The Hidden Price of Altruism: the Sense of Sacrifice

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Abstract

People in close relationships exchange more than money, such as trivial but timely assistance or a subjectively valued favor that is difficult to quantify. Typically, one party pays more than the other and appears willing to do so. However, this seemingly stable relationship can abruptly break down but gradually recover, owing partly to the hidden price of altruism: the accumulation of a sense of sacrifice. It can be a double-edged sword, encouraging people to put more effort into altruistic behaviors to gain more recognition and reputation while also instilling a sense of unfairness due to the relatively higher contribution to the relationship. In this paper, we construct a model to explain the persistence of this altruistic behavior and suggest a way to prevent one party from being obsessed with giving. Furthermore, we propose an interaction model to explain both the infrequent and frequent communication cases. Our theory also implies that frequent communication allows the unilateral unstable state to converge to a steady state and can be used to explain the coexistence of compensatory behaviors and addictions.

1 Introduction

Interpersonal relationships, including intimate, familial, and other types, fulfill a range of physical and emotional needs. Since it is non-monetary, people are usually involved in implicit reciprocal favor exchange (Burger et al., 2006). Sometimes the exchange occurs sporadically, but sometimes frequently and continuously, especially in a close relationship.

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Specifically, parents who expect their children to have a bright future tend to sacrifice their own time and money to make an exchange. In this situation, although parents are seemingly willing to devote anything, it is natural to believe that they may have a sense of sacrifice. This phenomenon is also common in a romantic relationship where one person sacrifices more to maintain their relationship than the other. Unlike some simple reciprocal deals based on fair value where people clear accounts on time, people in close relationships appear more generous and do not argue about the details of each exchange as much. What is the mechanism of giving and repayment in the absence of a receivable account? How can people continue to engage in altruistic behaviors voluntarily? This paper wants to explore what favor exchange is, what motivates people to voluntarily sacrifice themselves, and how addictions and compensatory behaviors can coexist. This paper models these behaviors by introducing "sacrifice capital," "guilt capital," and depreciation factors.

To unravel the mechanism of giving and repayment, we need to first understand the psychological motives and outcomes of sacrifice. Not only the giver, the person who receives the sacrifice (i.e., the recipient) is also an essential factor in this relationship. According to the norm of reciprocity, people who receive an unexpected favor feel obligated to return that favor (Burger et al., 2006). Then the recipients will not only have their feeling but also affect the givers' satisfaction. Specifically, Visserman et al. (2017) showed that gratitude would not emerge unless people think that their partner sacrificed for partner-focused approach motives, presumably because these types of motives suggest that the partner had authentic altruistic intentions. If not, the recipients think that their giver's sacrifice comes from self-interest, and they did not feel grateful. Predictably, the indifferent attitude will hurt the givers. On the one hand, not all sacrifices are inherently joyful to provide. Sometimes, people experience negative emotions such as irritation, resentment, or anger when they sacrifice for their partner (Righetti and Impett, 2017). On the other hand, individualism and commitment are mutually required in a modern world. Then sacrificing oneself also has an opportunity cost, that is, personal development. Nonetheless, even if the givers are aware of the detrimental effect of the sense of sacrifice, they persist as if they are addicted to it. This phenomenon is partly due to their firm intention to maintain their relationship (Etcheverry and Le, 2005), and their relative power in the relationship (Righetti et al., 2015).

Given the possible motives and outcomes of sacrifice, now it is natural to ask the psychological scheme for those sacrifices. Past researches indicate that information and self-control are two important factors. Firstly, the timeliness of information transmission is one of the key factors of avoiding possible unbalanced contributions and potential malformed relationships. In Adlerian Individual Psychology, happiness can come from a feeling of contribution to the community (Kishimi and Koga, 2019). Kishimi and Koga (2018) also indicate that this feeling of contribution does not have to be visible or quantifiable, but the subjective sense that "I am of use to someone." Notably, despite its subjectiveness, this feeling should be recognized instead of self-deception. Therefore, no matter in the formation of community or recognizing contribution, communication plays a key role. During the communication, both sides will make clear the boundary and the separation of tasks (Kishimi and Koga, 2018). Secondly, self-control ability is also of great importance. Sociologists find that people with low self-control tend to engage in small sacrifices in an established long-term relationship (Clark and Jordan, 2002), while higher self-control is needed when the sacrifices are immense and costly (Findley et al., 2014).

> *Consider the following scenario: Maggie is Ben's mother. Maggie stayed at home as a full-time parent since Ben's birth until he entered junior high school. She assisted Ben with his homework and decided who he should befriend. Ben was moved by Maggie's sacrifice and worked hard in school. She returned to her job as a reporter after Ben graduated from junior high school. Since then, she has spent the majority of her time researching news and interviewing, and has spent less time focusing on Ben's friendship. They only talked on the phone after Ben graduated from high school and moved into a dorm. Because of the infrequency with which they communicated, they misunderstood each other. Maggie did not understand Ben's subtle emotional change and implicit repayment (e.g., squeezing some time every day to share interesting stories with his mother to make her happy despite his busy schoolwork). Ben was also oblivious to his mother's care and exhaustion after work. It's a common occurrence in our daily lives, but few papers explain the mechanism of giving and repayment in a close relationship.*

To characterize the frequency of information transmission, we use both a unilateral dynamic model and an interaction model. In terms of the self-control factor, economists have developed a number of theoretical models. Using the non-exponential discount function, Strotz (1956)'s model attributes people's deviation from their original optimal plan to the variation of preferences as people re-evaluate it from a different date. This theory implies the weakness of human beings; that is, there is a conflict between people's long-term interests and short-term desires. To complete the model, Thaler and Shefrin (1981) incorporate the conflicts into the two-self conceptualization. However, self-control theory believes that precommitment or long-termism can eliminate inconsistent behaviors, which cannot answer why sacrifice always happens to people with strong motivation and self-discipline. Becker and Murphy (1988) develop a framework to rationalize the addictive consumption behaviors. They found that binges are not the result of different personalities for control under the powerful adjacent complementarity assumption but the outcome of consistent maximization over time. This theory is the excellent basis for explaining the *sacrifice addiction*. Our analysis builds on the model of Becker and Murphy (1988)'s rational addiction theory and also brings in Thaler and Shefrin (1981)'s two-self concept.

The paper organizes as follows: in the second part, we will talk about the baseline model and use it to explain the sacrifice in close relationship and some other real-world cases; in the third part, we will extend to the interaction model and discuss different conditions as well as real-world applications; in the fourth part we will give a summary of the main conclusions, and the last part is the appendices where we include some derivation and detailed simulation.

The contribution of this paper is that it uses an economic approach to model the sense of sacrifice in the interpersonal relationship and further extends to the interaction between the giver and the recipient, which considers the communication between the two parties and can be helpful in a real-world application. This paper also helps explain some seemingly sudden emotional breakdowns in the behavior of generous people and the coexistence of compensatory behaviors and addictions.

2 The Baseline Model

2.1 The Setup

In the baseline model, the utility of an individual at any moment depends on the energy allocation between two goods, altruistic virtue (v) and personal development/"me-time" (m). The utility of good m only depends on current consumption, while the utility of good v not only depends on its current consumption but also depends on its past consumption S. S is denoted as the "sense of sacrifice" and affects current utility. Then we define the current utility at each period t as

$$u(t) = u[v(t), m(t), S(t)] \tag{1}$$

First, we assume that every time people engage in altruistic behaviors, it will leave a sense of sacrifice in the inner, and the sense will fade as time passes by. So the accumulation process (the law of motion) is defined as

$$\dot{S}(t) = v(t) - \delta S \tag{2}$$

Second, each individual only has limited energy (E) to be allocated to either altruistic virtues (v) or personal development (m). Therefore, balancing between the two goods is the critical decision for the individual. As usual, the decision for personal development correlates with one's long-term ambition(A). Each individual should devote enough energy to achieve their ambition in their lifetime (i.e., from 0 to T). The budget constraints are as follows (Equation (3)). Since individuals differ in development efficiency, we introduce the item φ to characterize this ability (Equation (4)).

$$v(t) + m(t) \leq E \tag{3}$$

$$\int_0^T \varphi m(t) dt \geq A \tag{4}$$

Now we can define our maximization problem:

$$max \int_0^T e^{-\sigma t} u[v(t), m(t), S(t)] dt \tag{5}$$

where 0 means the present time for the agent to make a decision and T is the end of planning horizon, σ is a constant to measure the agent's time preference which can also be interpreted as "subjective discount rate".
To solve this model, we can set up the current value Hamiltonian function:

$$H = u(v(t), m(t), S(t)] + \mu_1[E - v(t) - m(t)] + \mu_2[\varphi m(t) - A] + \lambda(t)[v(t) - \delta S(t)] \tag{6}$$

In this function, v, m are control variables, S is the state variable, $\lambda(t)$ is the co-state variable, and μ is the Lagrange multiplier. Denote $V(A, E, S_0, \varphi)$ as the value of the optimal solution for the utility. $\mu_1 = \frac{\partial V}{\partial E}$ is the shadow price of energy value, and $\mu_2 = -\frac{\partial V}{\partial A}$ is the shadow price of energy spent on self-development, where this paper assumes both to be constant. Constant μ_1 μ_2 means that the value of energy and ambition are constant given any two periods.

By solving Hamiltonian function, we can derive the following conditions:

$$\frac{\partial H}{\partial v} = u_v(t) + \lambda(t) - \mu_1 = 0 \tag{7}$$

$$\frac{\partial H}{\partial m} = u_m(t) - \mu_1 + \mu_2\varphi = 0 \tag{8}$$

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial S} = -[u_s(t) - \delta\lambda(t)] \qquad (Euler\ equation) \tag{9}$$

$$S(0) = S_0 \tag{10}$$

$$\lim_{t\to\infty} e^{-\sigma t}[S(t)]^2 = 0 \qquad (Transversality\ condition) \tag{11}$$

Then we can get

$$\lambda(t) = \int_t^T e^{-(\sigma+\delta)(\tau-t)} u_s(\tau)d\tau \tag{12}$$

$$u_v(t) = \mu_1 - \lambda(t) \tag{13}$$

This equation means that at the optimum, marginal benefit of devotion equals the marginal cost of devotion, including the shadow price of one unit of energy value less of one unit of sacrifice accumulation.

$$u_m(t) = \mu_1 - \mu_2\varphi \tag{14}$$

This equation indicates that the optimal point equalizes the marginal cost and the marginal utility brought by one unit of self-development m. The LHS is the utility gain brought by one unit of self-time contribution. The first item on the RHS stands for one unit of the energy loss, and the second item means the value gain by approaching ambition.

2.2 Dynamics

In order to see how the altruistic behavior correlates with sacrifice sense stock, this paper follows Becker and Murphy (1988) and makes the following additional assumptions:
(A1) $T = \infty$, such that the lifetime is infinite;

(A2) instantaneous utility function u_t is in quadratic forms of v, m, S.

$$U = \beta_v v + \beta_s S + \beta_m m + \frac{\beta_{vv}}{2} v^2 + \frac{\beta_{ss}}{2} S^2 + \frac{\beta_{mm}}{2} m^2 + \beta_{vs} vS + \beta_{vm} vm + \beta_{sm} Sm \quad (15)$$

where the interaction between m and v, or between m and S are zero (i.e. $\beta_{vm} = \beta_{sm} = 0$); the signs of marginal utility are as follows: $u_v > 0, u_m > 0, u_s < 0, u_{vv} < 0, u_{mm} < 0, u_{ss} < 0$;

(A3) By an adjacent complementary assumption that is similar to Becker and Murphy (1988), we assumes $u_{vs} > 0$, which means that the accumulation of sacrifice sense can make altruistic behaviors more appealing. (Giver would expect more future gain.)

By Equation (14) and (15), we can get the optimized allocation of self-time m,

$$m(v, S) = \frac{\mu_1 - \mu_2\varphi - \beta_m - \beta_{vm} v - \beta_{sm} S}{\beta_{mm}} = \frac{\mu_1 - \mu_2\varphi - \beta_m}{\beta_{mm}} \quad (16)$$

Then, we can get the value function:

$$V^* = k + \max_{v,S} \int_0^T [e^{-\sigma t} F(v, S)] dt \quad (17)$$

where k is a constant that is defined by $E, A, \mu_1, \mu_2, \sigma$, and F is in the following form,

$$F = \beta_v v + \beta_s S + \frac{\beta_{vv}}{2} v^2 + \frac{\beta_{ss}}{2} S^2 + \beta_{vs} vS - \mu_1 v \quad (18)$$

The maximization occurs subject to Equation (2) and Equation (11).

By substituting $v = \dot{S} + \delta S$ (derived from Equation (2)) into $e^{-\sigma t} F(v, S)$ and defining $I(S, \dot{S}, S^2, \dot{S}^2, \dot{S}S) \equiv e^{-\sigma t} F(v, S)$, we can know that

$$\begin{aligned} I &= I(S, \dot{S}, S^2, \dot{S}^2, \dot{S}S) \equiv e^{-\sigma t} F(v, S) \\ &= e^{-\sigma t}[\beta_v \delta + \beta_s - \mu_1 \delta) S + (\beta_v - \mu_1)\dot{S} + (\frac{\beta_{vv}}{2}\delta^2 + \frac{\beta_{ss}}{2} + \beta_{vs}\delta) S^2 + \frac{\beta_{vv}}{2}\dot{S}^2 + (\beta_{vv}\delta + \beta_{vs})\dot{S}S \\ &+ \frac{(\mu_1 - \mu_2\varphi)^2 - \beta_m^2}{2\beta_{mm}})] \end{aligned}$$

By the property of Euler equation, we can get

$$\ddot{S} - \sigma\dot{S} - JS = K \quad (19)$$

where

$$J = \delta(\delta+\sigma) + \frac{\beta_{vs}}{\beta_{vv}}(2\delta+\sigma) + \frac{\beta_{ss}}{\beta_{vv}}, \tag{20}$$

$$K = \frac{\beta_v(\delta+\sigma) + \beta_s - (\delta+\sigma)\mu_1}{\beta_{vv}} \tag{21}$$

There are two characteristic roots for this Euler equation: $\theta_1 = \frac{\sigma+\sqrt{\sigma^2+4J}}{2}, \theta_2 = \frac{\sigma-\sqrt{\sigma^2+4J}}{2}$. Since $\sigma^2 + 4J = [(\sigma+2\delta) - 2\sqrt{\frac{\beta_{ss}}{\beta_{vv}}}]^2 + 4(\sigma+2\delta)(\sqrt{\frac{\beta_{ss}}{\beta_{vv}}} + \frac{\beta_{vs}}{\beta_{vv}})$, the root exists only if $\sqrt{\frac{\beta_{ss}}{\beta_{vv}}} \geq -\frac{\beta_{vs}}{\beta_{vv}}$. Due to the transversality condition, there is only one feasible root, that is,

$$\theta_2 = \frac{\sigma - \sqrt{\sigma^2+4J}}{2} \tag{22}$$

Then, from previous results and Equation (10) we can get the optimal path of the capital stock:

$$S(t) = (S_0 - S^*)e^{\theta_2 t} + S^*, \tag{23}$$

with $\theta_2 = \frac{\sigma-\sqrt{\sigma^2+4J}}{2}$ and S^* is a steady state. Thus, the optimal path for altruistic behaviors is

$$v(t) = (\theta_2+\delta)S - \theta S^*, or\ v(t) = \theta_2(S - S^*) + \delta S. \tag{24}$$

The sign of the correlation between the altruistic virtue (v) and the sense of sacrifice stock (S) depends on the sign of $(\theta_2 + \delta)$. It is the rate at which the system will return to equilibrium and maintain a steady state. Intuitively, we can regard it as an honor-motive multiplier of the sacrifice stock. It describes how much more time and effort individuals are willing to pay in response to each additional unit of sacrificing stock to earn a sense of honor of contributing to a better future for their community (i.e., a giver and his/her recipient). By substituting the expression of θ_2, this is equivalent to the sign of $[(2\delta+\sigma)\beta_{vs} + \beta_{ss}]$. If $(2\delta+\sigma)\beta_{vs} > -\beta_{ss}$, then v is positively correlated with S on the optimal path. [1] As for the movement direction of v on the optimal path, it depends on the sign of the root θ_2 according to Equation (24) and (2), which is equivalent to the sign of $-J$.[2] Specifically, if $J > 0$ or $\theta_2 < 0$, the v will decrease as $S > S^*$ and increase as $S < S^*$.

Under this assumption ($\theta_2 + \delta > 0$ and $\theta_2 < 0$), we characterize the relationship in the

[1] Since we assume that $\beta_{vs} > 0$ by adjacent complementarity, all the three conditions can happen; if $\beta_{vs} < 0$, the relationship must be negative.

[2] By combining the two equations, we will have $\dot{S} = \theta_2(S - S^*)$.

Figure 1. As is shown in this figure, the altruistic virtue (v) is positively correlated with the sense of sacrifice stock (S). Since $\theta_2 < 0$, a person with higher than steady state v^* will decrease his/her altruistic activities. Intuitively, as the individual accumulates the sense of sacrifice too rapidly ($v > v^*$) the rate at which his/her memory of the individual's sense of sacrifice fades (δS) cannot keep up. This leads to the overall negative feeling, so the individual will less participate in altruistic activities. On the contrary, if the individual starts with a low engagement in altruistic virtue ($v < v^*$), the adjacent complementary effect still dominates the direct negative effect of sense of sacrifice accumulation. In this situation, the individual has incentive to increase their altruistic activities. In both cases, the individual will finally reach (v^*, S^*). At the optimal steady state where $\ddot{S} = \dot{S} = 0$, we have $S^* = -\frac{\beta_v(\delta+\sigma)+\beta_s-(\delta+\sigma)\mu_1}{J\beta_{vv}}$, and $v^* = -\delta\frac{\beta_v(\delta+\sigma)+\beta_s-(\delta+\sigma)\mu_1}{J\beta_{vv}}$.[3] These numerical results generally match with Becker and Murphy (1988)'s results. Then we will discuss additional findings in the next part.

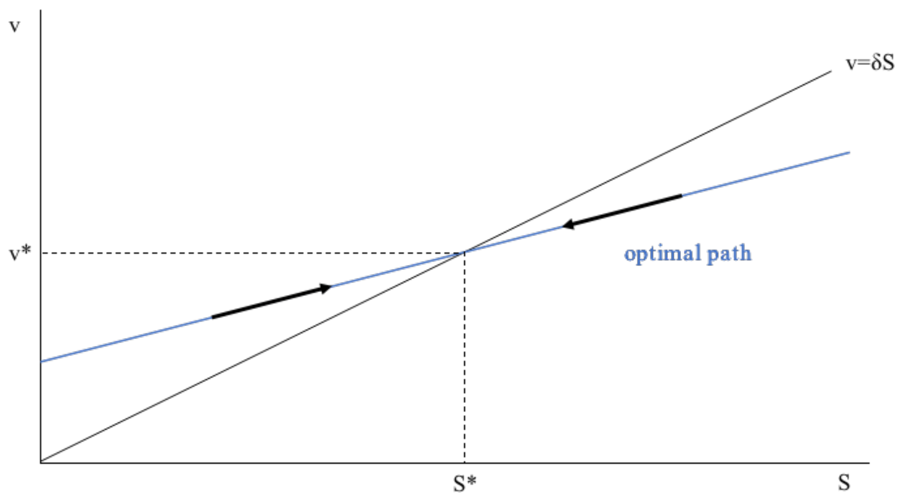


Figure 1: Positive relationship with converging trend, as $\beta_{vs} \in (-\frac{\beta_{ss}}{2\delta+\sigma}, \frac{-\delta(\delta+\sigma)\beta_{vv}-\beta_{ss}}{2\delta+\sigma})$

[3]Other different situations are summarized in the Appendix (Table A).

2.3 Discussion

2.3.1 The Effect of Efficiency Change

In our model, although the accumulation of the sense of sacrifice has a direct negative effect on the individual's utility, it also has an indirect positive effect that it can increase the marginal utility of engaging in altruistic behaviors. In other words, the sense of sacrifice can encourage receivers to repay the givers in order to feel a relief. Thus, this targeted group of people will be willing to devote enough time to altruistic behaviors if they can achieve their ambition. Here we want to talk about the effect of ability: how people change their energy allocation between altruistic virtue and self-development as they have a positive efficiency shock at the optimum. Here we define the *altruistic intensity*, which is the ratio of v^* over m^*, to measure the relative proportion of energy allocation on two different activities.

$$\textit{altruistic intensity } (AI) = \frac{v^*}{m^*} = \frac{-\delta \frac{\beta_v(\delta+\sigma)+\beta_s-(\delta+\sigma)\mu_1}{J\beta_{vv}}}{\frac{\mu_1-\mu_2\varphi-\beta_m}{\beta_{mm}}} \tag{25}$$

From the above equation, we can derive

$$\frac{\partial AI}{\partial \varphi} = \mu_2 \beta_{mm} v^* (\mu_1 - \mu_2\varphi - \beta_m)^{-2} < 0 \tag{26}$$

Additionally,

$$\frac{\partial^2 AI}{\partial \varphi^2} = 2\mu_2^2 \beta_{mm} v^* (\mu_1 - \mu_2\varphi - \beta_m)^{-3} > 0 \tag{27}$$

The inequality signs follow that $v^* > 0, m^* > 0$. Therefore, as people's self-investment efficiency increases, they will spend less time on altruistic behaviors at an increasing rate.

> *As with the previous story, Maggie finds her sense of achievement after returning to her work, thus reducing the time spent on Ben's movement.*

This conclusion is applicable in our daily life that highly productive people usually have deeper exploration in their fields instead of meddling others' business. What needs attention is that people's efficiency is not always increasing; thus, people changing their relative energy allocation as efficiency varies. Intuitively, we have a core result here that needs further empirical tests: economically prosperous people will behave less altruistically than others. This is also a sophisticated behavioral economics justification of Communist ideology. As Marx predicted, successful capitalists ignore externalities and the social good, leaving workers and peasants to struggle to survive at a subsistence level while the bourgeoisie and rapacious landlords thrive.

This conclusion is also apposite when relating to the close relationship between parents and children. On the one hand, this model offers a possible answer for explaining the existence of stay-at-home parents. Except for individual preferences, the non-stay-at-home one in this couple might have higher self-development efficiency, which decreasing his/her utility for a family commitment. This matches with Becker (1981)'s theory of labor division in multi-adult household that the one with comparative advantage at a specific market will specialize in that market. On the other hand, this brings us an approach to prevent parents from excessive concern for their children. Specifically, we could divert their attention to their own life and help them gain efficiency in their development. For instance, college for the aged is a good choice for the retired parents. When many countries face the aging population, providing new motives and paths of self-development for the elderly requires further attention.

2.3.2 The Disillusionment: Permanent Change in Marginal Utility

As is discussed previously, at the optimum we have $v^* = -\delta\frac{\beta_v(\delta+\sigma)+\beta_s-(\delta+\sigma)\mu_1}{J\beta_{vv}}$. If there is a sudden shock that reverses the marginal utility such that $\beta_v < 0$, then the optimal steady-state will turn to be negative (i.e., $v^{**} < 0$ and $S^{**} < 0$).[4] Recall that $J = \delta(\delta+\sigma) + \frac{\beta_{vs}}{\beta_{vv}}(2\delta+\sigma) + \frac{\beta_{ss}}{\beta_{vv}}$, so J will be greater than zero and thus $\theta_2 < 0$. This equation means that the individual will abruptly abort his/her altruistic behaviors, which he/she previously regards as a virtue ($v^* \to v^c$) but then gradually reach a relative less negative optimal level (v^{**}, S^{**}) as is shown in Figure 2. It can explain the sudden breakdown of some seemingly stable relationship, say divorce. In the social media time, an interpersonal relationship can also extend to some virtual version. In a wider scope, fan economy is a good example. Usually, fans selflessly buy merchandise to support their idols, such as albums, concert tickets, and posters. Also, some of them pay much energy to some invisible products like boosting the position in various ranking lists. This kind of behavior can be regarded

[4]In close relationships, out of simple faith or expectation on the other's likely bright future that can somehow benefit himself/herself (e.g., blood relation) one party is willing to sacrifice himself/herself to contribute to the other's development. However, if this expectation is not met due to events involving the receiver, the utility of voluntary devotions will be diminished and even turn negative.

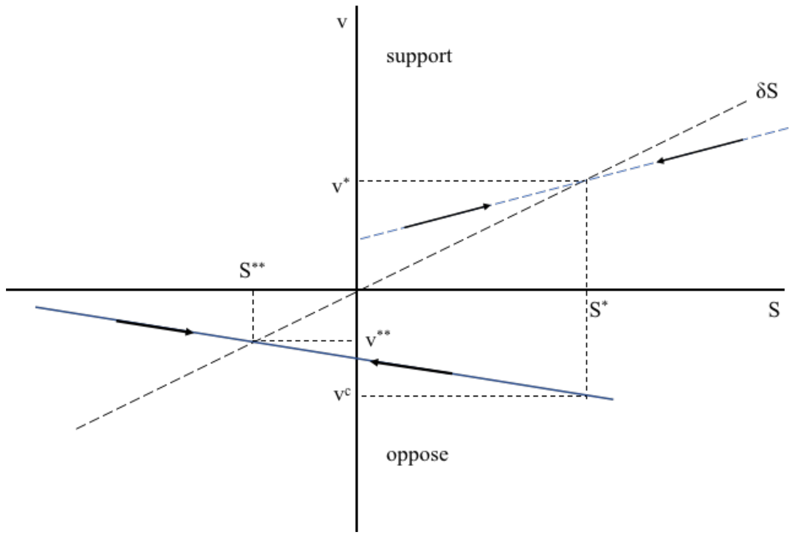


Figure 2: The Disillusionment

as the altruistic virtue in our model, by doing which people gain utility from expectations and a sense of achievement as if they were investing in some promising projects. However, if something extremely negative happens that can reverse the idolaters' recognition of the worship, they will give up supporting their idols and even become extreme anti-fans. As time passes by, their fierce opposition will gradually subside.

3 The Interaction Model

3.1 The Setup and Dynamics

After discussing the unilateral model for altruistic behaviors, we introduce the interaction model in this part. The interaction model considers both the active giver and the recipient. Now we firstly define the utility maximization problem for the recipients (abbreviated as RP).

(RP)

$$max \int_0^T e^{-\sigma t} u[r(t), m(t), G(t)] \tag{28}$$

$$s.t. \quad \dot{G}(t) = v(t) - \gamma G(t) - \rho r(t) \tag{29}$$

$$r(t) + m^{RP}(t) \leq E \tag{30}$$

$$\int_0^T \varphi m^{RP}(t)dt \geq A \tag{31}$$

where recipients choose how much to repay (r) and self-time devotion (m^{RP}). The recipient also accumulates guilt stock (G), resulting from exogenous altruistic virtue (v) given by the giver, fades as time passes by, and can be decreased by repayment. Here, the depreciation rate (γ) can be regarded as how quickly people's memory of guilt fades, and it is also heterogeneous among people depending on their personal characteristics (e.g. age). For example, young children may have bigger γ, that is, they are easy to forget the guilt.

The giver's maximization problem (abbreviated as GP) is:
(GP)

$$max \int_0^T e^{-\sigma t} u[v(t), m(t), S(t)] \tag{32}$$

$$s.t.\ \ \dot{S}(t) = v(t) - \delta S(t) - \eta r(t) \tag{33}$$

$$v(t) + m^{GP}(t) \leq E \tag{34}$$

$$\int_0^T \varphi m^{GP}(t)dt \geq A \tag{35}$$

where the recipient's repayment choice (r) is exogenous in this GP. Givers choose how much self-time (m^{GP}) to devote and how much altruistic virtue (v) to give, which can add up to a sense of sacrifice (S). Simultaneously, their sense of sacrifice (S) will be diminished by memory fading at a δ rate and the recipient's repayment behavior (r).[5]

[5] In close relationships, repayment usually exists as a subjective recognition rather than a real gain. Only when the person receiving the repayment views it as a recognition of his or her altruistic virtue will the repayment take effect and reduce the accumulation of the sense of sacrifice. As a result, we want repayment behavior to affect only the giver's giving motivation rather than the utility gain. That is why, rather than the utility function, we include the repayment factor in the change of accumulation variable, i.e. the law of motion equation. Similarly, we do not include v in recipient's utility function because the recipient mainly cares about their repayment and guilt. In this simplified setting, we can focus on the mechanism of giving and repayment behaviors. In addition, identity swapping cannot occur in a short period of time because devotion addiction results from an imbalanced power/strength in the relationship, which will not change without enough time.

By the similar assumption and argument, we can get

$$\ddot{S} - \sigma \dot{S} - J_1 S = K_1 \tag{36}$$

where

$$J_1 = \delta(\delta + \sigma) + \frac{\beta_{vs}}{\beta_{vv}}(2\delta + \sigma) + \frac{\beta_{ss}}{\beta_{vv}}, \tag{37}$$

$$K_1 = \frac{\beta_v(\delta + \sigma) + \beta_s - (\delta + \sigma)\mu_1^{GP} + \beta_{vs}\eta r}{\beta_{vv}} + \eta r(\delta + \sigma) \tag{38}$$

$$S(t) = (S_0 - S^*)e^{\theta_2 t} + S^*, \tag{39}$$

with $\theta_2 = \frac{\sigma - \sqrt{\sigma^2 + 4J_1}}{2}$. The optimal path for the altruistic behavior is

$$v(t) = (\theta_2 + \delta)S - \theta_2 S^* + \eta r, \ \ or \ v(t) = \theta_2(S - S^*) + \delta S + \eta r \tag{40}$$

To guarantee the existence of the root, we need $\sqrt{\frac{\beta_{ss}}{\beta_{vv}}} \geq -\frac{\beta_{vs}}{\beta_{vv}}$.
At the optimum where $\ddot{S} = \dot{S} = 0$, we have

$$S^* = -\frac{\beta_v(\delta + \sigma) + \beta_s - (\delta + \sigma)\mu_1^{GP} + \beta_{vs}\eta r}{J_1 \beta_{vv}} - \frac{\eta r(\delta + \sigma)}{J_1} \tag{41}$$

$$v^* = -\delta\frac{\beta_v(\delta + \sigma) + \beta_s - (\delta + \sigma)\mu_1^{GP} + \beta_{vs}\eta r}{J_1 \beta_{vv}} - \delta\frac{\eta r(\delta + \sigma)}{J_1} + \eta r \tag{42}$$

Similarly, we can derive

$$\ddot{G} - \sigma \dot{G} - J_2 G = K_2 \tag{43}$$

where

$$J_2 = \gamma(\gamma + \sigma) - \frac{\beta_{rG}(2\gamma + \sigma)\rho}{\beta_{rr}} + \frac{\beta_{GG}\rho^2}{\beta_{rr}} \tag{44}$$

$$K_2 = -v(\gamma + \sigma) - \frac{\beta_r(\gamma + \sigma)\rho}{\beta_{rr}} + \frac{(\mu_1^{RP}\gamma + v\beta_{rG})\rho + \beta_G\rho^2}{\beta_{rr}} \tag{45}$$

$$G(t) = (G_0 - G^*)e^{\omega_2 t} + G^*, \tag{46}$$

with $\omega_2 = \frac{\sigma - \sqrt{\sigma^2 + 4J_2}}{2}$. The optimal path for the repayment behavior is:

$$r(t) = -\frac{\gamma + \omega_2}{\rho}G + \frac{\omega_2}{\rho}G^* + \frac{v}{\rho}, \ \ or \ r(t) = -\frac{\omega_2}{\rho}(G - G^*) - \frac{\gamma}{\rho}G + \frac{v}{\rho} \tag{47}$$

To guarantee the existence of the root, we need $\sqrt{\frac{\beta_{GG}}{\beta_{rr}}} \geq \frac{\beta_{rG}}{\beta_{rr}}$ (as $\rho \geq 0$). [6] At the optimum where $\ddot{G} = \dot{G} = 0$, we have

$$G^* = -\frac{-v(\gamma+\sigma)\beta_{rr} - \beta_r(\gamma+\sigma)\rho + (\mu_1^{RP}\gamma + v\beta_{rG})\rho + \beta_G\rho^2}{J_2\beta_{rr}} \tag{48}$$

$$r^* = \frac{v}{\rho} + \gamma\frac{-v(\gamma+\sigma)\beta_{rr} - \beta_r(\gamma+\sigma)\rho + (\mu_1^{RP}\gamma + v\beta_{rG})\rho + \beta_G\rho^2}{\rho J_2\beta_{rr}} \tag{49}$$

3.2 The Recipient Problem: Adjacent Substitution

For the recipient problem, we have a different assumption from the giver problem: adjacent substitution between the stock variable G and the control variable r. The intuition behind it is straightforward. Initially, people regard repayment as a virtue, but as the guilt accumulates, they feel more pressure and take the repayment behavior as a responsibility instead of a choice. Therefore, they gain fewer marginal utility from repayment behavior as guilt accumulates (i.e. $u_{rG} < 0$ or $\beta_{rG} < 0$).

Under this key assumption, we have three conditions: 1) negative and diverging; 2) negative and converging; 3) positive and diverging (see discussion about different conditions in Appendix-Table B.). Concerning the reasonable correlation, we only consider the first two cases and combine the discussions with the GP in the simple setting we assumed before.[7] For an initial start, we will take the GP equilibrium as given in this section, which can be interpreted as a lack of communication or comprehension with each other. In other words, the giver cannot update information about r^* very quickly as the recipient changes the repayment behavior.

3.2.1 Negative Diverging RP

If we assume that $\beta_{rG} \in (\frac{\beta_{rr}\gamma(\gamma+\sigma)+\beta_{GG}\rho^2}{(2\gamma+\sigma)\rho}, \frac{\beta_{GG}\rho}{2\gamma+\sigma})$, then we will have the first case.
In this case (Figure 3), except for caring about the altruistic giver's behavior, we are more

[6] $\sigma^2 + 4J_2 = [(\sigma+2\gamma) - 2\sqrt{\frac{\beta_{GG}}{\beta_{rr}}}]^2 + 4\rho(\sigma+2\gamma)(\sqrt{\frac{\beta_{GG}}{\beta_{rr}}} - \frac{\beta_{rG}}{\beta_{rr}})$

[7] We assume $\beta_{vs} \in (-\frac{\beta_{ss}}{2\delta+\sigma}, \frac{-\delta(\delta+\sigma)\beta_{vv}-\beta_{ss}}{2\delta+\sigma})$, where the GP is positive and converging.

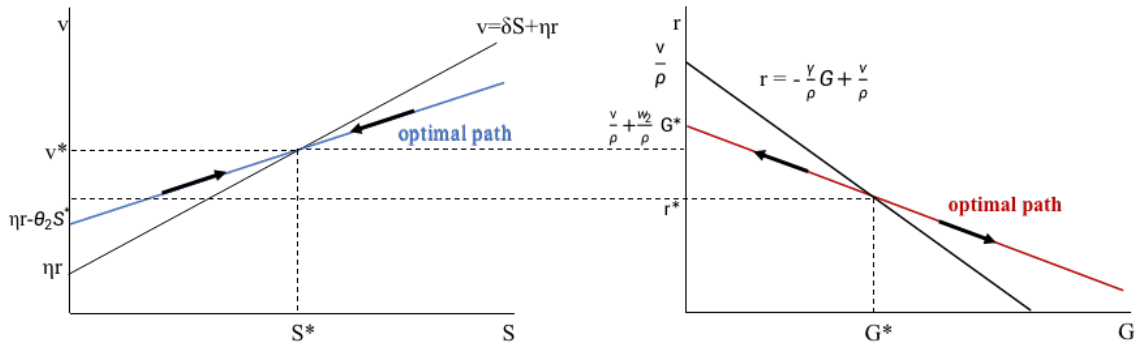


Figure 3: The optimal path for the giver and the recipient: negative diverging RP

curious about the recipient's reactions. For givers, the analysis is similar to the baseline model. The key difference here is that the intercept is not at the original point, which means repayment from the recipient decreases the sense of sacrifice stock to zero as the altruistic giver performs ηr units of altruistic virtues. They receive an unexpected favor for the recipient, so they feel obligated to return that favor according to the norms of reciprocity (Burger et al., 2006). For a given level of giving v, the recipient's guilt level will decrease if the accumulation of guilt is lower than the optimal G^* and will increase otherwise. Following the optimal path, the recipient will increase the repayment level r if G decreases since the decrease in the accumulation of guilt increased the marginal utility of repayment. Similarly, the recipient will decrease the repayment level r as G increases due to the decrease of marginal utility of repayment. In other words, when people regard the repayment more as an obligation than a virtue, they tend to gain a lower level of happiness from repayment and decrease their commitment level.

> *Assume Ben is a pessimist. When he and his mother had already established a balanced relationship after he lived in a dorm, his mother's unintentional emphasis on how much she did for Ben threw that balance off. Ben feels guilty for not caring for his mother enough, despite the fact that he makes time every day to talk with her, even when his schoolwork is demanding. Now that he feels tired to repay more and decreases his sharing to please his mother. Maggie, on the other hand, only sees his hard work as a repayment, so she is unaware that Ben's repayment actually decreases. Just like smashing a cracked pot to pieces, Ben continues to reduce his repayment level. The less he chat with his mother, the more guilt he accumulates.*

To summarize, in a single recipient problem analysis, as we fix the giving level v, we can only reach stable equilibrium if the starting point is (G^*, r^*). We need to notice that r is exogenous to (GP) and v is exogenous to (RP). Under this situation, for any given r, the (GP) could reach an equilibrium $(v^*(r), S^*(r))$. However, for any given v, the (RP) could not reach a stable equilibrium $(r^*(v), G^*(v))$ unless the starting point is the optimal point.

3.2.2 Negative Converging RP

If we assume that $\beta_{rG} \in (-\infty, \frac{\beta_{rr}\gamma(\gamma+\sigma)+\beta_{GG}\rho^2}{(2\gamma+\sigma)\rho})$, then we will have the second case.

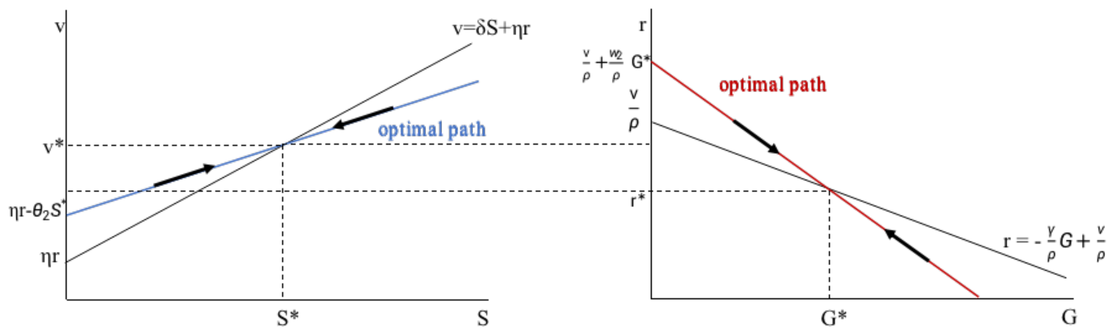


Figure 4: The optimal path for the giver and the recipient: negative converging RP

Similarly, in this case (Figure 4), we will take the GP outcome as fixed and mainly talk about the RP. For a given level of giving v, the recipient's guilt level will increase if the accumulation of guilt is lower than the optimal G^* and will decrease otherwise. Therefore, the movement of r on the optimal path still converges to the steady-state (r^*, G^*). In the end, they will reach an equilibrium (v^*, S^*, r^*, G^*) and keep at this level if there is no other shock.

> *In this case, we can regard Ben as a positive person. When he realizes he has not adequately repaid her mother, he chooses to repay more in order to feel relieved, and the balance gradually returns.*

Furthermore, we are also interested in when the optimal giving and repayment equalize. Since the variables are not necessarily the same at optimum, we denote the giving and repayment level under equalization as $v^*_\bullet$ and $r^*_\bullet$ respectively in case of confusion. After simple calculations, we can derive that the equalization reaches under both assumptions as

$$v_{\bullet}^{*} = r_{\bullet}^{*} = \frac{\gamma}{1-\rho\eta} G^{*} + \frac{\delta\rho}{1-\rho\eta} S^{*}, \tag{50}$$

where G^* and S^* are the stock levels at the optimum. It implies that if they choose a level of giving to each other such that it is equal to the combination of guilt and sense of sacrifice with proper weight, they will reach fair exchanges.

3.3 Discussion

3.3.1 The Link Between Two Dynamics

This section will loosen the assumption that the giver cannot update information quickly and talk deeply about the interaction between the two dynamic processes to better analyze the fluctuation in response to some exogenous shocks. Since the two cases follow the same steps, we will only discuss the first case for nontriviality.

Suppose at the beginning, the two parties both reach the equilibrium $(v^*(r^*), S^*(r^*), r^*(v^*), G^*(v^*))$. In preparation for the following discussion of both parties' reactions, we need first to compare the marginal change of the two variables – G^* and r^* – under a steady-state in (RP). From Equation (48) and (49), we can derive that

$$\frac{\partial G^*}{\partial v} = \frac{\gamma+\sigma}{J_2} + \frac{\rho\beta_{rG}}{J_2\beta_{rr}} \tag{51}$$

where the sign of this partial derivative has the opposite sign with $Q = (\gamma+\sigma)\beta_{rr} + \rho\beta_{rG}$,

$$\frac{\partial r^*}{\partial v} = \frac{1}{\rho} - \frac{\gamma(\gamma+\sigma)}{\rho J_2} + \frac{\gamma\beta_{rG}}{J_2\beta_{rr}} \tag{52}$$

where the sign of the partial derivative has the opposite sign with $J_2\beta_{rr} - \gamma Q$.
There are three conditions [8]:

[8] Since we assume $\beta_{rG} < 0$ by adjacent substitution, the last condition $Q > 0$ will not be possible.

condition	equilibrium (G^*, r^*)	
$Q < \frac{J_2\beta_{rr}}{\gamma}$	G^* increases and r^*decreases as v increases.	$(G^* \uparrow, r^* \downarrow)$
$\frac{J_2\beta_{rr}}{\gamma} < Q < 0$	G^* and r^* both increase as v increases.	$(G^* \uparrow, r^* \uparrow)$
$Q > 0$	G^* decreases and r^* increases as v increases.	$(G^* \downarrow, r^* \uparrow)$

Then, we also need to compare the marginal change of the two variables – S^* and v^* – under steady state in (GP). From Equation (41) and (42), we can derive that

$$\frac{\partial S^*}{\partial r} = -\frac{\eta}{J_1}(\frac{\beta_{vs}}{\beta_{vv}} + \delta + \sigma) \tag{53}$$

where the sign of this partial derivative has the opposite sign with $Z = \frac{\beta_{vs}}{\beta_{vv}} + \delta + \sigma$,

$$\frac{\partial v^*}{\partial r} = -\delta\frac{\eta}{J_1}(\frac{\beta_{vs}}{\beta_{vv}} + \delta + \sigma) + \eta \tag{54}$$

where the sign of the partial derivative has the opposite sign with $1 - \frac{\delta}{J1}Z$.
There are also three conditions:

condition	equilibrium (S^*, v^*)	
$Z < 0$	S^* and v^* both increase as r increases.	$(S^* \uparrow, v^* \uparrow)$
$0 < Z < \frac{J1}{\delta}$	S^* decreases and v^* increases as r increases.	$(S^* \downarrow, v^* \uparrow)$
$Z > \frac{J1}{\delta}$	S^* and v^* both decrease as r increases.	$(S^* \downarrow, v^* \downarrow)$

Here we want to elaborate one case that$\frac{J_2\beta_{rr}}{\gamma} < Q < 0$ and $Z < 0$. In reality, the responses from both sides usually happen simultaneously. To better understand the model, we will divide them into several steps. Since stock variables passively depend on the change of other control variables and control variables v and r are chosen intentionally, we assume that the stock variable will change immediately as the other changes the control variable in our following analysis.

Step 0: At the beginning, there is a shock that makes the giver offer a higher than optimal level of care (e.g., the mother, Maggie, stays up late to supervise her kid to do homework). Since the depreciation of stock cannot catch up with the increasing speed, then the stock of

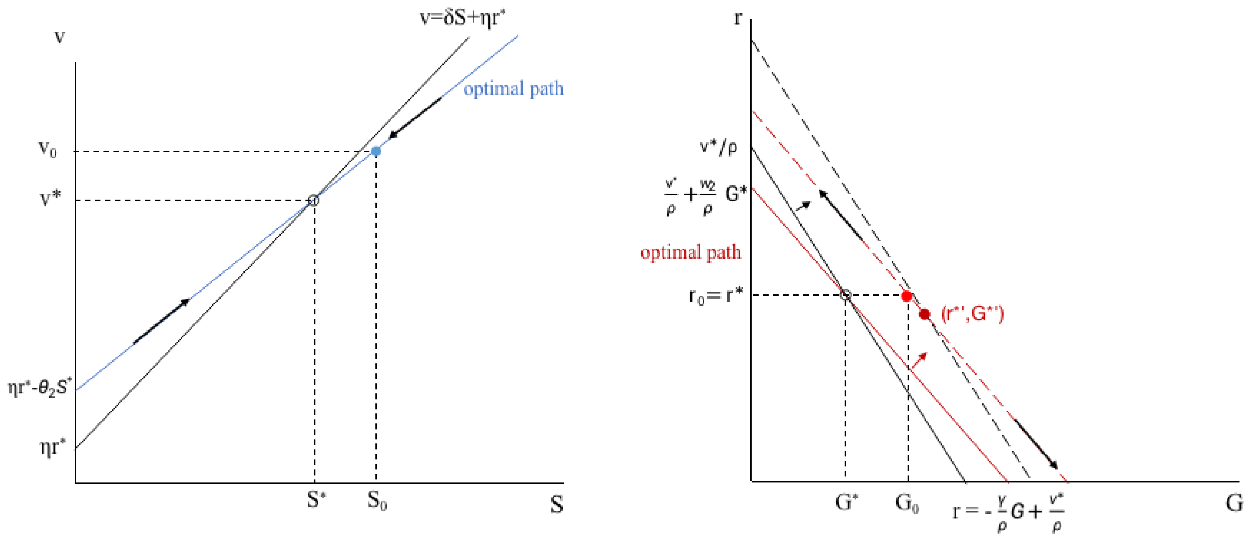


Figure 5: Small increase of v in the interaction model: Step 0

the sense of sacrifice suddenly increases to a level that is higher than the optimal $S_0 > S^*$ (e.g., recently the mother has paid too much attention to the kid, and she feels too tired to offer more). Following the optimal path, the giver plans to decrease the care (as the arrow in the left graph shows).

At this step 0, the recipient's repayment level will not suddenly change, but the additional care will accumulate a higher level of guilt ($G^* \to G_0$) (e.g., keeping at the past level of repayment cannot offset the accumulation of guilt due to the higher giving per period). At the same time, the optimal path curve (the solid red line) will move upward (the red dotted line), so will the calibration line (the black line). Thus, we can see in the figure that the optimal guilt level and repayment level (r^*, G^*) move to a new position $(r^{*'}, G^{*'})$ where G^* increases but r^* decreases (e.g., the kid, Ben, feels a greater debt to his mother but the perceived obligation makes his feeling of behaving obediently, as usual, less valuable). The recipient is at the light red point (r_0, G_0) or (r^*, G_0). With lower than optimal level of guilt ($G_0 < G^{*'}$), the guilt stock tends to further decrease since $\dot{G} = -\omega_2(G - G^{*'}) < 0(\omega_2 > 0)$. Then the decrease of G increases the marginal utility of repayment so that the recipient will increase the repayment at the next step 1 (e.g., although the kid feels less urge for repayment, the decrease of guilt stock made his marginal repayment happier, thus increasing the repayment).

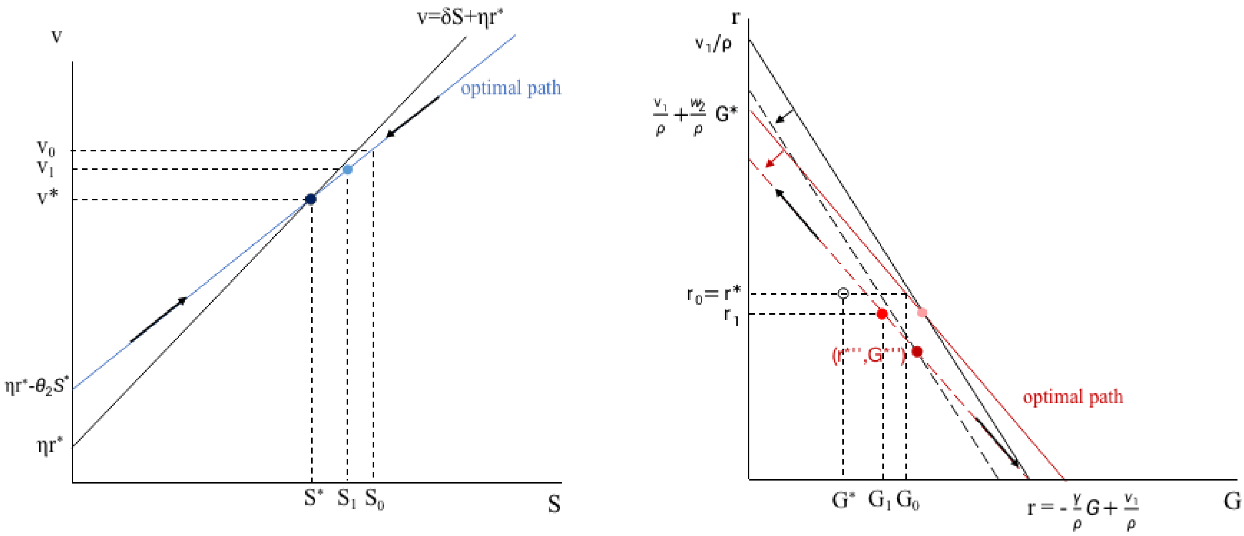


Figure 6: Small increase of v in the interaction model: Step 1

Step 1: Following the optimal path, the giver has decreased the sacrifice stock to a new level ($S_0 \to S_1$), and the recipient has decreased its guilt stock to a new level ($G_0 \to G_1$). Since the repayment level (r_0) has not changed yet at Step 0, the optimal path and calibration line will stay the same on the left graph in Figure 6 and the steady-state point is still (v^*, S^*). Therefore, the giver's sacrifice stock (S_1) will decrease ($\dot{S} = \theta_2(S - S^*) < 0$) so the giver will choose a lower level of altruistic behavior v_1 to optimize the utility (e.g. this time Maggie realizes Ben's decrease of chatting with her and then tells Ben that he does not need to be guilty of not enough repayment and she has a lot of personal time). As v decreases, the recipient's curve will move downward from the previous stage. Then the recipient will choose the repayment level (r_1) according to the new optimal path (the dotted light red line), so the current position is (r_1, G_1) (e.g. Maggie's words decreases Ben's guilt. Since Maggie offers less care, Ben temporarily chooses a repayment level that is lower than before). Since the current position (r_1, G_1) is on the left side of the new steady-state $(r^{*''}, G^{*''})$, the recipient's guilt level will continue to decrease, and then the recipient will choose to further increase the repayment level at the next step 2.

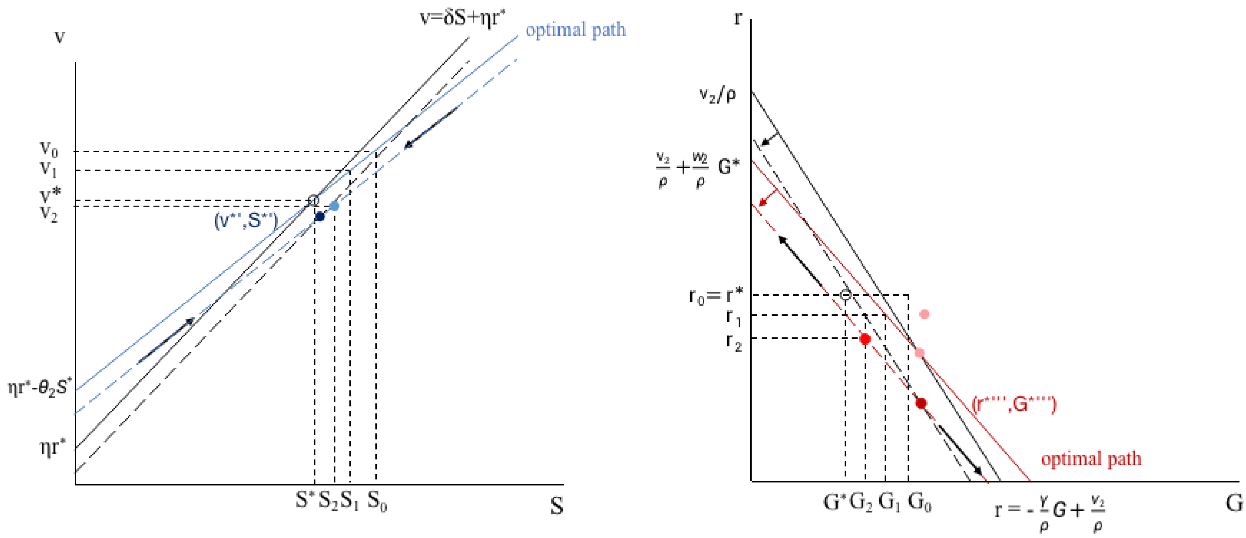


Figure 7: Small increase of v in the interaction model: Step 2

Step 2: Again, the giver decreased the sacrifice stock to a new level ($S_1 \to S_2$), and the recipient decreased its guilt stock to a new level ($G_1 \to G_2$). Since this time, the altruistic behavior has decreased to a lower level (r_1), the optimal path and the calibration line will both move downward (Figure 7) for GP. So the giver and the recipient will choose v_2 and r_2 respectively. Since the targeted steady-state for GP is still on the left side of the current point (v_2, S_2), the giver will decrease S and v. Since the targeted steady-state for RP is still on the right side of the current point (r_2, G_2), the recipient will decrease G and increase r.

By a similar argument, we can also explore the movement under the negative converging case. What is more, these are simply the initial few steps for the interaction model in response to the shock. In order to be more specific, we simulate the possible outcomes under this condition.

3.3.2 Simulations

This section will show two simulative outcomes for the first case, where we explore the interaction between negative diverging RP and positive converging GP. There are two main possibilities: one is overall converging (Setting 1), and the other is overall diverging (Setting 2).

Setting 1: overall converging case

In setting 1, we get the overall converging equilibrium.[9] As is shown in Figure 8, as we increase the simulation number n from 30 to 300, the current points will first oscillate for a while and then gradually converge to the center and become stable in both two graphs (left: GP; right: RP). Intuitively, by frequent communication, both the giver and the recipient will finally go back to the initial equilibrium: $(r^*, G^*) = (0.351, 1.412) \rightarrow (r^{*F}, G^{*F}) = (0.351, 1.412)$ where r and G return back; $(v^*, S^*) = (1.446, 1.489) \rightarrow (v^{*F}, S^{*F}) = (1.446, 1.489)$ where v and S also return back. [10]

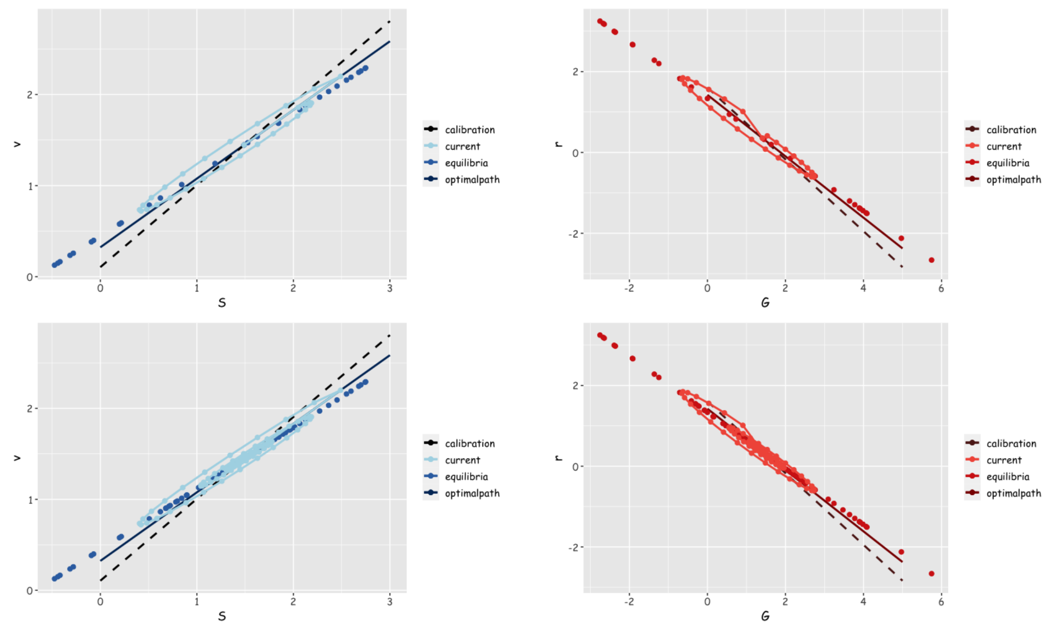


Figure 8: Setting 1: Stable joint dynamics with positive converging GP and negative diverging RP (upper: n=30; lower: n=350)

Setting 2: overall diverging case

However, there is also another possibility under the interaction between negative diverging RP and positive converging GP. In the setting 2,[9] we reach the overall diverging condition. As Figure 9 shows, the tracks of current points will diverge sharply in both graphs (left:

[9]See detailed parameters settings in the Appendix (Table C).

[10]The superscript $*F$ means the final equilibrium.

GP; right: RP) when we increase the simulation number from 20 to 30. Since here we set a positive shock to v, the v and S keep increasing, and the G keep increasing while the r keeps decreasing.

Intuitively, an abrupt increase in caring may cause the recipient to abandon the conscientiousness if they cannot afford the guilt-induced suffering, and they may continue to lose the motivation to repay. It is consistent with Glass (1964)'s finding that "the aggressor would reduce dissonance by becoming more unfriendly toward his victim." Generally, the defensive scheme in our mind will alleviate the marginal cost of continuous suffering, thus making oneself more indifferent and apathetic. As time passes, the recipient will adapt to the mode, take it for granted, and even ask for more (i.e., $r < 0$). If the sense of sacrifice makes the giver feel good about his/her reputation, then he/she will keep giving. To put it generally, social programs can be designed to engage people to contribute to society, but if poorly designed, they can create a sense of apathy and helplessness in a sullen, listless recipient population.

> *We can also use Ben and Maggie's story to compare this frequent communication case with the infrequent communication case. This time Ben chooses to tell his mother that he has done more than just study hard. When Maggie realizes his increasing guilt and decreasing care for her, she decides to reduce her caring level to strike a balance. In setting one, we can see that they will achieve a balanced relationship after several rounds of adjustment. However, setting two predicts that the situation will be similar to the infrequent communication case. This disparity is related to their sensitivity to guilt. If Ben is pretty sensitive to changes in guilt, even frequent communication cannot help pull the self-abandonment force back.*

3.3.3 Applications in Compensatory Behaviors

Broadly speaking, the giving and repayment interaction can also explain the addiction to compensatory behaviors. Binge eating accompanied by excessive exercise can be a good example. Specifically, exercise is generally perceived as a virtue. For most people, it is beneficial to their physical and mental health. Those who exercise routinely are often praised for their motivation and self-discipline. However, a series of research papers in psychiatry (Lichtenstein et al., 2013; Dittmer et al., 2018) find that compulsive exercise

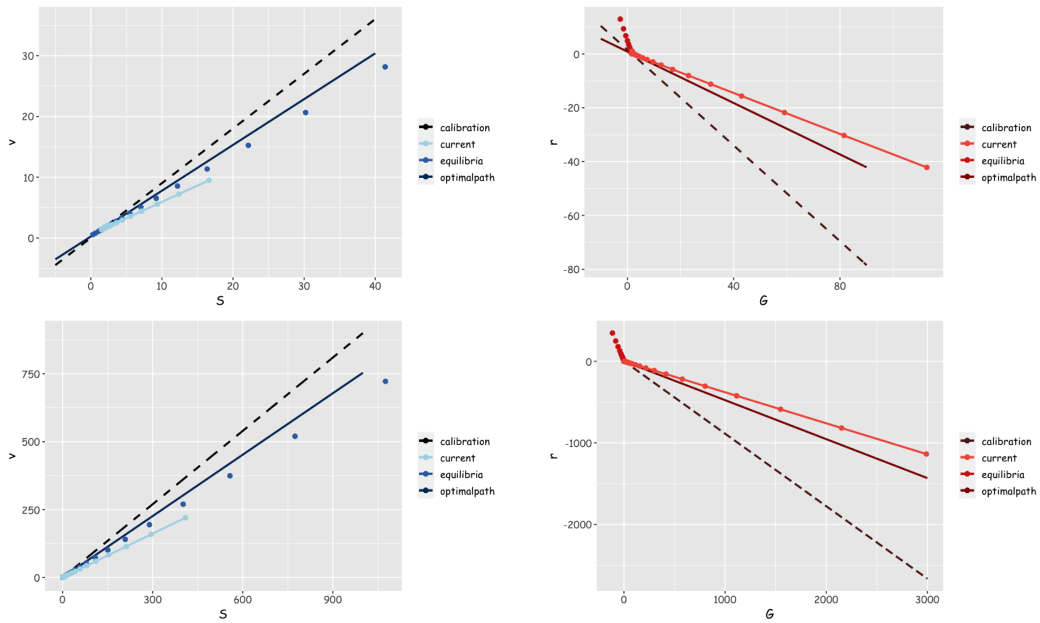


Figure 9: Setting 2: Unstable joint dynamics with positive converging GP and negative diverging RP (upper: n=20; lower: n=30)

usually coexists with eating disorders, a general designation of a class of mental distress related to abnormal eating habits in psychopathology. It is hardly acceptable that a well self-disciplined person cannot control his/her eating habit.

Dittmer et al. (2018) reveal that excessive exercise is a common symptom and sometimes even plays a role in the development and maintenance of the disorder. Under this situation, exercise is no longer a pure, healthy lifestyle but a compensatory resort from the patients' perspective. Except for excessive exercise, self-induced vomiting and excessive use of laxatives or diuretics are also some types of compensatory behaviors (Stiles-Shields et al., 2012). They are simply things people with eating disorders do to make up for having eaten and consumed calories. In our model, this kind of compensatory behavior can be interpreted as repayment behavior. People resort to them to erase shame, anxiety, guilt, or other "bad" feelings about the food eaten and the act of eating it. Importantly, binge eating does not come from nowhere. It usually results from the anxiety derived from a harsh self-disciplined lifestyle, so people need an exit to release pressure. Induced by the habit developed from past eating experiences, the addict will gain a higher level of happiness from marginal eating behavior. At the same time, it will also accumulate guilt in the addict's mind because of, say, disappointment towards the weight gain. Thus, excessive exercise usually follows behind binge eating disorder to alleviate such guilt, which results from the fact that people crazily eat a considerable amount within a short period to release stress.

In this way, we regard the addict as two-self: sense and sensitivity. Indulgence is the altruistic behavior for the ascetic self, and exercise is the repayment for indulgence. These two behaviors can sustain each other, which is consistent with the outcome of Setting 1. Therefore, compensatory behaviors may not substantially prevent people from binge eating. In a bigger sense, major effort today may compensate for past sin or shame, by oneself or one's family.

3.3.4 Swapping Identities

In most bilateral relationships players are switching between being givers and recipients over time. Swapping identities, in the context of our model, is more than just changing the sign of v and r or switching to each other's function, because sacrifice addiction does not use an easily cleared account, and givers and recipients are two distinct roles. In most cases, sacrifice addiction occurs when one party pays more than the other, at least in a specific field. There are two possible ways for the two parties to swap their identities. The first option necessitates a stage change, which takes a long time for the two parties to switch their relative strengths. Children, for example, will become more powerful than parents as they grow older. Adult children will gradually take over as the payment-dominant party. So a new relationship will form, and the old relationship will generally disappear or decrease to a minor level as they connect less (r approaches 0 in GP and v approaches 0 in RP) and givers feel less accomplishment in devotion (β_v decreases) as well as greater suffering in sacrifice ($|\beta_S|$ increases) while recipients feel less relieved from repayment (β_r decreases) as well as less suffering from guilt ($|\beta_G|$ decreases). The second option is to form a new interaction relationship between the same two parties at the same time, such as a wife and husband. They may have a comparative advantage in different fields. Individual A is the payment-dominant party in one field, and individual B is the payment-dominant party in the other. Both options require the creation of a new psychological account to deal with the giving and repayment, simulating the complexities of human nature.

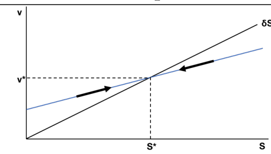
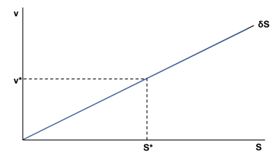
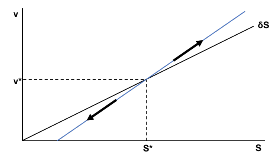
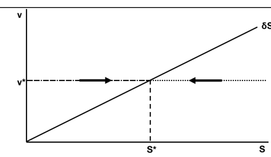
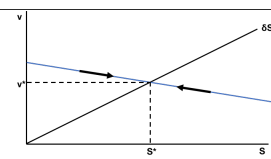
4 Summary and Conclusions

In this paper, we construct a theoretical model to explain the maintenance of sacrifice behavior in a close relationship. The giver takes the sacrifice as an altruistic behavior and can gain more marginal utility from accumulating the sense of sacrifice. It can be regarded as the sense of achievement from doing the right things. We also discuss the effect of efficiency change and find that people will pay less effort to altruistic behavior if they have higher efficiency in self-development, consistent with the comparative advantage theory. In terms of practical implication, we suggest that the offspring can expose their nagging parents more to self-development opportunities, which can divert their attention from offering unnecessary care to their children. For elderly parents, college for the aged can be a good choice. This model is also applicable to the currently widespread phenomenon of fan economy, where fans can suddenly stop their supporting activities if their worship collapses, but the extreme antipathy will finally calm down .

Additionally, we extend the one-sided model to an interaction model between the giver and the recipient to help explain different situations. We find that the recipient problem (RP) can be negatively diverging under the adjacent substitution assumption when we fix the giver problem (GP) at the optimal level. It can explain less frequently communicative situations where two subjects do not exchange information timely. If we allow both RP and GP to change their giving and repayment levels simultaneously after a small positive shock, the diverging RP can reach either an overall diverging outcome or an overall converging outcome from simulations. This converging result is applicable to explain why some compensatory behaviors like excessive exercise coexist with addictive behaviors like binge eating. Therefore, we suggest that resorting to compensatory behaviors is not a good choice to stop binge eating, and "cold turkey" is more effective as suggested by Becker and Murphy (1988).

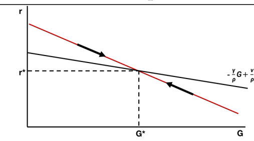
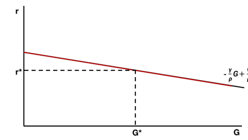
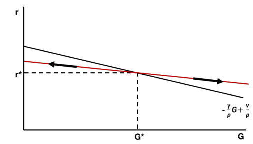
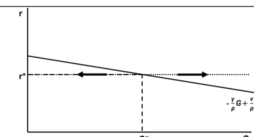
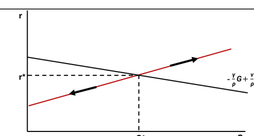
Appendix

Table A: The optimal path under different assumptions for baseline model

Correlationship	Movement direction	β_{vs}	Graph	
positive ($\theta_2 + \delta > 0$)	converge ($\theta_2 < 0$)	$(-\frac{\beta_{ss}}{2\delta+\sigma}, \frac{-\delta(\delta+\sigma)\beta_{vv}-\beta_{ss}}{2\delta+\sigma})$		(a)
	static ($\theta_2 = 0$)	$\frac{-\delta(\delta+\sigma)\beta_{vv}-\beta_{ss}}{2\delta+\sigma}$		(b)
	diverge ($\theta_2 > 0$)	$(\frac{-\delta(\delta+\sigma)\beta_{vv}-\beta_{ss}}{2\delta+\sigma}, \infty)$		(c)
uncorrelated ($\theta_2 + \delta = 0$)	converge ($\theta_2 < 0$)	$-\frac{\beta_{ss}}{2\delta+\sigma}$		(d)
negative ($\theta_2 + \delta < 0$)	converge ($\theta_2 < 0$)	$(0, -\frac{\beta_{ss}}{2\delta+\sigma})$		(e)

Note: To guarantee the existence of the root, we need $\beta_{vs} \geq -\sqrt{\beta_{vv}\beta_{ss}}$, which is naturally satisfied.

Table B: The optimal path under different assumptions for (RP)

Correlationship	Movement direction	β_{rG}	Graph
negative $(\omega_2 + \gamma > 0)$	converge $(\omega_2 > 0)$	$(-\infty, \frac{\beta_{rr}\gamma(\gamma+\sigma)+\beta_{GG}\rho^2}{(2\gamma+\sigma)\rho})$	 (a)
	static $(\omega_2 = 0)$	$\frac{\beta_{rr}\gamma(\gamma+\sigma)+\beta_{GG}\rho^2}{(2\gamma+\sigma)\rho}$	 (b)
	diverge $(\omega_2 < 0)$	$(\frac{\beta_{rr}\gamma(\gamma+\sigma)+\beta_{GG}\rho^2}{(2\gamma+\sigma)\rho}, \frac{\beta_{GG}\rho}{2\gamma+\sigma})$	 (c)
uncorrelated $(\omega_2 + \gamma = 0)$	diverge $(\omega_2 < 0)$	$\frac{\beta_{GG}\rho}{2\gamma+\sigma}$	 (d)
positive $(\omega_2 + \gamma < 0)$	diverge $(\omega_2 < 0)$	$(\frac{\beta_{GG}\rho}{2\gamma+\sigma}, 0)$	 (e)

Note: The existence of the root requires $\beta_{rG} \leq \sqrt{\beta_{rr}\beta_{GG}}$, which is naturally satisfied under adjacent substitution.

Table C: Simulation Parameters

Parameter	*Setting* 1	*Setting* 2
β_v	1.3	1.3
β_s	-1.3	-1.3
β_r	1.2	1.2
β_G	-1.2	-1.2
β_{vv}	-2	-2
β_{ss}	-2	-2
β_{rr}	-2	-2
β_{gg}	-2	-5
β_{vs}	1.9	1.9
β_{rG}	-1.9	-3
δ	0.9	0.9
γ	0.8	0.8
η	0.3	0.3
ρ	0.9	0.9
σ	0.4	0.4
μ_1^{RP}	2	5
μ_1^{GP}	0.06	0.1

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