Product and process innovation when considering changes in consumer dispersion

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Abstract

This paper constructs a two-period model where the monopolist invests in product and process flexibilities and research capabilities in the first period and decides whether to innovate in the second period. By considering that the product design innovation will not only shift but also rotate the demand curve, this paper re-characterizes the conditions for the investment choices to be complementary to each other. Also, by introducing the decision variable that determines how much and in which direction the consumer dispersion will change after the product design innovation, this model provides a method for the firm to decide how to design its product characteristics in the first period so as to achieve the most favorable dispersion level in the second period.

1 Introduction

Several important papers have analyzed the class of supermodular games which has a similar idea as the "strategic complementary" in Bulow et al. (1985). In a supermodular game, the marginal payoffs to increase one's strategy rise as the competitors' strategies increase, and if a player's strategies are multidimensional, then an increase in one component increases the marginal returns of increasing another component. The concept of supermodular games was first introduced by Topkis (1979) and later was formally analyzed by Milgrom and Roberts (1990). The model in Milgrom and Roberts (1990) incorporates complementarities between many organizational and technological variables. Athey and Schmutzler (1995) further identify the complementarities between the short-term and long-term decision variables under an innovation environment and explicitly model the dynamics in a firm's decision problem.

Athey and Schmutzler (1995) build a two-period model with a monopolist producing a single product. They categorize the firm's long-run choices into four categories, including product and process flexibilities as well as product and process research capabilities. In brief, product and process flexibilities refer to how adaptable the manufacturing is to the possible changes in product design and production process, they determine how costly the adjustment would be. Product and process research capabilities refer to the capabilities of conducting research in innovating the product design and production process, they determine how profitable the adjustment would be. Athey and Schmutzler (1995) identify that those four long-run choices exhibit complementarities, with the increase in one choice increasing the return to increase the others. They also identify that the short-run decisions,

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including whether or not to innovate, as well as the production quantity, are complementary to each other. Further, they show that the short-run complementarities lead to long-run complementarities.

Although Athey and Schmutzler (1995) provide sharp insights about the complementarities between the monopolist's decision variables in an innovative environment, their results are based on the strict assumptions that the adjustment of product design innovation always raises prices and the marginal revenues. Putting it in the terminologies in Johnson and Myatt (2006), that is, the shift of the distribution of consumer valuation always outweighs the change in its shape, resulting in the inverse demand curve that always moves outward after the adjustment and does not cross with the original one. This is the main force that leads to the complementarities in the short-run choices and further leads to the complementarities in the long-run variables.

These assumptions seem reasonable for those tech-heavy products such as computers, whose demand roots in the product's functionality. For this type of product, the product design innovation mostly focuses on improving the functionality and thus making most of its consumers prefer such an innovation, shifting the distribution of consumer valuation but not changing its dispersion. However, this may not be the case for those fast-moving consumer goods, especially the fashion industry, whose demand roots highly in product's sensory characteristics, e.g., fragrance and color. For those products, for example, perfume, the product design innovation will be very likely to make those who love it do so even more intensely, and those who dislike it very much dislike it, or conversely, make everyone has similar perceptions.

In Johnson and Myatt (2006), this phenomenon is called the change in the dispersion of consumer valuation. Such changes will make the distribution of consumer valuation more or less dispersed and result in a rotation of the inverse demand curve and marginal revenue curve, thus affecting the firm's profit. The authors also suggest that a firm's product design decisions, for example, the combination of different characteristics, will affect the dispersion of consumer valuation.

Therefore, this paper tries to incorporate the change in consumer dispersion into the analysis of the complementarities among manufacturing choices. As in Athey and Schmutzler (1995), I set up a two-period model with a monopolist producing a single product. In the first period, the monopolist invests in product and process flexibilities as well as process research capabilities. For its investment in product research capabilities, I split it into two parts, one is for shifting the mean of the distribution of consumer valuation, which is most related to factors that determine the product's functionality; the other part is for changing the dispersion, which is most related to factors that determine the product's sensory characteristics. In the second period, the monopolist decides whether to perform product design and process innovation as well as the production quantity, depending on the realization of the return to adjustment and how much it will change the consumer dispersion.

The main contribution of this paper is that, by considering product innovation will not only shift the distribution of consumer valuation but also increase its this dispersion, this paper incorporates the possibility that the prices and marginal revenues might decrease for some production quantity after adjustment, and therefore re-characterizes the conditions for the investment choices to be complementary in each other.

The remainder of the article is organized as follows. Section 2 sets up the model and analyzes the short-run decision problem; section 3 analyzes the long-run decision problem; section 4 concludes this paper.

2 The second-period decision problem

Following the definition in Athey and Schmutzler (1995), I denote the product and process adjustment as $\eta \in \{D, T\}$, where D refers to the product design dimension and T refers to the process dimension. The monopolist faces a binary adjustment decision in each dimension: it either innovates or does not innovate. Its decision choice is denoted by $a_\eta \in \{0, 1\}$, where 0 means not to innovate while 1 means to innovate. I also assume that, in the second period, together with a_D and a_T, the monopolist also chooses the production quantity z.

Though lots of factors affect the monopolist's decision, for simplicity, as Athey and Schmutzler (1995) suggests, I only consider the most relevant factor, the returns from adjustment. The returns can be decomposed into two aspects: how will the product adjustment change the consumer demand and how will the process adjustment change the production cost. As discussed in the introduction, the change in consumer demand can be further decomposed into how the adjustment will change the product's functionality and sensory characteristics that correspond to the change in mean and dispersion of consumer valuation. Therefore, I introduce three variables, r_T, r_D, and γ^M to parameterize how the production cost and consumer demand will react to the adjustment decision. More detailed explanations are as follows.

Returns to process innovation. In this paper, process innovation directly relates to the reduction in the cost of production. In terms of the production cost, for simplicity, I assume that the monopoly has no fixed cost and faces a constant marginal cost $C(a_T, r_T)$, which is also the constant average cost. As in Athey and Schmutzler (1995), I impose two assumptions on this constant average cost. First, a process innovation would reduce the constant marginal cost:

$$C(1, r_T) - C(0, r_T) \leq 0. \tag{1}$$

Second, the variable $r_T \in \mathbb{R}^+$ parameterizes the average cost in a way that:

$$\frac{\partial}{\partial_{r_T}}[C(1, r_T) - C(0, r_T)] \leq 0 \text{ and } \frac{\partial}{\partial_{r_T}}C(a_T, r_T) \leq 0, \tag{2}$$

meaning that a higher r_T is associated with a higher cost reduction induced by implementing process innovation.

Consumer demand. In terms of consumer demand, I model how the consumer demand will react to the product design adjustment following the notion of changing in the shape of the inverse demand curve in Johnson and Myatt (2006). Assume there is a unit mass of consumers, and a consumer is willing to pay up to θ for a single unit of the monopolist's product. A consumer's willingness to pay follows a distribution $F_s(\theta)$ with support on $(\underline{\theta}_s, \bar{\theta}_s)$. The parameter $s \in S = [s_L, s_H]$ indexes a family of distributions, which will be explained more in Definition 1. Further assume $F_s(\theta)$ is twice continuously differentiable in θ and s on its support and denote $f_s(\theta)$ as the strictly positive density of $F_s(\theta)$. For a given price p, consumers with $\theta > p$ will choose to buy the product, then the demand will be $z \equiv \mathbb{P}(\theta \geq p) = 1 - F_s(p)$. Then the inverse demand function is $p = P_s(z) \equiv F_s^{-1}(1 - z)$, meaning that if z units are to be sold, the market clearing price should satisfy $P_s(z) \equiv F_s^{-1}(1 - z)$. Thus, $s \in S$ also indexes a family of inverse demand curves.

Now comes to how the parameter s parameterizes the distribution of consumer valuation. Johnson and Myatt (2006) define it in a way that the change in s corresponds to the change in the shape of the cumulative distribution function (CDF). In particular, they consider the case that an increase in s results in a distribution that is "more dispersed." Specifically, an increase in s will result in a spread in a sense that it moves density away from the center of $f_s(\theta)$ and toward the upper and lower tails (not necessarily mean-preserving). Such a spread will result in a clockwise rotation of the CDF: there is a rotation point $\theta_s^\dagger$ such that $F_s(\theta)$ is increasing in s for $\theta < \theta_s^\dagger$ and decreasing in s for $\theta > \theta_s^\dagger$. A formal definition is as follows.

Definition 1. *A local change in s leads to a rotation of $F_s(\theta)$ if, for some $\theta_s^\dagger$ and each $\theta \in (\underline{\theta}_s, \bar{\theta}_s)$,*

$$\theta \gtrless \theta_s^\dagger \Leftrightarrow \frac{\partial F_s(\theta)}{\partial s} \lesseqgtr 0.$$

If this holds for all s, then $\{F_s(\theta)\}$ is ordered by a sequence of rotations.

Following this definition, in this paper, I will characterize the distribution of consumer valuation with its mean and standard deviation. I use $r_D \in \mathbb{R}^+$ to measure how much the product design innovation would shift the mean of the distribution of consumer valuation and γ^M where $\gamma \in [1, \bar{\gamma}]$ and $M \in \{-1, 1\}$ to measure how much and in which direction the adjustment would change the consumer dispersion. Assume that before the adjustment, consumers' valuation towards the product follows a distribution $F_{\mu_0,s_0}(\theta)$ with mean μ_0 and standard deviation s_0. The adjustment will shift the mean to $\mu_0 + r_D$ and scale the standard deviation upward to $\gamma^1 s_0$ or scale it down to $\gamma^{-1} s_0$. This means that the decision variable γ^M is actually a joint outcome of how much and in which direction the consumer dispersion would be scaled. When $\gamma^M = \gamma^1$, the product adjustment would increase the consumer dispersion, and when $\gamma^M = \gamma^{-1}$, it will decrease the dispersion. A formal definition is as follows.

Definition 2. *The family of distributions of consumer valuation is determined by its mean μ and standard deviation s in a way such that*

$$F_{\mu,s}(\theta) = F\left(\frac{\theta - \mu}{s}\right),$$

where $F(\cdot)$ is a continuous distribution with zero mean, unit variance, and strictly positive density. The corresponding inverse-demand curve satisfies $P_{\mu,s}(z) = \mu + sP(z)$ where $P(z) = F^{-1}(1 - z)$.

As discussed earlier, before any adjustment, θ follows the distribution $F_{\mu_0,s_0}(\theta)$, and the corresponding inverse demand function is $P_{\mu_0,s_0}(z) = \mu_0 + s_0 P(z)$. After the adjustment, the distribution of θ becomes $F_{\mu_0+r_D,s_0\gamma^M}(\theta)$ with mean $\mu_0 + r_D$ and standard deviation $s_0\gamma^M$. The inverse demand function after adjustment would then become $P_{\mu_0+r_D,s_0\gamma^M}(z) = (\mu_0 + r_D) + s_0\gamma^M P(z)$. To ease notation burden, I denote the inverse demand curve as $P_{a_D}(z) = (\mu_0 + a_D r_D) + s_0\gamma^{a_D M} P(z)$, thus when $a_D = 0$, $P_{a_D}(z) = P_{\mu_0,s_0}(z)$ and when $a_D = 1$, $P_{a_D}(z) = P_{\mu_0+r_D,s_0\gamma^M}(z)$.

Lemma 1 presents how Definition 2 falls in the class of distribution family in Defintion 1.

Lemma 1. *The family of consumer distribution is rotation-ordered. When $\gamma^M = \gamma^1$, the CDF rotates clockwise at the rotation point $\theta^\dagger$ after adjustment and when $\gamma^M = \gamma^{-1}$, the CDF rotates counter-clockwise after adjustment. In a special case when $\gamma = 1$, meaning that the adjustment only shifts the mean of consumer valuation but does not change its dispersion, the CDF after adjustment will lie below the original one, which is the case of first-order stochastic dominance.*

Proof. Take partial derivative of $F_{\mu,s}$ with respect to s, $\frac{\partial F_{\mu,s}}{\partial s} = f(\cdot)(\theta - \mu)(-\frac{1}{s^2})$. Therefore, as the standard deviation increases, there exists a rotation point $\theta^\dagger$ such that when $\theta < \theta^\dagger$, $\partial F_{\mu,s}(\theta)/\partial s$ is positive and when $\theta > \theta^\dagger$, $\partial F_{\mu,s}(\theta)/\partial s$ is negative, meaning that the cumulative distribution function (CDF) rotates clockwise as the variance increases. Observe that when $\gamma^M = \gamma^1$, the variance will increase after adjustment, and it will decrease if $\gamma^M = \gamma^{-1}$, so the proof of the first part is complete. The second part of the lemma can be obtained directly from Definition 2. □

Since the inverse demand functions are inverse functions of the CDF, the rotation results of the CDF immediately apply to the inverse demand curve.

Corollary 1. *The inverse demand function after adjustment rotates clockwise at the rotation point $z^\dagger$ when $\gamma^M = \gamma^1$, and counter-clockwise when $\gamma^M = \gamma^{-1}$. That is,*

$$z \gtreqless z^\dagger \Leftrightarrow \frac{\partial P_{a_D}(z)}{\partial a_D} \lesseqgtr 0 \text{ when } \gamma^M = \gamma^1$$
$$z \gtreqless z^\dagger \Leftrightarrow \frac{\partial P_{a_D}(z)}{\partial a_D} \gtreqless 0 \text{ when } \gamma^M = \gamma^{-1}$$

In a special case when $\gamma = 1$, the inverse demand curve will only shift outward but does not rotate.

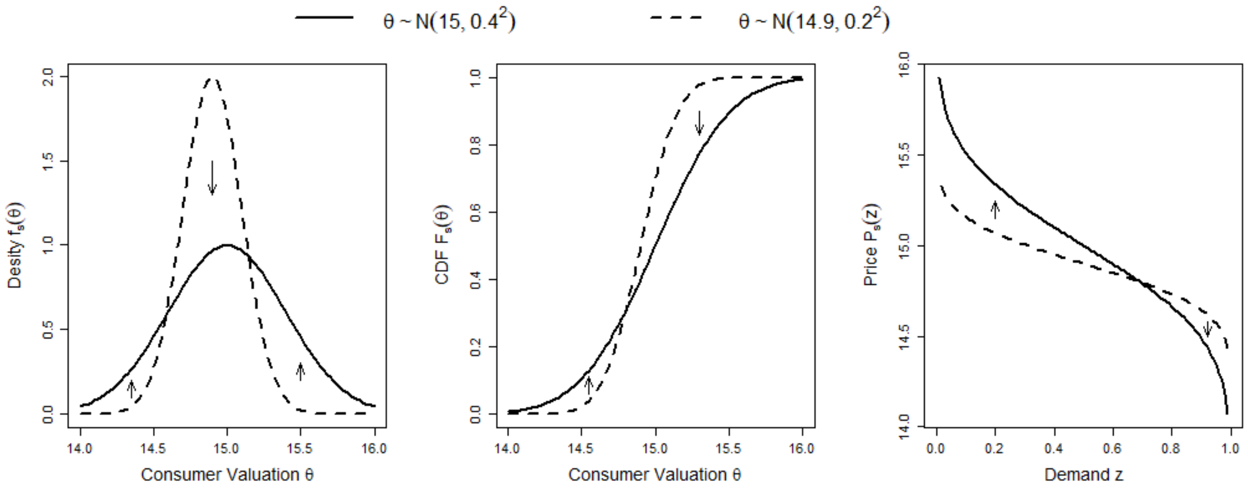
Corollary 1 establishes similar results in Johnson and Myatt (2006) and states that if the product innovation increases consumer dispersion, it will rotate and steepen the demand curve, making the price more sensitive to quantity. The following lemma gives the expressions of the rotation pair and its relationship with the scale magnitude.

Lemma 2. *The rotation pair is $\theta^\dagger = \mu_0 - \frac{r_D}{\gamma^M - 1}$ and $z^\dagger = \left\{z : P(z) = \frac{r_D}{s_0(1-\gamma^M)}\right\}$. The rotation quantity $z^\dagger$ is increasing in the original dispersion s_0 and decreasing in the scale magnitude γ if $\gamma^M = \gamma^1$ while increasing in γ if $\gamma^M = \gamma^{-1}$.*

Proof. Observe that at the rotation point the value of $F(\cdot)$ and price do not change after rotation. Therefore, $F\left(\frac{\theta^\dagger - \mu_0}{s_0}\right) = F\left(\frac{\theta^\dagger - \mu_0 - r_D}{s_0 \gamma^M}\right)$ and $(\mu_0 + r_D) + s_0\gamma^M P(z^\dagger) = \mu_0 + s_0 P(z^\dagger)$. Solving these equations gives the expression of $\theta^\dagger$ and $z^\dagger$. Since $P(z)$ is decreasing in z and $\gamma \in [1, \bar{\gamma}]$, observing the expression of $z^\dagger$ gives the second part of the lemma. □

The intuition behind Lemma 2 is that, since the rotation point $z^\dagger$ is the quantity at which the price remains unchanged after rotation, and because the fraction of consumers with extremely high willingness-to-pay would be higher if the dispersion increases more, therefore the same price could be obtained with a lower quantity. Thus the rotation quantity would be decreasing in the scale magnitude in the case where the dispersion is scaled upward. Similar logic applies to the case when the dispersion is scaled downward.

Below is an example illustrating the rotation of the inverse demand curve as the consumer dispersion changes. In this example, consumer valuation $\theta \sim N(\mu, s^2)$, the initial distribution is $\theta \sim N(14.9, .2^2)$, $\gamma^M = \gamma^1 = 2$ and $r_D = 0.1$. Thus, after adjustment, $\theta \sim N(15.0, .4^2)$. This adjustment results in an increase in consumer dispersion, corresponding with a distribution that is more widely spread, as illustrated in the first figure. The resulting rotation of the CDF and inverse demand function is shown in Figure 2 and Figure 3, which is an illustration of Lemma 1 and Corollary 1.



Marginal Revenue. Following the establishment of the inverse demand function, I derive the expression of the marginal revenue curve. From the inverse demand function $P_{\mu,s}(z) = \mu + sP(z)$, standard calculation yields the expression of the monopolist's marginal revenue $MR_{\mu,s}(z) = \mu + s[P(z) + zP'(z)] = \mu + sMR(z)$, where $MR(z)$ is the marginal revenue associated with the inverse demand curve $P(z)$. To ease notation burden, write $MR_{a_D}(z) = (\mu_0 + a_D r_D) + s_0\gamma^{a_D M} MR(z)$ to synthesize the notation of the marginal revenue before and after adjustment. Assume the marginal revenue is decreasing in z.

With the rotation of the inverse demand curve, the marginal revenue curve will also rotate in a similar way. Lemma 3 describes how the marginal revenue curve rotates as the consumer dispersion changes.

Lemma 3. *The marginal revenue curve after adjustment will rotate clockwise when $\gamma^M = \gamma^1$, and counter-clockwise when $\gamma^M = \gamma^{-1}$. That is,*

$$z \gtrless z^{\ddagger} \Leftrightarrow \frac{\partial MR_{a_D}(z)}{\partial a_D} \lesseqgtr 0 \text{ when } \gamma^M = \gamma^1$$
$$z \gtrless z^{\ddagger} \Leftrightarrow \frac{\partial MR_{a_D}(z)}{\partial a_D} \gtrless 0 \text{ when } \gamma^M = \gamma^{-1}$$

In a special case when $\gamma = 1$, the marginal revenue curve will only shift outward but not rotate. The rotation quantity $z^{\ddagger} = \left\{ z : MR(z) = \frac{r_D}{s_0(1-\gamma^M)} \right\}$ is increasing in the original dispersion s_0 and decreasing in the scale magnitude γ if $\gamma^M = \gamma^1$ while increasing in γ if $\gamma^M = \gamma^{-1}$.

Proof. Observe that $\frac{\partial^2 MR_{\mu,s}(z)}{\partial z \partial s} = \frac{\partial MR(z)}{\partial z} < 0$ since I assume marginal revenue is decreasing in z. Therefore, as the standard deviation increases, there exists a rotation point $z^{\ddagger}$ such that when $z < z^{\ddagger}$, $\partial MR_{\mu,s}(\theta)/\partial s$ is positive and when $z > z^{\ddagger}$, $\partial MR_{\mu,s}(\theta)/\partial s$ is negative, meaning that the marginal revenue curve rotates clockwise as the variance increases. Observe that when $\gamma^M = \gamma^1$, the variance will increase after adjustment, and it will decrease if $\gamma^M = \gamma^{-1}$, so the proof of the first part is complete. Observe that the value of marginal revenue at the rotation point remains unchanged after the adjustment, thus, $\mu_0 + r_D + s_0\gamma^M MR(z) = \mu_0 + s_0 MR(z)$. Solving the equation gives the expression of the rotation quantity. The second part of this lemma can be obtained by observing this expression and by the assumption that $MR(z)$ is decreasing in z. □

The underlying logic that generates Lemma 3 is exactly the same as the one in Lemma 2. The results in Lemma 2 and 3 that the rotation quantity of the inverse demand curve and the marginal revenue curve would be higher if the original dispersion level is higher are the same results as in Johnson and Myatt (2006). What is new here is that the location of the rotation quantity is shown to also depend on the magnitude and the direction of rotation.

Observe that $MR(z^{\ddagger}) = P(z^{\dagger})$, then $z^{\ddagger}P'(z^{\ddagger})+P(z^{\ddagger}) = P(z^{\dagger})$. By the fact that the inverse demand curve is downward sloping, Corollary 2 can be immediately obtained.

Corollary 2. *The rotation quantity of the marginal revenue curve $z^{\ddagger}$ is always less than the rotation quantity of the inverse demand curve $z^{\dagger}$.*

The intuition behind Corollary 2 is that, when the dispersion increases, at the rotation quantity $z^{\dagger}$, the inverse demand curve becomes steeper, and thus the marginal revenue decreases at this point. Thus it must lie to the right of the marginal revenue rotation quantity $z^{\ddagger}$.

Returns to product innovation in functionality. Observing the expression of the marginal revenue and inverse demand function gives the important properties of the parameter r_D, which represents the shift in the mean of consumer valuation as discussed earlier. First, given quantity and the change in dispersion, a higher r_D is associated with a higher incremental price induced by implementing product design innovation:

$$\frac{\partial}{\partial_{r_D}}\left[P_{a_D=1}(z) - P_{a_D=0}(z)\right] = 1 \geq 0. \tag{3}$$

Second, given quantity and the change in dispersion, a higher r_D is associated with a higher incremental marginal revenue induced by implementing product design innovation:

$$\frac{\partial}{\partial_{r_D}}\left[MR_{a_D=1}(z) - MR_{a_D=0}(z)\right] = 1 \geq 0. \tag{4}$$

Adjustment costs. The last pieces to introduce before establishing the first-period profit function are the costs of adjustment. Denote the adjustment cost function as $A_\eta(a_\eta, f_\eta)$ where f_η represents product or process flexibility. Notice that the product design innovation cost is a joint cost of product functionality and sensory characteristic innovation because they are assumed to be adjusted together in this paper. Following Athey and Schmutzler (1995), I assume that flexibility lowers the costs of adjustment:

$$\frac{\partial}{\partial f_\eta}\left[A_\eta(1, f_\eta) - A_\eta(0, f_\eta)\right] \leq 0. \tag{5}$$

Second-period profits. Combining together, the second-period profit function is:

$$\mathbf{\Pi}_2(\boldsymbol{a}, z, \boldsymbol{f}; \boldsymbol{r}, \gamma^M) \equiv \left[P_{a_D}(z, a_D; r_D, \gamma^M) - C(a_T, r_T)\right] \cdot z - A_D(a_D, f_D) - A_T(a_T, f_T) \tag{6}$$

This profit function helps establish the following lemmas and propositions about the characteristics of the second-period decision variables.

Lemma 4. *When $\gamma^M = \gamma^1$, $\mathbf{\Pi_2}$ is supermodular in (a_D, a_T, z) for $z \leq z^{\ddagger}$. When $\gamma^M = \gamma^{-1}$, $\mathbf{\Pi_2}$ is supermodular in (a_D, a_T, z) for $z \geq z^{\ddagger}$. When $\gamma = 1$, $\mathbf{\Pi_2}$ is supermodular in (a_D, a_T, z) for all z.*

Proof. The supermodularity in (a_D, a_T) is obtained for free because they do not have interaction terms in $\mathbf{\Pi_2}$. The supermodularity in (a_T, z) for all z is obtained from condition (1). The supermodularity in (a_D, z) is obtained from Lemma 3. □

Lemma 4 presents the complementarities in the second-period choice variables as in Athey and Schmutzler (1995). The decision of product design innovation and process innovation are mutually reinforcing, meaning that the increase in one variable increases the return of increasing the others. Where Lemma 4 is different from the result in Athey and Schmutzler (1995) is that the supermodularity results are not applicable for all ranges of production quantity. This means that jointly expansionary innovation is not optimal all the time. Such differences come from the change in the shape of the inverse demand curve. By considering that product innovation will not only shift the consumer valuation but also increase its dispersion, this paper considers the possibility that the marginal revenue might decrease for some production quantity after adjustment, and therefore increasing quantity might reduce the return to implement innovations.

Following the establishment of the complementarities in the second-period choice variables, Proposition 1 characterizes the range of monotonicity of these variables.

Proposition 1. *When $\gamma^M = \gamma^1$, for all $z^* \leq \{z : MR(z) = 0\}$, the optimal second-period choices of product adjustment, process adjustment, and quantity (a_D^*, a_T^*, z^*) are monotonically non-decreasing in the returns associated with product functionality and process innovation, r_D and r_T, and product and process flexibility, f_D and f_T, as well as the magnitude of dispersion change γ. The same arguments hold for (a_D^*, a_T^*, z^*) when $\gamma^M = \gamma^{-1}$ and $z^* \geq \{z : P(z) = 0\}$.*

Proof. See Appendix A. □

Proposition 1 implies that under the specified condition, if one of the flexibility, the return to process innovation, the scale magnitude of consumer dispersion, or the shift in the mean of consumer valuation increases, then the firm will be more likely to implement innovations in both dimensions, and further, the firm produces a larger quantity. For similar reasons in Lemma 4, compared to Athey and Schmutzler (1995), the monotonicity in Proposition 1 is not applicable for all ranges of production quantity, because the innovation might decrease the marginal revenue at some production quantity.

One important implication from Proposition 1 is that it coincides with the "monopolist's preference for extreme" in Johnson and Myatt (2006). Proposition 1 states that when the operation quantity is less than a certain threshold, which is similar to the "niche market player", its decision choices are non-decreasing if the dispersion is scaled upward, meaning that she is more willing to innovate so as to increase dispersion. This coincides with the result in Johnson and Myatt (2006) which shows that niche-market monopolist prefers an extremely high level of dispersion. When the operation quantity is higher than a certain threshold, which is similar to the "mass market player", its decision choices are non-decreasing if the dispersion is scaled downward, meaning that she is more willing to innovate to decrease the dispersion. This coincides with the result in Johnson and Myatt (2006) which shows that mass-market monopolist prefers an extremely low level of dispersion.

The following lemma formally characterizes the relationship between the optimal operation quantity and the scale magnitude.

Lemma 5. *When $\gamma^M = \gamma^1$, the optimal production quantity z^* is non-decreasing in γ. When $\gamma^M = \gamma^{-1}$, the optimal production quantity z^* is non-increasing in γ.*

Proof. The monopolist sets the production quantity based on the condition that marginal revenue equals marginal cost. Thus, $(\mu_0 + a_D r_D) + s_0 \gamma^{a_D M} MR(z^*) = C(a_T, r_T)$. Taking the derivative with respect to γ on both sides and treating z^* as a function of γ give $s_0 a_D M \gamma^{a_D M - 1} MR(z^*) + s_0 \gamma^{a_D M} MR'(z^*) \frac{\partial z^*}{\partial \gamma} = 0$, and $\frac{\partial z^*}{\partial \gamma} = -a_D M \gamma^{-1} MR'(z^*)^{-1} MR(z^*)$. Since $MR(z)$ is decreasing in z, then when $\gamma^M = \gamma^1$, z^* is non-decreasing in γ, and when $\gamma^M = \gamma^{-1}$, z^* is non-increasing in γ. □

The intuition behind Lemma 5 is that, for example, in the case where the monopolist decides to increase consumer dispersion and set $\gamma^M = \gamma^1$, it is profitable to increase the output since the right tail of the distribution will become thicker, meaning that more consumers will have high willingness-to-pay after the innovation. Thus the optimal production quantity z^* is non-decreasing in γ when $\gamma^M = \gamma^1$. Similar logic applies to the case where $\gamma^M = \gamma^{-1}$.

When $\gamma^M = \gamma^1$, after adjustment, the inverse demand curve will rotate clockwise. When the optimal quantity z^* is less than the rotation quantity of the inverse demand function $z^\dagger$, profit will increase after rotation because the price level will rotate up. The higher the γ is, the more it will rotate up. Therefore, profit is increasing in γ if $\gamma^M = \gamma^1$ and $z^* < z^\dagger$, as well as when $\gamma^M = \gamma^{-1}$ and $z^* > z^\dagger$. Lemma 2 gives the relationship between the rotation quantity on the inverse demand curve $z^\dagger$ and the scale magnitude γ, and Lemma 5 gives the relationship between the optimal operation quantity z^* and the scale magnitude γ. With these results, it is straightforward to list all the possible situations about the relative magnitude of z^* and $z^\dagger$ and obtain the results in the following proposition.

Proposition 2. *When $\gamma^M = \gamma^1$, z^* is a non-decreasing function in γ while $z^\dagger$ is decreasing in γ, then $\mathbf{\Pi_2}$ would be a quasi-concave (inverse-U-shape) function and thus the optimal scale magnitude will be either 1 or $\bar{\gamma}$ or an interior point in $[1, \bar{\gamma}]$. When $\gamma^M = \gamma^{-1}$, z^* is a non-increasing function in γ while $z^\dagger$ is increasing in γ, then $\mathbf{\Pi_2}$ would be a quasi-concave (inverse-U-shape) function and thus the optimal scale magnitude will be either 1 or $\bar{\gamma}$ or an interior point in $[1, \bar{\gamma}]$.*

Proposition 2 lists different situations that the monopolist might face under different behavior of the optimal quantity and the rotation quantity. Which case that the monopolist will face depends on the exact distribution of consumer valuation in the market. In practice, if the monopolist knows the exact distribution of consumer valuation in the first period, and the exact form of the marginal cost function, it is possible for her to determine whether γ^M should be γ or γ^{-1} as well as the optimal level of γ by comparing the profits in different cases in Proposition 2. Then by this backward induction, the monopolist can obtain the most favorable γ^M and set it as a goal when investing in the first period.

Considering that a monopolist typically would have been operated in the market for years and knows the taste of her customers. From this point on, for simplicity, assume the monopolist can always accurately control the direction of the change in consumer dispersion, that is, she can fully determine whether the realization of M equals to 1 or -1. Therefore, in the first period, when the monopolist invests in the product design research capabilities in terms of sensory characteristics, she only needs to be concerned with the uncertainty of how much the innovation would scale the consumer dispersion, but not concerned in which direction that the scale would happen, because she can fully determine it. Therefore, from this point on, I drop the superscript of γ^M and denote the monopolist's favorite level of γ as $\hat{\gamma}$.

3 The first-period decision problem

In the first period, the monopolist faces uncertainty regarding the realization of r_D, r_T, and γ. The distribution of the possible realizations can be characterized by the joint distribution denoted as $G(r_D, r_T, \gamma)$. The monopolist can achieve a more favorable probability distribution by investing in its research capabilities. Mathematically speaking, that is, she invests in shifting the parameters that govern $G(r_D)$, $G(r_T)$ and $G(\gamma)$. Denote these three parameters as $i_D, i_T, i_\gamma \in \mathbb{R}^+$ and the corresponding marginal distribution as $G(r_D; i_D)$, $G(r_T; i_T)$ and $G(\gamma; i_\gamma)$. Denote the second-period value function as $\mathbf{\Pi}_2^*(\boldsymbol{f};, r_D, r_T, \gamma) = \mathbf{\Pi}_2^*(\boldsymbol{a}^*, z^*, \boldsymbol{f};, r_D, r_T, \gamma)$, then the expected profits in the second period can be written as:

$$E_{r_D,r_T,\gamma}[\mathbf{\Pi}_2^*(\boldsymbol{f};,r_D,r_T,\gamma)] = \int_{r_D,r_T,\gamma}\Big[[P(z^*,a_D^*,r_D,\gamma,)-C(a_T^*,r_T)]\cdot z^*- A_D(a_D^*,f_D)-A_T(a_T^*,f_T)\Big]\cdot dG(r_D,r_T,\gamma;i_D,i_T,i_\gamma)$$

For simplicity, as in Athey and Schmutzler (1995), I assume that the joint distribution $G(r_D,r_T,\gamma)$ is separable. That is, $G(r_D,r_T,\gamma)=G(r_D;i_D)G(r_T;i_T)G(\gamma;i_\gamma)$. Also, I parameterize the marginal distribution of research capabilities in i_T and i_D in a sense of first-order stochastic dominance (FOSD):

$$\frac{\partial}{\partial_{i_\eta}}G_\eta(r_\eta;i_\eta)\le 0 \text{ for } \eta\in\{D,T\} \tag{7}$$

This assumption means that the more effort that the monopolist puts into research, the more likely that she can obtain a higher realization of r_D or r_T.

For the research capabilities that determines γ, recall as discussed in Section 2, the monopolist has an exact favorite level, it is not necessarily the higher the better. Thus, I parameterize the marginal distribution of $G(\gamma;i_\gamma)$ in a way that a higher i_γ moves the mean of γ closer to the optimal $\hat{\gamma}$, and also shrinks the variance of γ. By this assumption, the more effort that the monopolist puts into research in designing the product's sensory characteristics with the aim of achieving her favorite $\hat{\gamma}$, the more likely that she can achieve the optimal realization of γ. This assumption is similar to Definition 2, and by applying similar arguments as in Lemma 1, it can be shown that the characterization of $G(\gamma;i_\gamma)$ fits in Definition 1. Therefore, there exists a $\gamma^\dagger$ such that

$$\gamma\gtreqless\gamma^\dagger\Leftrightarrow\frac{\partial G(\gamma;i_\gamma)}{\partial i_\gamma}\gtreqless 0.$$

As in Athey and Schmutzler (1995), for simplicity, I assume all costs of investment in the first period are separable. Assume the costs of investments into flexibility are given as $F_D(f_D)$ and $F_T(f_T)$. Further, assume the costs of the firm's investments into research capabilities are given as $I_D(i_D)$, $I_T(i_T)$, and $I_\gamma(i_\gamma)$. Then the first-period profit function can be written as

$$\mathbf{\Pi_1}(f_D,f_T,i_D,i_T,i_\gamma)\equiv\int_{r_D}\int_{r_T}\int_\gamma[\mathbf{\Pi}_2^*(\boldsymbol{f};r_D,r_T,\gamma)]dG(r_D;i_D)G(r_T;i_T)G(\gamma;i_\gamma) - F_D(f_D)-F_T(f_T)-I_D(i_D)-I_T(i_T)-I_\gamma(i_\gamma)$$

The following proposition establishes the conditions for this first-period profits to be supermodular in the first-period decision variables.

Proposition 3. $\mathbf{\Pi_1}(\boldsymbol{f},\boldsymbol{i})$ *is supermodular in* $(f_D,f_T,i_D,i_T,i_\gamma)$ *when* $\gamma<\gamma^\dagger$ *and* $z\le\{z:MR(z)=0\}$ *if* $\gamma^M=\gamma^1$ *or when* $\gamma<\gamma^\dagger$ *and* $z\ge\{z:P(z)=0\}$ *if* $\gamma^M=\gamma^{-1}$.

Proof. In Appendix A, it is shown that when $z\le\{z:MR(z)=0\}$ if $\gamma^M=\gamma^1$ or when $z\ge\{z:P(z)=0\}$ if $\gamma^M=\gamma^{-1}$, $\mathbf{\Pi}_2$ is supermodular in $(a_D,a_T,z,f_D,f_T,r_D,r_T,\gamma)$. Then by Theorem 2 in Appendix B, the value function $\mathbf{\Pi}_2^*(\boldsymbol{f};,r_D,r_T,\gamma)$ is supermodular in (f_D,f_T,r_D,r_T,γ) when $z\le\{z:MR(z)=0\}$ if $\gamma^M=\gamma^1$ or when $z\ge\{z:P(z)=0\}$ if $\gamma^M=\gamma^{-1}$. Then by Lemma 6 in Appendix B, $\mathbf{\Pi_1}(\boldsymbol{f},\boldsymbol{i})$ is supermodular in $(f_D,f_T,i_D,i_T,i_\gamma)$ when $\gamma<\gamma^\dagger$ and $z\le\{z:MR(z)=0\}$ if $\gamma^M=\gamma^1$ or when $\gamma<\gamma^\dagger$ and $z\ge\{z:P(z)=0\}$ if $\gamma^M=\gamma^{-1}$. □

Athey and Schmutzler (1995) establish the result that increases in product and process flexibility and research capability are mutually reinforcing and therefore conclude that the short-run complementarities leads to the long-run complementarities. However, in this paper, these choice variables are only complementary under a certain output range, and therefore the first-period complementaries are based on the outcome in the second period. However, in the first period, the monopolist cannot foresee the outcome and therefore it is difficult to use Proposition 3 as guidance for investment. However, it is important for the firm to notice that as the first-period profit is not always supermodular in her investment, it is possible that the increase in one research dimension will reduce the return to increasing the other.

4 Conclusion

This paper constructs a two-period model with a monopolist making short-run innovation decisions and long-run investment decisions. In the long run, the monopolist invests in product and process flexibilities as well as product and process research capabilities. Firms' investments usually interact with each other, and it is not necessarily that investing in one dimension is always favorable for investments in other dimensions. As a result, understanding in what situations investment choices would be complementary to each other is important for a firm to maximize its return to investment.

Athey and Schmutzler (1995) show that these four investment variables are complementary to each other if the product design innovation always raises prices and marginal revenues. However, since product design innovation might result in a change in the dispersion of consumer valuation, it is possible that the innovation will reduce prices and marginal revenues. To model such changes, this paper introduces a decision variable that determines how much and in which direction the consumer dispersion will change after the adjustment, allowing product design innovation to not only shift but also rotate the demand curve.

With this model, this paper re-characterizes the conditions for the investment choices to be complementary in each other and provides a framework for the firm to decide how to design its product characteristics related to functionality and sensory in the first period so as to achieve the most favorable dispersion level in the second period.

A Omitted Proof

Proof of Proposition 1. The proof can be split into two parts, first prove that $\mathbf{\Pi}_2$ is supermodular in $(a_D, a_T, z, f_D, f_T, r_D, r_T, \gamma)$ when $z \leq \{z : MR(z) = 0\}$ if $\gamma^M = \gamma^1$ or when $z \geq \{z : P(z) = 0\}$ if $\gamma^M = \gamma^{-1}$, then prove the monotonicity of (a_D^*, a_T^*, z^*).

Observing the expression of $\mathbf{\Pi}_2$, to prove the supermodularity in $(a_D, a_T, z, f_D, f_T, r_D, r_T, \gamma)$, we need to show the supermodularity in (a_D, z), (a_D, f_D), (a_D, r_D), (a_D, γ), (a_T, z), (a_T, f_T), (a_T, r_T), (z, r_D), (z, r_T), (z, γ). The suupermodularity in (a_T, r_T) and (a_D, r_D) can be obtained from condition (2) and condition (3). Supermodularity in (a_D, f_D) and (a_T, f_T) can be obtained from condition (5). Condition (1) guarantees the supermodularity in (a_T, z). Condition (2) and (4) gives supermodularity in (z, r_T) and (z, r_D). Conditions for the supermodularity in (a_D, z), (z, γ) and (a_D, γ) are shown below.

When $\gamma^M = \gamma^1$, by Lemma 4, $\mathbf{\Pi_2}$ is supermodular in (a_D, z) when $z \leq z^{\ddagger}$. To find the condition of supermodularity in (z, γ), we need to find the condition that $\partial MR_{a_D}(z)/\partial\gamma = s_0 MR(z) a_D \gamma^{a_D - 1} \geq 0$. Thus, to ensure the supermodularity in (z, γ), we need $MR(z) \geq 0$ and thus $z \leq \{z : MR(z) = 0\}$. To ease notation burden, denote $z^0_{MR} = \{z : MR(z) = 0\}$. To find the condition of supermodualrity in (a_D, γ), we need to find the condition that $\frac{\partial}{\partial \gamma}\left(\frac{\partial P_{a_D}(z)}{\partial a_D}\right) = P(z)s_0 \geq 0$. Then we need $P(z) \geq 0$ and thus $z \leq \{z : P(z) = 0\}$. To ease notation burden, denote $z^0_P = \{z : P(z) = 0\}$. From the equality $MR(z) = zP'(z) + P(z)$, we can see that $z^0_{MR} < z^0_P$. From the inequality $MR(z^\ddagger) = \frac{r_D}{s_0(1-\gamma)} \leq 0$, we can see that $z^0_{MR} \leq z^\ddagger$. Therefore, when $\gamma^M = \gamma^1$, the condition that can ensure the supermodularity in (a_D, z), (z, γ) and (a_D, γ) is that $z \leq z^0_{MR}$.

When $\gamma^M = \gamma^{-1}$, by Lemma 4, $\mathbf{\Pi_2}$ is supermodular in (a_D, z) when $z \geq z^\ddagger$. To find the condition of supermodularity in (z, γ), we need to find the condition that $\partial MR_{a_D}(z)/\partial\gamma = -s_0 MR(z) a_D \gamma^{-a_D-1} \geq 0$. Thus, to ensure the supermodularity in (z, γ), we need $MR(z) \leq 0$ and thus $z \geq \{z : MR(z) = 0\}$. To find the condition of supermodualrity in (a_D, γ), we need to find the condition that $\frac{\partial}{\partial \gamma}\left(\frac{\partial P_{a_D}(z)}{\partial a_D}\right) = -P(z)s_0\gamma^{-2} \geq 0$. Thus, to ensure the supermodularity in (a_D, γ), we need $P(z) \leq 0$ and thus $z \geq \{z : P(z) = 0\}$. From the inequality $MR(z^\ddagger) = \frac{r_D}{s_0(1-\gamma^{-1})} \geq 0$, we can see that $z^0_{MR} \geq z^\ddagger$. Therefore, the condition that can ensure the supermodularity in (a_D, z), (z, γ) and (a_D, γ) is that $z \geq z^0_P$.

Applying Theorem 1 in Appendix B completes the proof. □

B Definition, Theorems, and Lemmas

Definition 3. *A function $f : \mathbb{R}^n \to \mathbb{R}$ is supermodular if for all $\boldsymbol{x}, \boldsymbol{x'} \in \mathbb{R}^n$, $f(\boldsymbol{x}) + f(\boldsymbol{x'}) \leq f(\boldsymbol{x} \vee \boldsymbol{x'}) + f(\boldsymbol{x} \wedge \boldsymbol{x'})$.*

Theorem 1 (Topkis (1978))**.** *Let $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ such that $f(\boldsymbol{x}, \boldsymbol{t})$ is supermodular in $(\boldsymbol{x}, \boldsymbol{t})$. Then if S is a sublattice of $\mathbb{R}^n$, $argmax_{\{\boldsymbol{x} \in S\}} f(\boldsymbol{x}, \boldsymbol{t})$ is monotone nondecreasing in $\boldsymbol{t}$ (in the strong set order).*

Theorem 2 (Milgrom and Roberts (1990))**.** *Let S^2 be a sublattice of $\mathbb{R}^{n2}$. Further, let $f : \mathbb{R}^{n1} \times \mathbb{R}^{n2} \times \mathbb{R}^m \to \mathbb{R}$ such that $f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t})$ is supermodular in $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t})$. Then $g(\boldsymbol{x}, \boldsymbol{t}) \equiv max_{\{\boldsymbol{y} \in S^2\} f((\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t})}$ is supermodular in $(\boldsymbol{x}, \boldsymbol{t})$*

Lemma 6 (Athey (1995))**.** *The following two conditions are equivalent: (i)*

$$\int_{\boldsymbol{r}} \boldsymbol{\pi}(\boldsymbol{x}, \boldsymbol{r})\, dG(r_D; i_D) G(r_T; i_T)$$

is supermodular in $(\boldsymbol{x}, \boldsymbol{i})$ for all payoff function $\pi : \mathbb{R}^n \times \mathbb{R}^2 \to \mathbb{R}$ that are supermodular. (ii) $\partial G_\eta(r_\eta; i_\eta)/\partial i_\eta \leq 0$ for all r_η $(\eta \in \{D, T\})$.

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