DKU-NUSRI Joint Workshop on Pure and Applied Mathematics 2022

Talk Information

6th-9th Jan. Duke Kunshan University

Zoom ID： 913 0849 7114 **Passcode：** 471585

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Thursday 6th Jan. Theme: Pure Math.

Title: Random walks in the general linear group

Speaker: **DINH Tien-Cuong**

Affiliation: National University of Singapore

Abstract: We consider products of N randomly chosen dxd matrices. The aim of the theory is to study statistical properties of these products when N goes to infinity. By introducing a new method using complex analysis, we prove several equidistribution properties and limit theorems in the case of SL(2,C). The general case of GL(d,R) will be also discussed. The talk is based on a joint work with Lucas Kaufmann and Wu Hao.

Title: Symplectomorphism groups of rational surfaces

Speaker: **Weiwei Wu**

Affiliation: University of Georgia

Abstract: The symplectomorphism groups of a given symplectic manifold is, in general, a difficult subject. Thanks to Gromov's J-holomorphic techniques, the four-dimensional cases can sometimes be understood completely. However, despite the availability of these powerful tools, there are still lots of mysteries. The symplectic mapping class group, given by the connected components of symplectic automorphisms, is a higher-dimensional generalization of mapping class groups of a surface. A longstanding problem in four-dimensional symplectic topology asks whether the symplectic mapping class groups of all Kahler surfaces are generated by Seidel's Dehn twists. We give an affirmative answer to a large class of symplectic forms for an arbitrary number of blow-ups. Our result has two features: one can reduce the problem of symplectic mapping class groups for all symplectic forms using our method to a problem of existence (and non-existence) of certain holomorphic curves, while the latter problem implies Nagata conjecture. On the other hand, we confirm the existence of a homologically trivial Z/2-action on a certain rational surface that does not extend to an S^1-action, answering a question due to Polterovich.

Title: Anisotropic Dynamical Horizons Arising in Gravitational Collapse

Speaker: Xinliang An

Affiliation: National University of Singapore

Abstract: Black holes are predicted by Einstein's theory of general relativity, and now we have ample observational evidence for their existence. However theoretically there are many unanswered questions about how black holes come into being and about the structures of their inner spacetime singularities. In this talk, we will present several results in these directions. First, in a joint work with Qing Han, with tools from scale-critical hyperbolic method and non-perturbative elliptic techniques, with anisotropic characteristic initial data we prove that: in the process of gravitational collapse, a smooth and spacelike apparent horizon (dynamical horizon) emerges from general (both isotropic and anisotropic) initial data. This result extends the 2008 Christodoulou's monumental work and it connects to black hole thermodynamics along the apparent horizon. Second, in joint works with Dejan Gajic and Ruixiang Zhang, for the spherically symmetric Einstein-scalar field system, we derive precise blow-up rates for various geometric quantities along the inner spacelike singularities. These rates obey polynomial blow-up upper bounds; and when it is close to timelike infinity, these rates are not limited to discrete finite choices and they are related to the Price's law along the event horizon. This indicates a new blow-up phenomenon, driven by a PDE mechanism, rather than an ODE mechanism.

Title: Suppression of explosion by mixing

Speaker: Xiaoqian Xu

Affiliation: Duke Kunshan University

Abstract: In the study of incompressible fluid, one fundamental phenomenon that arises in a wide variety of application is dissipation enhancement by so-called mixing flow. In this talk, I will give a brief introduction to the idea of mixing flow and the role it plays in the field of advection-diffusion-reaction equation. More specifically, I will explain why the presence of fluid can enhance the dissipation and prevent the singularity formation for some types of evolution equations, even with degeneracy.

Title: On the Hausdorff dimension of the graph of a Weierstrass-type function

Speaker: Weixiao Shen

Affiliation: Fudan University

Abstract： The graphs of Weierstrass-type nonwhere differentiable continuous functions are well-studied as fractal sets. We will review the history of research in this direction and present a joint work with Haojie Ren where the following dichotomy is proved: For b>1 integer and \phi real analytic and periodic, either $W(x) =$ \sum _{n\ge 0} b^{-t}\phi(b^n x) is real analytic or its graph has Hausdorff dimension 2-t

Title: Singular Lagrangian fibrations and integrable Hamiltonian systems

Speaker: Konstantinos Efstathiou

Affiliation: Duke Kunshan University

Abstract： Understanding the topology and geometry of the singular Lagrangian fibrations of integrable Hamiltonian systems is important for problems as disparate as the structure of the joint spectrum of integrable quantum systems and the SYZ conjecture in mirror symmetry.

In this talk we will review results on the topological and symplectic classification of integrable Hamiltonian systems, including Duistermaat's work on global action-angle coordinates, Delzant's classification of systems with a Hamiltonian torus action of maximal dimension, and more recent results on classification. Then we will focus on Hamiltonian monodromy and its generalizations and present recent results associating monodromy to the topology of a certain circle bundle (for standard monodromy) or a Seifert fibration (for generalized monodromy).

Title: Cusped Hitchin representations and Anosov representations for geometrically finite Fuchsian groups

Speaker: Tengren Zhang

Affiliation: National University of Singapore

Abstract： We develop a theory of Anosov representation of geometrically finite Fuchsian groups in SL(d,R) and show that cusped Hitchin representations are Borel Anosov in this sense. We establish analogues of many properties of traditional Anosov representations. In particular, we show that our Anosov representations are stable under type-preserving deformations and that their limit maps vary analytically. This is joint work with Richard Canary and Andrew Zimmer.

Title: Lusztig's a-functions for classical Weyl groups

Speaker: Zhanqiang Bai

Affiliation: Soochow University

Abstract： In the representation theory of Lie groups and Lie algebras, Gelfand-Kirillov dimension is a very important invariant to measure the size of infinite-dimensional representations. Lusztig's **a**-functions can be used to compute the Gelfand-Kirillov dimensions of simple highest weight modules. In this talk, I will present some algorithms of Lusztig's **a**-functions for classical Weyl groups. This is a joint work with Wei Xiao and Xun Xie.

Friday 7th Jan.Theme: Applied Math.

Title: Nonlinear Path-Dependent PDEs and Nonlinear Expectation

Speaker: Shige Peng

Affiliation: Shandong University

Abstract: This talk presents some key concepts between the framework of quasi-linear and fully nonlinear PDE and the corresponding expectations. Recent rapid developments in the domain of high dimensional PDE through backward stochastic differential equations become an important engine to this research domain.

Title: Viscosity Solutions of Stochastic Hamilton-Jacobi-Bellman Equations

Speaker: Jinniao Qiu

Affiliation: University of Calgary

Abstract: Fully nonlinear stochastic Hamilton-Jacobi-Bellman (HJB) equations that were first proposed by Prof. Shige Peng would be discussed for the optimal stochastic control problem of stochastic differential equations with random coefficients. The notion of viscosity solution would be introduced, and the value function of the optimal stochastic control problem is the unique viscosity solution to the associated stochastic HJB equation. Applications and some recent developments will be reported as well.

Title: Set values of non-zero sum games and mean field game

Speaker: Jianfeng Zhang

Affiliation: University of Southern California

Abstract: A non-zero sum game may typically have multiple Nash equilibria and different equilibria could lead to different values. We propose to study the set of values over all equilibria, which we call the set value of the game. The set value will play the role of the standard value function in the optimal control problem. In particular, we shall establish two major properties of the set value: (i) the dynamic programming principle (or say time consistency); and (ii) the stability. The results are extended to mean field games without monotonicity conditions, for which we shall also establish the convergence of the set values of the corresponding N-player games. Some subtle issues on the choices of controls will also be discussed. The talk is based on two works, one with Feinstein and Rudloff, and the other with Iseri.

Title: A Time-Inconsistent Dynkin Game

Speaker: Zhou Zhou

Affiliation: The University of Sydney

Abstract: We consider a nonzero-sum Dynkin game in discrete time under non-exponential discounting. For both players, there are two levels of game-theoretic reasoning intertwined. First, each player looks for an intra-personal equilibrium among her current and future selves, so as to resolve time inconsistency triggered by non-exponential discounting. Next, given the other player's chosen stopping policy, each player selects a best response among her intra-personal equilibria. A resulting inter-personal equilibrium is then a Nash equilibrium between the two players, each of whom employs her best intra-personal equilibrium with respect to the other player's stopping policy. Under appropriate conditions, we show that an inter-personal equilibrium exists, based on concrete iterative procedures along with Zorn's lemma. To illustrate our theoretic results, we investigate a two-player real options valuation problem: two firms negotiate a deal of cooperation to initiate a project jointly. By deriving inter-personal equilibria explicitly, we find that coercive power in negotiation depends crucially on the impatience levels of the two firms.

Title: A principal-agent problem with exit contract

Speaker: Xiaolu Tan

Affiliation: The Chinese University of Hong Kong

Abstract: We study a principal-agent problem with one principal and multiple agents. The principal provides an exit contract which is identical to all agents, then each agent chooses her/his optimal exit time with the given contract. The principal looks for an optimal contract in order to maximize her/his reward value which depends on the agents' choices. Under a technical monotone condition, and by using Bank-El Karoui's representation of stochastic process, we are able to decouple the two optimization problems, and to reformulate the principal's problem into an optimal control problem. The latter is also equivalent to an optimal multiple stopping problem and the existence of the optimal contract is obtained. We then show that the continuous time problem can be approximated by a sequence of discrete time ones, which would induce a natural numerical approximation method. We finally discuss the principal-agent problem if one restricts to the class of all Markovian and/or continuous contracts.

Title: Does the leverage effect affect the return distribution?

Speaker: Dangxin Chen

Affiliation: Duke Kunshan University

Abstract: There is broad agreement in the literature that the leverage effect should be present for theoretical reasons, and it has been consistently found in empirical settings. However, some papers have pointed out a puzzling observation: the return distributions of many assets do not appear to be affected by the leverage effect in some specific models. In this paper, we test the hypothesis that the return process is heavily affected by the leverage effect in a general setup. In practice, it is difficult to test this hypothesis without introducing a model. Therefore, we utilize a general stochastic volatility model to fit the return process and provide a joint hypothesis testing framework. To estimate the model parameters, we consider three methods: The method of moments, maximum likelihood estimation, and a minimum-distance method. Through theoretical arguments and empirical experiments, we demonstrate that the minimum-distance estimation method is preferable, due to its robustness, which is essential for return data. We applied our methodology to empirical data and found that the return data are not necessarily affected by the leverage effect. Thus, it can be considered safe to drop the leverage effect assumption, consequently simplifying the model structure.

Title: TBA

Speaker: Cyril Benezet

Affiliation: Université Paris-Saclay

Abstract: TBA

Title: TBA

Speaker: Simone Scotti

Affiliation: Université Paris Diderot

Abstract: TBA

Saturday 8th Jan. Theme: Applied Math.

Title: Deep Approximation via Deep Learning

Speaker: Zuowei Shen

Affiliation: National University of Singapore

Abstract: The primary task of many applications is approximating/estimating a function through samples drawn from a probability distribution on the input space. The deep approximation is to approximate a function by compositions of many layers of simple functions, that can be viewed as a series of nested feature extractors. The key idea of deep learning network is to convert layers of compositions to layers of tuneable parameters that can be adjusted through a learning process, so that it achieves a good approximation with respect to the input data. In this talk, we shall discuss mathematical theory behind this new approach and approximation rate of deep network; we will also show how this new approach differs from the classic approximation theory, and how this new theory can be used to understand and design deep learning network.

Title: Generalization Analysis of Deep-Learning-based PDE Solvers

Speaker: Haizhao Yang

Affiliation: Purdue University

Abstract: Deep learning has been a powerful tool in scientific computing. One of the main recent interests is deep learning-based PDE solves. We will discuss several types of PDE solvers via deep learning and introduce their corresponding generalization error analysis. We will discuss the conditions under which these solvers would admit error rates independent of the dimension of the PDE, which could provide the theoretical justification of deep learning-based PDE solvers in high dimensions.

Title: Understanding Loss Landscapes of Neural Network Models in Solving Partial Differential Equations

Speaker: Jingrun Chen

Affiliation: Soochow University

Abstract: Solving partial differential equations (PDEs) by parametrizing its solution by neural networks (NNs) has been popular in the past a few years. However, different types of loss functions can be proposed for the same PDE. For the Poisson equation, the loss function can be based on the weak formulation of energy variation or the least squares method, which leads to the deep Ritz model and deep Galerkin model, respectively. But loss landscapes from these different models give arise to different practical performance of training the NN parameters. To investigate and understand such practical differences, we propose to compare the loss landscapes of these models, which are both high dimensional and highly non-convex. In such settings, the roughness is more important than the traditional eigenvalue analysis to describe the non-convexity. We contribute to the landscape comparisons by proposing a roughness index to scientifically and quantitatively describe the heuristic concept of ``roughness'' of landscape around minimizers. This index is based on random projections and the variance of (normalized) total variation for one dimensional projected functions, and it is efficient to compute. A large roughness index hints an oscillatory landscape profile as a severe challenge for the first order optimization method. We apply this index to the two models for the Poisson equation and our empirical results reveal a consistent general observation that the landscapes from the deep Galerkin method around its local minimizers are less rough than the deep Ritz method, which supports the observed gain in accuracy of the deep Galerkin method.

Title: Anomaly detection via robust autoencoders

Speaker: Dongmian Zou

Affiliation: Duke Kunshan University

Abstract: Anomaly detection aims to identify data points that "do not conform to expected behavior". It can be done either unsupervised (outlier detection) or semi-supervised (novelty detection). In this talk, we will discuss using robust reconstruction methods for both outlier detection and novelty detection. For outlier detection, we propose an autoencoder with a robust subspace recovery layer (RSR layer). This layer seeks to extract the underlying subspace from a latent representation of the given data and removes outliers that lie away from the subspace. Specifically, the encoder maps data into a latent space, from which the RSR layer extracts the subspace. The decoder then smoothly maps back the underlying subspace to a manifold close to the original inliers. Inliers and outliers are distinguished according to the distances between the original and mapped positions. For novelty detection, we propose a robust VAE with the following components: 1. Extracting crucial features of the latent code by a carefully designed dimension reduction component for distributions; 2. Modeling the latent distribution as a mixture of Gaussian low-rank inliers and full-rank outliers, where the testing only uses the inlier model; 3. Applying the Wasserstein-1 metric for regularization, instead of the KL-divergence; and 4. Using a least absolute deviation error for reconstruction. We illustrate state-of-the-art results for anomaly detection tasks on standard benchmarks.

Title: Exploring the Second-order Solution Sparsity in Statistical Optimization Problems

Speaker: Defeng Sun

Affiliation: The Hong Kong Polytechnic University

Abstract: It is widely believed by many researchers, in particular by those outside the traditional optimization community, that the second-order methods such as Newton's method are no longer applicable for solving large scale optimization problems. This is partially true for optimization models that neither need a good optimal solution nor need to be solved quickly. In this talk, we shall first use large scale statistical optimization problems arising from machine learning to explain why the second-order methods, in particular the proximal point dual Newton methods (PPDNA), if wisely used, can be much faster than the first-order methods. The key point is to make use of the second order solution sparsity of the optimal solutions in addition to the data sparsity so that, at each iteration, the computational costs of the second order methods can be comparable or even lower than those of the first order methods. Equipped with the PPDNA, we shall then introduce adaptive sieving methodologies to generate solution paths of very large sparse statistical optimization problems of particular importance in applications. Finally, we shall illustrate the high efficiency of our approach with extensive numerical results.

Title: On the near-viability property of controlled mean-field flows

Speaker: Juan Li

Affiliation: Shandong University

Abstract: We aim at studying the property of controlled stochastic flows with mean-field dynamics to comply with some (closed) state restrictions. This property, known as (near)-viability, is tackled via (quasi-)tangency methods. Law restrictions and mixed state-law restrictions are considered as is the interplay between the two classes. Explicit conditions for the coefficient functions are provided in the invariance context. Moreover, specific applications to comparison in the convex order to illustrate the theoretical results.

Based on the joint work with Rainer Buckdahn (Brest, France), Dan Goreac (SDU, Weihai).

Title: Non-convex Factorization and Manifold Formulations in Low-rank Matrix Optimization

Speaker: Xudong Li

Affiliation: Fudan University

Abstract: In this talk, we consider the geometric landscape connection of the widely studied manifold and factorization formulations in low-rank positive semidefinite (PSD) and general matrix optimization. We establish an equivalence on the set of first-order stationary points (FOSPs) and second-order stationary points (SOSPs) between the manifold and the factorization formulations. We further give a sandwich inequality on the spectrum of Riemannian and Euclidean Hessians at FOSPs, which can be used to transfer more geometric properties from one formulation to another. We also discuss applications of our findings to some machine learning problems.

Title: Numerical methods for Mean field Games based on Gaussian Processes and Fourier Features

Speaker: Xianjin Yang

Affiliation: Tsinghua University

Abstract: In this talk, we present two numerical methods, the Gaussian Process (GP) method and the Fourier Features (FF) algorithm, to solve mean field games (MFGs). The GP algorithm approximates the solution of a MFG with maximum a posteriori probability estimators of GPs conditioned on the partial differential equation (PDE) system of the MFG at a finite number of sample points. The main bottleneck of the GP method is to compute the inverse of a square gram matrix, whose size is proportional to the number of sample points. To improve the performance, we introduce the FF method, whose insight comes from the recent trend of approximating positive definite kernels with random Fourier features. The FF algorithm seeks approximated solutions in the space generated by sampled Fourier features. In the FF method, the size of the matrix to be inverted depends only on the number of Fourier features selected, which is much less than the size of sample points. We give the existence and the convergence proofs for both algorithms. We show the efficacy of our algorithms through experiments on a stationary MFG with a non-local coupling and on a time-dependent planning problem.

Sunday 9th Jan. Theme: Applied Math.

Title: How Math and AI are revolutionizing biosciences

Speaker: Guowei Wei

Affiliation: Michigan State University

Abstract: Mathematics underpins fundamental theories in physics such as quantum mechanics, general relativity, and quantum field theory. Nonetheless, its success in modern biology, namely cellular biology, molecular biology, biochemistry, genomics, and genetics, has been quite limited. Artificial intelligence (AI) has fundamentally changed the landscape of science, technology, industry, and social media in the past few years and holds a great future for discovering the rules of life. However, AI-based biological discovery encounters challenges arising from the structural complexity of macromolecules, the high dimensionality of biological variability, the multiscale entanglement of molecules, cells, tissues, organs, and organisms, the nonlinearity of genotype, phenotype, and environment coupling, and the excessiveness of genomic, transcriptomic, proteomic, and metabolomic data. We tackle these challenges mathematically. Our work focuses on reducing the complexity, dimensionality, entanglement, and nonlinearity of biological data in AI. We have introduced evolutionary de Rham-Hodge, persistent cohomology, persistent Laplacian, and persistent sheaf theories to model complex, heterogeneous, multiscale biological systems and thus significantly enhance AI's ability to handle biological datasets. Using our mathematical AI approaches, my team has been the top winner in D3R Grand Challenges, a worldwide annual competition series in computer-aided drug design and discovery for years. Using over two million genomes isolates from patients, we discovered the mechanisms of SARS-CoV-2 evolution and transmission and accurately forecast emerging SARS-CoV-2 variants.

Title: Topological data analysis based machine learning for drug design

Speaker: Kelin Xia

Affiliation: Nanyang Technological University, Singapore

Abstract: Effective molecular representation is key to the success of machine learning models for molecular data analysis. TDA-based featurization and feature engineering have demonstrated great power in structure representations. In this talk, we will discuss a series of persistent models, including persistent homology, persistent spectral models, and persistent Ricci curvature and their combination with machine learning models. Unlike traditional graph and network models, these filtration-induced persistent models can characterize the multiscale topological and geometric information, at the same time significantly reduce molecular data complexity and dimensionality. Features are obtained from various persistent attributes derived from mathematical invariants, such as homology, cohomology, eigenvalues, and Ricci curvature. They are combined with learning models, in particular, random forest, gradient boosting tree and convolutional neural network. Our persistent representation based molecular fingerprints can significantly boost the performance of learning models in drug design.

Title: Polymeric Bilayer Interfaces and Their Elastic Properties

Speaker: Yongqiang Cai

Affiliation: Beijing Normal University

Abstract: Bilayer membranes self-assembled from amphiphilic molecules are ubiquitous in biological and soft matter systems. The mechanical response and shape of self-assembled bilayer membranes depend crucially on their elastic properties characterized by a set of elastic moduli. This talk will provide numerical methods to calculate the elastic moduli of self-assembled bilayers within the framework of the self-consistent field theory.

Title: Asymptotic analysis of long-term investment with two illiquid and correlated assets

Speaker: Cong Qin

Affiliation: Soochow University

Abstract: We consider a long-term portfolio choice problem with two illiquid and correlated assets and formulate it as an eigenvalue problem in the form of a variational inequality. The eigenvalue is associated with the portfolio's optimal long-term growth rate, and the free boundaries implied by the variational inequality correspond to the optimal trading strategy. After proving the existence and uniqueness of viscosity solutions for the eigenvalue problem, we perform an asymptotic expansion in terms of small correlations and obtain semi-analytical approximations of the free boundaries and the optimal growth rate. Our leading order expansion implies that the free boundaries are orthogonal to each other at four corners and have C^1 regularity. We propose an efficient numerical algorithm based on the expansion, which proves to be accurate even for large correlations and transaction costs. Moreover, following the approximate trading strategy, the resulting growth rate is very close to the optimal one.

Title: Modelling moving contact lines on elastic membrane

Speaker: Weiqing Ren

Affiliation: National University of Singapore

Abstract: We consider the system of two immiscible fluids on an elastic membrane, where the fluid interface intersects with the membrane at a contact line. In the first part of the talk, we study the static profiles of the interfaces by minimising the total energy of the system, which consists of the interfacial energies and the membrane bending energy. Asymptotic solutions are obtained in the limits as the bending modulus tends to infinity (stiff limit) and zero (soft limit), respectively. In the second part, we consider the dynamical problem and derive the hydrodynamic model, particularly the boundary conditions, from generalised thermodynamics. Numerical solutions are presented for the relaxation of droplets on a membrane and transport of droplets by bendotaxis.

Title: Effective boundary conditions for dynamic contact angle hysteresis

Speaker: Zhen Zhang

Affiliation: National University of Singapore

Abstract: We study the time averaging of the apparent advancing and receding contact angles on surfaces with periodic chemical patterns. We first derive a Cox-type boundary condition for the apparent dynamic contact angle on homogeneous surfaces using Onsager Variational principle. Based on this condition, we derive a reduced model for moving contact line problems on chemically inhomogeneous surfaces by multiscale expansion and averaging techniques. We obtain a quantitative formula for the averaged dynamic contact angles. It gives explicitly how the advancing and receding contact angles depend on the velocity and the chemical inhomogeneity of the substrate. Numerical simulations are presented to validate the analytical results.

Title: Low-regularity integrators for KdV-type equations

Speaker: Xiaofei Zhao

Affiliation: Wuhan University

Abstract: In this talk, I will talk about the numerical integrators for solving the KdV-type equations. Existing methods suffer from convergence order reductions when the solution of the PDE is not ideally smooth. A class of exponential-type integrators for the KdV and the mKdV equations will be introduced to overcome order reduction issues under the rough solution case.

Title: Sharp-Interface Approaches for Simulating Solid-State Dewetting Problems

Speaker: Wei Jiang

Affiliation: Wuhan University

Abstract: Solid thin films are usually thermodynamically unstable in the as deposited state. Due to the very large surface-area-to-volume-ratio, such films can be unstable to particle formation (dewetting or agglomeration), especially at high temperature and/or on long time scales. The process is driven by the minimization of the total interfacial energy of the system. In this talk, I will talk about how to use sharp-interface models for simulating solid-state dewetting problems. The sharp-interface model is governed by surface diffusion and contact line migration, and it belongs to a highly nonlinear, high-order geometric PDE. We propose a parametric finite element method (PFEM) for numerically solving the sharp-interface model. Numerical results demonstrate the high performance of the numerical schemes, as well as the ability of the model to capture many of the complexities that have been observed in the experimental dewetting of thin films on substrates.