Optimal environmental reporting and the role of regulations

Rajas Pandey*

Abstract

Environmental accounting is a nascent field and there is considerable flexibility in the kind of disclosures a firm can provide. This paper proposes a model with two agents - the sender (firm) and receiver (investor) - where the receiver forms beliefs about the sender's value after observing disclosures. Upon observing non-disclosure of an environmental characteristic, the receiver places probabilities on the following two events - poor performance along that dimension or that the characteristic is not *material* or relevant to the firm's value. Ambiguity in the scope of what is a *material* characteristic can thus be leveraged by the sender to induce a higher posterior belief about the valuation. An optimal reporting structure is uninformative about the precise value of the high states. The paper further discusses the role of competition and regulation in inducing more informative reporting. Finally, the paper specifies conditions for regulatory action which leads to more informative reporting by the firms.

1 Introduction

On March 21, 2022, the US Securities and Exchange Commission (SEC) proposed rule changes that would require firms to disclose material climate risks and audited climate-related financial metrics in their periodic disclosures to the regulator [SEC, 2022]. The new rules will require firms to disclose information about the firm's climate-related risks and the risk management practices it is adopting, the materiality of climate-related events to the firm's business, and its financial impact. In addition, the new rules will also require the firms to report their direct and indirect greenhouse gas (GHG) emissions in their operations (Scope 1 and 2 emissions).

The rule announcement mentions that these disclosures will enable better risk assessment on part of the investors. While it is not overt in the SEC proposal, Governments and Regulators often want to use policy instruments like mandatory disclosures to meet their international climate commitments [van Soest, 2005]. Another potential reason from a Government's (or social planner's) perspective is that full and verifiable disclosures by everyone increase the social welfare through efficient capital allocation towards firms that are better performing on this dimension [van Soest, 2005]. Environmental disclosures and accounting also lead to innovation around environmentally friendly products [Ferreira et al., 2010]

The SEC proposed ruling does not indicate the compliance timelines for the firms or the nature of liability in the case of non-compliance. The rules also do not specify the exact nature of disclosures expected from the firms, also leaving some ambiguity around the *materiality* of the disclosure to the firm's business. Compliance with these rules will require firms to invest in monitoring, reporting,

*rajas.pandey@duke.edu. I would like to thank Prof. Charles M. Becker and Prof. Edward Tower for very useful comments and suggestions. All errors and omissions are my own.

and verification infrastructure, along with third-party attestation. With the law and practices around reporting being nascent, the firms have flexibility in the kind of information they choose to disclose. This can be thought of as the firms choosing how to monitor characteristics, deciding which attributes are reasonably material to the firm's business, and choosing specific accounting frameworks.

In this paper, a model is developed using the Bayesian persuasion framework to see if the firms can leverage information design to their advantage. The firms are the senders of information and have to invest in an information system - accounting practice and characteristics to disclose - ahead of time. In the next period, the signals are realized and the receivers of the signal (investors) form beliefs about the value of the firm. This paper studies the design of an optimal disclosure policy from a firm's perspective. Further, the regulator's role is analysed in the game and how they can induce maximum disclosure.

This paper studies the firm's benefit of using persuasion in information disclosure, which is relevant to the firm manager, the investor as well as the regulator. The information design gives gives context to the firm's disclosures. In particular, the investor forms posterior beliefs about the importance and nature of missing information. The firm gets a higher payoff when the investor expects the true value to be high. The regulator can use this information to design an optimum liability structure to ensure maximum compliance and disclosures by the firm, leading to maximum welfare. The lessons from this paper can be extended to any setting where more disclosure leads to social welfare but is not always beneficial for the sender. Examples include drug trials, election campaigning, and advertisements on product quality.

Related literature - This paper draws heavily from accounting practices literature, in particular optimum disclosure under uncertainty. [Grossman and Hart, 1980] suggest that in a setting with purely voluntary disclosures (no mandatory disclosures), it is optimum for the sender to disclose all the information they have. They base their argument on the fact that the receiver assumes the worst outcome when they do not observe any disclosures, so the seller cannot do any worse by not disclosing information. [Dye, 1985] and [Jung and Kwon, 1988] consider a setting where there is ambiguity in the receiver about the availability of information to the sender, and suggest that the senders can benefit in this setting through partial disclosures. This paper extends this to a setting of Bayesian Persuasion of [Kamenica and Gentzkow, 2011] where the firm commits to a reporting framework such that the expected value of the report yields more optimal payoffs for the sender.

More recent literature on these themes includes [Friedman et al., 2020], which discusses optimal disclosure commitments before realizing the true value of the signal, and how they might vary under different accounting and payoff structures. [Cianciaruso et al., 2021] talks about disclosures in a financial market setting where the price is discovered through trade and information disclosure influences the value the traders put on the firm. This paper considers a framework where the value of the firm is dependent on multiple characteristics, and the firm gets to choose which characteristics are important. Such a framework has previously been discussed by [Shin, 2003] in a financial market setting and by [Milgrom, 2008] in a goods quality setting.

Lastly, this paper contributes to the environmental regulation literature by discussing strategies through which a regulator can induce better and more informative disclosures. This has been studied

by [Friedman et al., 2021] who argue that ambiguous regulation induces more voluntary disclosure of private information. The idea of probabalisitc investigation used in this paper is similar to the extension proposed by [Yang, 2021]

2 The economic setting

In the paper, a model of firm value as proposed in [Milgrom, 2008] is adapted where the value of the firm is the sum of N multidimensional and statistically independent characteristics[1]. Formally, the value of the firm $\theta = \omega_1 + \omega_2 + \cdots + \omega_N$, where ω_i is the total normalised value of the ith characteristic. Each characteristic ω_i can be thought of as the firm's environmental performance on different dimensions like carbon footprint, waste disposal, supply chain efficiency, etc.

The value of the firm can be thought of as its reputation for environmental activities, or the value of its share price for an environmentally-conscious investor. The only condition is that both the sender's and receiver's payoffs are directly proportional and increasing in θ. Higher θ values are good for the receiver. An investor interested in the overall environmental performance of the firm considers the performance along each of the dimensions and forms and overall assessment of the firm quality. The firm benefits from higher assessment of quality by the investor.

This setting is modelled as a Bayesian persuasion game where the firm is the sender of information and the investors are the receivers. The payoff of the firm depends on the action of the investors, and the firms choice of signals for the firm is a disclosure structure that maximizes their payoff. The investors form beliefs about the expected value of the firm after observing some disclosures by the firm. The firm has some discretion over what information it wants to disclose and how it wants to disclose it, such that the value of the firm is maximized in the investor's eyes. In the environmental accounting setting, the discretion is choosing which characteristics should be considered material to the firm's business.

There are two sets of information that the sender shares with the receiver. First, the firm tests each of the N characteristics and shares a report with value r_i with the investor. The firm chooses this reporting structure at the outset. Choosing the reporting structure is akin to choosing the kind of monitor to install to measure some characteristic. Based on the underlying true value of characteristic i, the reporting structure realises a value r_i and shares with the receiver. This report does not always reveal the exact value of the state to the receiver but induces a posterior belief about the value. In the setting of environmental accounting, an example of this this reporting structure is a pollution monitor, which only reports value above a certain threshold. Below the threshold, all the values are considered good [2].

The second is communication about a *material* characteristic. With probability p_i, the firm decides that a characteristic i is material to the firm's business. It privately observes the true value of the state and chooses a message m_i to communicate the value $m_i \in \{\emptyset, s_i\}$ to the receiver. Intuitively, the

[1] The statistical independence assumption is strong but not unreasonable. Two statistically correlated characteristics can be notionally clubbed into one value without affecting the rest of the model

[2] This type of reporting system is called an impairment type reporting and is commonly used in financial accounting to report unrealised loss of assets but not unrealised gains [Göx and Wagenhofer, 2009]

firm realises the materiality of a characteristic, and then puts additional effort to test the true value of the characteristic. It then makes a decision whether to communicate this exact value s_i to the receiver or $\emptyset$. This message is in addition to the report and leaves no ambiguity in the receiver about the value of the state. However, when the receiver does not get the additional message, $m_i = \emptyset$, the receiver cannot distinguish whether the characteristic is immaterial or the value of the state is low. This is the central idea which creates scope for persuasion by the firm.

An important assumption here is that the sender cannot misrepresent the message s and lie about its value. In the real world, it can be thought that material values are more closely audited than immaterial values. This assumption makes solving for optimal disclosure policy tractable. An extension to the model could do away with this assumption and impose a regulatory cost of misrepresenting the value.

The firm incurs a cost C from the investor if a characteristic falls below a threshold k determined exogenously by the investor. This is similar to a loss of reputation along that dimension, which may lead to a higher cost of capital in the future. Thus, the payoff to the firm on characteristic i after sending report r_i and message m_i is

$$\pi_i(r, m) = E[\omega_i \mid r_i, m_i] - C_i \mathbb{1}_{E[\omega_i|r_i,m_i]<k_i} \tag{1}$$

Such a payoff structure is outlined in [Friedman et al., 2020] and is helpful in this setting as it is linear in the expected value of the characteristic. Each characteristic contributes to the overall value. At the same time, there is a discontinuity in terms of cost. If any characteristic falls below a certain level, it imposes a penalty to the overall value as well. In climate finance setting, this is a realistic payoff structure for an investor with specific interests like firms with low coal usage. If a particular characteristic fails to meet the threshold, the investor can impose a cost.

3 Model

In this section, we formalise the model and infer some results. As mentioned in section 2, the value of the firm is $\theta = \omega_1 + \omega_2 + \cdots + \omega_N$, where ω_i is the value of the ith characteristic.

Each $\omega_i \in [\omega_i^l, \omega_i^h]$ such that $\omega_i^l \leq \omega_i^h$. These values are distributed according to a CDF $F_i(\omega)$ which is continuous and differentiable over the state space. The PDF is $f_i(\omega) > 0, \forall \omega$ which is defined over all states. These are universally shared prior distributions.

The firm shares a report $\mathbf{r} = (r_1, r_2, \ldots, r_N)$ related to the value of each characteristic. This is similar to a firm's annual report. Each $r_i \in \mathbf{R}$ and induces posterior beliefs about the true value of the state. Denote the posterior beliefs by $F_i(\omega_i \mid r_i)$.

In the discretionary part of the disclosure, the firm believes that a characteristic is material to the firm's business with probability $p \in [0, 1]$. [3] It determines the exact state of the world s and communicates message $m \in \{s, \emptyset\}$. With probability $1 - p$, it does not consider the state to be material and sends message $m = \emptyset$. These values are present for each characteristic so the message m shared by the

[3]For example, water contamination is a material characteristic for a beverage company but not for an an airline. Companies have discretion in determining which characteristics they consider material to their business

firm is a list of values.

At time T = 0, the firm commits to a reporting structure[4].
At time T = 1, firm observes $\mathbb{r}$ and s.
At time T = 2, firm communicates $\mathbb{r}$ and message $m \in \{s, \emptyset\}$ to receiver.
At time T = 4, firm gets payoffs.

3.1 Optimum message under any report

The game is solved using backward induction. In order to determine the optimum reporting structure, first goal is to determine the optimum message the firm should send for a material characteristic given any report r. Then, the firm optimises the reporting structure given the optimum message.

As each characteristic i is statistically independent and additive, the optimum for each characteristic is independent. The analysis computing the optimum for one characteristic can be extended to compute the optimum for the other $N-1$ characteristics. Thus, in this subsection, we compute the optimum for one characteristic given any report. In terms of notation, I drop the subscript i everywhere as we are dealing with a single characteristic. The analysis follows from [Jung and Kwon, 1988].

The firm has already observed report r and formed posterior beliefs $f(\omega \mid r)$ (with CDF $F(\omega \mid r)$) about the value of the characteristic. These beliefs are the new priors for the optimal message game.

Let $E[\omega \mid r]$ be the prior expected value of ω (given r).

$$E[\omega \mid r] = \int_{\omega^l}^{\omega^h} \omega f(\omega \mid r) d\omega$$

There are 3 possible cases here -

- Case 1 - The firm considers the characteristic to be material. It investigates for the true value and chooses to report it
- Case 2 - It investigates the true value of a material characteristic but chooses to not report it
- Case 3 - It does not consider a characteristic to be material

Continuing the example given above, suppose there is a beverage company. It knows with probability p that water contamination is a material characteristic. It investigates to find the true value s of the contamination. In case 1, it reports this value accurately. In case 2, it does not report it. Case 3 is when the firm does not find the characteristic to be material.

More abstractly, with probability p, the firms considers a characteristic to be material. The first case is when the firm investigates further and finds the true value of the state to be s. Now the firm must decide whether to disclose this value or not. Suppose the firm has an implicit cutoff $y \in [w^l, w^h]$

[4] An example of reporting structure is report some value r^g when $\omega > \hat{\omega}$ and r^b otherwise. This is a binary reporting structure which works for presence/absence type characteristics.

above which it discloses the value exactly and below which it sends an empty message. Formally

$$m = \begin{cases} s, & \text{if } s \geq y \\ \emptyset, & \text{if } s < y \end{cases}$$

The cutoff y is drawn from the same distribution as ω, $y \sim F(\omega \mid r)$. Thus probability that a characteristic is material and merits disclosure is $p \times (1 - F(y \mid r))$.

In the second case, the firm does not disclose the value of a material characteristic. The probability of that is $p \times F(y \mid r)$. In the third case, the firm does not consider the characteristic to be material and thus has no disclosures with probability $1 - p$.

The receiver observes disclosure and forms posterior beliefs about the characteristic value. With report r and message s, the receiver knows certainly that

$$E[\omega \mid r, m = s] = s$$

Upon non disclosure, the receiver does not know whether the non-disclosure is from immateriality or low value. Let nd be the event of non-disclosure. The receiver calculates

$$\begin{aligned} E[\omega \mid nd] &= E[\omega \mid \text{case2}]P(\text{case2} \mid nd) + E[\omega \mid \text{case3}]P(\text{case3} \mid nd) \\ &= \int_{\omega^l}^{y} \frac{\omega dF(\omega \mid r)}{F(y)} \times \frac{p \times F(y)}{(1-p) + p \times F(y)} + \int_{\omega^l}^{\omega^h} \omega dF(\omega) \times \frac{1-p}{(1-p) + p \times F(y)} \\ &= \frac{1}{(1-p) + p \times F(y)} \times \Big[p \int_{\omega^l}^{y} \omega dF(\omega \mid r) + (1-p).E[\omega \mid r] \Big] \end{aligned}$$

The firm wants to choose the cutoff y such that it maximises the expected value of the state given non disclosure. The best a firm could get on non disclosure is y, thus the firm would want that $E[\omega \mid nd] = y$. Substituting this value in the equation above, we find

$$y = \frac{1}{(1-p) + p \times F(y)} \times \Big[p \int_{\omega^l}^{y} \omega dF(\omega \mid r) + (1-p).E[\omega \mid r] \Big]$$

Using integration by parts, we get

$$y = E[\omega \mid r] - \frac{p}{1-p} \int_{\omega^l}^{y} F(\omega \mid r) d\omega \tag{2}$$

The firm (truthfully) discloses all values above y through message $m = y$ and sends message $m = \emptyset$ for all other values.

Proposition 3.1.1 *As the materiality of the characteristic increases, it is optimal for the firm to disclose more information.*

Consider the trivial case first, where the characteristic is immaterial and $p = 0$. In this case, the disclosure cutoff is $y = E[\omega \mid r]$. Thus, the firm discloses information only when it is above the expected value and has the potential to increase the payoff.

When $p = 1$ or completely material report, $y = \omega^l$. All observed values of the underlying state must be reported by the firm. When a characteristic is surely material, non-disclosure leads the receiver to expect a very low value. This is similar to the adverse selection case discussed in [Grossman and Hart, 1980] when it is optimal for the firm to provide all the information.

Taking the derivative of 2, we find that $\frac{dy}{dp} < 0$ [5]. Thus, the disclosure threshold decreases as the materiality of a characteristic increases. The mechanism through which this works is the scepticism about non-reporting among the receivers increases as the materiality increases.

Another thing to note is that the disclosure cut-off is independent of k, the investor determined threshold for cost. This is true as long as the firm's payoffs are linear in the expected value of the underlying state. In the subsequent section, we shall see that in equilibrium, it is optimal for the sender to report all material values above k truthfully.

3.2 Optimum reporting structure

In equation 1, the firm incurs a cost when the posterior expected value is higher than an exogenously determined threshold k by the receiver. Also, from section 3.1, the firm always discloses the true value of a material characteristic above some threshold y.

Proposition 3.2.1 *An optimum reporting structure is uninformative about the true value of the state above a cutoff, i.e $r = r^g$ whenever $\omega \geq \hat{\omega}$.*

This proposition implies that there exists some cutoff $\hat{\omega} \in [\omega^l, \omega^h]$ above which the reporting structure produces a constant uninformative report r^g. Uninformative here means that observing the report r does not help the receiver pin down the value of the state. The intuition behind this proposition is that when the firm does not voluntarily disclose a message $m = \emptyset$, clubbing the higher values together creates maximum ambiguity about the true value of the state. The firm wants to set level $\hat{\omega}$ such that the receiver always forms posterior beliefs about the firm value above the cost threshold k.

Consider the three cases:

1. Case 1 - The true value of a material characteristic is above k. The firm always reports the true value and gets payoff $E[\omega \mid r^g, m = k] \geq k$. There is no cost. In this reporting structure, the firm always sends a good report above k. There are no gains from persuasion as the true state is always revealed.

2. Case 2 - The value of a material characteristic is below k. The firm chooses to disclose $m = \emptyset$.

3. Case 3 - The firm needs to report the value of an immaterial characteristic. The message sent by the firm is $m = \emptyset$

In cases 2 and 3, the firm wants to induce the highest expected payoffs given the message $\emptyset$. In other words, the firm wants the report to generate $E[\omega \mid r, m = \emptyset] \geq k$ as that reduces the cost. Consider an informative reporting structure which gives a higher value proportionate to the true value $r \propto \omega$. Similar to Case 1, values of $\omega > k$ will induce a high expected value and no cost. However, the firm

[5] The proof is similar to the proof of Proposition 2 in [Jung and Kwon, 1988], and hence omitted

will incur cost for all values below k, equal to $CF(k)$.

There exists a value $\hat{\omega} \in (\omega^l, k)$ which induces an expected value of exactly k, given an uninformative report and non disclosure. This dominates any informative disclosure as it increases the payoff in case 2 while keeping the payoffs of the other cases constant. Any informative report will restrict the choice set and lead to lower expected value. The finding is similar to result in [Friedman et al., 2020].

Proposition 3.2.2 *The firm's expected payoff under the optimal reporting structure is* $E[\omega] - CF(\hat{\omega})$

From equation 1, the payoff to the firm for a characteristic i is

$$\pi(r, m) = E[\omega \mid r, m] - C\mathbb{1}_{E[\omega|r,m]<k}$$

Taking expectation -

$$\begin{aligned} E[\pi(r, m)] &= E[E[\omega \mid r, m]] - C(Pr(E[\omega \mid r, m] < k)) \\ &= E[\omega] - C(Pr(E[\omega \mid r, m] < k)) \text{ (Law of iterated expectations)} \end{aligned}$$

Minimising the probability that the posterior expected value is lower than k is equivalent to maximising the probability that the expectation is above k. From the discussion above, this value is maximised when the firm reports $r = r^g$ for all $\omega \geq \hat{\omega}$. The probability that the posterior expectation is below k is the same as prior probability of ω lying below $\hat{\omega}$, equal to $F(\hat{\omega})$. Thus, the maximum expected value for any characteristic of the firm is

$$\pi^*(r, m) = E[\omega] - CF(\hat{\omega})$$

3.3 Discussion of optimum reporting structure

Under the optimum disclosure structure, the sender does two things - (i) Reports all material values when the value of the state is more than k and not disclose otherwise. (ii) Send an uninformative report r^g about the state when the state lies above $\hat{\omega} \in (\omega^l, k)$. This will lead to maximum payoff for the firm when the payoff is linear in the expected value of the state.

We can now partition the N characteristics of quality into 4 groups

1. **Group 1** - characteristics which are material and have values above the cost threshold k. For these characteristics, the firm will optimally report the observed value of the state under the assumption that the firm cannot misrepresent the value.

2. **Group 2** - characteristics which are material and have values more than $\hat{\omega}$ as defined in 3.2. The firm will not send any message, disclose a good report r^g and lead the receiver to an expected value greater than or equal to k.

3. **Group 3** - characteristics which are not material and have values more than $\hat{\omega}$, leading to a good report r^g. The firm cannot send any message and the receiver's expected value is again greater than or equal to k.

4. **Group 4** - characteristics which have values less than $\hat{\omega}$, leading to a bad report $r \neq r^g$. The expected value of the firm is $E[\omega] - C$. In this case, the materiality of the characteristic is not a factor. In all cases, the firm faces a cost from the investors.

The model reinforces the results in [Milgrom, 2008] in that the sender communicates the exact value of the state when it is verifiable and when it is sufficiently high. In this setting, such characteristics fall under group 1. All other characteristics induce uninformative reports and non-disclosures, and the receiver has to rely heavily on their prior valuation.

The model here is simplistic on two dimensions. First, the model assumes that searching for and verifying the true value of the state is costless. This is not true in real settings where additional investments are required to accurately monitor characteristics and there is also a cost associated with verification through third parties. This model however does give some indications to which characteristics should be prioritised when investing in the reporting infrastructure. Group 1 characteristics lead to the highest payoffs in expectation and are a natural priority for investment in reporting and verification.

This also leads us to some insights on interests of lobbying groups trying to influence beliefs about materiality of characteristics. Lobbying firms have an interest in ensuring that characteristics where their performance is better are considered material.

Competition can resolve this issue to some extent, whereby two competing firms in the same industry can present different characteristics as "material" to the investors. The investors can then demand the firms to disclose their values of the characteristic the other firm has reported as material. This will lead to more disclosure, even when the true values for a characteristic are less than k. This conclusion does not assume any knowledge on part of the investor about the actual materiality of characteristics.

The second simplistic assumption is that the firm can decide to not disclose a material value without any cost. This is where a regulatory authority can ensure the firm faces some cost for not disclosing an important material value. The subsequent section addresses this issue further.

3.4 Can regulator lead to better disclosures?

So far, the setting has only considered two agents, the sender and the receiver, whose payoffs are linked. Conditions for optimal payoffs from the firm's perspective have been described. The conclusion of the model that the value must only be disclosed when high is contrary to the idea of social welfare. The role of competition in increasing disclosure and welfare has been discussed. However, competition alone is not sufficient as it leaves scope for a characteristic which is actually material to the investors, but does not have a good value for the competing firms. Such a characteristic will never be discovered by the investor and demanded to be disclosed.

Thus, there is scope for a regulatory body with pecuniary authority which can penalise the firms for non disclosures on material characteristics. In particular, the firm should be penalised for misleading investors by sharing a good report r^g when the value is lower than the investor threshold of quality.

Let α denote the probability that a regulator investigates the disclosures of a firm. Such a probabilistic model is reflective of a real setting where the regulator does not have the capacity to investigate

every disclosure. An ideal regulator will have $\alpha = 1$.

Upon investigating the material disclosures, the regulator finds a misleading report when the true state of the world is between $(\hat{\omega}, k)$. In this state, the firm does not reveal the true value of the state and sends a message $m = \emptyset$. At the same time, the firm also shares a report $r = r^g$ influencing the posterior beliefs of the investors about the true value. In such a scenario, the regulator imposes a fine Z denominated in the same unit as the cost C imposed by investors. The probability of the penalty being imposed on the firm is $\alpha \times p \times (F(k) - F(\hat{\omega}))$. The last term denotes the probability of a material value being between $(\hat{\omega}, k)$.

The payoff for the firm on some characteristic is now

$$\pi(r, m, Z) = -\alpha \times p \times (F(k) - F(\hat{\omega})) \times Z + E[\omega] - CF(\hat{\omega}) \tag{3}$$

Proposition 3.4.1 *A higher penalty makes a good report less likely but makes it more informative.*

This result hinges on the relationship between $\hat{\omega}$, which induces a good report and the penalty imposed by the regulator. Using the implicit function theorem and equation 3, we have

$$\begin{aligned}
\frac{d\hat{\omega}}{dZ} &= -\frac{d\pi}{dZ} \Big/ \frac{d\pi}{d\hat{\omega}} \\
&= -\frac{-\alpha p(F(k) - F(\hat{\omega}))}{\alpha p Z f(\hat{\omega}) - C f(\hat{\omega})} \\
&= \frac{\alpha p(F(k) - F(\hat{\omega}))}{\alpha p Z f(\hat{\omega}) - C f(\hat{\omega})}
\end{aligned}$$

This fraction is increasing when $\alpha p Z > C \implies Z > \frac{C}{\alpha p}$ as $F(k) - F(\hat{\omega}) > 0$

When the penalty due to non-disclosure of material information is higher than the cost due to missing the investor threshold in expectation, the penalty will lead to a higher value of $\hat{\omega}$. This means that for the same k, the firm will be less likely to report a characteristic as good, and a good report will be more meaningful in conveying the true state. This is because of the cost of the penalty upon being caught lying about the true value.

If however, the cost from investors outweighs the penalty, the firm will have a higher payoff in disclosing material information with a lower $\hat{\omega}$ and paying the penalty to the regulator. The regulator's penalty in this model is intrinsically linked to the cost imposed by the investors. The firm trades off between suffering the cost C at the hands of the investors or facing a penalty at the hands of the regulator. Another insight is that a low penalty leads to a lower payoff for the firm and does not improve societal welfare. This is the case for a high level of penalty by the regulator.

Proposition 3.4.2 *A higher probability of being investigated by the regulator leads to more informative disclosures.*

Similar to the previous proposition, we consider the derivative of $\hat{\omega}$ with respect to α.

$$\begin{aligned}\frac{d\hat{\omega}}{d\alpha} &= -\frac{d\pi}{d\alpha}/\frac{d\pi}{d\hat{\omega}} \\ &= -\frac{-pZ(F(k)-F(\hat{\omega}))}{\alpha pZf(\hat{\omega})-Cf(\hat{\omega})} \\ &= \frac{pZ(F(k)-F(\hat{\omega}))}{\alpha pZf(\hat{\omega})-Cf(\hat{\omega})}\end{aligned}$$

This fraction is increasing when $\alpha > \frac{C}{pZ}$

The regulator thus faces a trade-off between increasing the penalty and increasing the rate of investigation. If the regulator is short staffed, a very high value of Z is sufficient to induce firms to have more information disclosure through a higher $\hat{\omega}$. This reflects the reality of regulations where not every case is investigated but the large penalties act as a deterrent to misrepresentation.

Another important link is the value of p, the materiality of the characteristic which influences both the penalty as well as the rate of investigation. The regulator can determine a characteristic to be material $p \to 1$, inducing higher scepticism among investors upon non-disclosure of information. This automatically leads to better reporting by raising the cost of non-disclosure. In practice, this is the easiest thing for the regulator to implement, which makes better reporting incentive compatible for the firms.

4 Conclusion and extension

The model of this paper can be extended to several directions, some of which have been discussed in the paper. One of key assumptions underlying the paper is that the firms do not face any cost of information discovery. The model can be extended to include the setting of costly search with mandatory disclosures of the discovered value to be more reflective of the real world. The analysis around competition and regulation will remain valid and the firms will want to first discover the value of the most material indices.

This model considers the setting of a single sender and a single receiver. The natural extension is a model with one firm and multiple receivers which can trade with each other to discover the true price of the firm. This has been discussed in the stock price setting by [Cianciaruso et al., 2021]. This model with multiple characteristics for each firm can reflect a setting where different receivers have different values for characteristics and discover prices through trade.

Empirical analyses can further consider the role of environmental accounting and investments in reporting, monitoring and verification framework on other aspects of the industrial performance. [Ferreira et al., 2010] suggests that these practices lead to higher process innovation. Governments and regulators may be interested in the supermodular relations between accounting practices and innovation in the economy. If there is a supermodular relationship between stricter regulations around climate accounting and innovation, society can expect more process innovation towards sustainable practices by investing in regulatory capacity.

This paper studies the problem of optimum disclosure in the setting where some characteristics are more material to the firm and investors. This has a natural application in environmental accounting settings where the regulations around disclosures are nascent and there is insufficient consensus about the materiality of characteristics to industry. This paper outlines a model where firms communicate with the investors about the true value of certain characteristics. The payoffs of the investors and firms depends upon the induced beliefs among the investors about the true value, and the uncertainty around materiality of a characteristic creates scope for persuasion. The paper discusses the optimum disclosure strategies and payoffs while reporting material and non-material information. Further, the role of competition and regulation in encouraging more beneficial disclosure practices from the perspective of social welfare is discussed.

References

[Cianciaruso et al., 2021] Cianciaruso, D., Marinovic, I., and Smith, K. (2021). Information Design in Financial Markets. SSRN Scholarly Paper 3686089, Social Science Research Network, Rochester, NY.

[Dye, 1985] Dye, R. A. (1985). Disclosure of Nonproprietary Information. *Journal of Accounting Research*, 23(1):123–145. Publisher: [Accounting Research Center, Booth School of Business, University of Chicago, Wiley].

[Ferreira et al., 2010] Ferreira, A., Moulang, C., and Hendro, B. (2010). Environmental management accounting and innovation: an exploratory analysis. *Accounting, Auditing & Accountability Journal*, 23(7):920–948. Publisher: Emerald Group Publishing Limited.

[Friedman et al., 2020] Friedman, H. L., Hughes, J. S., and Michaeli, B. (2020). Optimal reporting when additional information might arrive. *Journal of Accounting and Economics*, 69(2):101276.

[Friedman et al., 2021] Friedman, H. L., Hughes, J. S., and Michaeli, B. (2021). A Rationale for Imperfect Reporting Standards. *Management Science*. Publisher: INFORMS.

[Grossman and Hart, 1980] Grossman, S. J. and Hart, O. D. (1980). Disclosure Laws and Takeover Bids. *The Journal of Finance*, 35(2):323–334. Publisher: [American Finance Association, Wiley].

[Göx and Wagenhofer, 2009] Göx, R. F. and Wagenhofer, A. (2009). Optimal impairment rules. *Journal of Accounting and Economics*, 48(1):2–16.

[Jung and Kwon, 1988] Jung, W.-O. and Kwon, Y. K. (1988). Disclosure When the Market Is Unsure of Information Endowment of Managers. *Journal of Accounting Research*, 26(1):146–153. Publisher: [Accounting Research Center, Booth School of Business, University of Chicago, Wiley].

[Kamenica and Gentzkow, 2011] Kamenica, E. and Gentzkow, M. (2011). Bayesian Persuasion. *American Economic Review*, 101(6):2590–2615.

[Milgrom, 2008] Milgrom, P. (2008). What the Seller Won't Tell You: Persuasion and Disclosure in Markets. *Journal of Economic Perspectives*, 22(2):115–131.

[SEC, 2022] SEC (2022). SEC.gov | SEC Proposes Rules to Enhance and Standardize Climate-Related Disclosures for Investors.

[Shin, 2003] Shin, H. S. (2003). Disclosures and Asset Returns. *Econometrica*, 71(1):105–133. Publisher: [Wiley, Econometric Society].

[van Soest, 2005] van Soest, D. P. (2005). The impact of environmental policy instruments on the timing of adoption of energy-saving technologies. *Resource and Energy Economics*, 27(3):235–247.

[Yang, 2021] Yang, R. (2021). The Role of Nuisance Arrest in Fighting Crime. *Duke Journal of Economics*, XXXII(Spring 2020).