

Involution in Competition: Upgraded Efforts Yield Declining Results

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Abstract The term “involution” refers to the prevalent phenomenon of “effort inflation”, where effort is less effective for achieving success in escalating competition over limited resources. The paper constructs a competition model illustrating a repeated game in which students of different types maximize their reference-dependent utility. The key findings are as follows: (1) learners and non-competitors sustain a fixed level of effort; (2) competitors dynamically update reference points, resulting in increasing effort; (3) involution is a self-reinforcing spiral with impetus from learner’s ability accumulation, greater effort by competitors in the loss domain, and the screening effect; (4) involution has an unsustainable nature: competitors who generate involution ultimately will be eliminated.

Keywords Prospect theory, Reference dependence, Involution, Social comparisons

“I am a slow walker, but I never walk backwards.” — Abraham Lincoln

1 Introduction

The term “involution” originated from Immanuel Kant’s *Critique of Judgment*, where he conceptualized it as an internal replication and complication—an inherent regression compared with the concept of “evolution”. Subsequently, American anthropologist Goldenweiser (1936) adapted this term to delineate a distinctive cultural pattern characterized by a society unable to progress into a new stage or stabilize at its current status, but engaged in continuous internal refinement. Geertz (1963) further

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appropriated this expression to elucidate Indonesia's agricultural system. Despite external limitations such as restricted land and capital, Indonesia successfully absorbed surplus labor without causing a decline in average income by "becoming internally more complicated". However, it failed to achieve substantive breakthroughs within this "labor-stuffed" situation. In essence, "involution" denotes a self-conquering process (absorbing more input) coupled with simultaneous self-lock-in (lack of genuine development) under external constraints.

In the contemporary context of China, the term "involution" or "neijuan" has become a buzzword, commonly interpreted as an "effort inflation" phenomenon. The intensifying competition for limited resources, such as ranks and job opportunities, leads to escalating costs for individuals, encompassing efforts and time. This dynamic suggests that a certain level of effort is becoming less effective for achieving "success", while the increased effort level aggravates the intensity of competition, creating a self-reinforcing spiral. Though deviating slightly from the original meanings, today's "involution" still encapsulates the simultaneous processes of self-lock-in (resulting in the same level of resources) and self-conquest (investing more efforts).

Furthermore, the phenomenon of "voluntold involution" is particularly pronounced in China's education system. It is termed "voluntold" because a significant majority of participants are compelled to engage in this intensifying competition among peers, facing the risk of substantial setbacks if they choose not to keep up with the involution. For K-12 education, parents often resort to tiger parenting to push their children hard; otherwise, children may gradually lose all opportunities for success in the educational competition, resulting in degradation of the ranks and quality of schools to which they gain admission, which in turn fuels an educational arms race. In the college setting, where students face limited post-graduate recommendation quotas, a slim success rate in graduate exams, and a highly competitive job market, undergraduates often find themselves ensnared in the pursuit of GPA enhancement. This paper, rather than examining generic educational involution, specifically focuses on the "voluntold" involution phenomenon among college undergraduates, excluding considerations of parents, teachers, and other related parties.

Indeed, involution poses a significant detriment to education in China. The country's education assessment system remains predominantly academic-performance-oriented, and many colleges have adopted a curved GPA that computes GPA based on scores and ranks (percentiles). Therefore, each point in the academic performance assessment holds considerable significance. To obtain a higher exam score, students can participate in two types of effort-consuming activities. The first one is to endeavor to achieve *real learning*, contributing to ability accumulation, termed as *ability-accumulation* activities. The second type is investing effort in activities merely effective for raising exam scores but devoid of benefits to ability growth, termed as *non-ability-accumulation* activities – for instance,

exam preparation. Sometimes, non-ability-accumulation activities prove more effective and efficient in raising test scores, drawing students to invest more effort in these activities in the involution environment. Therefore, academic performance may misrepresent actual ability.

As students engaging in non-ability-accumulation activities easily boost their performance, the overall performance level rises, placing other students at a disadvantage. This situation compels them to invest more effort or also resort to non-ability-accumulation activities. Consequently, involution undermines the signal function of academic performance, causing considerable inefficiencies in education.¹ Compounding this issue is the comprehensive grading rule, initially designed for a more balanced consideration of both the learning process and outcomes. However, it inadvertently provides more opportunities for non-ability-accumulation activities,² thereby amplifying the impact of involution.

This paper constructs a competition model that describes students' decision-making in the involution environment. The model reveals a significant impetus behind the involution phenomenon, shedding light on why individuals find it hard to actively disengage from this escalating competition.

The remainder of the paper is organized as follows: Section 2 provides an overview of existing literature on social comparisons and competitions, which inspires the model. In Section 3, we establish our baseline model and analyze the best response at the first stage across different types of students. Section 4 generalizes these best responses and elucidates the involution dynamic. Section 5 involves simulations that demonstrate the conclusions. Additionally, we explore potential responses for educators to mitigate this issue. In Section 6, we conclude and suggest avenues for further research.

2 Literature Review

One fundamental element of involution is the competition among peers. The competition, or social comparison within a competitive culture, which also serves as an important component of peer effects, has been extensively studied in the economics literature. When peers' performance is partially observed or public, social comparisons present profound impacts on an individual's beliefs, preferences,

¹If we consider the goal of education is cultivating high-quality labor, real learning accumulates human capital and enhances productivity in the future labor market, while involution diverts an individual's effort to meaningless non-ability-accumulation activities.

²For example, academic performance assessments commonly include class participation, computed by the frequency of speaking in class rather than the quality. Therefore, a "rational" student may choose to express simple thoughts in class, avoiding more meaningful contributions that require careful consideration due to the effort involved.

utilities and behaviors, which contributes to the exacerbation of involution.

First, when individuals have incomplete information about themselves, social comparisons deepen their self-concept (Ertac, 2005). In an educational context, perceptibly similar and attainable peers provide aspiration windows (Ray, 2006) for individuals to develop perceived self-efficacy - the belief that one can take actions to achieve desired results (Bandura, 1977). Moreover, the success of a peer can strengthen an individual's self-confidence and motivate them to exhibit greater perseverance (Buechel, Mechtenberg and Petersen, 2018). However, in an involution environment, where performance no longer perfectly represents ability, perceptibly similar (based on perceived ability) peers may yield disparate performance - often higher than expected from perceived ability. Consequently, this misleads individuals to form elevated beliefs about themselves and be over-optimistic.

According to prospect theory (Kahneman and Tversky, 1979), individuals are risk-averse in the gain domain and risk-seeking in the loss domain. However, when the social comparison is integrated into individuals' utility function, it alters reference points, thereby influencing individuals' risk preferences and decision-making. In situations involving social comparison, individuals not only have multiple reference points - where both absolute value and relative position matter (Lu et al., 2015; Lindskog, Martinsson and Medhin, 2022) - but also exhibit heterogeneous endogenous or exogenous social reference points (SRP) (Terzi et al., 2016), such as the average and the best. In alignment with these findings, our model accommodates the allowance for different types of individuals to adopt various reference points.

Numerous studies have found that social comparison modifies individuals' risk preferences. Both upward and downward adjustments in risk preferences are possible as long as they prove beneficial for surpassing others (Lindskog, Martinsson and Medhin, 2022; Schwerter, 2023). Fafchamps, Kebede and Zizzo (2015) report that individuals with a relatively low endowment are more risk-seeking, demonstrating a tendency to track the winner. Lu et al. (2015) propose a detailed conceptual framework and conducted laboratory experiments to elaborate that under financial gain scenarios, individuals are risk-seeking in both social gain and loss situations. To avoid unnecessary complexity, this paper does not model the changes in individuals' risk preferences. However, these facts will strengthen our model's explanatory power for involution. The risk preferences adjustment due to social comparison makes student below their reference point more risk-seeking than in isolation, which potentially increases their expected marginal benefit of effort and ultimately results in greater effort input.

Furthermore, social comparisons influence individuals' future or subsequent behaviors. Buechel, Mechtenberg and Petersen (2018) point out that when individuals are observed by others, they persist longer on task, driven by a reputation effect. Using data on amateur tennis players, Haenni (2019)

reports that losing against opponents with lower rankings results in significant demotivational effects that undermine an individual's future effort and involvement in the competition. Additionally, when individuals face multiple tasks with inferiority in some, according to self-affirmation theory (Steele, 1988), the social comparison may cause effort distortions and inefficiencies as individuals irrationally allocate effort to excel in certain tasks while performing worse in others (Hannan et al., 2013; Villeval, 2020). Also, Roels and Su (2014) develop a model illustrating that social comparison generates output clustering in a behind-averse culture and output polarization in an ahead-seeking culture. In the Discussion Section, we propose the potential extension of our model based on the aforementioned literature.

Moreover, note that the influences of social comparisons on individuals significantly depend on the information structure or reference structure. In other words, different reference points have different effects on individuals in similar positions; and the same reference point can also have different effects on individuals in different positions. For instance, performance information of average or median peers spurs under-performers to improve (Allcott, 2011), while potentially causing over-performers to slack off (Chen et al., 2010). Conversely, providing performance information of the top quartile stimulates better performance for individuals above the median, but diminishes the performance for those below the median (Eyring and Narayanan, 2018). The impact of social comparisons is highly contingent on the reference structure, presenting an opportunity for educators to intervene in involution by deciding on or influencing the revealed information. In the Extension Section, we consider educators' concerns about involution and provide simple corresponding solutions based on educators' objectives, which can be further investigated as an agent-principal problem.

While competition and social comparison have been widely studied, to the best of our knowledge, no existing model adequately describes and explains the phenomenon of involution. Works by Liu and Neilson (2011), Gill and Prowse (2012), Roels and Su (2014) and Thiemann (2017) are most relevant to this paper's topic. However, Liu and Neilson (2011) introduce a simple model differentiating real learning from exam preparation to explain the "high score but low skill" phenomenon, but does not delve into the substance of reference points in social comparisons. Gill and Prowse (2012) develop a sequential move tournament model to analyze the response of disappoint-aversion individuals after observing other's efforts, but this model set the second mover's expectation of success as the reference point, which is just a strand of SRP (social reference points). And our model is structured as a repeated game with simultaneous moves. Roels and Su (2014)'s linear model does not explicitly consider the cost of effort, but the paper provides rich insights into reference structure. Thiemann (2017) is most relevant to our model, offering thorough discussions about a student's best response under different reference points within a detailed parameter range. However, it assumes simple relationships

among effort, performance and ability, neglecting the effort on non-ability-accumulation activities and comprising limited dynamics.

3 The Baseline Model

3.1 Setup

Each student i possesses their own aptitude $a_{i,0} > 0$, which can be deemed as initial ability. Assume the aptitude of this continuum of students is uniformly distributed on $[\underline{a}, \bar{a}]$. This initial distribution is known to all students, while subsequent ability $a_{i,t>0}$ is unobservable. At each time period $t \geq 1$, a student can allocate effort $e_{i,t}^A$ to ability-accumulation activities and $e_{i,t}^N$ to non-ability-accumulation activities. Define $e_{i,t} = e_{i,t}^A + e_{i,t}^N \leq e_{i,t}^{max} = e_i^{max}$. Here e_i^{max} is the maximum effort level for individual i . Let e_i^{max} be sufficiently large, and we will discuss the constraint from e_i^{max} in the later part. We assume aptitude and $e_{i,t}^A$ are complements in ability distribution, given by the relationship:

$$a_{i,T} = a_{i,0} + \sum_{t=1}^T \Delta a_{i,t} \text{ and } \Delta a_{i,t} = a_{i,0} \cdot e_{i,t}^A \quad (1)$$

Furthermore, we assume a linear relationship of ability, $e_{i,t}^N$ and performance $p_{i,t}$ shown in equation 2. Here $e_{i,t}^A$ and $e_{i,t}^N$ are substitutes in performance; θ_i is the parameter representing the effectiveness of $e_{i,t}^N$ for i on performance enhancement. Let $\theta_i > a_{i,0}$, indicating that there is a ‘‘shortcut’’ to success (high performance).

$$p_{i,t} = a_{i,t} + \theta_i \cdot e_{i,t}^N = a_{i,t-1} + e_{i,t}^A + \theta_i \cdot e_{i,t}^N \quad (2)$$

Assume each individual is short-sighted, meaning they only focus on the current stage without discounting future expected utility, then the utility maximization problem for each individual at the time period t is:

$$\max_{e_{i,t}^A, e_{i,t}^N} u_{i,t} = (1 - g_i) \cdot a_{i,t} + g_i \cdot [(1 - s_i)p_i + s_i \cdot v(p_{i,t} - r_{i,t})] - c(e_{i,t}, a_{i,0}) \quad \text{s.t. } e_{i,t}^A + e_{i,t}^N \leq e_i^{max} \quad (3)$$

The utility of each student consists of 3 parts: a value from absolute ability, a value from absolute as well as relative performance among peers and a cost of effort. g_i is a parameter, which indicates an individual’s goal or drive of study: to enhance ability or to improve performance. s_i is a parameter measuring individual i ’s sense of competition, in other words, the importance of social comparison. $r_{i,t}$ is the reference point that student i sets at time t . Note that g_i , s_i and a_i on the right side are private information as ability and types are unobservable by others, while peers’ performances are partially public and therefore, serve as a reference point. Let $F_{P,t}$ denote the cdf or performance distribution at time t , and $Q_{P,t}$ is the quantile function. In the following part, the subscript t may be omitted to avoid

clutter in the notation. According to Tversky and Kahneman (1992), the reference-dependence value function is given by:

$$v(p_i - r_i) = \begin{cases} -\lambda \cdot (r_i - p_i)^\beta & \text{if } p_i < r_i \\ (p_i - r_i)^\beta & \text{if } p_i \geq r_i \end{cases} \quad (4)$$

$\lambda > 1$, which captures the loss-aversion. In this baseline model, we set $\beta = 1$, so the value function is linear. We also assume each individual has the same coefficient of loss aversion λ . Additionally, the cost function is decreasing in $a_{i,0}$, increasing in both kinds of effort, and convex in (e_L, e_E) , with $c'(0) = c(0) = 0$. To simplify the computation results, we adopt a specific function form of cost, where b is a parameter:

$$c_{i,t} = \frac{b(e_{i,t}^A + e_{i,t}^N)^2}{a_{i,0}} \quad (5)$$

Next, we categorize students into two types - learner or non-learner - based on their study motivations g_i . If $g_i = 0$, then i is classified as a *learner*, showing an exclusive concern for their absolute ability with no gains or losses from performance. In the following part, we can see that learners inherently possess an implicit reference point—their ability from the previous stage. For $g_i = 1$, i is identified as a *non-learner*³, who study hard for higher performance. Additionally, non-learners can be further divided into competitors and non-competitors based on their sense of competition s_i . If $g_i = 1$ but $s_i = 0$, then i is a *non-competitor*, as he/she only cares about the absolute value of performance, for example, scores. If $g_i = 1, s_i > 0$, then i is a *competitor*, indicating the existence of emphasis on their relative performance among peers and they take both ranks and scores into account. Let competitors set their reference points at the $100\tau_{i,t}\%$ of the population's performance from the previous period, that is $r_{i,t} = Q_{P,t-1}(\tau_{i,t})$.⁴

The model describes a repeated game with simultaneous moves. At each round, individuals set their *prior* reference points r_i according to the information of previous performance distribution, then choose e_i^A and e_i^N to maximize their expected utility. At the end of the round, the performance

³Alternatively, a student with $\frac{1}{2} \leq g_i \leq 1$ could be designated as a non-learner, otherwise a learner. For simplicity, here we only consider extreme cases.

⁴Note individuals maximize their expected utility based on the prior reference point $r_{i,t}$, but their utility will be realized based on the post reference point determined by the realized and revealed performance information. As ambiguity about the population's s_i and g_i exists, a student can set their prior reference point higher or lower than their real aspiration about the position. For example, students with a target of post falling above 75% can set their prior reference point at performing above 80% of the previous period, if he/she worries that the possible intensifying competition among peers will increase the performance threshold. But in this paper, for simplicity, we just assume individuals set their prior reference point $r_{i,t}$ the same as their real aspirations.

information of this round is revealed and their utility is realized based on the *post* reference point. At the next round, they update their reference points and make effort choices.

3.2 One-shot Game Analysis for $t = 1$

In this section, we examine individuals' behaviors in the initial stage. Competitors establish their reference points based on the population's performance, which, in the first round, corresponds to the $100\tau_{i,1}\%$ of the known initial ability distribution: $r_{i,1} = \tau_{i,1}\bar{a}_0 + (1 - \tau_{i,1})\underline{a}_0$. Note that τ_i varies across i , allowing each competitor to set their own reference point. Additionally, we assume that at each stage, an individual's single performance will not influence the performance level of their reference points.

3.2.1 Learners' Best Response

By definition, for a learner, the utility maximization problem at the first stage is as follows:

$$\max_{e_{i,1}^A, e_{i,1}^N} u_{i,1} = a_{i,0}(1 + e_{i,1}^A) - \frac{b(e_{i,1}^A + e_{i,1}^N)^2}{a_{i,0}} \quad \text{s.t. } e_{i,1}^A + e_{i,1}^N \leq e_i^{max}$$

Proposition 1. *At first stage, learners' best response is $(e_{i,1}^A, e_{i,1}^N) = (\frac{a_{i,0}^2}{2b}, 0)$.*

This is the result directly from Karush-Kuhn-Tucker (K-T) conditions of the Lagrange multiplier, with the assumption that e_i^{max} is sufficiently large. Furthermore, for learners, the above best response can be generalized to every stage.

Corollary 1. *At each stage, learners' best response is $(e_{i,t}^A, e_{i,t}^N) = (\frac{a_{i,0}^2}{2b}, 0)$.*

First, learners abstain from investing effort in non-ability-accumulation activities, as they contribute to a reduction in utility due to increased costs, so $e_{i,t}^N = 0$. Then, the marginal benefit (*MB*) of $e_{i,t}^A$ at any stage is $a_{i,0}$, while the marginal cost (*MC*) of $e_{i,t}^A$ at each stage is $\frac{2be_{i,t}^A}{a_{i,0}}$. Consequently, the best choice of effort at each stage remains consistent, i.e., the solution to $MB = MC$. Moreover, *MB* is derived from $\Delta a_{i,t} = a_{i,0}e_{i,t}^A$ at each stage, indicating that only the incremental ability matters in FOC. Hence, the absolute ability from the previous stage can be deemed as the implicit reference point of learners.

3.2.2 Non-learners' Best Response

Note $e_{i,1}^A$ and $e_{i,1}^N$ are identical in the cost function, but for non-learners, these two types of efforts have different *MB*. At the first stage, as $\theta_i > a_{i,0}$, $e_{i,1}^N$ is more attractive for non-learner and they only put effort into non-ability-accumulation activities. This situation also holds in the later stage.

Remark 1. If $\theta_i > a_{i,0}$, non-learners only put effort in non-ability-accumulation activities. For non-learners, $a_{i,t} = a_{i,0}$.

For non-competitors, their utility maximization problem is:

$$\max_{e_{i,1}^A, e_{i,1}^N} u_{i,1} = a_{i,0} + a_{i,0}e_{i,1}^A + \theta_i e_{i,1}^N - \frac{b(e_{i,1}^A + e_{i,1}^N)^2}{a_{i,0}} \quad \text{s.t. } e_{i,t}^A + e_{i,t}^N \leq e_i^{max}$$

Non-competitors' best response is similar to that of learners, as both of them pay no attention to social comparison. Therefore, we can easily get:

Proposition 2. At each stage, non-competitors' best response is $(e_{i,t}^A, e_{i,t}^N) = (0, \frac{a_{i,0}\theta_i}{2b})$.

When it comes to competitors, the situation becomes more complicated. The competitor's reference-dependence utility function at the first stage is given by:

$$u_{i,1} = (1 - s_i)(a_{i,0} + a_{i,0}e_{i,1}^A + \theta_i e_{i,1}^N) + s_i \cdot v(a_{i,0} + a_{i,0}e_{i,1}^A + \theta_i e_{i,1}^N - r_{i,1}) - \frac{b(e_{i,1}^A + e_{i,1}^N)^2}{a_{i,0}}$$

Proposition 3.

- (i) For competitors with $a_{i,0} \in [a_0, \frac{2b \cdot r_{i,1}}{\theta_i^2[1-s_i+\lambda s_i]+2b}]$, their best response is $(e_{i,1}^A, e_{i,1}^N) = (0, \frac{a_{i,0}\theta_i(1-s_i+\lambda s_i)}{2b})$;
- (ii) For competitors with $a_{i,0} \in [\frac{2b \cdot r_{i,1}}{\theta_i^2[1-s_i+\lambda s_i]+2b}, \frac{2b \cdot r_{i,1}}{\theta_i^2+2b}]$, their best response is $(e_{i,1}^A, e_{i,1}^N) = (0, \frac{r_{i,1}-a_{i,0}}{\theta_i})$;
- (iii) For competitors with $a_{i,0} \in [\frac{2b \cdot r_{i,1}}{\theta_i^2+2b}, \bar{a}_0]$, their best response is $(e_{i,1}^A, e_{i,1}^N) = (0, \frac{a_{i,0}\theta_i}{2b})$.

We provide a detailed proof in Appendix 7.2.1. Proposition 3 is an application of Thiemann (2017)'s conclusions. Figure 1 adapted from that paper shows a similar analysis. MC_1 , MC_2 , and MC_3 respectively represent case 1 (low aptitude, completely loss domain), case 2 (medium aptitude, stopping at reference point), and case 3 (high aptitude, completely gain domain) of Proposition 3.

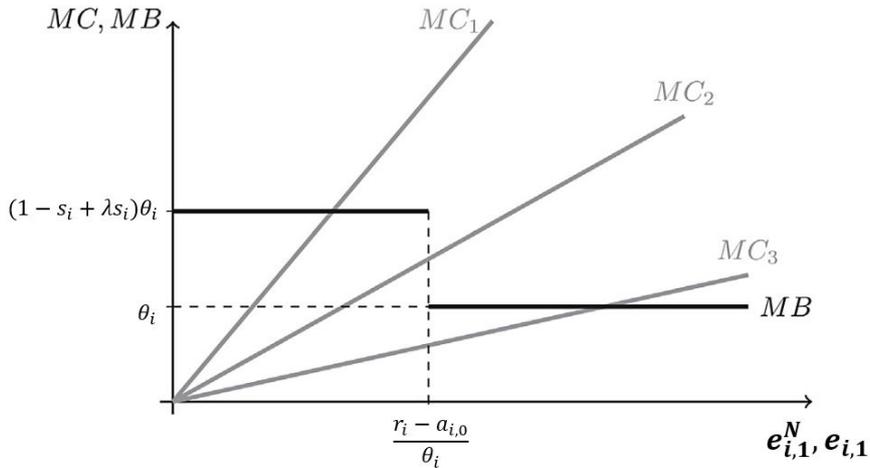


Figure 1: MB and MC for different initial ability

3.2.3 Comparison among Different Types

In this part, we explore the outcomes following simultaneous effort choices in the first stage, comparing effort choices, realized performance, and updated ability distribution among students of different types. For simplicity, we assume all competitors choose the same $\tau_i = \tau$.

Based on the preceding propositions, we can easily compare the best responses in the first stage among different types of students. It is important to note that, up to this point, we continue to assume that e_i^{max} is sufficiently large. *Given the same $a_{i,0}$* , a learner invests the least effort ($\frac{a_{i,0}^2}{2b}$), while a competitor inputs the most effort, especially in the loss domain case ($\frac{a_{i,0}\theta_i(1-s_i+\lambda s_i)}{2b}$)⁵; and non-competitors will choose effort levels the same as competitors under the gain domain case. The intuition is clear. All students have the same form of cost function, so the difference in their effort choices heavily depends on variance in MB for each type. Given that $\theta_i > a_{i,0}$ and $\lambda > 1$, non-ability-accumulation is tempting. Additionally, individuals in the loss domain exhibit a stronger drive to endeavor due to loss aversion.

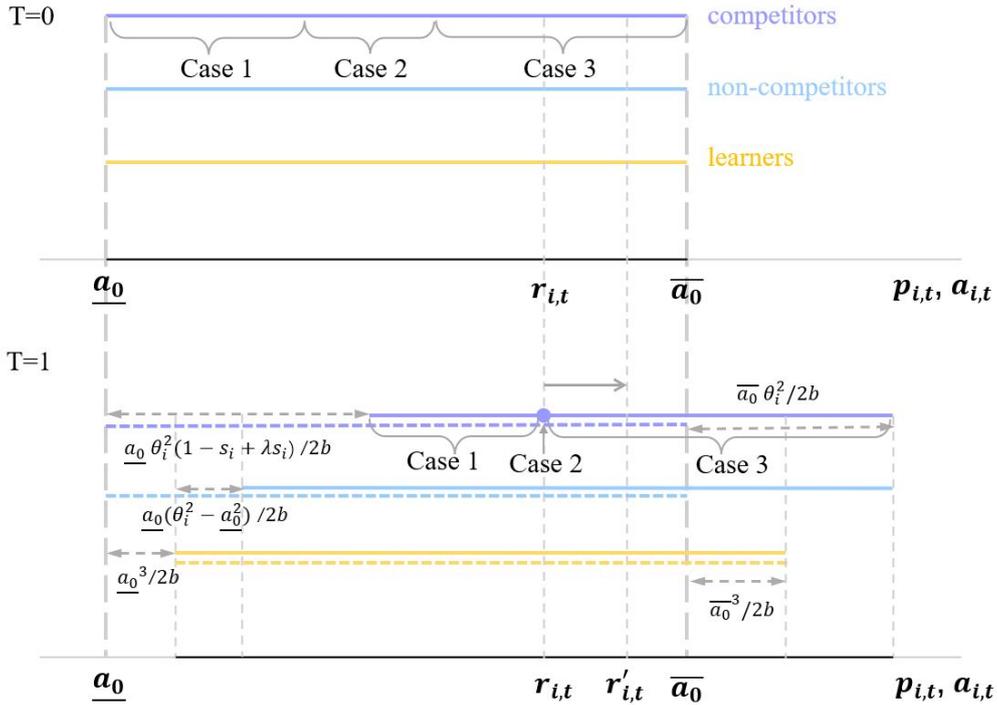


Figure 2: Changes in performance (solid) and ability (dashed) distribution at T=1

For simplicity, we assume that students' types s_i and g_i are independent of their aptitude $a_{i,0}$.

⁵This does *not* mean competitors completely in the loss domain are always more hard-working than competitors completely in the gain domain, because the latter always have higher aptitude and effort choice depends on both aptitude and reference point. For example, when $\frac{a_{j,0}^{case3}}{a_{i,0}^{case1}} \geq \frac{\theta_i(1-s_i+\lambda s_i)}{\theta_j}$, competitors in gain domain invest more efforts, and this situation is possible with proper parameter range.

Thus within each type, the aptitude is uniformly distributed on $[\underline{a}, \bar{a}]$. Figure 2 presents the realized performance (solid line) and updated ability (dashed line) after individuals make choice decisions at $T = 1$. There are several points worthy of note:

1) the spread of competitors' performance is shorter than that of non-competitors, due to greater effort input from individuals completely in the loss domain (case 1) and the concentration of students at prior reference point r_i (case 2).

2) With the realized performance distribution shifting rightward than the original one, the post reference point $r'_i = Q_{P,1}(\tau)$ is larger than the prior one $r_i = Q_{P,0}(\tau)$. The difference between these two reference points depends on the percentage of each type. Consequently, some competitors will have negative realized utility, and we define that once an individual suffers negative realized utility, they will *opt out* from this competition, indicating they are crashed out by the involution. If we restrict $e_i^{max} \leq \frac{a_{i,0}\theta_i(1-s_i+\lambda s_i)}{2b}$ or even smaller, then more competitors will opt out. Additionally, within each case, it is easier for competitors with low aptitude to opt out⁶, which "refines" the group of students and further increases the performance level at the reference point, worsening the situation at the next stage. We refer to this refinement as "screening effect" in the later part.

3) the updated ability distribution displays an opposite situation to the realized performance: the overall level of learners' ability is the highest, though their overall performance level is the lowest.

4 Dynamics

In this part, we discuss situations when $t > 1$. We have already generalized learners' best responses in Corollary 1 and non-competitors in Proposition 2.

4.1 Comparison between Learners and Non-competitors

According to Corollary 1, for learners:

$$a_{i,T} = a_{i,0}\left(1 + T\frac{a_{i,0}^2}{2b}\right), \quad p_{i,T} = a_{i,T} \quad (6)$$

Based on Proposition 2, for non-competitors:

$$a_{i,T} = a_{i,0}, \quad p_{i,T} = a_{i,0} + \frac{a_{i,0}\theta_i^2}{2b} = a_{i,0}\left(1 + \frac{\theta_i^2}{2b}\right) \quad (7)$$

⁶In this part, we assume all competitors have the same τ , which seems unreasonable for individuals with lower initial ability. However, even though they set a lower reference point, it will not alter the essence of this phenomenon. Lower reference points just allow them to crash out at a slower rate.

When $t \geq \lceil \frac{\theta_i^2}{a_{i,0}^2} \rceil$, given the same $a_{i,0}$, a learner will surpass a non-competitor even in terms of performance. Once they outperform non-competitors, they will keep performing better forever.

To some extent, non-competitors are very similar to learners as they share almost the same behavior, investing a fixed amount of effort (though in different activities) at each period. Though non-competitors excel at the beginning, the situation reverses very soon. As time goes by, the discrepancy grows, and ultimately, non-competitors lag far behind learners. Under the assumptions of short-sightedness and $\theta_i > a_{i,0}$, non-competitors are induced into a trap stemming from the temporarily more efficient growth of performance by non-ability-accumulation activities, ending up with decisions that harm long-term benefits (Platt, 1973).

4.2 Competitors' Best Response

At each stage, competitors' best responses share the same form as Proposition 3, while plugging in the updated reference point $r_{i,t} = Q_{P,t-1}(\tau)$. When the reference point is updated to a higher level, students previously in case 2 (case 3) may fall into case 1 (case 2), which pushes them to put in more effort. Despite the increasing effort level, when $t \geq \lceil \frac{\theta_i^2(1-s_i+\lambda s_i)}{a_{i,0}^2} \rceil$, a learner will excel relative to competitors with the same aptitude, even those in the loss domain thus investing the most efforts.

4.3 Involution

Follow the discussion on 3.2.3, in this dynamic repeated game, three factors shift $F_{P,t}$ rightward than $F_{P,t-1}$, elevating the performance level at the reference point: 1) the increase in performance of learners contributed by ability accumulation; 2) the screening effect arising from the opting-out of competitors with relatively low aptitude, which reduces the weight of low-performance level; 3) the greater effort invested by competitors in the loss domain than their non-competitor counterparts. Furthermore, the increasing level at the reference point strengthens the second factor (enhances the effort threshold for non-negative realized utility) and the third factor (places more students in the loss domain), generating a self-reinforcing spiral. This passively increasing performance level at the reference point also compels individuals to “voluntoldly” devote more effort to involution.

However, the outcome of such a self-accelerating procedure is self-annihilation, demonstrating that involution is unsustainable for obtaining long-run success. When $r_{i,t} = Q_{P,t-1}(\tau) \geq \frac{\bar{a}_0(\theta_i^2+2b)}{2b}$, it becomes impossible for competitors to fall into case 3, and this situation is also perilous for cases 1 and 2. The time when all competitors have crashed out depends on the shares of each type in the

population and various other parameters⁷ making it complex to be explicitly expressed. However, the simulation in the following part will illustrate this process.

5 Extension

5.1 Simulation

5.1.1 Involution Factor: Self-reinforcing Involution

In this part, we present an alternative method to describe the involution phenomenon, which is more convenient for comparing individuals' best responses without scrutinizing the population's performances. As mentioned earlier, involution is characterized by relatively limited resources and fiercer competition. We introduce γ^{t-1} as the involution factor with $\gamma > 1$, influencing the updating of the reference point by $r_{i,t} = \gamma^t \cdot r_{i,t-1}$. This equation explicitly illustrates the escalating reference points, signifying that within the involution environment, given the same resource (fixed top percentile for A or A-), the corresponding performance level experiences exponential growth.

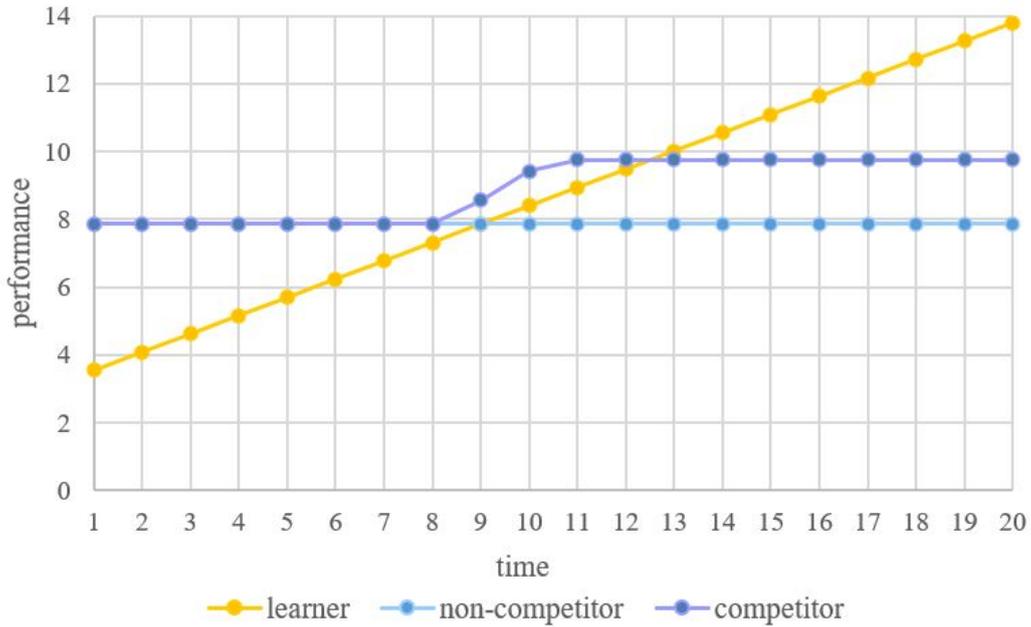


Figure 3: Changes in performance level of different types with the same aptitude

⁷It is possible that with relatively low τ or relatively small share of learners in the population, some competitors can survive over time. For example, when the lowest-aptitude learner's performance slightly exceeds $100\tau\%$, i.e. $\underline{a}_0(1 + T\frac{a_0^2}{2b}) > Q_{P,t}(\tau)$, the competitor with highest performance can still pass the post reference point $Q_{P,t}(\tau)$, and this competitor will survive over time, in the situation, say, with sufficient non-competitors. This is another equilibrium, but in our opinion less realistic, as competitors will not set reference points so low.

For the simulation, we set $\bar{a}_0 = 5, \underline{a}_0 = 1, \gamma = 1.1, b = 25, r_0 = 4, \theta = 3a_{i,0}, \lambda = 2.3$. Figure 3 depicts the performance trends of learners, non-competitors, and competitors with the same aptitude $a_0 = 3$, which is the population average. And let competitor's $s_i = 0.3$. At $T = 9, r_{i,9} = 4 \cdot \gamma^8 = 8.57$, and $\frac{2b \cdot r_{i,1}}{\theta_i^2 + 2b} = 3.27 > a_{i,0} = 3$. This marks the competitor's transition from case 3 to case 2, prompting increased effort to boost performance and the gap between competitors and non-competitors ensues. At $T = 11, r_{i,11} = 4 \cdot \gamma^{10} = 10.38, \frac{2b \cdot r_{i,1}}{\theta_i^2 [1 - s_i + \lambda s_i] + 2b} = 3.19 > 3$, the competitor descends to case 1 (the completely loss domain). At $T = 13$, the competitor's realized utility is -0.66, leading to opting out of the competition for the next round. In contrast, the learner surpasses both the non-competitor and competitor at $T = 9$ and $T = 13$, respectively. The learner's effort level remains at $e_{i,t}^A = 0.18$, while the competitor's maximum effort level is $e_{i,t \in [11,13]}^N = 0.75$, four times more than that of learners. However, such a substantial effort fails to guarantee long-term success.

Moreover, at each stage, the non-competitor chooses $e_{i,t}^N = 0.54$, three times the learner's effort level. Initially, non-competitors outperform learners, but this advantage is not sustained.

5.1.2 Self-annihilation Involution

The above simplified simulation illustrates the disparities in best responses among different types. Back to our model, we use the quantile function to update the reference point and conduct another simulation to enhance understanding of the preceding conclusions that involution is self-annihilation. For simplicity, we consider a population only consisting of competitors and learners, as non-competitors decelerate the rate of destruction.

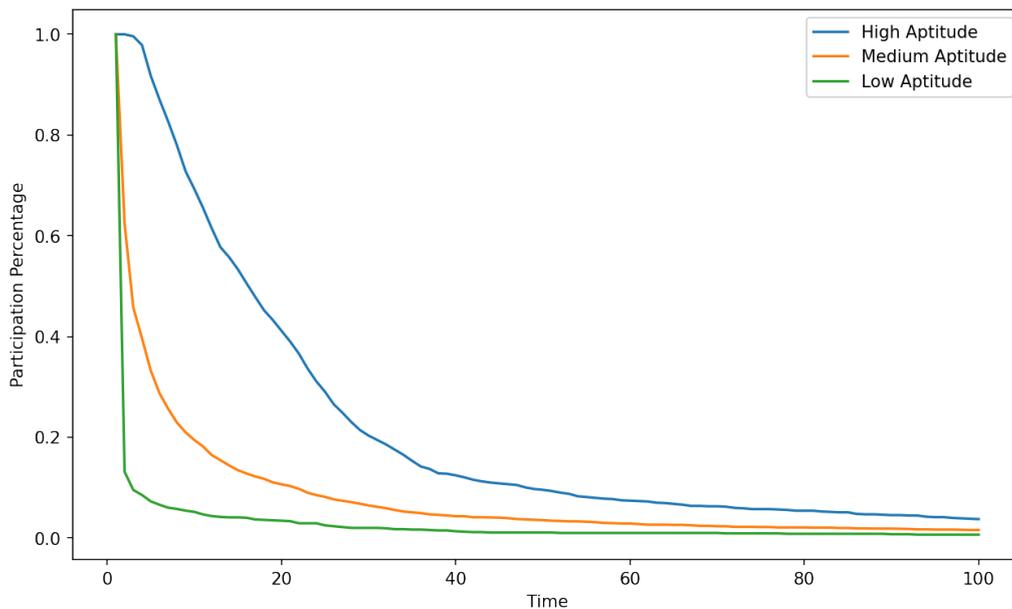


Figure 4: Participation percentages of competitors over time

All the parameters are the same as in the previous involution factor simulation. We set $N_{students} =$

10000, $\tau = 0.75$ and $N_{learners} : N_{competitors} = 1 : 1$. We further categorize competitors into 3 groups: “high aptitude” if $a_{i,0} \geq 4$, “medium aptitude” if $2 \leq a_{i,0} < 4$, “low aptitude” otherwise. Figure 4 presents the trend of participation percentages of competitors over time, where the participation percentage is calculated by $1 - N_{opt-out} / N_{group}$ within each group. Three points to note here:

1) All three lines are decreasing to 0 over time, which confirms the self-annihilation nature of the involution.

2) At the very first stage, there’s a drop in the medium aptitude and low aptitude groups, as the line is almost vertical. This is caused by the large discrepancy between prior reference point (based on $a_{i,0}$) and post reference point (based on $p_{i,1}$) at the first stage due to $\theta_{i,0} > a_{i,0}$, so the screening effect is considerable. And under this situation, generally, competitors with lower aptitude are easier to opt out, as the line from the lower aptitude group is steeper.

3) When we ignore the leftmost part of the figure, the decreasing rate of competitors with lower aptitude is slower, partially owing to competitors with high aptitude facing the risk of falling into case 2 (case 1) from case 3 (case 2), which may cause greater unexpected loss once they fail to reach post reference point, hence a higher possibility to opt-out.

Furthermore, we set $N_{learners} : N_{competitors} = 1 : 9$ and vary τ to assess the consistency and robustness of the conclusions. Figure 5 displays the comparisons between different τ .

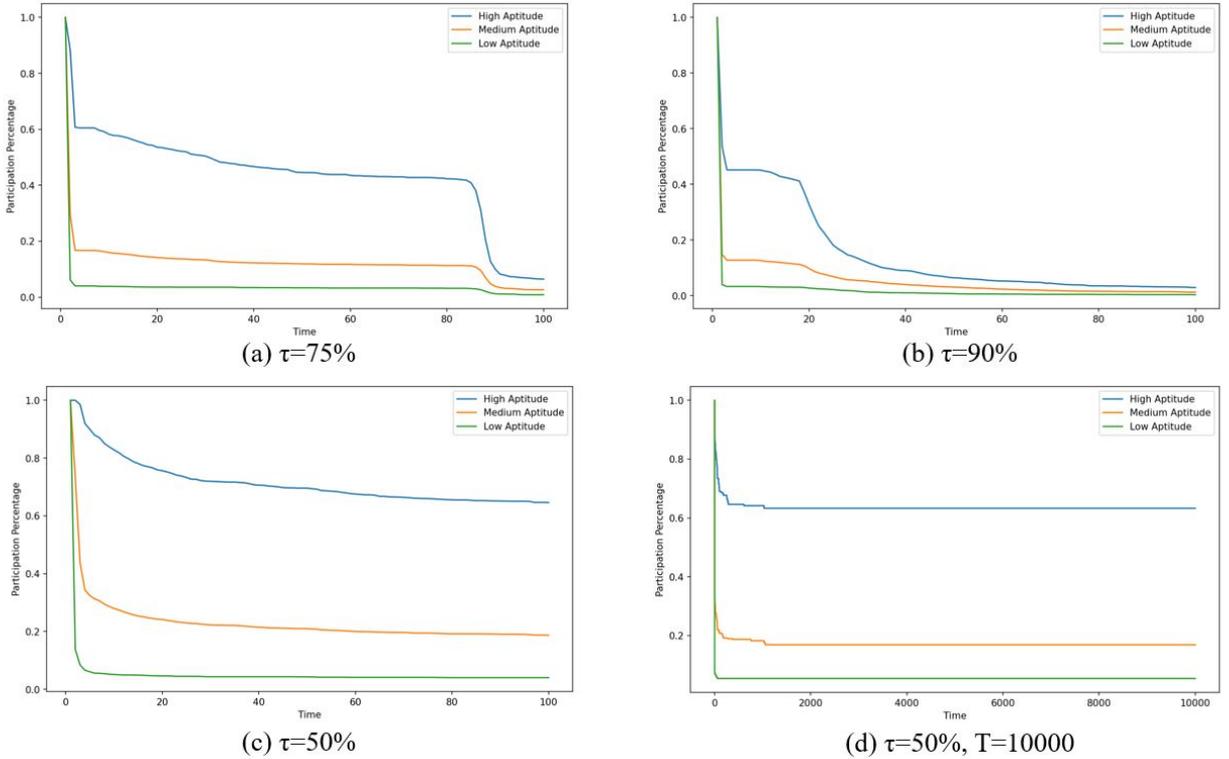


Figure 5: Comparisons of different τ : participation percentages of competitors over time

By comparing Figure 5 (a) (b) (c), we can tell from the leftmost and rightmost part of each line

that higher the τ , steeper the line, fiercer the competition, earlier the time of annihilation. It is quite intuitive, as higher τ means fewer “winners”, and the initial screening effect is stronger.

Moreover, the comparison between Figure 4 and Figure 5 (a) reveals several interesting points:

1) At the very first stages, learners in the population can be considered as a buffer against the sharp increase in post reference points contributed by competitors, as learners always fall in the lower percentile due to $\theta_i > a_{i,0}$ when T is small. Therefore, the leftmost part of the line in Figure 5 (a) is much steeper than that in Figure 4.

2) The middle parts of the blue line and orange line are much flatter than those in Figure 4. Since the share of learners is relatively small in Figure 5, the boost of reference points from learner’s ability accumulation is limited and requires more time to show up. Note that, consistent with the previous analysis, the flat part almost represents competition within competitors, which is very similar to the special equilibrium we are going to discuss.

3) After the flat part, a second sharp decline arises, indicating the roles of learners who steadily accumulate and make a significant impact upon reaching a certain ability threshold.

Additionally, as we discuss in the previous footnotes 7, figure 5 (c) and (d) depict another potential equilibrium where some competitors survive over time. Since with relatively low τ and relatively low share of learners, when the lowest-aptitude learner’s performance surpasses $Q_{P,t}(\tau)$, the impetus from learner’s ability accumulation - one of the primary drives of involution - will no longer be able to increase reference point. Consequently, it becomes a competition within competitors themselves, hence another equilibrium is reached, though perhaps not so realistic.

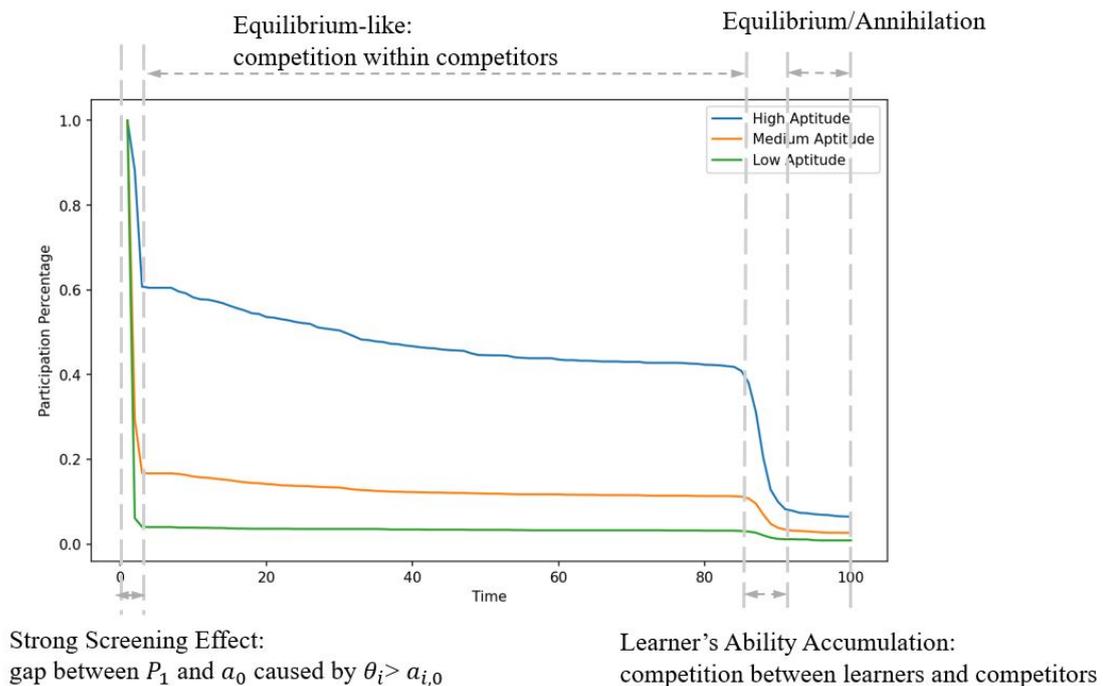


Figure 6: Impetus of each stage

5.2 Educator's Concerns

5.2.1 Predictable test - θ_i

As highlighted in Remark 1, the assumption $\theta_i > a_{i,0}$ is pivotal to our analysis. However, for educators aiming to mitigate the observed phenomenon and guide individuals back on track, this assumption presents an opportunity.

In the model setup, we refer to θ_i as a parameter measuring non-ability-accumulation's effect on performance. Moreover, educators can influence θ_i through various exogenous factors, making it an effective tool for intervention. For instance, to decrease θ_i , teachers reduce the predictability of exams by providing less information about the test content. Notably, maintaining a certain level of predictability is crucial to prevent complete frustration among students who might otherwise study inadequately (Lazear, 2006). Therefore, educators can strike a balance by emphasizing key concepts while maintaining a moderate yet effective level of unpredictability. The revealed information ensures that students reach a basic performance level p_t^* (e.g., passing the exam), where $\theta_i \cdot e_{i,t}^N$ only contributes additively to p_i when $p_{i,t} < p_t^*$. Consequently, competitors aspiring to perform above the reference point will find the enhancement from $e_{i,t}^N$ limited, which compels them to turn to $e_{i,t}^A$ for further increase. This approach helps mitigate the misrepresentation of performance on ability and addresses inefficiencies in education.

The above discussion also parallels with the literature on the agent-principal problem of incentivizing teachers and “teaching to the test”.

5.2.2 Reference Structure - r_i

The model hasn't explicitly mentioned the outcomes for competitors after they opt-out, which limits further discussion on educators' responses aligned with their goals. Additionally, the baseline model assumes that the realized performance distribution (at least $Q_{i,t}(\tau)$) at each period is public information. But in this part, we constraint that information can only be partially public within *groups* or directly revealed by educators. We explore how educators can influence involution by modifying reference groups and information structures, or “granularity of reference points” (Roels and Su, 2014).

Firstly, let's consider a scenario where competitors, due to significant losses from opting out, decrease their s_i or even change their types to non-competitors or learners. Educators can strategically group competitors and learners with high aptitude together, accelerating the involution process and prompting competitors' transitions to other types.

Alternatively, if students who opt out tend to tank or drop out, educators may aim to minimize

such outcomes. Naturally, educators can slow down the involution by disclosing statistics about the performance distribution and revealing reference points at lower levels, such as median or mean performance. Additionally, grouping competitors with non-competitors can also influence the dynamics of the involution.

6 Conclusions and Discussions

Involution is generally interpreted as “effort inflation”. The paper presents a competition model elucidating the involution phenomenon in college students. It categorizes students as learners, non-competitors, and competitors, each navigating distinct utility maximization problems. Learners and non-competitors maintain a fixed effort investment at each stage, whereas competitors dynamically update their reference points, resulting in escalating effort. Through a comparative analysis of type-specific best responses at each stage, the paper unveils the driving force behind the self-reinforcing involution spiral and underscores its unsustainable nature: competitors who create the atmosphere of intensifying competition ultimately will be crushed by involution itself.

For academic involution, several potential extensions for future research are identified to enrich and refine the current framework:

1) Discounted future utility: Lift the constraints that all students only care about the utility at the current stage by adding the discounted utility of the near future to the utility maximization problem. This is reasonable as during a study circle (e.g. a semester), there may exist phased outcomes entering into the final GPA, such as midterms and weekly quizzes. In this setting, within the proper parameter range, it is possible for non-competitors to transit into a learner even if they have varied study motivations.

2) Decreasing cost function in ability: Consider a cost function that decreases in ability, not just aptitude. This adjustment incentivizes individuals to choose $e_{i,t}^A$, which may alter non-learners’ behaviors.

3) Heterogeneous updating in reference points: Introduce heterogeneity in the reference-point updating process of competitors. Different updating strategies, influenced by loss aversion parameter λ ⁸ can also be explored (Sun et al., 2020). This extension aligns with evolutionary optimization algorithms in computer science and could accommodate more dynamics (Lin et al., 2023).

4) Multitask and effort-distortion: Extend this model to a multi-task setting, for example, mul-

⁸Individuals with larger λ is more loss-averse, thus may be more pessimistic about the increasing performance threshold, and updates the reference point more aggressively.

tiple subjects or courses j . Different students have different aptitudes $a_{i,j}$ (for simplicity, high or low) and different $\theta_{i,j}$ for task j . We can explore how each type of student allocates efforts over easy tasks (high aptitudes or high $\theta_{i,j}$) and challenging tasks; and how they change their subsequent behaviors when the performance is revealed, according to self-affirmation theory (Hannan et al., 2013; Villeval, 2020); also, we can investigate the impact of involution on effort distribution and potential polarization or clustering phenomena (Roels and Su, 2014). Additionally, educators' influence on outcomes by controlling task structures or information can be studied (Holmstrom and Milgrom, 1991; Hannan et al., 2013; Villeval, 2020).

5) Expectations, random shocks and demotivation: Develop an expectation model that considers beliefs or perceived ability to better address the ambiguity stemming from unknown distributions of student types, allowing individual set expectations as a reference point like Gill and Prowse (2012)'s model for disappoint-averse individuals. Moreover, in this case, future studies can introduce common shocks (e.g. exam difficulty) and idiosyncratic shocks (e.g. sickness) into the performance function (equation 2) (Ertac, 2005), which may also incorporate with findings on the demotivational effects of unexpected failures to achieve reference points (Haenni, 2019).

These extensions aim to capture additional dimensions of the academic competition environment, providing a more nuanced understanding of students' decision-making processes and educators' potential interventions. Although we discuss involution in the context of China's education system, its applicability extends to other scenarios, including the labor market and employee motivation design. Furthermore, the involution is not unique to China; it may manifest in any setting where there is intensifying competition for limited resources, especially common in densely populated countries.

7 Appendix

7.1 Glossary

Table 1: Primary Notations

Notation	Explanations
$a_{i,t}$	the ability level of individual i at time t ; $a_{i,0}$ is the aptitude
$e_{i,t}^A$	the effort level i allocates to ability-accumulation activities at time t
$e_{i,t}^N$	the effort level i allocates to non-ability-accumulation activities at time t
$p_{i,t}$	i 's realized performance level at time t
θ_i	the parameter representing the effectiveness of $e_{i,t}^N$ for i on $p_{i,t}$
g_i	i 's weight on study goal: to improve performance
s_i	i 's weight on social comparison
$r_{i,t}$	the reference point set by i prior to effort choice at time t
$\tau_{i,t}$	the percentile of the population's performance distribution chosen by i to set $r_{i,t}$
λ	the coefficient for loss-aversion
$c_{i,t}$	the cost of effort for i at time t
b	a coefficient in cost function
γ	the involution factor, which influences reference point updating

7.2 Proofs

7.2.1 Proposition 3

Proof. For competitors, $MC = \frac{2be_{i,t}^N}{a_{i,0}}$, $e_{i,t}^A = 0$. We slightly change the order: we prove case 1 and case 3 first, then is case 2.

(i) Case 1 - low aptitude, completely loss domain: no way to achieve prior reference point

In this case,

$$u_{i,t} = (1 - s_i)(a_{i,0} + \theta_i e_{i,t}^N) - s_i \lambda (r_{i,t} - a_{i,0} - \theta_i e_{i,t}^N) - \frac{b(e_{i,t}^N)^2}{a_{i,0}}$$

Therefore, $MB_1 = (1 - s_i + \lambda s_i)\theta_i$. Let $MB_1 = MC \implies e_{i,t}^N = \frac{a_{i,0}\theta_i(1 - s_i + \lambda s_i)}{2b}$

For individuals falling in this case, we have constraint: $p_{i,t} \leq r_{i,t}$, i.e.

$$\begin{aligned} a_{i,0} + \theta_i \cdot e_{i,t}^N &= a_{i,0} + \theta_i \cdot \frac{a_{i,0}\theta_i(1 - s_i + \lambda s_i)}{2b} \\ &= \left(1 + \frac{\theta_i^2(1 - s_i + \lambda s_i)}{2b}\right)a_{i,0} \leq r_{i,t} \end{aligned}$$

$$\implies a_{i,0} \leq \frac{2b \cdot r_{i,1}}{\theta_i^2 [1-s_i + \lambda s_i] + 2b}$$

(ii) Case 3 - high aptitude, completely gain domain: perform above prior reference point

In this case,

$$u_{i,t} = (1-s_i)(a_{i,0} + \theta_i e_{i,t}^N) + s_i(a_{i,0} + \theta_i e_{i,t}^N - r_{i,t}) - \frac{b(e_{i,t}^N)^2}{a_{i,0}} = a_{i,0} + \theta_i e_{i,t}^N - s_i r_{i,t} - \frac{b(e_{i,t}^N)^2}{a_{i,0}}$$

Therefore $MB_3 = \theta_i$. Let $MB_3 = MC \implies e_{i,t}^N = \frac{a_{i,0}\theta_i}{2b}$ For individuals falling in this case, we have constraint: $p_{i,t} \geq r_{i,t}$, i.e.

$$\begin{aligned} a_{i,0} + \theta_i \cdot e_{i,t}^N &= a_{i,0} + \theta_i \cdot \frac{a_{i,0}\theta_i}{2b} \\ &= \left(1 + \frac{\theta_i^2}{2b}\right) a_{i,0} \geq r_{i,t} \end{aligned}$$

$$\implies a_{i,0} \geq \frac{2b \cdot r_{i,1}}{\theta_i^2 + 2b}$$

(iii) Case 2 - medium aptitude, stop at the prior reference point

In this case, individual endeavors until their performance increases to the prior reference point,

$$p_{i,t} = a_{i,0} + \theta_i \cdot e_{i,t}^N = r_{i,t}$$

$$\implies e_{i,t}^N = \frac{r_{i,t} - a_{i,0}}{\theta_i}$$

For individual falling in this case, we have constraint: $MC \in [MB_3, MB_1]$, i.e.

$$\begin{aligned} \theta_i &\leq \frac{2b \frac{r_{i,t} - a_{i,0}}{\theta_i}}{a_{i,0}} \leq (1-s_i + \lambda s_i) \theta_i \\ \frac{\theta_i^2}{2b} &\leq \frac{r_{i,t} - a_{i,0}}{a_{i,0}} \leq \frac{(1-s_i + \lambda s_i) \theta_i^2}{2b} \end{aligned}$$

$$\implies a_{i,0} \in \left[\frac{2b \cdot r_{i,1}}{\theta_i^2 [1-s_i + \lambda s_i] + 2b}, \frac{2b \cdot r_{i,1}}{\theta_i^2 + 2b} \right]$$

□

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