Persuasion: Multiple Rounds of Search and Matching in the Real Estate Market

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Abstract

This paper considers an information design model where a real estate broker selects the extent of information disclosure to his client(s), who then take an action that affects the benefits of both the broker and the client(s). In the housing market context, the author characterizes the optimal information structure and the benefit from persuasion for the broker in a search game and in a matching game, respectively. The paper also compares two types of prior beliefs; i.e., a constant prior throughout all rounds of persuasion, and an evolving prior consisting of a series of updated priors in each round. The results show that a client with an evolving prior will be more easily disheartened and more challenging to persuade.

Keywords: Bayesian persuasion, information design, search and matching, evolving prior

1 Introduction

Despite its importance in the decision-making process in the real estate market, uncertainty in the state of the property has not received enough attention. The standard search and matching models cannot adequately explain the property rental or purchase decisions under uncertainty. Therefore, a Bayesian persuasion model describing the search and matching behaviors under Bayesian updating is essential for understanding housing market behaviors.

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This persuasion model introduces a new role for a real estate broker in revealing information to his clients, thereby influencing the clients' decisions. There are two types of persuasion problems in the housing market: a search problem and a matching problem. The former involves a broker who is the information sender, and a renter (or buyer) who is the information receiver. The latter, in contrast, involves a broker, a renter (or buyer), and a landlord (or seller), with both the renter and the landlord being information receivers.

In the Bayesian persuasion problems, I consider two types of prior beliefs about the housing unit. The first type is a constant prior belief, held by people who are confident in their beliefs, because they have a good knowledge of the housing market and have observed abundant historical data. The second type is an evolving prior belief, which consists of a series of updated priors. This means that the person updates his/her prior after observing the outcome of every round of the search/ matching. This belief is held by less confident people in their beliefs because of their limited prior information about the market.

This paper applies an information design model in the housing market context, focusing on the benefit from persuasion by the real estate broker. It is of theoretic and practical values to investigate the optimal extent of information disclosure that maximizes the probability of reaching a deal, thereby reducing information asymmetry and allocating housing resources more efficiently under uncertainty. Therefore, the analysis of persuasion problems in the housing market is crucial for improving real estate broker services and housing market efficiency. Although this paper mainly focuses on the housing market, the persuasion model proposed in the paper could have many other interesting, real-life applications, such as matchmaking services in the marriage market and directors of graduate studies' role in matching graduate students and professors as RAs or TAs.

The study aims to investigate the optimal information structure and benefit from persuasion for the real estate broker. The paper examines and compares the equilibrium outcomes of a search problem and a matching problem in the real estate market. Moreover, the paper compares the scenarios of constant prior and evolving prior and discusses the underlying intuition.

Related literature. This paper belongs to the growing literature on Bayesian persuasion started and enriched by Kamenica & Gentzkow (2011), Kamenica (2019), and Bergemann & Morris (2019). Building on the previous research, this study aims to extend the persuasion model to explore multiple rounds of persuasion and evolving prior belief.

This study also relates to the literature on search and matching models. Courant (1978) examines the higher search costs under racial discrimination in the urban housing market in a search model. Yinger (1981) constructs a search model and analyzes how uncertainty and costly search influence real estate brokers' search and matching behavior. Wheaton (1990) proposes a housing market matching model with households that are both buyers and sellers and explains "structural" vacancy. The previous literature does not involve strategic interactions between the broker and the renter/ landlord (or buyer/ seller). This paper, by contrast, adopts a belief-based approach to investigate the optimal extent of information disclosure that optimizes the real estate broker's objective under uncertainty about whether the renter and the landlord match.

Another closely related strand of research is the Bayesian persuasion literature with a focus on coalitions. Vosooghi (2017) examines the formation of multiple coalitions in a public-bad game setting and investigates the unique stable coalition structure. Proposing a model in the social learning context, Meng (2021) explores the coalitions of communication partners between agents with heterogeneous prior beliefs. Bardhi & Guo (2018) compare the benefits from different persuasion modes in a voting context where a group of receivers votes on a collective decision under a unanimous decision rule. The contribution of this paper is to explore the difference between the optimal information structure for a coalition of the renter and the landlord under a constant prior and its counterpart under an evolving prior.

The remainder of the paper is organized as follows. Section 2 presents the general setup of the model; Section 3 formalizes the broker's optimal information structure in a search problem and compares the benefits from persuasion under constant and evolving priors; Section 4 characterizes the optimal information structure and the benefit from persuasion in a matching problem; Section 5 discusses an extension of the model. Section 6 concludes; All proofs are contained in the Appendices.

2 Model

Actions and states. Consider the persuasion problem with a real estate broker (he) who is the information sender and a renter (she) who is the information receiver in multiple rounds of housing search.[1] The renter needs to take an action $d \in D$ in each round of the search, where $D = \{a, r\}$. Let $d = a$ denote accepting the housing unit and $d = r$ denote rejecting the housing unit. The state of the world $\theta \in \Theta$ is binary, where $\Theta = \{f, nf\}$. Let $\theta = f$ represent the housing unit is a right fit for the renter's preferences, and $\theta = nf$ represent the housing unit is not a right fit. The state only

[1] In Section 3, the problem is extended to a matching game with a sender and two receivers (i.e., the renter and the landlord).

reflects a match between the property and the renter, and does not necessarily indicate the overall quality of the property. Also, the duration of the rental contract is not considered in the model.

Payoffs. Suppose the broker only receives the full commission after the renter finds a housing unit that she accepts. Also, he faces a fixed time cost from each round of the search. Therefore, the broker's objective is to minimize his time cost, which could also be interpreted as maximizing the probability of the renter accepting the housing unit in each round.

The renter receives utility 1 from accepting the property which is a right fit, utility -1 from accepting the property which is not a right fit, and utility 0 from rejecting the property in each round. Also, the renter faces an increasing marginal time cost c_n in the n^{th} round of search, where $0 < c_1 < \cdots < c_n < \cdots < 1$. Therefore, the renter's utility in the n^{th} round is

$$u(d,\theta) = \begin{cases} 1 - \sum_{i=1}^{n} c_i & \text{if d = a and } \theta = \text{f} \\ -1 - \sum_{i=1}^{n} c_i & \text{if d = a and } \theta = \text{nf} \\ -\sum_{i=1}^{n+1} c_i & \text{if d = r and keep on searching} \\ -\overline{U} - \sum_{i=1}^{n} c_i & \text{if d = r and give up searching} \end{cases}$$

In what follows, it is assumed that the disutility from giving up searching, $\overline{U} \geqslant 1$, is large enough to dissuade the renter from giving up searching. This assumption is natural since renting a housing unit is usually a rigid demand, especially when people can not afford a property of their own.

Constant prior and evolving prior. The paper considers two types of prior beliefs, namely a constant prior and an evolving prior. The difference between the two types of prior beliefs reflects people's confidence in their prior information.

A constant prior implies that the broker or the renter is confident in the prior information and finds it unnecessary to update his/ her prior throughout the search. A renter with a constant prior of the state being "a right fit", $Pr(\theta = f) = \mu_0$, where $0 < \mu_0 < 0.5$, is assumed to have good knowledge of the housing market and has observed sufficient data of previous successful and unsuccessful searches. Therefore, she would not be disheartened by her first several unsuccessful rounds of the search.

An evolving prior can also be referred to as a series of updated priors. It implies that the renter is not confident in the prior belief, and therefore would like to combine the prior information and the sample information (i.e., the results of her previous rounds of the search) to update her belief, because she is not familiar with the housing market, and her prior belief is based on limited data.[2]

[2]These updated beliefs are, in nature, posterior beliefs (in terms of all housing units on the market as a

Formally, before the first round of the search, the renter has an initial prior distribution, f_0, of the probability of state being "a right fit", μ, i.e.,

$$f_0(\mu|n^{'}, r^{'}) = \frac{\Gamma(n^{'})}{\Gamma(r^{'})\Gamma(n^{'} - r^{'})} \cdot \mu^{r^{'}-1} \cdot (1-\mu)^{n^{'}-r^{'}-1}$$

where $n^{'}, r^{'}$ are two positive parameters of the initial prior distribution f_0, which is a natural-conjugate prior;[3] the probability of the state being "a right fit" is $\mu \in [0, 1]$; and the Gamma function formula is $\Gamma(x) = (x-1)!$ for positive integer x. Note that the initial prior can be regarded as "equivalent sample information". That is, before the search begins, the renter has observed the prior information, which is equivalent to a limited sample of size $n^{'}$ with $r^{'}$ successes from a Bernoulli process, where $n^{'} > r^{'} > 0$. The mean of the initial prior is $\mathbb{E}_0(\mu) = \int_0^1 \mu f_0(\mu) \mathrm{d}\mu = \frac{r^{'}}{n^{'}}$. Also, the model further assumes that $\mathbb{E}_0(\mu) = \mu_0$ to keep consistency with the constant belief. Hence $r^{'} = n^{'}\mu_0$.

Before the n^{th} round begins, where $n = 1, 2, \ldots,$ the renter has gone through $n-1$ unsuccessful rounds of the search, and has observed a sample of size $n-1$ with zero success. According to the property of the natural-conjugate family, the updated prior belief for the n^{th} round can be constructed as below.

Definition 2.1 (Evolving priors)**.** *The prior belief is an evolving priors if, before the n^{th} round of the search, where $n = 1, 2, \ldots,$ the updated prior distribution is,*

$$f_{n-1}(\mu|n^{''}, r^{''}) = \frac{\Gamma(n^{''})}{\Gamma(r^{''})\Gamma(n^{''} - r^{''})} \cdot \mu^{r^{''}-1} \cdot (1-\mu)^{n^{''}-r^{''}-1}$$

where $n^{''} = n^{'} + n - 1$, $r^{''} = r^{'}$. The mean of the updated prior is,

$$\mathbb{E}_{n-1}(\mu) = \int_0^1 \mu f_{n-1}(\mu) \mathrm{d}\mu = \frac{r^{''}}{n^{''}} = \frac{n^{'}\mu_0}{n^{'} + n - 1}$$

Information structure. The broker has private information of whether the state is "a right fit" or "not a right fit" in each round, and can commit to an information structure π,[4] which specifies

whole). However, I refer to them as a series of updated beliefs to not confuse them with the posterior belief (in terms of a specific housing unit) that the renter forms after receiving the information disclosed by the broker in a particular round of search.

[3] In Bayesian probability theory, if the posterior distributions are in the same probability distribution family as the prior distribution, the prior and posterior are then called natural-conjugate distributions, and the prior is called a natural-conjugate prior.

[4] An information structure is also referred to as a signal. The two terms are usually used interchangeably.

a family of state-contingent distributions $\{\pi(\cdot|\theta)\}_{\theta\in\Theta}$ over a finite realization space S. To illustrate, $s \in S$ denotes the signal realization sent to the renter, and $\pi(s|\theta)$ denotes the probability that the realization s is drawn by nature when the state is θ.

Moreover, the broker is assumed to have commitment power. That is, he would truthfully send the signal realization to the renter. This assumption is natural and necessary because usually in reality the signal is observable and the signal realization is verifiable ex post for the renter, therefore it would be costly for the broker to distort the signal realization.

Direct obedient and full-support signal. A signal is direct obedient if the realization space S equals the action space D and if the equilibrium action that the signal produces coincides with the signal realization. Let $\hat{d} \in \{a, r\}$ denote the broker's action recommendation to the renter.[5] The direct obedient rule, which is analogous to the revelation principle (Myerson, 1986), implies that under a direct obedient signal, the renter receives a "recommended action", $\hat{d}$, which she always follows.[6] Moreover, a signal π is full-support if $\pi(\hat{d}|\theta) \in (0, 1)$ for any $\hat{d} \in \{a, r\}$ and for any $\theta \in \Theta$. That is, for any given state, all possible recommendations are sent with non-zero probability.[7]

Equilibrium concept and timeline. The equilibrium concept of the problem is Sender-preferred subgame perfect equilibrium. If the renter is indifferent between accepting and rejecting the housing unit under a specific belief, she would always take the action that the broker prefers. Moreover, the timeline for each round of the search game is as follows.

Time 0: Before the n^{th} round starts, where $n = 1, 2, \ldots$, the renter's prior belief is μ_0 under a constant prior, or $f_{n-1}(\mu)$ under an evolving prior. The broker's prior belief is μ_0.

Time 1: The broker commits to a signal, π, which can be observed by the renter.

Time 2: Nature draws the state, $\theta \in \Theta$.

Time 3: Nature draws the signal realization, $s \in S$, according to the signal, π.

Time 4: The renter chooses an action, $d \in D$, based on the signal, the signal realization that she receives, and her prior belief.

Time 5: The payoffs are realized according to the renter's action, d.

Time 6 (only under an evolving prior): If the game continues to the next round, the renter forms an updated prior, $f_n(\mu)$, after observing the unsuccessful result of the n^{th} round.

[5] Or the broker's recommendation profile to the renter and the landlord, in the case of the matching game.

[6] The proof of the direct obedient rule follows directly from Kamenica & Gentzkow (2011) and Bardhi & Guo (2018) and is therefore omitted.

[7] For the case of the matching game, the full-support definition requires that for any given state profile, all possible recommendation profiles are sent with non-zero probabilities.

3 A search problem

In a search problem with a broker and a renter, the broker's objective in each round of search is to design an optimal information structure which maximizes the probability of the renter accepting the property.

3.1 Constant prior

As a benchmark, the paper first considers the scenario where the renter had a good knowledge of the housing market and is confident in her prior belief, μ_0.

For the broker to benefit from persuasion, it must be the case that the renter does not take the sender's preferred action by default. That is, the renter would not accept the property under her prior belief. The following assumption formalizes this condition.

Assumption 3.1. *There exists some n such that the renter strictly prefers to reject the property under her prior belief in the n^{th} round, i.e.,*

$$\mu_0(1-\sum_{i=1}^{n} c_i) + (1-\mu_0)(-1-\sum_{i=1}^{n} c_i) \leqslant -\sum_{i=1}^{n+1} c_i$$

where the left-hand side is the expected payoff of accepting under the renter's prior belief, with the first term representing the expected payoff when the state is "a right fit" and the second term representing the expected payoff when the state is "not a right fit". The right-hand side is the expected payoff of rejecting, that is, the total time cost for all the previous n rounds of search and the following $(n+1)^{th}$ round after the n^{th} housing unit is rejected. This cost is certain regardless of the state realization. The inequality implies that $\mu_0 \leqslant \frac{1}{2}(1-c_{n+1})$, where $\frac{1}{2}(1-c_{n+1})$ is the critical threshold at which the renter is indifferent between accepting and rejecting the property.

The threshold $\frac{1}{2}(1-c_{n+1})$ decreases in n. Therefore, when n is large enough such that $\mu_0 > \frac{1}{2}(1-c_{n+1})$, Assumption 3.1 no longer holds. This implies that the renter has gone through so many rounds, and her marginal time cost for the next round, c_{n+1}, is so large that she would accept the property in the n^{th} round even without persuasion. Thus, there would be no room for improvement for the broker if Assumption 3.1 does not hold.

3.1.1 The broker's optimization problem

According to the direct obedient rule, let $\hat{d}_a$ be the recommendation for accepting the property and $\hat{d}_r$ be the recommendation for rejecting the property. After observing a recommendation $\hat{d} \in \{\hat{d}_a, \hat{d}_r\}$ from the broker, the renter's posterior belief of the state being "a right fit" is

$$Pr(\theta = f|\hat{d}) = \frac{\mu_0 \pi(\hat{d}|f)}{\mu_0 \pi(\hat{d}|f) + (1-\mu_0)\pi(\hat{d}|nf)} \tag{1}$$

Conditional on receiving a recommendation $\hat{d}$, the renter's expected payoff for accepting the property is

$$Pr(\theta = f|\hat{d}) \cdot (1 - \sum_{i=1}^{n} c_i) + (1 - Pr(\theta = f|\hat{d})) \cdot (-1 - \sum_{i=1}^{n} c_i) \tag{2}$$

where the first term is the expected payoff when the state is "a right fit", and the second term is the expected payoff when the state is "not a right fit", both under the renter's posterior belief. Similarly, the renter's expected payoff for rejecting the property is

$$-\sum_{i=1}^{n+1} c_i \tag{3}$$

which represents the total time cost for the first $n+1$ rounds of search.

Given equations (1)-(3), one can formally define the Incentive Compatibility (IC hereafter) constraints as follows. It is optimal for the renter to obey the recommendation of accepting, $\hat{d}_a$, if

$$\begin{aligned} &\mu_0 \pi(\hat{d}_a|f) \cdot (1 - \sum_{i=1}^{n} c_i) + (1-\mu_0)\pi(\hat{d}_a|nf) \cdot (-1 - \sum_{i=1}^{n} c_i) \geqslant \\ &\qquad [\mu_0 \pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf)] \cdot (-\sum_{i=1}^{n+1} c_i) \end{aligned} \tag{4}$$

Similarly, it is optimal for the renter to obey the recommendation of rejecting, $\hat{d}_r$, if

$$\begin{aligned} &\mu_0 \pi(\hat{d}_r|f) \cdot (1 - \sum_{i=1}^{n} c_i) + (1-\mu_0)\pi(\hat{d}_r|nf) \cdot (-1 - \sum_{i=1}^{n} c_i) \leqslant \\ &\qquad [\mu_0 \pi(\hat{d}_r|f) + (1-\mu_0)\pi(\hat{d}_r|nf)] \cdot (-\sum_{i=1}^{n+1} c_i) \end{aligned} \tag{5}$$

As long as the information structure, π, satisfies the IC constraints (4) and (5), accepting and rejecting obediently produces a subgame perfect equilibrium. Also, recall that the broker's objective is to maximize the probability of the renter accepting the property in each round. Hence, for a given round n, and the set of information structures, Π, the broker's optimization problem is

$$\underset{\pi \in \Pi}{max} \; \mu_0 \pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf)$$
$$s.t.\ IC\ constraints\ (4)\ and\ (5)\ and\ \pi(\hat{d}|\theta) \in [0,1],\ \forall \theta \in \{f, nf\}$$

3.1.2 The optimal information structure

The following proposition establishes the properties of the optimal information structure for the n^{th} round of the search, where $n = 1, 2, \ldots$.

Proposition 3.1. *The optimal solution of the broker's problem, in terms of the probability of sending a recommendation for accepting, is*

$$\pi(\hat{d}_a|f) = 1,\ \pi(\hat{d}_a|nf) = \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0} \in (0,1)$$

Please find the proof of Proposition 3.1 in Appendix A. Proposition 3.1 specifies that when the state is "a right fit," the broker will send a recommendation for accepting with certainty, i.e., $\pi(\hat{d}_a|f) = 1$. However, when the state is "not a right fit," the broker will send a recommendation for accepting neither with certainty nor with zero probability, i.e., $\pi(\hat{d}_a|nf) \in (0,1)$.

One intuition for the results is that, $\pi(\hat{d}_a|nf) \in (0,1)$ implies the optimal information structure is neither fully informative nor completely uninformative. This result obtains because when $\pi(\hat{d}_a|f) = 1$ and $\pi(\hat{d}_a|nf) = 0$, the posterior belief of the state being "a right fit" conditional on receiving a recommendation for accepting is $Pr(\theta = f|\hat{d}_a) = \dfrac{\mu_0 \cdot 1}{\mu_0 \cdot 1 + (1-\mu_0)\cdot 0} = 1$, which leaves no uncertainty about the true state for the renter and is therefore fully informative. But the broker prefers not to do so because he would like to maximize the probability of the renter accepting the housing unit. In contrast, when $\pi(\hat{d}_a|f) = 1$ and $\pi(\hat{d}_a|nf) = 1$, the posterior belief of the state being "a right fit" conditional on receiving a recommendation for accepting is $Pr(\theta = f|\hat{d}_a) = \dfrac{\mu_0 \cdot 1}{\mu_0 \cdot 1 + (1-\mu_0)\cdot 1} = \mu_0$, which is the same as the prior belief of the renter and is therefore completely uninformative. Thus, the broker would not send a recommendation for accepting all the time, because if he does so, the renter would understand the bias and discount recommendation. Hence, the optimal information structure provides an extent of information disclosure between full information disclosure and no information disclosure.

In terms of comparative statics, $\pi(\hat{d}_a|nf)$ increases in μ_0. That is, as the renter becomes more optimistic about the state being "a right fit", it is easier to persuade her into accepting the housing unit. Moreover, $\pi(\hat{d}_a|nf)$ increases in n, since c_{n+1} increases in n and $\pi(\hat{d}_a|nf)$ increases in c_{n+1}. That is, as the number of rounds increases, the marginal time cost for the renter also increases, so it is easier to persuade the renter into accepting.

3.1.3 Benefit from persuasion

Before persuasion, the probability of the renter accepting the property is precisely the prior belief of the state being "a right fit," μ_0. However, after persuasion, the maximal probability of the renter accepting the property is,

$$\mu_0\pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf) = \frac{2}{1-c_{n+1}} \cdot \mu_0 > \mu_0$$

The next proposition formalizes this observation.

Proposition 3.2. *Under the optimal information structure, the broker's benefit from persuasion is strictly positive. The improvement in the probability of the renter accepting the property is given by* $\dfrac{1+c_{n+1}}{1-c_{n+1}} \cdot \mu_0$.

3.2 Evolving prior

Another potential, if not more probable, real-life scenario is that the renter does not have a deep understanding of the housing market and is not confident in her initial prior distribution, $f_0(\mu)$. Therefore, after $n-1$ rounds of unsuccessful search, she would update her prior belief to $f_{n-1}(\mu)$, which serves as her updated prior distribution for the n^{th} around of the search, where $n = 1, 2, \ldots$.

The following assumption guarantees that the broker benefits from persuasion.

Assumption 3.2. *There exists some n such that the renter strictly prefers to reject the property under her prior belief in the n^{th} round, i.e.,*

$$\int_0^1 [\mu(1-\sum_{i=1}^{n} c_i)] \cdot f_{n-1}(\mu)\mathrm{d}\mu + \int_0^1 [(1-\mu)(-1-\sum_{i=1}^{n} c_i)] \cdot f_{n-1}(\mu)\mathrm{d}\mu \leqslant \int_0^1 (-\sum_{i=1}^{n+1} c_i) \cdot f_{n-1}(\mu)\mathrm{d}\mu$$

where the left-hand side is the expected payoff of accepting under the renter's prior belief, with the first term representing the expected payoff when the state is "a right fit" and the second term representing the expected payoff when the state is "not a right fit". The right-hand side is the expected payoff of rejecting, that is, the total time cost for the first $n+1$ rounds of the search. The cost is certain regardless of the state realization. The inequality implies that $\mu_0 \leqslant \frac{1-c_{n+1}}{2} \cdot \frac{n^{'}+n-1}{n^{'}}$, where $\frac{1-c_{n+1}}{2} \cdot \frac{n^{'}+n-1}{n^{'}}$ is the critical threshold at which the renter is indifferent between accepting and rejecting the property.

Unlike under a constant prior, the change in the critical threshold as n increases is ambiguous under an evolving prior. On the one hand, as the number of rounds, n, increases, the marginal time cost, c_{n+1}, also increases, and therefore lowers the threshold and makes the renter more easily persuaded into accepting. On the other hand, as n increases, the fraction $\frac{n^{'}+n-1}{n^{'}}$ also increases, and therefore increases the threshold and makes the renter harder to persuade. This is because in the fraction, the denominator is the size of the equivalent sample of the prior information, while the numerator is the sum of the sizes of prior information and sample information, which increases in n. An increase in the size of sample information implies that the renter places more weight on the unsuccessful results of the previous rounds, and therefore becomes more pessimistic and harder to persuade. Hence an increase in n can either increase or decrease the critical threshold. In what follows, the analysis only considers the set of n that satisfies Assumption 3.2.

3.2.1 The broker's optimization problem

Let $\hat{d}_a$ and $\hat{d}_r$ be the recommendations for accepting and rejecting the property, respectively. After observing a recommendation $\hat{d} \in \{\hat{d}_a, \hat{d}_r\}$, the renter's posterior of the state being "a right fit" is

$$\begin{aligned} Pr(\theta = f|\hat{d}) &= \frac{\int_0^1 (\mu\pi(\hat{d}|f)) \cdot f_{n-1}(\mu)\mathrm{d}\mu}{\int_0^1 [\mu\pi(\hat{d}|f) + (1-\mu)\pi(\hat{d}|nf)] \cdot f_{n-1}(\mu)\mathrm{d}\mu} \\ &= \frac{\mathbb{E}_{n-1}(\mu) \cdot \pi(\hat{d}|f)}{\mathbb{E}_{n-1}(\mu) \cdot \pi(\hat{d}|f) + (1-\mathbb{E}_{n-1}(\mu)) \cdot \pi(\hat{d}|nf)} \\ &= \frac{n^{'}\mu_0\pi(\hat{d}|f)}{n^{'}\mu_0\pi(\hat{d}|f) + (n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}|nf)} \end{aligned} \tag{6}$$

Given equations (2), (3) and (6), one can formally define the IC constraints as follows. It is optimal for the renter to obey the recommendation of accepting, $\hat{d}_a$, if

$$n^{'}\mu_0\pi(\hat{d}_a|f)\cdot(1-\sum_{i=1}^{n}c_i)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_a|nf)\cdot(-1-\sum_{i=1}^{n}c_i)\geqslant$$
$$[n^{'}\mu_0\pi(\hat{d}_a|f)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_a|nf)]\cdot(-\sum_{i=1}^{n+1}c_i) \tag{7}$$

where the left-hand side is the expected payoff of accepting the housing unit, and the right-hand side is the expected payoff of rejecting the housing unit.

Similarly, it is optimal for the renter to obey the recommendation of rejecting, $\hat{d}_r$, if

$$n^{'}\mu_0\pi(\hat{d}_r|f)\cdot(1-\sum_{i=1}^{n}c_i)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_r|nf)\cdot(-1-\sum_{i=1}^{n}c_i)\leqslant$$
$$[n^{'}\mu_0\pi(\hat{d}_r|f)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_r|nf)]\cdot(-\sum_{i=1}^{n+1}c_i) \tag{8}$$

For a given n, and the set of information structures, Π, the broker's optimization problem is

$$\underset{\pi\in\Pi}{max}\ \ \mu_0\pi(\hat{d}_a|f)+(1-\mu_0)\pi(\hat{d}_a|nf)$$
$$s.t.\ IC\ constraints\ (7)\ and\ (8)\ and\ \pi(\hat{d}|\theta)\in[0,1],\ \forall\theta\in\{f,nf\}$$

3.2.2 The optimal information structure

The following results establish the properties of the optimal information structure for the n^{th} round of the search, where $n = 1, 2, \ldots$.

Proposition 3.3. *The optimal solution of the broker's problem, in terms of the probability of sending a recommendation for accepting, is*

$$\pi(\hat{d}_a|f)=1,\ \ \pi(\hat{d}_a|nf)=\frac{1+c_{n+1}}{1-c_{n+1}}\cdot\frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n^{'}}}\in(0,1)$$

Please find the proof of Proposition 3.3 in Appendix A. As a counterpart to Proposition 3.1, Proposition 3.3 specifies that, when the state is "a right fit", the broker will send a recommendation for accepting with certainty, i.e., $\pi(\hat{d}_a|f)=1$, and when the state is "not a right fit", the broker will send a recommendation for accepting neither with certainty nor with zero probability, i.e., $\pi(\hat{d}_a|nf)\in(0,1)$. One intuition for the results is that, the optimal information structure provides an extent of information disclosure between full information disclosure and no information disclosure, as illustrated in Section 3.1.

In terms of comparative statics, $\pi(\hat{d}_a|nf)$ increases in μ_0, which is similar to the conclusion in Proposition 3.1. However, it is ambiguous how $\pi(\hat{d}_a|nf)$ would change as n increases because there are two competing effects. On the one hand, $\pi(\hat{d}_a|nf)$ increases in c_{n+1}, which increases in n. This implies that the marginal cost increases over time, which forces the renter to become less picky. Therefore, the renter has a lower expectation threshold for the housing unit to be a good fit, and thereby becomes easily persuaded into accepting. On the other hand, however, as n increases, the fraction, $\frac{n-1}{n'}$, also increases, and therefore lowers $\pi(\hat{d}_a|nf)$. Note that the fraction is, in fact, the ratio of the size of sample information to the size of the equivalent sample of the prior information. This implies that as the sample size increases (i.e., the number of previous unsuccessful rounds increases), the renter becomes more pessimistic and difficult to persuade. Hence an increase in n can either increase or decrease the $\pi(\hat{d}_a|nf)$.

3.2.3 Benefit from persuasion

Before persuasion, the probability of the renter accepting the property is precisely the prior belief of the state being "a right fit," $\mathbb{E}_0(\mu) = \mu_0$. However, after persuasion, the maximal probability of the renter accepting the property is

$$\mu_0\pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf) = \mu_0 + \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0(1-\mu_0)}{1-\mu_0+\frac{n-1}{n'}}$$

The next proposition formalizes this observation.

Proposition 3.4. *Under the optimal information structure, the broker's benefit from persuasion is strictly positive. The improvement in the probability of the renter accepting the property is* $\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0(1-\mu_0)}{1-\mu_0+\frac{n-1}{n'}}$.

3.2.4 Comparison between constant and evolving priors

A comparison can be made between a constant prior and an evolving prior in terms of optimal information structure. When the state is "a right fit", the probability of sending a recommendation for accepting, $\pi(\hat{d}_a|f) = 1$, is the same under both types of priors. However, when the state is "not a right fit", the proposition below establishes a comparison between the two types of priors.

Proposition 3.5. *The probability of sending a recommendation for accepting when the state is "not a right fit", is weakly lower under an evolving prior than under a constant prior, i.e.,*

$$\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}} \leqslant \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}$$

Equality holds if and only if $n = 1$.

The inequality suggests that, in the evolving prior case, as n increases, the ratio of the sample information (including only previous unsuccessful attempts of the renter) to the prior information (including both successful and unsuccessful cases in the market), $\dfrac{n-1}{n'}$, also increases. In essence, the previous rejections provide information that housing is bad/ flawed, thereby reducing the odds of the state being "a right fit." This makes the renter more disheartened and pessimistic then before, and therefore more challenging to persuade. In order to persuade such disheartened renter, the sender needs to provide more precise information about what the true state is by designing a more informative information structure with a lower probability of sending a recommendation for accepting when the state is "not a right fit".

Moreover, the benefit from persuasion under an evolving prior is also weakly lower than its counterpart under a constant prior, i.e.,

$$\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0(1-\mu_0)}{1-\mu_0+\dfrac{n-1}{n'}} \leqslant \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \mu_0$$

Equality holds if and only if $n = 1$. This conclusion directly results from the weakly lower probability of sending a recommendation for accepting under an evolving prior. The intuition is that the renter with evolving prior is less confident in her prior information and is therefore more easily disheartened by the negative results of the previous rounds of the search. Such renter is harder to persuade and requires more information disclosure, which lowers the sender's benefit from persuasion.

4 A matching problem

In a matching problem, the model is extended to a game with a sender (i.e., a broker, he) and two receivers (i.e., a renter and a landlord, she). Suppose that the broker has two pools of clients, one with many renters and another with many landlords, and that he matches one renter with one landlord at a time. Therefore, the matching problem is equivalent to forming a coalition of two receivers, in which a deal can be tied up only when both receivers accept the matching.

Also, it is natural to assume that the landlord would not rent her property to just any person who could afford the rent. Instead, she would like to match with a renter who is a right fit for her. For example, she would prefer a renter who is a tidy person, does not have a pet, and can rent the property for at least a year. Suppose that the landlord has the same utility function and prior belief as those of the renter. The only heterogeneity between the two receivers is that, when a renter and a landlord are matched with each other, the renter is in her n^{th} round of the matching, while the landlord is in her m^{th} round of the matching, where $n, m = 1, 2, \ldots$ and n does not necessarily coincide with m. Therefore, they may have different time costs (and updated priors in the evolving prior case.)

Moreover, suppose that the binary state of the renter, $\theta_r \in \{f, nf\}$, and the binary state of the landlord, $\theta_l \in \{f, nf\}$, are perfectly correlated with each other. That is, either $\theta_r = \theta_l = f$, which implies that the two receivers are a good match for each other, or $\theta_r = \theta_l = nf$, which implies that the two receivers are not a good match for each other.

Finally, the broker's objective in each round of the matching is to design an optimal information structure which maximizes the probability of the renter and the landlord reaching a deal in each round. The timeline for each round of the matching game is as follows.

Time 0: Before the n^{th} round for the renter and the m^{th} round for the landlord start, where $n, m = 1, 2, \ldots$, the receivers' prior beliefs are μ_0 under a constant prior, or $f_{n-1}(\mu)$ and $f_{m-1}(\mu)$ under an evolving prior. The broker's prior belief is μ_0;

Time 1: The broker commits to a signal, π, which can be observed by the receivers;

Time 2: Nature draws the state profile, (θ_r, θ_l), where $\theta_r, \theta_l \in \Theta$;

Time 3: Nature draws the signal realization profile, (s_r, s_l), where $s_r, s_l \in S$, according to the signal, π;

Time 4: The receivers respectively choose an action, $d \in D$, based on the signal, the signal realization profile that they receives, and their prior beliefs.

Time 5: The payoffs are realized according to the receivers' action profile.

Time 6 (only under an evolving prior): If the renter and the landlord each continue to the next round, they will each form an updated prior, $f_n(\mu)$ and $f_m(\mu)$, respectively, after observing the unsuccessful result of this round.

4.1 Constant prior

As in Section 3, the paper first analyzes the case of constant prior, μ_0. To ensure that the broker benefits from persuasion, it must be the case that at least one of the receivers would not accept the matching under her prior belief. This condition is formalized in the following assumption.

Assumption 4.1. *There exists some n and m such that at least one of the receivers strictly prefers to reject the matching under her prior belief, i.e.,*

$$\mu_0(1-\sum_{i=1}^{n} c_i) + (1-\mu_0)(-1-\sum_{i=1}^{n} c_i) \leqslant -\sum_{i=1}^{n+1} c_i \quad \textit{or}$$
$$\mu_0(1-\sum_{i=1}^{m} c_i) + (1-\mu_0)(-1-\sum_{i=1}^{m} c_i) \leqslant -\sum_{i=1}^{m+1} c_i$$

where for both inequalities, the left-hand side is the expected payoff of accepting under the receiver's prior belief, while the right-hand side is expected payoff of rejecting (the time cost for the first $n+1$ or $m+1$ rounds of the matching). The inequality implies that $\mu_0 \leqslant \max\{\frac{1}{2}(1-c_{n+1}), \frac{1}{2}(1-c_{m+1})\}$, where $\max\{\frac{1}{2}(1-c_{n+1}), \frac{1}{2}(1-c_{m+1})\}$ is the critical threshold at which either the renter or the landlord is indifferent between accepting and rejecting the matching.

The threshold depends only on the critical receiver who has gone through a smaller number of rounds of matching, $\min\{n, m\}$.[8] Because a smaller number of rounds means a smaller marginal time cost, which allows the receiver to be pickier, and therefore critical in the persuasion. Moreover, the threshold, $\max\{\frac{1}{2}(1-c_{n+1}), \frac{1}{2}(1-c_{m+1})\}$, decreases in $\min\{n, m\}$. Therefore, when $\min\{n, m\}$ is large enough such that $\mu_0 > \max\{\frac{1}{2}(1-c_{n+1}), \frac{1}{2}(1-c_{m+1})\}$, Assumption 4.1 no longer holds. That is, the critical receiver has gone through so many rounds that she would accept the matching even without persuasion. Then there would be no room for improvement for the broker.

For simplicity of exposition, the broker's optimization problem is omitted, and can be found in online appendix B.1.

4.1.1 The optimal information structure

The following proposition establishes the properties of the optimal information structure for the matching between the renter (who is in her n^{th} round of matching), and the landlord (who is in her

[8] The heterogeneity between the receivers lies in the difference of the round number. However, if $n = m$, then the game would degenerate to a matching problem with homogeneous receivers. Then the conclusions would not vary a lot from those in the search problem.

m^{th} round of matching), where $n, m = 1, 2, \ldots$.

Proposition 4.1. *The optimal solution of the broker's problem, in terms of the probability of sending a recommendation for accepting, is*

$$\pi(\hat{d}_a|f) = 1,\ \ \pi(\hat{d}_a|nf) = \min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0}\} \in (0,1)$$

Please find the proof of Proposition 4.1 in Appendix A. Proposition 4.1 specifies that when the state is "a good match," the broker will send a recommendation for accepting with certainty, i.e., $\pi(\hat{d}_a|f) = 1$. However, when the state is "not a good match," the broker will send a recommendation for accepting neither with certainty nor with zero probability, i.e., $0 < \pi(\hat{d}_a|nf) < 1$. That is, the optimal information structure is between full information disclosure and no information disclosure. Moreover, it is evident that the information structure $\pi(\hat{d}_a|nf)$ depends only on the critical receiver who is more challenging to persuade, that is, who has gone through a smaller number of rounds, $\min\{n, m\}$ and has a smaller marginal time cost, $\min\{c_{n+1}, c_{m+1}\}$.

In terms of comparative statics, $\pi(\hat{d}_a|nf)$ increases in μ_0 and $\min\{n, m\}$. That is, as the receivers become more optimistic about the state being "a good match," or as the critical receiver goes through a large number of rounds, it is easier to persuade the receivers into reaching a deal.

4.1.2 Benefit from persuasion

Before persuasion, the probability of the receivers accepting the matching is precisely the prior belief of the state being "a good match," μ_0. However, after persuasion, the maximal probability of the receivers accepting the matching is

$$\mu_0\pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf) = \min\{\frac{2}{1-c_{n+1}} \cdot \mu_0\ ,\ \frac{2}{1-c_{m+1}} \cdot \mu_0\} > \mu_0$$

The next proposition formalizes this observation.

Proposition 4.2. *Under the optimal information structure, the broker's benefit from persuasion is strictly positive. The improvement in the probability of the renter accepting the property is* $\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \mu_0\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \mu_0\}$*, which depends only on the critical receiver who has gone through a smaller number of rounds of matching,* $\min\{n, m\}$.

4.2 Evolving priors

Under the assumption of updated priors, the renter's updated prior distribution for her n^{th} rounds of the matching is $f_{n-1}(\mu)$, and the landlord's updated prior distribution for her m^{th} rounds of the matching is $f_{m-1}(\mu)$, where $n, m = 1, 2, \ldots$.

The following assumption guarantees that the broker benefits from persuasion.

Assumption 4.2. *There exists some n and m such that at least one of the receivers strictly prefers to reject the matching under her prior belief, i.e.,*

$$\int_0^1 [\mu(1-\sum_{i=1}^{n} c_i)] f_{n-1}(\mu)\mathrm{d}\mu + \int_0^1 [(1-\mu)(-1-\sum_{i=1}^{n} c_i)] f_{n-1}(\mu)\mathrm{d}\mu \leqslant \int_0^1 (-\sum_{i=1}^{n+1} c_i) f_{n-1}(\mu)\mathrm{d}\mu \text{ or}$$
$$\int_0^1 [\mu(1-\sum_{i=1}^{m} c_i)] f_{m-1}(\mu)\mathrm{d}\mu + \int_0^1 [(1-\mu)(-1-\sum_{i=1}^{m} c_i)] f_{m-1}(\mu)\mathrm{d}\mu \leqslant \int_0^1 (-\sum_{i=1}^{m+1} c_i) f_{m-1}(\mu)\mathrm{d}\mu$$

where for both inequalities, the left-hand side is the expected payoff of accepting under the receiver's updated belief, while the right-hand side is expected payoff of rejecting. The inequality implies that $\mu_0 \leqslant \min\{\frac{1-c_{n+1}}{2} \cdot \frac{n' + n - 1}{n'}, \frac{1-c_{m+1}}{2} \cdot \frac{m' + m - 1}{m'}\}$, where $\min\{\frac{1-c_{n+1}}{2} \cdot \frac{n' + n - 1}{n'}, \frac{1-c_{m+1}}{2} \cdot \frac{m' + m - 1}{m'}\}$ is the critical threshold at which either the renter or the landlord is indifferent between accepting and rejecting the matching.

Similar to the conclusions in Section 3.2, the change in the critical threshold as n or m increases is ambiguous under an evolving prior. Hence it is ambiguous whether the critical receiver who determines the value of the critical threshold would be the receiver who has gone through a larger number of rounds, $\max\{n, m\}$, or the receiver who has gone through a smaller number of rounds, $\min\{n, m\}$.

For simplicity of exposition, the broker's optimization problem is omitted, and can be found in online appendix B.2.

4.2.1 The optimal information structure

The following proposition establishes the properties of the optimal information structure for the matching between the renter (who is in her n^{th} round of matching), and the landlord (who is in her m^{th} round of matching), where $n, m = 1, 2, \ldots$.

Proposition 4.3. *The optimal solution of the broker's problem, in terms of the probability of sending* *a recommendation for accepting, is*

$$\pi(\hat{d}_a|f) = 1, \quad \pi(\hat{d}_a|nf) = \min\{\frac{1+c_{n+1}}{1-c_{n+1}}\frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}}\frac{\mu_0}{1-\mu_0+\dfrac{m-1}{m'}}\} \in (0,1)$$

Please find the proof of Proposition 4.3 in Appendix A. As the counterpart to Proposition 4.1, Proposition 4.3 specifies that when the state is "a good match," the broker will send a recommendation for accepting with certainty, i.e., $\pi(\hat{d}_a|f) = 1$. However, when the state is "not a good match," the broker will send a recommendation for accepting neither with certainty nor with zero probability, i.e., $0 < \pi(\hat{d}_a|nf) < 1$. That is, the optimal information structure is between full information disclosure and no information disclosure.

In terms of comparative statics, $\pi(\hat{d}_a|nf)$ increases in μ_0, which is similar to the conclusion in Proposition 4.1. However, it is ambiguous whether $\pi(\hat{d}_a|nf)$ is determined by the receiver who has gone through a larger or a smaller number of rounds of matching. This result obtains because an increase in n or m can either increase or decrease $\pi(\hat{d}_a|nf)$, as demonstrated in Proposition 3.3.

4.2.2 Benefit from persuasion

Before persuasion, the probability of the receivers accepting the matching is precisely the prior belief of the state being "a good match," $\mathbb{E}_0(\mu) = \mu_0$. However, after persuasion, the maximal probability of the receivers accepting the matching is

$$\begin{aligned}
&\mu_0\pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf) = \\
&\qquad \min\{\mu_0 + \frac{1+c_{n+1}}{1-c_{n+1}}\cdot\frac{\mu_0(1-\mu_0)}{1-\mu_0+\dfrac{n-1}{n'}}\ ,\ \mu_0 + \frac{1+c_{m+1}}{1-c_{m+1}}\cdot\frac{\mu_0(1-\mu_0)}{1-\mu_0+\dfrac{m-1}{m'}}\}
\end{aligned}$$

The next proposition formalizes this observation.

Proposition 4.4. *Under the optimal information structure, the broker's benefit from persuasion is strictly positive. The improvement in the probability of the renter accepting the property is*

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}}\cdot\frac{\mu_0}{1+\dfrac{n-1}{n'}\cdot\dfrac{1}{1-\mu_0}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}}\cdot\frac{\mu_0}{1+\dfrac{m-1}{m'}\cdot\dfrac{1}{1-\mu_0}}\}$$

4.2.3 Comparison between constant and evolving priors

A comparison can be made between a constant prior and an evolving prior in terms of optimal information structure. When the state is “a right fit,” the probability of sending a recommendation for accepting, $\pi(\hat{d}_a|f) = 1$, is the same under both types of priors. However, when the state is “not a right fit,” the proposition below establishes a comparison between the two types of priors.

Proposition 4.5. *The probability of sending a recommendation for accepting when the state is “not a right fit,” is weakly lower under an evolving prior than under a constant prior, i.e.,*

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{n-1}{n'}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{m-1}{m'}}\} \leqslant \min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0}\}$$

Equality holds if and only if $x = 1$ *and* $\frac{1+c_{x+1}}{1-c_{x+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{x-1}{x'}} \leqslant \frac{1+c_{y+1}}{1-c_{y+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{y-1}{y'}}$, *where* $x = \min\{n, m\}$ *and* $y = \max\{n, m\}$.

Please find the proof of Proposition 4.5 in Appendix A. The inequality suggests that, as n and m increase, the ratios of the sample information (including only previous unsuccessful attempts of the receiver) to the prior information (including both successful and unsuccessful cases in the market) of the two receivers, $\frac{n-1}{n'}$ and $\frac{m-1}{m'}$, also increase. In essence, the previous rejections provide information that housing is bad/ flawed, and therefore reduce the odds of the state being “a right fit.” This makes the receivers more disheartened, and therefore harder to persuade. In order to persuade such disheartened renter, the sender needs to design a more informative information structure with a lower probability of sending a recommendation for accepting when the state is “not a right fit”.

Moreover, the benefit from persuasion under an evolving prior is also weakly lower than its counterpart under a constant prior, i.e.,

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1+\frac{n-1}{n'} \cdot \frac{1}{1-\mu_0}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1+\frac{m-1}{m'} \cdot \frac{1}{1-\mu_0}}\} \leqslant \min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \mu_0\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \mu_0\}$$

Equality holds if and only if $x = 1$ and $\frac{1+c_{x+1}}{1-c_{x+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{x-1}{x'}} \leqslant \frac{1+c_{y+1}}{1-c_{y+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{y-1}{y'}}$, where $x = \min\{n, m\}$ and $y = \max\{n, m\}$. This conclusion directly results from the weakly lower probability of sending a recommendation for accepting under an evolving prior. The intuition is that the renter with evolving prior is less confident in her prior information and is therefore more easily disheartened by the negative results of the previous rounds of the search. Such renter is harder to persuade and requires more information disclosure, which lowers the sender's benefit from persuasion.

5 Extension: a discouraged renter

The analysis thus far was restricted to the case in which the demand for renting a housing unit is rigid. In this section, the paper relaxes this assumption and considers the case of a discouraged renter. That is, the renter may give up searching because she does not necessarily suffer from a large disutility from quitting. She may still have other options, such as purchasing a property of her own or living with relatives or friends. Formally, the renter's utility in the n^{th} round of the search is as follows:

$$u(a,\theta) = \begin{cases} 1-\sum_{i=1}^{n} c_i & \text{if d = a and } \theta = \text{f} \\ -1-\sum_{i=1}^{n} c_i & \text{if d = a and } \theta = \text{n} \\ -\sum_{i=1}^{n+1} c_i & \text{if d = r and keep on searching} \\ -\overline{U}-\sum_{i=1}^{n} c_i & \text{if d = r and give up searching} \end{cases}$$

where the disutility from giving up searching, $\overline{U} \in (0,1)$, is relatively small. In particular, there exists an integer k such that $c_k < \overline{U} < c_{k+1}$. This implies that the renter would search for k rounds at most. After k unsuccessful rounds of search, she would become a discouraged renter and leave the housing market.

The broker's objective remains to be maximizing the probability of the renter accepting the property in each round. The conclusions for the previous $k-1$ rounds remain the same. In what follows, the analysis focuses on the optimal information structure and the benefit from persuasion for the k^{th} round of the search under a constant prior.[9] To ensure that the broker benefits from persuasion, the following assumption is needed.

[9] The conclusions could also be easily generalized to the scenario of evolving prior and the matching game.

Assumption 5.1. *The renter strictly prefers to reject the property under her prior belief in the k^{th} round, i.e.,*

$$\mu_0(1-\sum_{i=1}^{k}c_i)+(1-\mu_0)(-1-\sum_{i=1}^{k}c_i)\leqslant -\overline{U}-\sum_{i=1}^{k}c_i$$

where the left-hand side is the expected payoff of accepting under the renter's prior belief, and the right-hand side is the expected payoff of rejecting, that is, the total time cost for the first $k+1$ rounds of the search. The inequality implies that $\mu_0 \leqslant \frac{1}{2}(1-\overline{U})$, where $\frac{1}{2}(1-\overline{U})$ is the critical threshold at which the renter is indifferent between accepting and rejecting the property.

The threshold $\frac{1}{2}(1-\overline{U})$ decreases in $\overline{U}$. Therefore, when $\overline{U}$ is relatively large enough such that $\mu_0 > \frac{1}{2}(1-\overline{U})$, Assumption 5.1 no longer holds. That is, the disutility from giving up searching is relatively large, thereby making the renter lower her selection standard, and accept the housing unit of the k^{th} even without persuasion. Thus, there would be no room for improvement for the broker.

5.1 The broker's optimization problem

In the k^{th} round, the IC constraints are as follows. It is optimal for the renter to obey the recommendation of accepting, $\hat{d}_a$, if

$$\mu_0\pi(\hat{d}_a|f)\cdot(1-\sum_{i=1}^{k}c_i)+(1-\mu_0)\pi(\hat{d}_a|nf)\cdot(-1-\sum_{i=1}^{k}c_i)\geqslant$$
$$[\mu_0\pi(\hat{d}_a|f)+(1-\mu_0)\pi(\hat{d}_a|nf)]\cdot(-\overline{U}-\sum_{i=1}^{k}c_i) \tag{9}$$

where the left-hand side is the expected payoff of accepting the housing unit, and the right-hand side is the expected payoff of rejecting the housing unit.

It is optimal for the renter to obey the recommendation of rejecting, $\hat{d}_r$, if

$$\mu_0\pi(\hat{d}_r|f)\cdot(1-\sum_{i=1}^{k}c_i)+(1-\mu_0)\pi(\hat{d}_r|nf)\cdot(-1-\sum_{i=1}^{k}c_i)\leqslant$$
$$[\mu_0\pi(\hat{d}_r|f)+(1-\mu_0)\pi(\hat{d}_r|nf)]\cdot(-\overline{U}-\sum_{i=1}^{k}c_i) \tag{10}$$

Therefore, for a given round, k, and the set of information structures, Π, the broker's optimization problem is

$$\underset{\pi \in \Pi}{max}\ \mu_0 \pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf)$$
$$s.t.\ IC\ constraints\ (9)\ and\ (10)\ and\ \pi(\hat{d}|\theta) \in [0,1],\ \forall \theta \in \{f, nf\}$$

5.2 The optimal information structure

The following proposition establishes the properties of the optimal information structure for the k^{th} round of search.

Proposition 5.1. *The optimal solution of the broker's problem, in terms of the probability of sending a recommendation for accepting, is*

$$\pi(\hat{d}_a|f) = 1,\ \pi(\hat{d}_a|nf) = \frac{1+\overline{U}}{1-\overline{U}} \cdot \frac{\mu_0}{1-\mu_0} \in (0,1)$$

Please find the proof of Proposition 5.1 in Appendix A. The implications are similar to those of Proposition 3.1. The broker sends a recommendation for accepting with certainty for the state which is "a right fit," and sends a recommendation for accepting, neither with certainty nor with zero probability, for the state which is "not a right fit". Besides, $\pi(\hat{d}_a|nf)$ increases in both μ_0 and $\overline{U}$. As the renter becomes more optimistic, or as the disutility from quitting searching, $\overline{U}$, increases and thereby lowering the renter's selection standard, the renter becomes more easily persuaded.

Also, note that in the k^{th} round, the broker treats the two types of renters differently. In particular, the probability of sending a recommendation for accepting when the state is "not a right fit" is lower for a discouraged renter than for a renter with a rigid demand for renting a house, i.e.,

$$\frac{1+\overline{U}}{1-\overline{U}} \cdot \frac{\mu_0}{1-\mu_0} < \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}$$

The intuition is that, since the renter with a rigid demand for renting a house has a higher motivation to accept a housing unit, the broker would take advantage of her by sending an ambiguous and less informative recommendation. While for the the discouraged renter who has a less rigid demand and a lower motivation to accept a property, the broker is less able to manipulate her and needs to provide more informative recommendation about the true state in order to persuade her into accepting. That is, the broker has a smaller probability of sending a recommendation of accepting when the state is actually "not a right fit".

Moreover, in the case where whether the renter is a discouraged type is private information and not known to the broker. Since the renter prefers to receive a more informative recommendation about the true state of the property, she would have the incentive to pretend to be the discouraged type and signal "willing to quit" even it is not true.

5.3 Benefit from persuasion

Before persuasion, the probability of the renter accepting the property is precisely the prior belief, μ_0. However, after persuasion, the maximal probability of the renter accepting the property is

$$\mu_0 \pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf) = \frac{2}{1-\overline{U}} \cdot \mu_0 > \mu_0$$

The next proposition formalizes this observation.

Proposition 5.2. *Under the optimal information structure, the broker's benefit from persuasion is strictly positive. The improvement in the probability of the renter accepting the property is* $\frac{1+\overline{U}}{1-\overline{U}} \cdot \mu_0$.

6 Concluding remarks

In this paper, the author studies the persuasion problems with multiple rounds in a search game and a matching game in the housing market context. The research characterizes the optimal information structure that maximizes the real estate broker's utility. The results indicate that, the optimal information structure provides an extent of information disclosure that is between full disclosure and no disclosure. In particular, when the state is "the property being a right fit", the broker will send a recommendation for accepting with certainty; when the state is "the property not being a right fit", the broker will send a recommendation for accepting neither with certainty nor with zero probability. Moreover, under the assumption that the receiver does not take the sender's preferred action by default, the broker's benefit from persuasion is strictly positive under the optimal information structure.

The paper compares the conclusions under a constant prior and those under an evolving prior. First, the optimal information structure is more informative under an evolving prior than under a constant prior. In particular, he probability of sending a recommendation for accepting when the state is "a right fit," $\pi(\hat{d}_a|f)$, is the same under the two types of prior belief. However, the probability of sending a recommendation for accepting when the state is "not a right fit," $\pi(\hat{d}_a|nf)$, is weakly lower under an evolving prior than under a constant prior. This is because, under an evolving prior, the receiver incorporates the previous unsuccessful search/ matching experiences into her prior, and therefore has a lower prior belief of the state being "a right fit". Such receiver becomes more pessimistic and harder to persuade and needs a more informative recommendation about the true state from the sender in order to achieve a successful search/ matching. Second, the benefit from persuasion under an evolving prior is also weakly lower than its counterpart under a constant prior, which directly results from the difference in the optimal information structure.

Moreover, there are some conclusions for the matching problem. The design of the optimal information structure depends only on a critical receiver who is more challenging to persuade than the other receiver. Under a constant prior, the critical receiver is the receiver who has gone through fewer rounds of the matching and has a smaller marginal time cost, which allows her to be pickier. Under an evolving prior, however, it is ambiguous whether the critical receiver is the receiver who has gone through more rounds or fewer rounds of matching.

This matching-making problem is not restricted to the housing market context. In particular, a characterizing feature of this matching model is that there are multiple rounds of matching, and that the heterogeneity between the two receivers in each matching lies in the number of previous rounds that they have gone through, which determines their time costs as well as their prior beliefs of the true state (in the evolving prior case). Therefore, this model contributes to understanding the persuasion problem in settings where there are multiple rounds of matching between two pools of receivers. Such settings apply to real-life scenarios. For example, a matchmaker sets one pair of singles up for a blind date at a time, and needs to decide the optimal extent of information disclosure to achieve a successful match. The settings also arise naturally when a director of graduate studies tries to match selective graduate students with picky professors as RAs/ TAs and would like to maximize the chance of a successful match.

Several important issues remain open to future research, such as multiple information senders (e.g., competing real estate brokers,) heterogeneous states (i.e., independent or imperfectly correlated state realizations,) and costly signals. The author hopes to incorporate these essential aspects in future research.

Appendix A Omitted proofs

Proof of Proposition 3.1. First, want to show that $\pi(\hat{d}_a|f) = 1$. For a contradiction, suppose the optimal information structure $\pi(\hat{d}_a|f) \in [0,1)$. However, the broker could always raise $\pi(\hat{d}_a|f)$ to strictly increase the broker's benefit from persuasion without violating the IC constraint (4). Thus, $\pi(\hat{d}_a|f) \in [0,1)$ cannot be the optimal solution. Contradiction!

Second, want to show that $\pi(\hat{d}_a|nf) = \dfrac{1+c_{n+1}}{1-c_{n+1}} \cdot \dfrac{\mu_0}{1-\mu_0} \in (0,1)$. The broker would like to increase $\pi(\hat{d}_a|nf)$ as much as possible, and to satisfy the IC constraint (4) at the same time. Thus, the broker will set $\pi(\hat{d}_a|nf)$ such that the IC constraint (4) is binding. Hence $\pi(\hat{d}_a|nf) = \dfrac{1+c_{n+1}}{1-c_{n+1}} \cdot \dfrac{\mu_0}{1-\mu_0} > 0$, since $c_{n+1} \in (0,1)$ and $\mu_0 \in (0, 0.5)$. Also, Assumption 3.1 specifies that $\mu_0 \leqslant \dfrac{1}{2}(1-c_{n+1})$. Plugging that inequality into the expression of $\pi(\hat{d}_a|nf)$, one arrives at $\pi(\hat{d}_a|nf) < 1$.

Note that I only solves the relaxed problem in which the IC constraint (5) is omitted without loss of generality. Because once I have solved the probabilities of sending a recommendation for accepting under the IC constraint (4), I can always conveniently specify the probabilities of sending a recommendation for rejecting such that the IC constraint (5) holds. To illustrate, the broker could set $\pi(\hat{d}_r|f) = 0$ and $\pi(\hat{d}_r|nf) = 1 - \pi(\hat{d}_a|nf)$, which always meets the IC constraint (5). □

Proof of Proposition 3.3. The proof proceeds exactly along the same lines as those of Proposition 3.1 and is therefore omitted. □

Proof of Proposition 4.1. First, want to show that $\pi(\hat{d}_a|f) = 1$. For a contradiction, suppose the optimal information structure is $\pi(\hat{d}_a|f) \in [0,1)$. However, the broker could always raise $\pi(\hat{d}_a|f)$ to strictly increase the broker's benefit without violating the IC constraints (11) and (13). Thus, $\pi(\hat{d}_a|f) \in [0,1)$ cannot be the optimal solution. Contradiction!

Second, want to show that $\pi(\hat{d}_a|nf) = \min\{\dfrac{1+c_{n+1}}{1-c_{n+1}} \cdot \dfrac{\mu_0}{1-\mu_0}, \dfrac{1+c_{m+1}}{1-c_{m+1}} \cdot \dfrac{\mu_0}{1-\mu_0}\} \in (0,1)$. The broker would like to increase $\pi(\hat{d}_a|nf)$ as much as possible, and to satisfy the IC constraints (11) and (13) at the same time. Thus, the broker will set $\pi(\hat{d}_a|nf)$ such that the more stringent one of IC constraints (11) and (13) is binding. Hence $\pi(\hat{d}_a|nf) = \min\{\dfrac{1+c_{n+1}}{1-c_{n+1}} \cdot \dfrac{\mu_0}{1-\mu_0}, \dfrac{1+c_{m+1}}{1-c_{m+1}} \cdot \dfrac{\mu_0}{1-\mu_0}\} > 0$, since $c_{n+1} \in (0,1)$, $c_{m+1} \in (0,1)$ and $\mu_0 \in (0, 0.5)$. Also, Assumption 4.1 specifies that $\mu_0 \leqslant \max\{\dfrac{1}{2}(1-c_{n+1}), \dfrac{1}{2}(1-c_{m+1})\}$. Plugging that inequality into the expression of $\pi(\hat{d}_a|nf)$, one arrives at $\pi(\hat{d}_a|nf) < 1$.

Note that I only solves the relaxed problem in which the IC constraints (12) and (14) are omitted without loss of generality. Because once I have solved the probabilities of sending a recommendation for accepting under the IC constraints (11) and (13), I can always conveniently specify the probabilities of sending a recommendation for rejecting such that the IC constraints (12) and (14) hold. To illustrate, the broker could always set $\pi(\hat{d}_{r,i}|f) = 0$ and $\pi(\hat{d}_{r,i}|nf) = 1 - \pi(\hat{d}_a|nf)$, where $i = \{r, l\}$, which meet the IC constraints (12) and (14). $\square$

Proof of Proposition 4.3. The proof proceeds exactly along the same lines as those of Proposition 4.1 and is therefore omitted. $\square$

Proof of Proposition 4.5. W.l.o.g, suppose $n \leqslant m$, then

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0}\} = \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}$$

Note that

$$\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}} \leqslant \frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}$$

Equality holds if and only if $\min\{n, m\} = n = 1$.
If $\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{n-1}{n'}} \leqslant \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{m-1}{m'}}$, then

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \frac{\mu_0}{1-\mu_0+\dfrac{m-1}{m'}}\} = \frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}} \leqslant \frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0} = \min\{\frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \frac{\mu_0}{1-\mu_0}\}$$

Equality holds if and only if $\min\{n, m\} = n = 1$.
If $\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{n-1}{n'}} > \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0+\frac{m-1}{m'}}$, then

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}}, \frac{1+c_{m+1}}{1-c_{m+1}} \frac{\mu_0}{1-\mu_0+\dfrac{m-1}{m'}}\} = \frac{1+c_{m+1}}{1-c_{m+1}} \frac{\mu_0}{1-\mu_0+\dfrac{m-1}{m'}} < \frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}} \leqslant \frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0} = \min\{\frac{1+c_{n+1}}{1-c_{n+1}} \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \frac{\mu_0}{1-\mu_0}\}$$

Let $x = \min\{n, m\}$ and $y = \max\{n, m\}$, then

$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0+\dfrac{n-1}{n'}}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0+\dfrac{m-1}{m'}}\} \leqslant$$
$$\min\{\frac{1+c_{n+1}}{1-c_{n+1}} \cdot \frac{\mu_0}{1-\mu_0}\ ,\ \frac{1+c_{m+1}}{1-c_{m+1}} \cdot \frac{\mu_0}{1-\mu_0}\}$$

Equality holds if and only if $x = 1$ and $\dfrac{1+c_{x+1}}{1-c_{x+1}} \cdot \dfrac{\mu_0}{1-\mu_0+\dfrac{x-1}{x'}} \leqslant \dfrac{1+c_{y+1}}{1-c_{y+1}} \cdot \dfrac{\mu_0}{1-\mu_0+\dfrac{y-1}{y'}}$. □

Proof of Proposition 5.1. The proof proceeds exactly along the same lines as those of Proposition 3.1 and is therefore omitted. □

Appendix B The broker's optimization problem

B.1 The broker's optimization problem under a constant prior in a matching problem

Let $\hat{d}_a$ be the recommendation profile under which both receivers receive a recommendation for accepting the matching. Let $\hat{d}_{r,r}$ be the recommendation profile under which the renter receives a recommendation for rejecting, while the landlord receives a recommendation for accepting. Finally, let $\hat{d}_{r,l}$ be the recommendation profile under which the landlord receives a recommendation for rejecting, while the renter receives a recommendation for accepting.

The IC constraints are specified as follows. For the renter, it is optimal to obey the recommendation for accepting, $\hat{d}_a$, if

$$\mu_0\pi(\hat{d}_a|f) \cdot (1 - \sum_{i=1}^{n} c_i) + (1-\mu_0)\pi(\hat{d}_a|nf) \cdot (-1 - \sum_{i=1}^{n} c_i) \geqslant$$
$$[\mu_0\pi(\hat{d}_a|f) + (1-\mu_0)\pi(\hat{d}_a|nf)] \cdot (-\sum_{i=1}^{n+1} c_i) \qquad (11)$$

It is optimal for the renter to obey the recommendation for rejecting, $\hat{d}_{r,r}$, if

$$\mu_0\pi(\hat{d}_{r,r}|f)\cdot(1-\sum_{i=1}^{n}c_i)+(1-\mu_0)\pi(\hat{d}_{r,r}|nf)\cdot(-1-\sum_{i=1}^{n}c_i)\leqslant$$
$$[\mu_0\pi(\hat{d}_{r,r}|f)+(1-\mu_0)\pi(\hat{d}_{r,r}|nf)]\cdot(-\sum_{i=1}^{n+1}c_i) \tag{12}$$

It is optimal for the landlord to obey the recommendation for accepting, $\hat{d}_a$, if

$$\mu_0\pi(\hat{d}_a|f)\cdot(1-\sum_{i=1}^{m}c_i)+(1-\mu_0)\pi(\hat{d}_a|nf)\cdot(-1-\sum_{i=1}^{m}c_i)\geqslant$$
$$[\mu_0\pi(\hat{d}_a|f)+(1-\mu_0)\pi(\hat{d}_a|nf)]\cdot(-\sum_{i=1}^{m+1}c_i) \tag{13}$$

It is optimal for the landlord to obey the recommendation for rejecting, $\hat{d}_{r,l}$, if

$$\mu_0\pi(\hat{d}_{r,l}|f)\cdot(1-\sum_{i=1}^{m}c_i)+(1-\mu_0)\pi(\hat{d}_{r,l}|nf)\cdot(-1-\sum_{i=1}^{m}c_i)\leqslant$$
$$[\mu_0\pi(\hat{d}_{r,l}|f)+(1-\mu_0)\pi(\hat{d}_{r,l}|nf)]\cdot(-\sum_{i=1}^{m+1}c_i) \tag{14}$$

Therefore, for given rounds, n and m, and the set of information structures, Π, the broker's optimization problem is

$$\underset{\pi\in\Pi}{max}\ \mu_0\pi(\hat{d}_a|f)+(1-\mu_0)\pi(\hat{d}_a|nf)$$
$$s.t.\ IC\ constraints\ (11)-(14)\ and\ \pi(\hat{d}|\theta_i)\in[0,1],\ \forall\theta_i\in\{f,nf\}, where\ i\in\{r,l\}$$

B.2 The broker's optimization problem under an evolving prior in a matching game

The IC constraints are specified as follows. For the renter, it is optimal to obey the recommendation for accepting, $\hat{d}_a$, if

$$n'\mu_0\pi(\hat{d}_a|f)\cdot(1-\sum_{i=1}^{n}c_i)+(n'+n-1-n'\mu_0)\pi(\hat{d}_a|nf)\cdot(-1-\sum_{i=1}^{n}c_i)\geqslant$$
$$[n'\mu_0\pi(\hat{d}_a|f)+(n'+n-1-n'\mu_0)\pi(\hat{d}_a|nf)]\cdot(-\sum_{i=1}^{n+1}c_i) \tag{15}$$

It is optimal for the renter to obey the recommendation for rejecting, $\hat{d}_{r,r}$, if

$$n^{'}\mu_0\pi(\hat{d}_r|f)\cdot(1-\sum_{i=1}^{n}c_i)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_r|nf)\cdot(-1-\sum_{i=1}^{n}c_i)\leqslant$$
$$[n^{'}\mu_0\pi(\hat{d}_r|f)+(n^{'}+n-1-n^{'}\mu_0)\pi(\hat{d}_r|nf)]\cdot(-\sum_{i=1}^{n+1}c_i) \quad (16)$$

It is optimal for the landlord to obey the recommendation for accepting , $\hat{d}_a$, if

$$m^{'}\mu_0\pi(\hat{d}_a|f)\cdot(1-\sum_{i=1}^{m}c_i)+(m^{'}+m-1-m^{'}\mu_0)\pi(\hat{d}_a|nf)\cdot(-1-\sum_{i=1}^{m}c_i)\geqslant$$
$$[m^{'}\mu_0\pi(\hat{d}_a|f)+(m^{'}+m-1-m^{'}\mu_0)\pi(\hat{d}_a|nf)]\cdot(-\sum_{i=1}^{m+1}c_i) \quad (17)$$

It is optimal for the landlord to obey the recommendation for rejecting, $\hat{d}_{r,l}$, if

$$m^{'}\mu_0\pi(\hat{d}_r|f)\cdot(1-\sum_{i=1}^{m}c_i)+(m^{'}+m-1-m^{'}\mu_0)\pi(\hat{d}_r|nf)\cdot(-1-\sum_{i=1}^{m}c_i)\leqslant$$
$$[m^{'}\mu_0\pi(\hat{d}_r|f)+(m^{'}+m-1-m^{'}\mu_0)\pi(\hat{d}_r|nf)]\cdot(-\sum_{i=1}^{m+1}c_i) \quad (18)$$

Therefore, for given rounds, n and m, and the set of information structures, Π, the broker's optimization problem is

$$\underset{\pi\in\Pi}{max}\ \mu_0\pi(\hat{d}_a|f)+(1-\mu_0)\pi(\hat{d}_a|nf)$$
$$s.t.\ IC\ constraints\ (15)-(18)\ and\ \pi(\hat{d}|\theta_i)\in[0,1],\ \forall\theta_i\in\{f,nf\}, where\ i\in\{r,l\}$$

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