the packet is determined by the types of information needed for the control purpose as well as whether the information can be accessed by local on-board sensors, such as radar and camera.

The schematic representation of the proposed distributed event-triggered scheme for platooning systems is shown in Fig. 3. In the diagram, the event triggered method (ETM) is used on the side of each preceding vehicle that decides when to transmit its state information (X_{i-1}, u_{i-1}) to its following

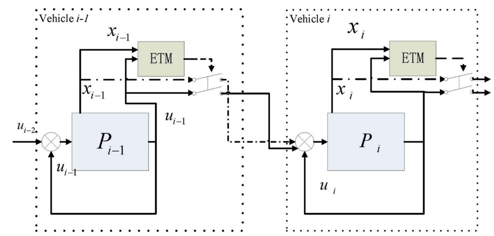

Fig. 3. Platooning systems under distributed event-triggered scheme.

vehicle. The transmission time instant will be determined by the ETM that will be developed in the following section.

As discussed in Section II-A, the controller needs the information of its tracking states x_i as well as the state u_{i-1} from its preceding vehicle. Thus, the information structure of the packet depends on whether the information of the tracking states x_i and preceding vehicle's state u_{i-1} are available to the controller. This paper discusses the information structure under two scenarios. The first scenario considers the case that both information need to be transmitted through wireless communication. In the second scenario, we assume that the tracking state x_i can be obtained by controller through local on-board sensors and the information of preceding vehicle u_{i-1} can only be accessed by wireless communication. This paper will then discuss the event-triggered control scheme under these two scenarios.

Scenario I: Since the information u_{i-1} and state x_i are obtained by wireless communication, let $\hat{u}_{i-1}(k)$ and $\hat{x}_i(k)$ denote the estimate of the u_{i-1} and x_i at time step k. The value of the estimate $\hat{u}_{i-1}$ and $\hat{x}_i$ will heavily depend on the channel conditions of the wireless communication. Consider a simple structure of the estimator for $\hat{u}_{i-1}$ and $\hat{x}_i$ as below, $\forall k \in \mathbb{Z}_{\geq 0}$

$$\hat{u}_{i-1}(k) = \begin{cases} u_{i-1}(k) & \text{if packet is successfully received} \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{x}_i(k) = \begin{cases} x_i(k) & \text{if packet is successfully received} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

one can clearly use other sophisticated estimator (e.g., zero-hold estimator that holds the previously available information). However, it is shown in [31] that there is no guarantee for performance improvement if either of these two strategies are used for even simple scalar systems. For this reason, this paper adopts the simple strategy of setting input to zero rather than the hold strategy that preserves the previous input. We further assume that the physical transmission time is relatively small to be safely neglected and one can decide whether the packet is received or dropped at transmission time step $k, \forall k \in \mathbb{Z}_{\geq 0}$. The controller can be obtained under the simple zero-set strategy $q_i = \alpha_i(k|s)(K_{1i}x_i + K_{2i}u_{i-1})$. The closed-loop subsystem i can be written as

$$x_i(k+1) = A_i x_i(k) + B_{1i}K_{1i}\hat{x}_i(k) \tag{10a}$$

$$+ B_{1i}K_{2i}\hat{u}_{i-1}(k) + B_{2i}u_{i-1}(k), \tag{10b}$$

$$y_i(k) = C_i x_i(k), \quad \forall k \in \mathbb{Z}_{\geq 0}. \tag{10c}$$

Scenario II: Similar to Scenario I, but only the information u_{i-1} is obtained by wireless communication. The estimate of $u_{i-1}(k)$ can be written as a compact form as $\hat{u}_{i-1}(k) = \alpha_i(k|s)u_{i-1}(k)$. With the estimate $\hat{u}_{i-1}$, the controller in (7) can be rewritten as $q_i = K_{1i}x_i + K_{2i}\hat{u}_{i-1}$. The closed-loop of tracking error subsystem i can then be obtained as

$$x_i(k+1) = (A_i + B_{1i}K_{1i})x_i(k) \tag{11a}$$

$$+ B_{1i}K_{2i}\hat{u}_{i-1}(k) + B_{2i}u_{i-1}(k), \tag{11b}$$

$$y_i(k) = C_i x_i(k), \quad \forall k \in \mathbb{Z}_{\geq 0}. \tag{11c}$$

Then, we present a distributed event-triggered control scheme under two different information structures discussed in previous section. In contrast to traditional time-triggered transmission scheme, event-triggered strategy determines the transmission time when the novelty of the variations of channel and vehicle states exceeds a predefined threshold. The key technical parts of event-triggered scheme lie in the event design of a state-dependent threshold.

A. Event Triggered Control Scheme Under User Datagram Protocol (UDP)

This subsection focuses on the design of an event-triggered control scheme under the UDP where the acknowledgement of the packet status is not available for the design.

1) Scenario I: Under the Scenario I, we first assume the expectation of $\alpha_i(k|s)$ is a constant parameter, and the event-triggered system model is written as follows:

$$x_i(k+1) = A_i x_i(k) + B_{1i}K_{1i}\hat{x}_i(j_k) + B_{1i}K_{2i}\hat{u}_{i-1}(j_k) + B_{2i}u_{i-1}(k),$$

and define

$$\delta_{x,i}(k) \triangleq x_i(k) - x_i(j_k),$$
$$\delta_{u,i-1}(k) \triangleq u_{i-1}(k) - u_{i-1}(j_k), \tag{12}$$

where $x_i(k)$ and $u_{i-1}(k)$ denote the state of the error system in (7) at time instant k, while j_k and $x_i(j_k)$, $u_{i-1}(j_k)$ denote the last released instant and the last released data of the transmitter before time instant k. Thus, $\delta_{x,i}(k)$ and $\delta_{u,i-1}(k)$ represent the estimation errors between the state of the plant and the last released state of the transmitter.

If the measurement of $x_i(k)$ and $u_{i-1}(k)$ satisfy a predefined triggering condition, then the packet containing $x_i(k)$ and $u_{i-1}(k)$ is then transmitted. Moreover, if the packet is successfully transmitted through the unreliable communication links, it becomes the new last released data, i.e., $x_i(j_{k+1}) = x_i(k)$. Otherwise, the last released state will not update and remain as the last value. Therefore, in a release period, $k \in [j_k, j_{k+1})$, and j_{k+1} is the next released instant of the transmitter, and $d_i(j_k) = k - j_k - 1$, where $d_i(j_k)$ denotes the number of the unreleased signal between current time and last released instant j_k. In order to reduce the release frequency of the transmitter, only the current measurement $x_i(k)$, $u_{i-1}(k)$ satisfying the following event-triggered condition will be released:

$$\delta_{x,i}^T(k)Q_i\delta_{x,i}(k) \geq \eta_i x_i^T(j_k)\, Q_i x_i(j_k),$$
$$\delta_{u,i-1}^T(k)Q_{ui}\delta_{u,i-1}(k) \geq \eta_i u_{i-1}^T(j_k)\, Q_{ui} u_{i-1}(j_k), \tag{13}$$

where η_i is a given parameter and Q_i and Q_{ui} are the designed matrices for event-triggered condition.

Let $\{\rho(j_k|s)\} = \{\rho(0|s), \rho(j_1|s), \rho(j_2|s), \ldots, \rho(j_k|s), \ldots\}$ denote a two-state Markov chain. Then the data sequence received by the buffer can be represented as follows:

$$\{\rho(j_k|s)\, x_i(j_k)\} = \left\{ \begin{array}{c} \rho(0|s)\, x_i(0), \rho(j_1|s)\, x_i(j_1), \\ \rho(j_2|s)\, x_i(j_2), \ldots, \rho(j_k|s)\, x_i(j_k), \ldots \end{array} \right\},$$

Consider the effectiveness of the buffer, the data of the buffer can be rewritten as

$$\{\hat{x}_i(j_k)\} = \left\{ \begin{array}{c} \overbrace{x_i(0), 0, \ldots, x_i(0)}^{d_i(0)+1}, \overbrace{x_i(j_1), \ldots, 0, x_i(j_1)}^{d_i(j_1)+1}, \\ \overbrace{x_i(j_2), \ldots, 0, \ldots, x_i(j_2)}^{d_i(j_2)+1}, \ldots \\ \overbrace{x_i(j_k), \ldots, 0, \ldots, x_i(j_k)}^{d_i(j_k)+1}, \ldots \end{array} \right\},$$

where $\hat{x}_i(j_k) = \rho(j_k|s)x_i(j_k)$. Consider another random sequence:

$$\{\alpha_i(k|s)\} = \left\{ \begin{array}{c} \alpha_i(0|s), \ldots, \alpha_i(0|s), \alpha_i(j_1|s), \ldots, \alpha_i(j_1|s), \ldots, \\ \alpha_i(j_k|s), \ldots, \alpha_i(j_k|s), \ldots \end{array} \right\},$$

where $\alpha_i(k|s)$ is defined in (6), then one has

$$\hat{x}_i(j_k) = \rho(j_k|s)x_i(j_k),\ k \in [j_k, j_{k+1}), \tag{14}$$

According to the definition of $\delta_{x,i}(k)$, we have

$$\hat{x}_i(j_k) = \alpha_i(j_k|s)\left(x_i(k) - \delta_{x,i}(k)\right).$$

According to the properties of the two-state Markov chain, we have:

$$\Pr\{\rho_i(j_k) = 1|s = 1\} = \bar{\alpha}_i(s = 1),$$
$$\Pr\{\rho_i(j_k) = 0|s = 1\} = 1 - \bar{\alpha}_i(s = 1),$$
$$\Pr\{\rho_i(j_k) = 1|s = 0\} = 1 - \bar{\alpha}_i(s = 0),$$
$$\Pr\{\rho_i(j_k) = 0|s = 0\} = \bar{\alpha}_i(s = 0),$$

where $\bar{\alpha}_i(s)$ is the expectation of $\rho_i(j_k|s)$, one can know that

$$\Pr\{\alpha_i(k) = 1|s = 1\} = \bar{\alpha}_i(s = 1),$$
$$\Pr\{\alpha_i(k) = 0|s = 1\} = 1 - \bar{\alpha}_i(s = 1),$$
$$\Pr\{\alpha_i(k) = 1|s = 0\} = 1 - \bar{\alpha}_i(s = 0),$$
$$\Pr\{\alpha_i(k) = 0|s = 0\} = \bar{\alpha}_i(s = 0),$$

for all k, which means that $\alpha_i(k|s)$ also follows a two-state Markov chain distribution, and $\bar{\alpha}_i(s)$ is also the expectation of $\alpha_i(k|s)$. Thus, the closed-loop system can be obtained as follows:

$$\begin{aligned} x_i(k+1) = &\ (A_i + \alpha_i(k|s)B_{1i}K_{1i})\, x_i(k) \\ &- \alpha_i(k|s)B_{1i}K_{1i}\delta_{x,i-1}(k) \\ &+ (\alpha_i(k|s)B_{1i}K_{2i} + B_{2i})\, u_{i-1}(k) \\ &- \alpha_i(k|s)B_{1i}K_{2i}\delta_{u,i-1}(k). \end{aligned} \tag{15}$$

The leader system in the platoon may be formulated as

$$x_0(k+1) = \bar{A}_0 x_0(k) + \bar{B}_0 u_0. \tag{16}$$

Remark 4: It is worth noting that if $x_i(j_k)$ is failed to be transferred, the input is set to be zero, which is the zero-input strategy in the form of (14). There is also another complex strategy called the hold-input strategy as follows:

$$\begin{aligned} \hat{x}_i(j_k) = &\ \alpha_i(k|s)x_i(j_k) \\ &+ (1 - \alpha_i(k|s))\hat{x}_i(j_{k-1}),\ k \in [j_k, j_{k+1}). \end{aligned}$$

In [31], it tells us that it is hard to say which one is better. To emphasize result of this paper, we choose the zero input strategy to facilitate the analysis.

2) Scenario II: To solve this problem, we first assume the expectation of $\alpha_i(k|s)$ is a constant parameter, and then, considering Scenario II, the event-triggered control model is as follows:

$$\begin{aligned} x_i(k+1) = &\ \left(\bar{A}_i + B_{1i}K_{1i}\right) x_i(k) + B_{1i}K_{2i}\hat{u}_{i-1}(j_k) \\ &+ B_{2i}u_{i-1}(k) \end{aligned}$$

Define

$$\delta_{u,i-1}(k) \triangleq u_{i-1}(k) - u_{i-1}(j_k), \tag{17}$$

where k is the current time, and $u_{i-1}(k)$ denotes the current state of the error system in (7). While, j_k and $u_{i-1}(j_k)$ denote the last released instant and the last released data of the transmitter. Further, $e_{u,i-1}(j_k)$ denotes the error between the current data of the plant and the last released data of the transmitter.

In order to reduce the release frequency of the transmitter, only the current measurement $u_{i-1}(k)$ satisfying the following event-triggered condition will be released:

$$\delta^T_{u,i-1}(k)Q_{ui}\delta_{u,i-1}(k) \geq \eta_i u^T_{i-1}(j_k)\, Q_{ui}u_{i-1}(j_k), \tag{18}$$

where η_i is a given parameter, and Q_{ui} is a weighting matrix with appropriate dimension, concerned with event-triggered condition. Then, borrow the data structure from Scenario I, the closed-loop system is as follows:

$$\begin{aligned} x_i(k+1) = &\ (A_i + B_{1i}K_{1i})\, x_i(k) \\ &+ (\alpha_i(k|s)B_{1i}K_{2i} + B_{2i})\, u_{i-1}(k) \\ &- \alpha_i(k|s)B_{1i}K_{2i}\delta_{u,i-1}(k). \end{aligned} \tag{19}$$

Theorem 4.1: Under the event-triggered strategy in (13), the closed-loop system given by (15) is strictly stochastically $\mathcal{L}_2$ string stable, if there exists a matrix $X_i > 0$, $\tilde{Q}_i > 0$, $Q_{ui} > 0$, and nonnegative constant γ_i, for $s \in S$, such that

$$\tilde{\Xi}_i(s) = \begin{bmatrix} \tilde{\Omega}_{1i} & \tilde{\Omega}_{2is} & \tilde{\Omega}_{3is} & X_iC_i^T \\ * & -X_i & 0 & 0 \\ * & * & -X_i & 0 \\ * & * & * & -I \end{bmatrix} < 0,$$

$$\gamma_1 < \infty,\ \gamma_i \leq 1,\ \forall i \in \mathbb{N}\backslash\{1\}, \tag{20}$$

where

$$\tilde{\Omega}_{1i} = \mathrm{diag}\left(\tilde{\Lambda}_{1i}, \tilde{\Lambda}_{2i}\right),$$

$$\tilde{\Lambda}_{1i} = \begin{bmatrix} -X_i + \eta_i\tilde{Q}_i & -\eta_i\tilde{Q}_i \\ * & (\eta_i - 1)\,\tilde{Q}_i \end{bmatrix},$$

$$\tilde{\Lambda}_{2i} = \begin{bmatrix} -\gamma^2 + \eta_iQ_{ui} & -\eta_iQ_{ui} \\ * & (\eta_i - 1)\,Q_{ui} \end{bmatrix},$$

$$\tilde{\Omega}_{2is} = \begin{bmatrix} \left(A_iX_i + \bar{\alpha}_i(s)B_{1i}\bar{K}_{1i}\right)^T \\ -\left(\bar{\alpha}_i(s)B_{1i}\bar{K}_{1i}\right)^T \\ \left(\bar{\alpha}_i(s)B_{1i}\bar{K}_{1i} + B_{2i}\right)^T \\ -\left(\bar{\alpha}_i(s)B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix},$$

$$\tilde{\Omega}_{3is} = \sqrt{\bar{\alpha}_i(s)\,(1 - \bar{\alpha}_i(s))} \begin{bmatrix} \left(B_{1i}\bar{K}_{1i}\right)^T \\ -\left(B_{1i}\bar{K}_{1i}\right)^T \\ \left(B_{1i}\bar{K}_{2i}\right)^T \\ -\left(B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix}.$$

The state-feedback controller gains and the event-triggered matrix are given by:

$$\begin{aligned} K_{1i}(s) = \bar{K}_{1i}(s)X_i^{-1}(s), K_{2i}(s) = \bar{K}_{2i}(s)\ Q_i(s) \\ = X_i^{-T}(s)\tilde{Q}_i(s)X_i^{-1}(s). \end{aligned}$$

Proof: Construct the Lyapunov functional as follows:

$$V_i(x_i(k)) = x_i^T(k)P_ix_i(k)$$

To facilitate the stability analysis, the sequences $\{\tilde{\alpha}(k|s)\}$ is obtained as

$$\begin{aligned} \tilde{\alpha}_i(k|s) = \alpha_i(k|s) - \bar{\alpha}_i(s),\ E\{\tilde{\alpha}_i(k|s)\} = 0, \\ E\{\tilde{\alpha}_i(k|s)\tilde{\alpha}_i(k|s)\} = \bar{\alpha}_i(s)(1 - \bar{\alpha}_i(s)), \end{aligned}$$

Then the following representations can be calculated as:

$$x_i(k+1) = (\Psi_{1i} + \tilde{\alpha}_i(k|s)\Psi_{2i})\,\bar{\xi}_i(k),$$

where

$$\Psi_{1i} = \begin{bmatrix} (A_i + \bar{\alpha}_i(s)B_{1i}K_{1i}), -\bar{\alpha}_i(s)B_{1i}K_{1i}, \\ \bar{\alpha}_i(s)B_{1i}K_{2i} + B_{2i}, -\bar{\alpha}_i(s)B_{1i}K_{2i} \end{bmatrix},$$

$$\Psi_{2i} = \begin{bmatrix} B_{1i}K_{1i} & -B_{1i}K_{1i} & B_{1i}K_{2i} & -B_{1i}K_{2i} \end{bmatrix},$$

$$\bar{\xi}_i(k) = \begin{bmatrix} x_i(k) & \delta_{x,i}(k) & u_{i-1}(k) & \delta_{u,i-1}(k) \end{bmatrix}.$$

Then we have

$$\begin{aligned} &E\{\Delta V_i(x(k))\} \\ &= \bar{\xi}_i^T(k)\left(\Psi_{1i} + \tilde{\alpha}_i(k|s)\Psi_{2i}\right)^T P_i \left(\Psi_{1i} + \tilde{\alpha}_i(k|s)\Psi_{2i}\right)\bar{\xi}_i(k) \\ &\quad - x_i^T(k)P_i x_i(k). \end{aligned}$$

On the other hand, define

$$X_i \triangleq P_i^{-1},\ F_i \triangleq \mathrm{diag}\{X_i, X_i, I, I, \ldots, I\},$$

performing a congruence transformation with F_i, to (21) and consider the event-triggered event in (13), we have

$$\begin{aligned} &E\{\Delta V_i(x(k))\} + \eta_i \left(x_i(k) - \delta_{x,i}(k)\right)^T Q_i \left(x_i(k) - \delta_{x,i}^T(k)\right) \\ &- \delta_{x,i}^T(k)Q_i\delta_{x,i}(k) \\ &+ \eta_i \left(u_{i-1}(k) - \delta_{u,i-1}(k)\right)^T Q_{ui}\left(u_{i-1}(k) - \delta_{u,i-1}^T(k)\right) \\ &- \delta_{u,i-1}^T(k)Q_{u,i}\delta_{u,i-1}(k) + y_i^T(k)y_i(k) \\ &- \gamma_i^2 u_{i-1}^T(k)u_{i-1}(k) < 0. \end{aligned}$$

Then, the following inequalities hold

$$E\{\Delta V_i(x(k))\} < 0,$$

$$E\left\{\sup_{u_{i-1}\neq 0}\frac{\|y_i(k)\|_{\mathcal{L}_2}}{\|u_{i-1}(k)\|_{\mathcal{L}_2}}\right\} \leq \gamma_i.$$

According to Theorem 1 and the conditions in (22), the strictly stochastic $\mathcal{L}_2$ string stability of the closed-loop system is guaranteed. This finishes the proof. ■

Theorem 2 gives a condition for Scenario I without acknowledgements of packet losses. If Scenario II is considered, the result is only a corollary of Theorem 2 as follows:

Corollary 1: Under the event-triggered strategy in (18), the closed-loop system given by (19) is strictly stochastically $\mathcal{L}_2$ string stable, if there exists a matrix $X_i > 0$, $Q_{ui} > 0$, and nonnegative constant γ_i, for $s \in S$, such that

$$\bar{\Xi}_i(s) = \begin{bmatrix} \bar{\Omega}_{1i} & \bar{\Omega}_{2is} & \bar{\Omega}_{3is} & X_iC_i^T \\ * & -X_i & 0 & 0 \\ * & * & -X_i & 0 \\ * & * & * & -I \end{bmatrix} < 0, \qquad (21)$$

$$\gamma_1 < \infty,\ \gamma_i \leq 1,\ \forall i \in \mathbb{N}\backslash\{1\}, \qquad (22)$$

where

$$\bar{\Omega}_{1i} = \begin{bmatrix} -X_i & 0 & 0 \\ * & -\gamma_i^2 + \eta_i Q_{ui} & -\eta_i Q_{ui} \\ * & * & (\eta_i - 1)Q_{ui} \end{bmatrix},$$

$$\bar{\Omega}_{2is} = \begin{bmatrix} \left(A_iX_i + \bar{\alpha}_i(s)B_i\bar{K}_{1i}\right)^T \\ \left(\bar{\alpha}_i(s)B_{1i}\bar{K}_{2i} + B_{2i}\right)^T \\ \left(-\bar{\alpha}_i(s)B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix},$$

$$\bar{\Omega}_{3is} = \sqrt{\bar{\alpha}_i(s)\left(1 - \bar{\alpha}_i(s)\right)}\begin{bmatrix} 0 \\ \left(B_{1i}\bar{K}_{2i}\right)^T \\ -\left(B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix}.$$

The state-feedback controller gains are given by:

$$K_{1i}(s) = \bar{K}_{1i}(s)X_i^{-1}(s), K_{2i}(s) = \bar{K}_{2i}(s).$$

B. Event Triggered Control Scheme With Acknowledgements of Packet Losses (TCP)

1) Scenario I: The event-triggered control model is as follows:

$$\begin{aligned} x_i(k+1) &= \tilde{A}_i x_i(k) - B_{1i}K_{1i}e_{x,i}(k) - B_{1i}K_{2i}e_{u,i-1}(k) \\ &\quad + (B_{1i}K_{2i} + B_{2i})u_{i-1}(k), \end{aligned} \qquad (23)$$

where

$$x_i(k) = \begin{bmatrix} e_{1,i}^T & e_{2,i}^T & e_{3,i}^T & u_i^T \end{bmatrix}^T,\ K_i = \begin{bmatrix} k & k_d & k_{dd} & 0 \end{bmatrix},$$
$$\tilde{A}_i = A_i + B_{1i}K_{1i}.$$

Define

$$e_{x,i}(k) \triangleq x_i(k) - \hat{x}_i(j_k), \qquad (24)$$

$$e_{u,i-1}(k) \triangleq u_{i-1}(k) - \hat{u}_{i-1}(j_k), \qquad (25)$$

where k is the current time, and $x_i(k)$, $u_{i-1}(k)$ denotes the current data of the error system in (7). While, j_k and $x_i(j_k)$ denote the last released instant and the last released data of the transmitter. $\hat{x}_i(j_k)$, $\hat{u}_{i-1}(j_k)$ denotes the last released estimation data of the transmitter after through the unreliable communication links. The relationship between $x_i(j_k)$, $u_{i-1}(j_k)$ and $\hat{x}_i(j_k)$, $\hat{u}_{i-1}(j_k)$ has been presented in (9). Further, $e_{x,i}(k)$, $e_{u,i-1}(k)$ denote the error between the current data of the plant and the last released estimation data of the transmitter after through the unreliable communication links.

In order to reduce the release frequency of the receiver, only the current measurement $x_i(k)$ $u_{i-1}(k)$ satisfying the following event-triggered condition will be released:

$$\begin{aligned} e_{x,i}^T(k)Q_ie_{x,i}(k) &\geq \eta_i\hat{x}_i^T(j_k)Q_i\hat{x}_i(j_k), \\ e_{u,i-1}^T(k)Q_{ui}e_{u,i-1}(k) &\geq \eta_i\hat{u}_{i-1}^T(j_k)Q_{ui}\hat{u}_{i-1}(j_k), \end{aligned} \qquad (26)$$

where η_i is a given arbitrary parameter, and Q_i and Q_{ui} are weighting matrices with appropriate dimension, concerned with event-triggered condition.

Thus, the closed-loop system can be obtained as follows:

$$\begin{aligned} x_i(k+1) = &\ (A_i + \alpha_i(k|s)B_{1i}K_{1i})\, x_i(k) \\ &- \alpha_i(k|s)B_{1i}K_{1i}e_{x,i}(k) \\ &+ (\alpha_i(k|s)B_{1i}K_{2i} + B_{2i})\, u_{i-1}(k) \\ &- \alpha_i(k|s)B_{1i}K_{2i}e_{u,i-1}(k). \end{aligned} \tag{27}$$

2) Scenario II: Borrow the similar idea from Scenario I, the event-triggered control model is as follows:

$$\begin{aligned} x_i(k+1) = &\ \left(\bar{A}_i + B_{1i}K_{1i}\right) x_i(k) - B_{1i}K_{2i}e_{u,i-1}(k) \\ &+ (B_{1i}K_{2i} + B_{2i})\, u_{i-1}(k). \end{aligned} \tag{28}$$

Define

$$e_{u,i-1}(k) = u_{i-1}(k) - \hat{u}_{i-1}(j_k), \tag{29}$$

where k is the current time, and the definition of $\hat{u}_{i-1}(k)$ j_k, $u_{i-1}(j_k)$ and $e_{u,i-1}(j_k)$ are the same as Scenario I.

In order to reduce the release frequency of the transmitter, only the current measurement $u_{i-1}(k)$ satisfying the following event-triggered condition will be released:

$$e^T_{u,i-1}(k)Q_{ui}e_{u,i-1}(k) \geq \eta_i \hat{u}^T_{i-1}(j_k)\, Q_{ui}\hat{u}_{i-1}(j_k), \tag{30}$$

where η_i is a given arbitrary parameter, and $Q_{u,i}$ is a weighting matrix with appropriate dimension, concerned with event-triggered condition.

Thus, the closed-loop system can be obtained as follows:

$$\begin{aligned} x_i(k+1) = &\ (A_i + B_{1i}K_{1i})\, x_i(k) \\ &(\alpha_i(k|s)B_{1i}K_{2i} + B_{2i})\, u_{i-1}(k) \\ &- \alpha_i(k|s)B_{1i}K_{2i}e_{u,i-1}(k). \end{aligned} \tag{31}$$

Theorem 4.2: Under the event-triggered strategy in (26), the closed-loop system given by (27) is strictly stochastically $\mathcal{L}_2$ string stable, if there exists a matrix $X_i > 0$, $\tilde{Q}_i > 0$, $Q_{ui} > 0$, and nonnegative constant γ_i, such that

$$\hat{\Xi}_i(s) = \begin{bmatrix} \hat{\Omega}_{1i} & \hat{\Omega}_{2is} & \hat{\Omega}_{3is} & X_iC_i^T \\ * & -X_i & 0 & 0 \\ * & * & -X_i & 0 \\ * & * & * & -I \end{bmatrix} < 0,$$

$$\gamma_1 < \infty,\ \gamma_i \leq 1,\ \forall i \in \mathbb{N}\backslash\{1\}, \tag{32}$$

where

$$\hat{\Omega}_{1i} = \mathrm{diag}\left(\hat{\Lambda}_{1i}, \hat{\Lambda}_{2i}\right),$$

$$\hat{\Lambda}_{1i} = \begin{bmatrix} -X_i - \tilde{Q}_i & \bar{\alpha}(s)\tilde{Q}_i \\ * & \bar{\alpha}(s)(\eta_i - 1)\,\tilde{Q}_i \end{bmatrix},$$

$$\hat{\Lambda}_{2i} = \begin{bmatrix} -\gamma_i^2 - Q_{ui} & \bar{\alpha}(s)Q_{ui} \\ * & \bar{\alpha}(s)(\eta_i - 1)\, Q_{ui} \end{bmatrix},$$

$$\hat{\Omega}_{2i} = \begin{bmatrix} A_i^T \\ (\bar{\alpha}_i(s)B_{1i}K_{1i})^T \\ B_{2i}^T \\ (\bar{\alpha}_i(s)B_{1i}K_{2i})^T \end{bmatrix},$$

$$\hat{\Omega}_{3i} = \sqrt{\bar{\alpha}_i(s)\,(1 - \bar{\alpha}_i(s))} \begin{bmatrix} 0 \\ \left(B_{1i}\bar{K}_{1i}\right)^T \\ 0 \\ \left(B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix}.$$

The state-feedback controller gains and the event-triggered matrix are given by: $K_{1i}(s) = \bar{K}_{1i}(s)X_i^{-1}(s)$, $K_{2i}(s) = \bar{K}_{2i}(s)$ $Q_i(s) = X_i^{-T}(s)\tilde{Q}_i(s)X_i^{-1}(s)$.

Proof: The proof follows closely the one in Theorem 2. Construct a Lyapunov function as $V_i(x_i(k)) = x_i^T(k)P_ix_i(k)$, and use the similar proof idea in Theorem 2. ■

In Theorem 3, it gives a stabilization condition for Scenario I with acknowledgements of packet losses. Using the similar idea about Corollary 1, we can easily derive the stabilization condition for Scenario II in the following corollary, because Scenario II is only a special case of Scenario I.

Corollary 2: Under the event-triggered strategy in (30), the closed-loop system given by (28) is strictly stochastically $\mathcal{L}_2$ string stable, if there exists a matrix $X_i > 0$, $Q_{ui} > 0$, and nonnegative constant γ_i, such that

$$\check{\Xi}_i(s) = \begin{bmatrix} \check{\Omega}_{1i} & \check{\Omega}_{2is} & \check{\Omega}_{3is} & X_iC_i^T \\ * & -X_i & 0 & 0 \\ * & * & -X_i & 0 \\ * & * & * & -I \end{bmatrix} < 0,$$

$$\gamma_1 < \infty,\ \gamma_i \leq 1,\ \forall i \in \mathbb{N}\backslash\{1\}, \tag{33}$$

where

$$\check{\Omega}_{1i} = \begin{bmatrix} -X_i & 0 & 0 \\ * & -\gamma_i^2 - Q_{ui} & \bar{\alpha}_i(s)Q_{ui} \\ * & * & \bar{\alpha}_i(s)(\eta_i - 1)\, Q_{ui} \end{bmatrix},$$

$$\check{\Omega}_{2is} = \begin{bmatrix} \left(A_iX_i + B_{1i}\bar{K}_{1i}\right)^T \\ (B_{2i})^T \\ (-\bar{\alpha}_i(s)B_{1i}K_{2i})^T \end{bmatrix},$$

$$\check{\Omega}_{3is} = \sqrt{\bar{\alpha}_i(s)\,(1 - \bar{\alpha}_i(s))} \begin{bmatrix} 0 \\ 0 \\ -\left(B_{1i}\bar{K}_{2i}\right)^T \end{bmatrix}.$$

The state-feedback controller gains are given by:

$$K_{1i}(s) = \bar{K}_{1i}(s)X_i^{-1}(s),\ K_{2i}(s) = \bar{K}_{2i}(s).$$

Remark 3: The key difference between these two protocols lies in the fact that under the TCP protocol, the event-triggered strategies will know whether the message sent out has been successfully received by the receiver (vehicle) while under the UDP protocol, such information will not be available for the design of event-triggered strategies. Because of such difference, the information used to design event-triggered strategies are different for TCP and UDP protocols. To be specific, under the TCP protocol, since the transmitter knows exactly whether the packet is successfully received by the receiver, the estimation of the state x_i at time instant t_k can be constructed on both sides of the transmitter and receiver and we use $\hat{x}_i(k)$ to denote the estimation of the state x_i at time instant t_k on both sides. Given such estimation, one can define the error state as $e_{x,i}(k) \triangleq x_i(k) - \hat{x}_i(k)$ that is used in the design of triggered condition. On the other hand, if the event-triggered strategies are designed under the UDP protocol, the transmitter will not know whether the packets are received by the receiver. Thus, in this paper, we use the information that was sent out in the previous time instant to define the error state, which is $\delta_{x,i}(k) \triangleq x_i(k) - x_i(j_k)$ where $x_i(j_k)$ is the state information that is transmitted in previous time instant. Such information may or may not be received. Because of the difference in defining the error states, this paper developed two event-triggered strategies based on the protocols adopted in the wireless communication.

C. Distributed Event-Triggered Control With Time-Varying Communication Condition

Theorem 2 and 3 are centralized controller design methods, because of the different allowable range of γ_i. But considering the structure of the theorem, the corresponding distributed controller design method can easily be derived by setting $\gamma_i = 1$, $i \in \mathbb{N}$ in Theorem 2 or 3.

Theorem 2, 3 and Corollary 1, 2 can stabilize the system with a constant $\Pr\{\alpha_i(k|s) = 1\} = \bar{\alpha}_i(s)$. Thus, we propose the following algorithm to find the feasible solution for different $\bar{\alpha}_i(s)$.

There are some key remarks about the algorithm in the following.

1) The step 1-9 can be calculated in advance.
2) In the jth probability interval $\left[\frac{j-1}{m}, \frac{j}{m}\right)$, the chosen K_{1i} and K_{2i} are always obtained with $\bar{\alpha}_i(s) = \frac{j-1}{m}$, which is the worst case across the whole interval.
3) For the first probability interval, there is no feasible solution for K_{1i} and K_{2i} with $\bar{\alpha}_i(s) = 0$. Thus, we given a small positive constant $\varepsilon < \frac{1}{m}$. In this algorithm, if $\bar{\alpha}_i(s) < \varepsilon$, the system is uncontrolled. We believe that in this extreme case, nearly no controller can stabilize the system, because nearly no feedback data is available.
4) In every probability interval, our presented theorems and corollaries can stabilize the system every time.

V. Simulation Results

An example of vehicle platooning system with 6 cars is used to demonstrate the effectiveness and advantages of the proposed event-triggered strategy over traditional periodic communication scheme. In the simulation, a Monte Carlo method is used to evaluate the vehicle states in probability. In particular, all the simulation examples are repeated 100 times from 0 second to 400 seconds. The average of these 100 samples is used to approximate the expectation of the vehicle states.

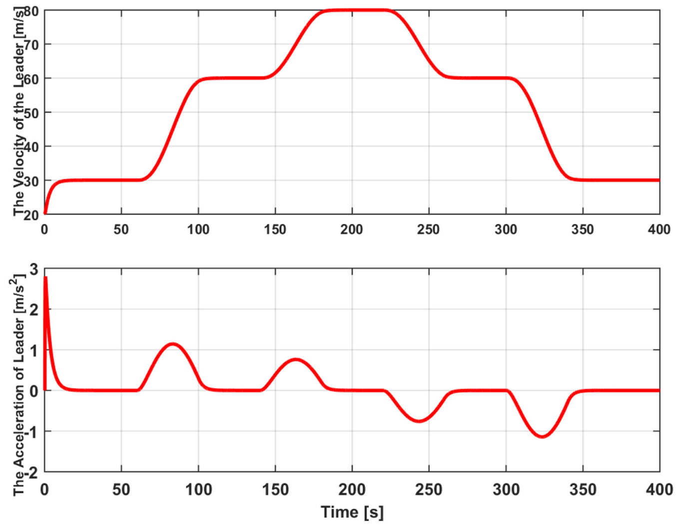

Fig. 4. The velocity and acceleration of the leader.

The control objective of the 6-vehicle platooning system is to have the five followers to track the leader's states by maintaining desired safety distance as well as the velocity and acceleration. The leader's model is given as below

$$\dot{x}_0(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} x_0(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix} u_0(t),$$

where

$$x_0(t) = \begin{bmatrix} d_0^T(t) & v_0^T(t) & a_0^T(t) \end{bmatrix}^T.$$

By selecting the parameters of time constant $\tau = 0.1$ and time headay $h = 0.7$, the continuous models of all the six vehicles are discretized with the sampling time $T = 0.1$ s. Note that the sampling time of $T = 0.1$ s is selected according to the standard of the V2V communication technology [32]. The desired velocities and acceleration of the leader are shown in Fig. 4 to simulate the scenarios when vehicle platoon is traveling in different road sections with different speed limits. Such a simulation setup is commonly adopted in literaure to verify the performance of the platoon systems [16]. The parameter η that defines the event-triggered threshold in (26) or (13) is selected to be $\eta = 0.1$. The selection of the parameter η reflects the trade off between the transmission time interval (communication bandwdith) and the system performance. In particular, a large η leads to an increase of the transmission time interval but a decrease of the tracking performance for the vehicle platoon system. Thus, by tuning the parameter η, one can strike a balance between communication efficiency and system performance. Other parameters in Algorithm 1 are selected as follows: the standstill distance $r_i = 15$, $\varepsilon = 0.001$ and the number of controller gains $m = 20$. In Algorithm 1, the value of m represents the number of intervals for which the controllers are generated. The larger m gives rise to finer selection of the controller gains. As stated in Algorithm 1, each of the controller gain K_{1i}, K_{2i} and the

TABLE I
THE CHOSEN FEEDBACK CONTROLLER GAINS K FOR DIFFERENT INTERVALS OF PROBABILITY

j	1	2	3	...
K	$\begin{bmatrix} 0.0081 \\ 20.3922 \\ 2.0384 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.3930 \\ 19.2531 \\ 2.1275 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.5946 \\ 17.3577 \\ 2.1435 \\ 0 \end{bmatrix}$	...

Algorithm 1: Stabilization Algorithm.

1: **procedure** $(K_{1i}, K_{2i}, Q_i, Q_{ui}) = \text{Choose}\bar{\alpha}_r(s)$
2: Divide the interval of [0, 1] to m intervals, then the jth interval is $[\frac{j-1}{m}, \frac{j}{m})$.
3: For a small positive constant ε, and set the first interval $[\varepsilon, \frac{1}{m})$.
4: set the expectation of $\alpha(k|s)$ as $\bar{\alpha}(s) = \varepsilon$,
5: Use Theorem 2 or Theorem 3 (Corollary 1 or 2) to calculate the feedback controller gain as $K_{1i,1}(s)$, $K_{2i,1}(s)$ and the event triggered matrix as $Q_{i,1}(s)$ and $Q_{ui,1}(s)$
6: **FOR** $j = 2, \ldots, m$ do
7: set the expectation of $\alpha(k|s)$ as $\bar{\alpha}(s) = \frac{j-1}{m}$,
8: Use Theorem 2 or Theorem 3 (Corollary 1 or 2) to calculate the feedback controller gains as $K_{1i,j}(s)$, $K_{2i,j}(s)$ and the event triggered matrices as $Q_{i,j}(s)$, $Q_{ui,j}(s)$.
9: **end FOR**
10: Calculate the real time the expectation of $\alpha(k|s)$ as $\bar{\alpha}_r(s)$, which is the input of this algorithm.
11: **IF** $\bar{\alpha}_r(s) \in \left[\varepsilon, \frac{1}{m}\right)$
12: $K_{1i} = K_{1i,1}(s)$, $K_{2i} = K_{2i,1}(s)$, $Q_{1i,1} = Q_1(s)$, $Q_{ui} = Q_{ui,1}(s)$.
13: Jump 20;
14: **end IF**
15: **FOR** $j = 2, \ldots, m$ do
16: **IF** $\bar{\alpha}_r(s) \in \left[\frac{j-1}{m}, \frac{j}{m}\right)$
17: $K_{1i} = K_{1i,j}(s)$, $K_{2i} = K_{2i,j}(s)$, $Q_i = Q_{i,j}(s)$, $Q_{ui} = Q_{ui,j}(s)$.
18: Break;
19: **end IF**
20: **end FOR**
21: **Output** the feedback controller gains and the event-triggered matrices are K_{1i}, K_{2i}, Q_i and Q_{ui}.
22: **end procedure**

corresponding event triggered matrices Q, Q_u are constructed based on the results in Theorems 2 or 3 (Corollary 1 or 2). Table I provides three out of 20 controller gains to demonstrate the results of applying Theorem 2 or 3. For the conciseness of the paper, we have omitted the actual value of the other 13 controller gains.

In this simulation, we adopted the realistic V2V channel characteristics provided in [6]. The actual V2V channel in vehicle platooning system is then characterized as below,

$$g(y_v, y_c) = g(d_i, y_c) * g(v_i, y_c). \quad (34)$$

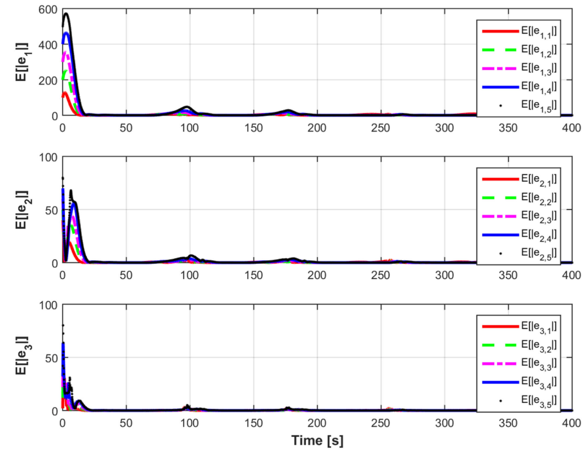

Fig. 5. The tracking errors of leader follower platoon under the event triggered strategy ($\eta = 0.1$) with TCP protocol.

We define

$$g(d_i, y_c) = \begin{cases} e^{-b} & d_i < R \\ e^{-cd_i} & d_i \geq R \end{cases},$$

$$g(v_i, y_c) = \begin{cases} e^{-e} & \tau_{ci} \geq \tau \\ e^{-\frac{f}{\tau_{ci}}} & \tau_{ci} < \tau \end{cases},$$

$$\tau_{ci} = \frac{c_1}{c_2\sqrt{v_{i-1}^2 + v_i^2} + c_3}, \ c_j > 0, \ j = 1, 2, 3,$$

where $b = 0.02$, $R = 25$, $c = b/R$, $c_1 = 0.3$, $c_2 = \frac{0.428}{\lambda\sqrt{2}}$, $c_3 = 11.5$, $\lambda = 0.02$, $f = 4 \times 10^{-5}$.

Then, according to the definitions above, we can obtain the packet loss probability $\alpha(k)$, through the two-state Markov chain model

$$P(k) = \begin{bmatrix} \alpha(k, s=1) & 1 - \alpha(k, s=1) \\ \alpha(k, s=0) & 1 - \alpha(k, s=0) \end{bmatrix}, \quad (35)$$

where $\alpha(k, s=1) = 1.05\alpha(k)$, $\alpha(k, s=0) = 0.95\alpha(k)$.

The initial states of six vehicles are set to be $x_0(0) = [1000\ 20\ 0]^T$, $x_1(0) = [100\ 40\ 2\ 0]^T$, $x_2(0) = [200\ 50\ 12\ 0]^T$, $x_3(0) = [300\ 60\ 22\ 0]^T$, $x_4(0) = [400\ 70\ 32\ 0]^T$, $x_5(0) = [500\ 80\ 42\ 0]^T$.

Simulation results in Fig. 5–6 show that the platooning system is stochastically string stable with the proposed distributed event triggered strategies under the TCP (Fig. 5) and UDP (Fig. 6). Specifically, under both TCP and UDP protocols, Fig. 5 and Fig. 6 show that the absolute values of the tracking errors of inter-vehicle distance $\mathbb{E}[|e_{1,i}|]$, velocity $\mathbb{E}[|e_{2,i}|]$ and acceleration $\mathbb{E}[|e_{3,i}|]$ for all five leader-follower pairs $i = 1, 2, \ldots, 5$ asymptotically converge to zero in expectation. These results imply that the platooning systems can achieve stochastic string stability under the proposed distributed event-triggered strategies. In addition, Fig. 8 shows the comparison of tracking performance of the last (fifth) leader-follower pair under event triggered strategies with TCP (marked by red solid line) and UDP (marked by the black dashed line) protocols. It is clear from the plots in Fig. 8, the tracking performance of the inter-vehicle

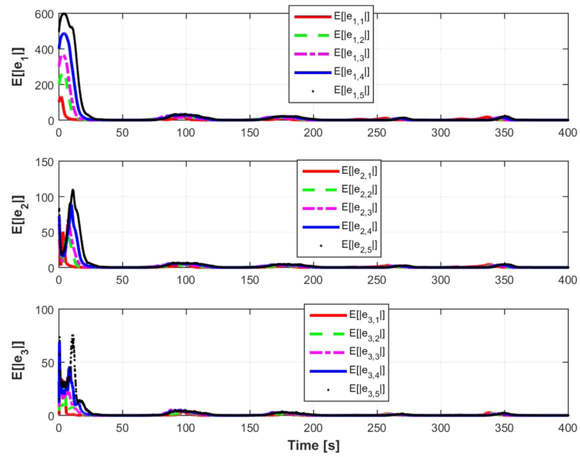

Fig. 6. The tracking errors of leader follower platoon under the event triggered strategy ($\eta = 0.1$) with UDP protocol.

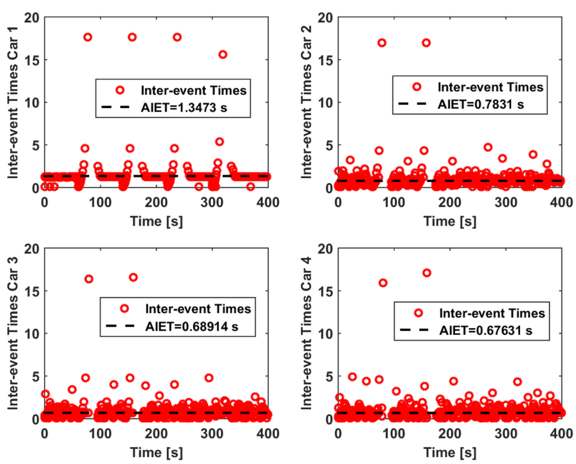

Fig. 7. The inter-event times for the first four vehicles under the event-triggered strategy ($\eta = 0.1$) with TCP protocol. AIET: Average Inter-Event Times.

distance (top plot) under both protocols is very close while the tracking errors of the velocity (middle plot) and acceleration (bottom plot) under TCP protocol are much smaller than that under the UDP protocol in their transient stages. This is because the event-triggered strategy under TCP protocol uses the feedback information from the receiver side (the acknowledgement of whether the transmitted packet is successfully received or not) to adaptively adjust the transmission interval that accommodates the changes on the vehicle states. In contrast, the UDP protocol does not have access to such feedback information and only rely on the changes on the states.

Fig. 7 shows the inter-event times (transmission time intervals) for the first four cars of the vehicle platoon under the event-triggered strategy ($\eta = 0.1$) with the TCP protocol. It is clear from the plots that the event-triggered strategy used in each vehicle generates a sequence of sporadic transmission time as a result of the changes on the vehicle states. In contrast to the traditional periodic transmission scheme that is triggered by constant time interval, the event-triggered strategy is driven by the "events" that correspond to the changes on the vehicle states. Besides the verification of the stochastic string stability under our proposed event-triggered strategies, we are also interested in showing

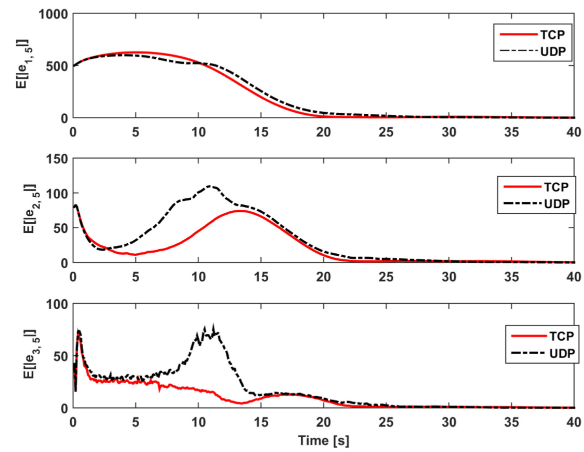

Fig. 8. The comparison of tracking errors of the $5th$ leader-follower pair under TCP and UDP protocols.

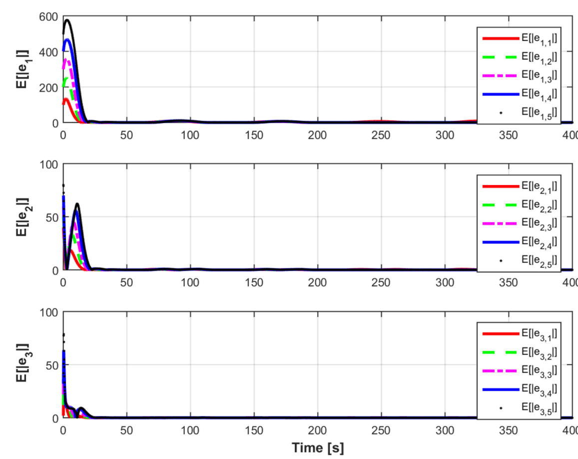

Fig. 9. The tracking errors of leader follower platoon under the periodic transmission scheme with $T = 0.2$ second.

the benefits and advantages of the proposed strategies by comparing it against the traditional periodic transmission scheme. Fig. 10 shows the comparison of tracking performance of the five leader-follower pairs in the 6-vehicle platooning in terms of the expected $\mathcal{L}_2$ performance metrics $\mathbb{E}[\|e\|_{\mathcal{L}_2}]$ under the event-triggered strategy with TCP protocol and periodic transmission scheme with $T = 0.2$ s. As shown in the Fig. 9 and Fig. 10, both strategies of event-triggered and periodic transmission achieve similar $\mathcal{L}_2$ tracking performance. However, as shown in Fig. 7, under the event-triggered strategy, the average transmission time intervals for each leader-follower pair in the vehicle platooning are 1.3473 seconds for Vehicle 1, 0.7831 seconds for Vehicle 2, 0.6891 seconds for Vehicle 3, 0.6763 seconds for Vehicle 4 and 0.6592 seconds for Vehicle 5. With similar tracking performance, the average transmission time intervals generated by the event-triggered strategy are clearly much larger (around 6 times larger for Vehicle 1 and 3 times larger for the rest) than the transmission time interval $T = 0.2$ second used in the periodic transmission scheme. These results clearly imply that the proposed event-triggered strategy indeed ensures efficient use of communication bandwidth while achieving similar system performance. It is worth noting that our further simulation results show that the 6-vehicle platooning system can not be stabilized

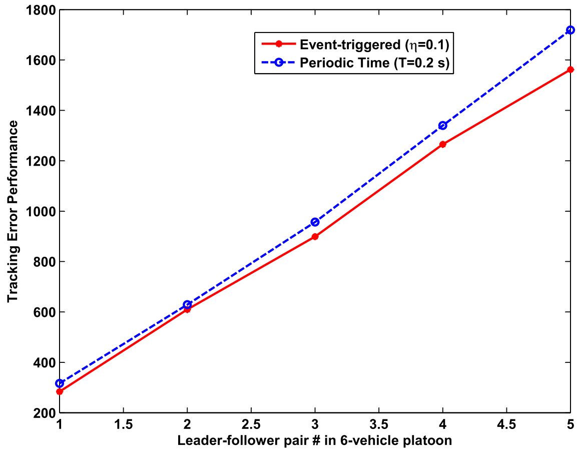

Fig. 10. Comparison of $\mathcal{L}_2$ tracking error performance in expectation for five leader-follower pairs $\mathbb{E}[\|e_i\|_{\mathcal{L}_2}], i = 1, 2, \ldots, 5$ under event-triggered strategy ($\eta = 0.1$) with TCP protocol and periodic transmission strategy with a period $T = 0.2$ second.

under the periodic transmission scheme if the transmission time interval is selected to be larger than $T = 1$ second.

VI. Conclusions

This paper has investigated the problems of $\mathcal{L}_2$ string stochastic stability analysis and controller design for VNS. First, a new VNS model has been proposed based on the communication fading channel model and the existing vehicle platoon model. Then, an event-triggered controller design method has been considered with unreliable communication links, and a distributed controller design condition has been proposed. Considering the changes of the probability of data received based on the fading model, a new algorithm has been also presented. At last, a practical VNS example has been given, and from the analysis of the result, the proposed method has been illustrated to be successful to stabilize the VNS.

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