Exploring an Alternative to Student Loans: Constructing and Exploring the Demand for an Income Share Agreement Market

Paul G. Zimmer III[1] & Alex S. Hilsenrath[2]

Professor Curtis R. Taylor, Faculty Advisor

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[1] Paul graduated from Duke University in May 2020 with High Distinction in Economics and a concentration in Finance. Beginning in July 2020, he will be working as a Private Equity Analyst at Ares Management in Los Angeles. He can be reached at pbrzimmer@gmail.com.
[2] Alex also graduated from Duke University in May 2020 with High Distinction in Economics and a concentration in Finance. Beginning in September 2020, he will be working as an Associate Consultant at Mars & Co. in New York. He can be reached at a.hilsenrath@gmail.com.

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Abstract

With the rapid growth in the US student loan market and rising default rates in the space, students and universities have begun to explore other methods to fund rising undergraduate education costs. This paper will seek to analyze a proposed financing solution, exploring a theoretical market for a financial mechanism known as an Income Share Agreement, or ISA. Currently, the market for ISAs is relatively unpenetrated and not institutionalized. Under a theoretical institutionalized market, the paper will pursue a framework which compares a student's preference for an ISA versus a private student loan. The objective is to determine under what circumstances and what variables determine when a student will prefer an ISA versus a private student loan, and vice versa. We first evaluate a student's CARA utility function under an ISA and under private debt. We find that under some degree of risk-aversion, the individual will strictly prefer the ISA financing option. Next, we introduce the concept of moral hazard in the student's optimization problem. When solving the optimization problem, we determine that a relationship exists between degree of risk aversion, variance of the income distribution, principal amount borrowed, and student's earning ability when choosing between an ISA and a private student loan. We then examine this relationship in the context of a mixed financing strategy and solve for the optimal mixed financing bundle. Finally, we introduce the concept of adverse selection based upon asymmetric information between the borrower and the financier.

JEL Classification: I22, I23, J31
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Introduction

With the US student loan market recently surpassing $1.5 trillion of outstanding obligations and default rates expected to hit 40% by 2023, students and universities have begun to explore other methods to fund rising undergraduate education costs. This paper will seek to analyze a new financial mechanism first proposed by Milton Friedman in 1955. Friedman's proposal, known as an Income Share Agreement, or ISA, is a financial instrument in which an individual or organization provides a valuable good or service to the recipient who, in exchange, agrees to pay back a percentage of their income to reimburse the investor over a set period of time. In the context of education, an ISA functions as an "investment" on behalf of a private investor or university which allows students to pay for their tuition costs and reimburse these investors in the future using a percentage of their income. This mechanism is different from a traditional loan in that repayment is not mandatory and occurs only if an individual's income exceeds a certain predetermined threshold, with the total amount of repayment usually bounded by some upper limit.

Currently, the market for ISAs is relatively undefined and unpenetrated. A consortium of various private investors and universities have started to experiment with different ISA structures as a complementary mechanism to traditional (government-backed) student loans to fund the remainder of students' tuition requirements. That is to say, at the moment and for the foreseeable future, Income Share Agreements will be used as an additional form of financing to fund college tuition for individuals that demand more capital than traditional (government-backed) student loans can provide. However, the problem lies in the fact that there is currently no formal market for ISAs; potentially interested students must attend a university which offers an ISA program or, less commonly, attempt to pursue an individual contract with a private investor. By virtue of the relatively undefined nature of the market, we are interested in what a theoretical ISA market would look like as an alternative to the current forms of tuition financing.

Due to the complementary nature of the relationship between ISAs and traditional (government-backed) student loans, the paper will instead seek to analyze the possible relationship of ISAs and their nearest competitive peer, the private student loan market. As stated, a current institutionalized market for ISA instruments does not exist, thus, we seek to evaluate if there would be demand for such a market by evaluating a theoretical ISA market in the context of consumer utility. The paper pursues a framework which compares a student's preference for an ISA versus a private student loan by evaluating (indirect) relative utility between the instruments on a stand-alone basis, followed by under moral hazard, and lastly under adverse selection. The objective is to determine under what circumstances and what variables determine when a student will prefer an ISA versus a private student loan, and vice versa. We first evaluate a student's CARA utility function under an ISA and under private debt. We find that under some degree of risk-aversion, the individual will strictly prefer the ISA financing option. Next, we introduce the concept of moral hazard in the student's optimization problem. We include an effort function which accounts for the cost a student must endure to achieve a higher wage. When solving the optimization problem, we determine that a relationship exists between degree of risk aversion, variance of the income distribution, principal amount borrowed, and student's earning ability when choosing between an ISA and a private student loan. We then examine this relationship in the context of a mixed financing strategy and solve for the optimal mixed financing bundle. Finally, we introduce the concept of adverse selection based upon asymmetric information between the borrower and the financier. We first explore this relationship in the continuous case by evaluating the certainty equivalents at an individual's theoretical indifference point. We find that a unique relationship exists between the principal amount borrowed, and the risk parameter. We then move to a discrete case with two individual types where we perform a similar evaluation and expand on our results from the previous section. Lastly, we consider both moral hazard and adverse selection concurrently. We introduce new exogenous parameters and establish a relationship for further study.

Literature Review

The existing literature surrounding ISAs is somewhat limited, and very recent. A majority of the papers explore the “concept” of more widespread adoption of ISAs and the inherent limitations of this adoption. A recent PhD candidate’s paper discusses the adverse selection problem associated with consumers that would pursue an ISA (Herbst, 2018). Other papers have explored the “need” for such an alternative by explaining the deficiency of the current traditional (government-backed) and private student loan markets (Palacios, DeSorrento, and Kelly, 2014). No current research or literature surrounding the topic discusses a theoretical framework for an ISA market, nor the feasibility of such a market in a quantitative context.

Some of the most promising literature on ISAs is a recent paper from Kevin Mumford, which is an empirical analysis that centers around students at Purdue University selecting into an ISA (Mumford, 2018). Mumford uses data from 431 ISA participants and 319 non-participants who applied for and were offered an ISA but chose not to participate, to estimate the extent of selection into an ISA by ability, academic performance, beliefs about future income, location preference, risk aversion, debt aversion, optimism, financial sophistication, and parent characteristics. Mumford’s goal was to use a linear regression to determine which (if any factors) contribute to selection into an ISA. Using a linear-probability model, Mumford was able to discern some unexpected results. Mumford hypothesized that there would be significant adverse selection of participants versus non-participants, however, in this limited set of data the found no evidence of adverse selection across a few key areas. For example, salary expectations, salary growth, and labor participation were all relatively similar, as were more subtle adverse effects such as age expectation for having children (if any). Ultimately, Mumford concluded was that there was no adverse selection into an ISA based on student ability (GPA, standardized test scores), and that selection is primarily driven by parent characteristics. This propensity for selection based on parent characteristics is posited to be based on the fact that the most common alternative to an ISA is a form of private loan known as a direct PLUS loan. These loans require parents to take out a loan on behalf of their children to cover tuition expense, thus, intuitively it makes sense that parent characteristics may impact the decision of a student to undertake an ISA.

Additionally, another important finding from Mumford's research was that students in higher-paying majors are less likely to participate than those in lower-paying majors. To combat this adverse selection, Purdue has developed a tiered income structure to their ISA agreements, with students in traditionally lower-paying majors required to pay a larger proportion of their income than students in higher-paying majors (students in lower-paying majors face a higher "price"). Overall, Mumford's work, while not directly applicable to our theoretical formwork, was helpful in educating our decision on which variables to include in our model of consumer choice between an ISA or a private student loan.

Going off of this idea of modeling a consumer utility function – which is the first step in our theoretical framework – are a few notable papers that discuss topics similar to ours as they model utility functions in order to map out selection. The first paper is that of Bechlioulis and Brissimis (2014) which seeks to model optimal consumption decisions of households in a micro framework that analyzes heterogeneity between borrowing and saving households, and endogenous default (Bechlioulis and Brissimis, 2014). Most interesting in the context of our work is their derivation of a consumer utility function which takes into account consumption, working hours, and unpaid debt, while also constructing a model of a household's cash inflows and outflows which serves as the constraint in the optimization problem. The model of the constraint considers 'state' variables such as wage rate, interest rate, price level, penalty premium on the interest rate and the percentage of unpaid debt which is refinanced. Ultimately, while we did not elect to pursue a consumer utility framework similar to that of Bechlioulis and Brissimis, we took inspiration from the inclusion of 'state' variables to inform our model.

A second paper which discusses consumer utility in the context of student loans seeks to measure the gains in consumer surplus arising from risked-based pricing in the private student loan market, as well as the re-distributional consequences arising from refinancing out of the public (government-backed) student pool into the private loan market (Cox, 2017). In relation to our research, we were most interested in Cox's work regarding consumer sensitivity to interest rate fluctuations. We chose to apply a similar approach in the context of ISAs rather than private student loans.

Theoretical Specification

1 Setup

Let $\frac{P}{1+r}$ be the cost of education to be financed, where $r > 0$ is the risk-adjusted interest rate. Then P is the amount the graduate will need to repay for financing his education. Suppose that after graduating the student obtains a job that pays w which is normally distributed with mean μ and variance σ^2, $w \sim \mathrm{N}(\mu, \sigma^2)$.[1] The student has constant-absolute risk-aversion Bernouli utility over wealth $y \in \mathbb{R}$:

$$U(y) \equiv -\exp(-\rho y)$$

where $\rho \geq 0$ is his Arrow–Pratt coefficient of absolute risk aversion (Arrow, 1965; Pratt, 1964).

Lemma 1 (Certainty Equivalent)**.** *Suppose y is normally distributed with mean μ and variance σ^2. Then,*

$$\mathbb{E}[-\exp(-\rho y)] = -\exp(-\rho(\mu - 0.5\rho\sigma^2))$$

Proof. Note that

$$\mathbb{E}[-\exp(-\rho y)] = -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\rho y - \frac{(y-\mu)^2}{2\sigma^2}\right) \mathrm{d}y$$

Now, the argument of the exponential function can be written as:

$$-\frac{y^2 - 2y\mu + \mu^2 + 2y\rho\sigma^2}{2\sigma^2}$$

Combining terms gives

$$-\frac{y^2 - 2y(\mu - \rho\sigma^2) + \mu^2}{2\sigma^2}$$

Completing the square in the numerator by adding and subtracting $(\mu - \rho\sigma^2)^2$ gives:

$$-\frac{(y - (\mu - \rho\sigma^2))^2 + \mu^2 - (\mu - \rho\sigma^2)^2}{2\sigma^2}$$

Combining terms gives:

$$-\frac{(y - (\mu - \rho\sigma^2))^2}{2\sigma^2} - \rho\mu + 0.5\rho^2\sigma^2$$

[1] While this specification is commonly used in finance models because of its tractability, an unfortunate implication is the possibility of negative wealth. If $\mu > 0$ is large relative to σ, then negative outcomes are unlikely and it is appropriate to regard the full normal distribution as a close approximation to its truncated counterpart.

Hence,

$$\mathbb{E}[-\exp(-\rho y)] = -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-(\mu-\rho\sigma^2))^2}{2\sigma^2} - \rho\mu + 0.5\rho^2\sigma^2\right) \mathrm{d}y$$

Decoupling the exponential terms and pulling the one that does not depend on y out of the integral gives

$$\mathbb{E}[-\exp(-\rho y)] = -\exp(-\rho(\mu - 0.5\rho\sigma^2) \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-(\mu-\rho\sigma^2))^2}{2\sigma^2}\right) \mathrm{d}y$$

The integrand in this expression corresponds to a normal density with mean $\mu - \rho\sigma^2$ and variance σ^2. Hence, it integrates to one, which establishes the claim. $\square$

This result is technically convenient and makes intuitive sense as well. It says that if an individual has CARA utility with risk parameter ρ and faces normally distributed wealth, y, with mean μ and variance σ^2, then his expected utility is

$$\mathbb{E}[u(y)] = u(\mu - 0.5\rho\sigma^2)$$

In other words, the individual's *certainty equivalent* wealth is $\mu - 0.5\rho\sigma^2$. This differs from $\mathbb{E}[y] = \mu$ by the term $-0.5\rho\sigma^2$, which is the risk premium the individual would be willing to pay to get full insurance against the shocks to his wealth. Note that the risk premium is larger (in absolute value) the larger is the coefficient of absolute risk aversion, ρ, and the larger is the variance in his wealth, σ^2.

Lemma 2 (Affine Transformations). *If $y \sim N(\mu, \sigma^2)$, then for any $a > 0$ and $b \in \mathbb{R}$, $ay + b \sim N(a\mu + b, (a\sigma)^2)$.*

Proof (from Taboga, 2017). Mutually independent normal random variables are jointly normal: $Kx1$ random vector X defined as,

$$X = [X_1 ... X_K]^T$$

has multivariate normal distribution with mean

$$\mathbb{E}[X] = [\mu_1 \dots \mu_k]^T$$

and covariance matrix

$$\begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_K^2 \end{bmatrix}.$$

We can write

$$Y = \sum_{i=1}^{K} b_i X_i = BX$$

where

$$B = [b_1 \dots b_k]$$

Therefore, according to the above proposition on linear transformations Y has a (multivariate) normal distribution with mean

$$\mathbb{E}[Y] = B\mathbb{E}[X] = \sum_{i=1}^{K} b_i^2 \mu_i^2$$

and variance

$$Var[Y] = BVar[X]B^T = \sum_{b_i^2}^{\sigma_i^2}$$

A special case of the above proposition obtains when X has dimension $1x1$ (i.e., it is a random variable). Consider a normal random variable X with mean μ and variance σ. Let a and b be two constants (with $b \neq 0$). Then random variable Y defined by

$$Y = a + bX$$

has normal distribution with mean

$$\mathbb{E}[Y] = a + b\mu$$

and variance

$$Var[Y] = b^2\sigma^2$$

□

2 The Baseline Model

Our baseline model offers students two financing options, private loan or ISA. We assume a perfectly competitive credit market. Therefore, debt financiers expect to receive the amount financed times $(1+r)$, or P as their only repayment. Below we simply refer to P as the principal. A student's utility function under a debt contract can be expressed as the principal subtracted from their wage expectation. Therefore, under the debt the lender will be payed back P.[2] So, by Lemmas 1 and 2, the certainty equivalent of the student under a pure debt contract is

$$CE_D = \mu - P - 0.5\rho\sigma^2 \tag{1}$$

Under an ISA the financier will get a share α of the graduate's income. Similar to debt financiers, ISA financiers expect to be returned the principal. Competition in the credit market then gives

$$P = \mathbb{E}[\alpha w] = \alpha \mathbb{E}[w] = \alpha\mu$$

Therefore, $\alpha = \frac{P}{\mu}$.[3] Applying Lemmas 1 and 2 to the ISA contract we see that the student's certainty equivalent is

$$CE_I = (1-\alpha)\mu - 0.5\rho(1-\alpha)^2\sigma^2$$

Substituting for α this becomes

$$CE_I = \mu - P - 0.5\rho\left(1 - \frac{P}{\mu}\right)^2 \sigma^2 \tag{2}$$

Proposition 1. *In the absence of moral hazard or adverse selection, the student strictly prefers to finance his education with an ISA if he is strictly risk-averse.*

Proof. Consider the following equivalent expressions

$$\begin{aligned} & CE_I > CE_D \\ \iff \quad & \mu - P - 0.5\rho\left(1 - \frac{P}{\mu}\right)^2 \sigma^2 > \mu - P - 0.5\rho\sigma^2 \\ \iff \quad & \rho\left(1 - \frac{P}{\mu}\right)^2 < \rho \end{aligned}$$

Because the last line holds iff $\rho > 0$, the claim is established. □

The logic holds under the implicit assumption that no student would be permitted to borrow more than their expected wage, formally: $\frac{P}{\mu} < 1$. Therefore, so long as a student exhibits some degree of risk aversion they will always choose to finance with the ISA. Under the baseline model

[2] In this model, there is implicitly no risk of default. See footnote 1 for justification.

[3] There is an implicit assumption here that the student cannot finance more than the expected value of his wealth: $P < \mu$

ISAs provide downside protection that a debt contract does not. Recall (1) and (2). Comparing the respective certainty equivalents, they differ only with respect to their risk premium. Under the ISA, this premium is smaller than under the debt due to the presence of the $(1-a)^2$ term. Once α is replaced by its exogenous pieces it is clear that under an ISA an individual is not penalized for sharing their income but instead is rewarded for sharing the risk.

3 Moral Hazard

Now Suppose that the mean of the wage distribution depends on the graduate's effort in searching for a job. In particular, to obtain a wage distribution with mean μ, the graduate must suffer effort cost $C(\mu) = 0.5\theta\mu^2$, where $\theta > 0$ is a parameter. θ can be thought of as a parameter measure for an individual's inherent efficiency or ability level (i.e. their effort needed to achieve a given wage). Certain proxies for θ could be IQ, undergraduate GPA, SAT/ACT score (etc.) all of which would inevitably weigh heavily in the underwriting process of ISA securities. As can be seen, $\mu\theta$ is the marginal cost of financing. The relationship between θ and "efficiency" is inverse, meaning that for two individuals with the same μ, the individual with a higher θ incurs a higher marginal cost of achieving a given wage. Therefore, factoring in effort cost, under a debt contract the student will choose μ to solve

$$\max_{\mu} \mu - 0.5\theta\mu^2 - P - 0.5\rho\sigma^2$$

The first-order condition gives $\mu_D = \frac{1}{\theta}$, yielding (indirect) certainty equivalent

$$0.5(1/\theta - \rho\sigma^2) - P$$

Under an ISA the graduate solves

$$\max_{\mu}(1-\alpha)\mu - 0.5\theta\mu^2 - 0.5\rho(1-\alpha)^2\sigma^2$$

The first-order condition reveals

$$\mu_I = \frac{1-\alpha}{\theta} < \frac{1}{\theta} = \mu_D$$

The graduate searches less aggressively because he must share his income with the financier. This effect is known as *moral hazard*. Substituting the first-order condition into the student's certainty equivalent yields

$$(1-\alpha)\mu_I - 0.5\theta\mu_I^2 - 0.5\rho(1-\alpha)^2\sigma^2 = 0.5(1-\alpha)^2(1/\theta - \rho\sigma^2)$$

Therefore in the presence of moral hazard the ISA is preferred to debt iff

$$0.5(1-\alpha)^2(1-\theta\rho\sigma^2) > 0.5(1-\theta\rho\sigma^2) - \theta P \tag{3}$$

We know that perfect competition in the ISA market implies

$$P = \alpha\mu_I$$

Substituting for μ_I from the first-order condition gives

$$\theta P = \alpha(1-\alpha) \tag{4}$$

Maximizing with respect to α:

$$\max_{\alpha} \alpha(1-\alpha) - \theta P$$

The first-order condition reveals

$$\alpha^* = \frac{1}{2}$$

Therefore the first-order condition implies

$$\alpha \in \left[0, \frac{1}{2}\right]$$

In other words, the ISA cannot claim more than half the graduate's income or he will not have incentive to search for a high-paying job. Note also that $\alpha \leq \frac{1}{2}$ implies $\theta P \leq \frac{1}{4}$, this bounds the amount the student can finance with an ISA. Now we must solve for α in terms of its exogenous variables. Solving the quadratic equation present in (4) gives

$$\alpha = \frac{1}{2} - \sqrt{\frac{1}{4} - \theta P} \tag{5}$$

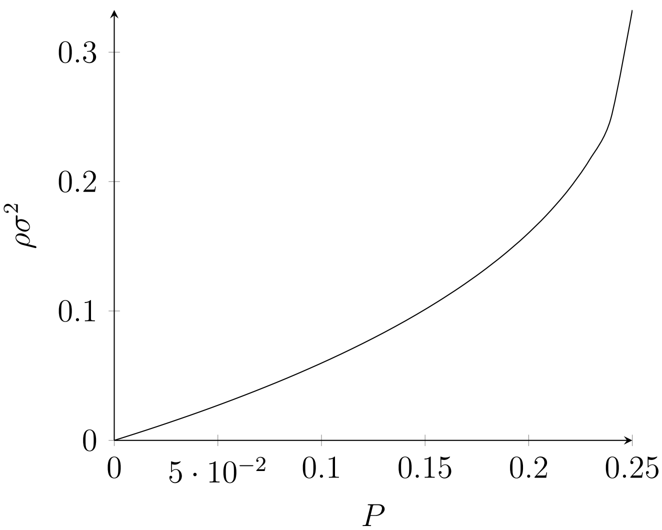
Substituting for α in (3) we see that the ISA is preferred to debt iff

$$\frac{1}{2}(1-\theta\rho\sigma^2)\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \theta P}\right)^2 > \frac{1}{2}(1-\theta\rho\sigma^2) - \theta P$$

or equivalently iff

$$\rho\sigma^2 > \frac{1}{\theta} - \frac{2P}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}} \tag{6}$$

Figure 1



The graph above represents an example of (6) holding θ constant ($\theta = 1$). If the student's risk premium for a given P lies in the area above the curve, they will prefer to finance with the ISA. Conversely, if their measured risk premium lies in the area below the curve, they will prefer to finance with debt.

Proposition 2. *As the student's Arrow-Pratt coefficient of absolute risk aversion increases, the ISA contract becomes more attractive relative to the debt contract.*

Proof. Consider the following equivalent expressions

$$\frac{\partial}{\partial \rho}[CE_I] > \frac{\partial}{\partial \rho}[CE_D]$$

$$\iff \quad \frac{\partial}{\partial \rho}\left[\rho\sigma^2\right] > \frac{\partial}{\partial \rho}\left[\frac{1}{\theta} - \frac{2P}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}}\right]$$

$$\iff \quad \sigma^2 > 0$$

Because the last line holds iff $\sigma^2 > 0$, the claim is established. □

This logic holds under similar intuition as in Proposition 1. Under the ISA, a student shares the risk between himself and the investor. Therefore, as the student's tolerance for risk decreases (as ρ increases) they value the contract that offers a risk-sharing component relatively more.

Proposition 3. *As the variance of the wage distribution increases, the ISA contract becomes more attractive relative to the debt contract.*

Proof. Consider the following equivalent expressions

$$
\begin{aligned}
& & \frac{\partial}{\partial \sigma^2}[CE_I] > \frac{\partial}{\partial \sigma^2}[CE_D] \\
\iff & & \frac{\partial}{\partial \sigma^2}\left[\rho\sigma^2\right] > \frac{\partial}{\partial \sigma^2}\left[\frac{1}{\theta} - \frac{2P}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}}\right] \\
\iff & & \rho > 0
\end{aligned}
$$

Because the last line holds iff $\rho > 0$, the claim is established. □

For similar reasons outlined above in Proposition 2, the individual is giving more weight to the risk parameter as σ^2 increases. This intuitively makes sense. For example, in a highly volatile wage environment, individuals will face high fixed costs under a debt contract which will pose greater uncertainty to wealth outcomes. Conversely, under an ISA in such market conditions, the individual will only be responsible for repaying a set proportion of the income they make, regardless of that amount; i.e. the amount owed for repayment under an ISA is variable (more flexible) compared to the fixed amount owed under debt (P). This yields downside protection as the increase in the risk premium under the ISA is smaller than the increase in the risk premium under the debt.

Proposition 4. *In the absence of adverse selection, as θ increases (as student ability decreases), the debt contract becomes more attractive relative to the ISA contract.*

Proof. Consider the following equivalent expressions

$$
\begin{aligned}
& & \frac{\partial}{\partial \theta}[CE_I] < \frac{\partial}{\partial \theta}[CE_D] \\
\iff & & \frac{\partial}{\partial \theta}\left[\rho\sigma^2\right] < \frac{\partial}{\partial \theta}\left[\frac{1}{\theta} - \frac{2P}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}}\right] \\
\iff & & 0 < -\frac{1}{\theta^2} + \frac{2P^2}{\left(\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}\right)^2}\left(1 + \frac{1}{2\sqrt{\frac{1}{4} - \theta P}}\right) \\
\iff & & \frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P} < \theta P\left(2 + \frac{1}{\sqrt{\frac{1}{4} - \theta P}}\right)^{\frac{1}{2}}
\end{aligned}
$$

This inequality holds so long as $0 < \theta P < \frac{1}{4}$. When maximizing α in (4) we found that θP always lies between 0 and $\frac{1}{4}$. Additionally, when the inequality above is flipped it has no solution.

Therefore, we can conclude that as θ increases the debt contract becomes more attractive; the claim is established. $\square$

At first this relationship does not appear to make intuitive sense. The outputted relationship implies that as an individual's ability level decreases (as θ increases), thereby decreasing expected earning potential, the debt contract will become more attractive. Intuitively, we would think the opposite would be occurring — that the ISA would become more attractive in this scenario. This logic is derived from the fact that as an individual's effort cost increases (as their θ increases), they begin to incur a higher cost to achieving a given wage, and thus experience a corresponding loss in utility from such an increase. In the case of the debt contract, the wage loss falls completely on the individual, whereas in the case of the ISA, this wage loss is shared between the individual and the financier. Thus, it would be the inherent assumption that the corresponding loss in utility is less under the ISA than under the debt contract. However, this is not the case. Why?

This is a result of multiple factors. First, this has to do with the assumption of perfect information within the underwriting market. By nature of the ISA, individuals must reveal their θ throughout the underwriting process (giving financiers access to their undergraduate GPA or test scores, taking an IQ test, etc.). Students themselves are incentivized to do so as it will allow them to achieve better financing terms (a lower proportion of income owed, α). Because of this perfect information, the financiers are able to effectively price a student's θ into their financing terms (α).

Although it is clear that a higher θ results in the debt contract becoming more attractive, the mechanics behind this are slightly more nuanced. For example, consider a student's effort cost function, $0.5\theta\mu^2$. There is a clear direct effect, as θ increases the effort cost increases. However, there is also an indirect effect. Recall, $\mu_I = \frac{1-\alpha}{\theta}$, as well as (5). Therefore, when written in terms of it's exogenous components, the effort cost function becomes

$$\frac{1}{2\theta}\left(1-\left(\frac{1}{2}-\sqrt{\frac{1}{4}-\theta P}\right)\right)^2$$

By taking the partial derivative of the above effort cost function with respect to θ,

$$\frac{\partial}{\partial\theta}\left[\frac{1}{2\theta}\left(1-\left(\frac{1}{2}-\sqrt{\frac{1}{4}-\theta P}\right)\right)^2\right] < 0$$

$$\iff \quad -\frac{1}{4\theta^2}-\frac{P}{4\theta\sqrt{\frac{1}{4}-\theta P}}-\frac{\sqrt{\frac{1}{4}-\theta P}}{2\theta^2} < 0$$

We can see that as θ increases the overall impact on a student's effort cost is negative - which is contrary to the direct effect and our initial intuition. However, we must examine the relationship between μ_I and θ. We see that as θ increases, a student's μ_I decreases. Therefore, a student's effort cost is experiencing a simultaneous direct effect - an increase in effort cost via an increase in θ - and an indirect decrease in effort cost through a decrease in μ_I. What we see is that this indirect effect on μ_I dominates the direct effect as shown above. With further analysis, this intuition does hold, as what is occurring is that the student now optimizes at a lower μ_I, therefore the cost associated with searching for a lower wage is also lower. This decrease lowers the effort of the student, dominating the direct effect increase.

Now, let us consider the impact on a student's risk premium, $0.5\rho(1-\alpha)^2\sigma^2$. A similar relationship exists, relative to the one explored above. Again, recall (5). Therefore, when written in terms of it's exogenous components, the risk premium becomes

$$0.5\rho\sigma^2\left(1-\left(\frac{1}{2}-\sqrt{\frac{1}{4}-\theta P}\right)\right)^2$$

By taking the partial derivative of the above risk premium with respect to θ,

$$\begin{aligned} & \frac{\partial}{\partial\theta}\left[0.5\rho\sigma^2\left(1-\left(\frac{1}{2}-\sqrt{\frac{1}{4}-\theta P}\right)\right)^2\right] < 0 \\ \iff \qquad & -0.5\rho\sigma^2\left(\frac{P}{\sqrt{\frac{1}{4}-\theta P}}+P\right) < 0 \end{aligned}$$

We can see that as θ increases, the impact on a student's risk premium is negative. Intuitively, as θ goes up α goes up. Therefore, from the student's perspective more risk is being shared with the ISA financier and the overall risk premium decreases.

Lastly, let us consider the impact on a student's financing terms, α. This can be shown below, where the student's proportion of income owed to the lender is a function of their ability (θ). Recall, (5). By taking the partial derivative with respect to θ, it can clearly be seen that α is increasing with θ.

$$\frac{\partial}{\partial\theta}[\alpha] = \frac{P}{2\sqrt{\frac{1}{4}-\theta P}} > 0$$

Therefore, as a student's θ increases they experience marginal utility loss due to worsening financing terms. This marginal utility loss dominates the marginal utility gain a student experiences through the decrease in their effort cost and risk premium. Thus, what is occurring in the ISA market is that the lenders are appropriately pricing the contracts to fit the risk profile of the individual - which they are able to do because of the robust underwriting process. Moreover, the pricing of the debt contract is not dependent on θ. Rather, a student is not penalized for being of low ability (high θ); i.e. you are not being penalized by the financier under the debt because they continue to underwrite you the same way (irrespective of the change in θ). However, it is important to note, that there is a corresponding loss in utility under the debt instrument from an increase in θ. Recall, that $\mu_D = \frac{1}{\theta}$, therefore an increase in θ leads to a decrease in μ_D, thereby lowering the certainty equivalent of the individual (loss in utility). Therefore, as a student's ability decreases (as θ increases), ISA financiers adjust repayment rates while debt financiers do not. Ultimately, this leads to a larger loss in utility under the ISA contract for the student.

For this reason, while initially not intuitive, we see that as θ increases, the debt contract becomes more attractive as students are not as heavily penalized for the increased effort cost under this arrangement, compared to under the ISA where the student is now forced to give up a larger proportion of their income (α increases).

Proposition 5. *As the student borrows more, the debt contract becomes more attractive relative to the ISA contract.*

Proof. Consider the following equivalent expressions

$$\begin{aligned}
& \frac{\partial}{\partial P}\left[CE_I\right] < \frac{\partial}{\partial P}\left[CE_D\right] \\
\iff \quad & \frac{\partial}{\partial P}\left[\rho\sigma^2\right] < \frac{\partial}{\partial P}\left[\frac{1}{\theta} - \frac{2P}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}}\right] \\
\iff \quad & 0 < -\frac{2}{\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}} + \frac{2P}{\left(\frac{1}{2} + \theta P - \sqrt{\frac{1}{4} - \theta P}\right)^2}\left(\theta + \frac{\theta}{2\sqrt{\frac{1}{4} - \theta P}}\right) \\
\iff \quad & 0 < \theta^2 P^2
\end{aligned}$$

Because the last line holds iff $\theta^2 P^2 > 0$, this claim is established. □

The intuition behind this relationship is similar to the case of θ. The pricing of the ISA contract is determined by α, or the proportion of income owed back to the financier. α is a function of P, where it can be shown that α is increasing in P. Again, recall (5). By taking the partial derivative with respect to P, it can clearly be seen that α is increasing with P.

$$\frac{\partial}{\partial P}\left[\alpha\right] = \frac{\theta}{2\sqrt{\frac{1}{4} - \theta P}} > 0$$

Thus, similar to above, it can be seen that the student is more penalized under the ISA (as the financing terms get disproportionately less favorable compared to the debt contract) and prefers the debt more as P increases.

4 Optimal Mixed Financing

Now suppose the individual is able to finance with a combination of debt and an ISA instrument. The new certainty equivalent is subject to the constraint

$$P = P_I + P_D$$

Applying Lemmas 1 and 2 and substituting the constraint into the objective, the mixed financing certainty equivalent becomes

$$\max_{P_I} \frac{1}{2}(1-\theta\rho\sigma^2)\left(\frac{1}{2} - \theta P_I + \sqrt{\frac{1}{4} - \theta P_I}\right) - \theta(P - P_I)$$

This is quasi-linear in P_I and hence an interior solution will not depend on P. Under an ISA contract, an individual's optimal wage is a function of P: $\mu_I = \frac{(1-\alpha)}{\theta}$, where $\alpha = \frac{1}{2} - \sqrt{\frac{1}{4} - \theta P}$. Comparatively, an individual's optimal wage under a debt contract is independent of P: $\mu_D = \frac{1}{\theta}$. Therefore, when solving the optimization problem outlined above, the individual will choose to optimize their wage with respect to the ISA wage constraint. The individual will first choose to maximize their funding under the ISA, with any excess funding fulfilled by the debt ($P_D = P - P_I$). Solving, the first-order condition reveals

$$P_I^* = \frac{\rho\sigma^2}{(1+2\theta\rho\sigma^2)^2}$$

Where

$$P_D^* = P - \frac{\rho\sigma^2}{(1+2\theta\rho\sigma^2)^2}$$

This is clearly decreasing in θ and can be shown to be increasing in $\rho\sigma^2$ for P_I^*, and conversely is increasing in θ and can be shown to be decreasing in $\rho\sigma^2$ for P_D^*.

These results closely follow the intuition of Proposition 4 from Section 3 (Moral Hazard). Regarding θ, recall that as a student's ability level decreases they are disproportionately penalized under the ISA relative to the debt. Because of this, the logic holds that under the optimal mixed bundle, P_I^* is decreasing and P_D^* is increasing in θ.

Again, in-line with Section 3, following the intuition of Propositions 2 and 3, we see that as

$\rho\sigma^2$ increases, this represents an increase in the risk premium for the individual. This increase makes the ISA contract relatively more attractive as it offers downside protection to the borrower by nature of the risk being shared between the lender and the student. This contrasts the greater risk undertaken with a debt contract which is levied entirely onto the individual. For this reason, we see that P_I^* is increasing and P_D^* is decreasing in $\rho\sigma^2$.

5 Adverse Selection

Abstracting from issues of moral hazard, suppose μ is uniformly distributed between 0 and 1 throughout the population. Student's privately know their own value of μ. This presents a case of asymmetric information on behalf of the lenders and borrowers. With the opportunity to exploit this asymmetric information,the borrowers give rise to the risk of adverse selection within the financing market. Due to the presence of asymmetric information, high earners may not be willing to pool with low earners on the same ISA contract.[4] Due to the risk being shared between borrower and lender under the ISA contract, suppose that all students with $\mu \leq \mu^*$ finance with an ISA and all with $\mu > \mu^*$ finance with debt. Note that

$$\mathbb{E}[\mu \mid \mu \leq \mu^*] = \int_0^{\mu^*} \frac{\mu}{\mu^*}\, d\mu = \frac{\mu^*}{2}$$

The market anticipates that only students with relatively low expected wages will finance with an ISA so

$$P = \alpha\mathbb{E}[\mu \mid \mu \leq \mu^*] \iff \alpha = \frac{2P}{\mu^*}$$

A student with $\mu = \mu^*$ must be indifferent between financing with an ISA and with debt so

$$(1-\alpha)\mu^* - \frac{1}{2}\rho(1-\alpha)^2\sigma^2 = \mu^* - P - \frac{1}{2}\rho\sigma^2$$

Substituting for α, and rearranging this becomes

$$\frac{{\mu^*}^2}{2\rho\sigma^2} - \mu^* + P = 0$$

$$\rho\sigma^2 = \frac{{\mu^*}^2}{2(\mu^* - P)}$$

[4]For this reason there must generally be one student type who is indifferent between pooling and choosing debt.

Rearranging and solving for μ^* in terms of the exogenous parameters via the quadratic formula yields

$$\mu^* = \rho\sigma^2 \pm \sqrt{\rho\sigma^2(\rho\sigma^2 - 2P)}$$

To observe when there is no adverse selection, we can set $\mu^* = 1$. We see that all students choose to finance with the ISA iff

$$\rho\sigma^2 \geq \frac{1}{2(1-P)}$$

However, by rearranging and flipping the inequality, we can see that if P is too large ($P \to 1$), the ISA market may not exist due to adverse selection.[5] Formally,

$$P \geq 1 - \frac{1}{2\rho\sigma^2}$$

As can be seen above, if P is sufficiently large, then adverse selection will occur. This intuitively makes sense. In the absence of risk-aversion (as ρ tends to 0) the right side of the equation tends to $-\infty$ as individuals will adversely select out of the ISA pool. The intuition behind this adverse selection holds, as one of the key benefits offered by ISA contracts is the risk-sharing afforded to borrowers. However, as demonstrated in the absence of risk-aversion, individuals may achieve higher utility from the debt.

6 Adverse Selection with Two Types

Ultimately we would like to consider both moral hazard and adverse selection. As a preliminary step, suppose that there are two types of students in the population, a fraction ϕ of them have high expected wages μ_H and the complementary fraction $1-\phi$ have low expected wages μ_L, where $\mu_H > \mu_L > 0$. The expected earnings over the entire population is

$$\mathbb{E}[\mu] \equiv \phi\mu_H + (1-\phi)\mu_L$$

Suppose that students know which type, H or L, they are but that financiers cannot observe this ahead of time.

As we know, other things equal, a risk-averse individual prefers to finance his education with an ISA and this is a socially preferable allocation of risk. In the current setting, however, the market will offer the same ISA contract to both high and low expected earners and the high types may prefer to finance with debt rather than pool with low types. In other words, there may be adverse selection.

[5] Referencing footnote 3, an inherent assumption is that a student cannot borrow more than their expected wage, μ. Therefore, setting $\mu^* = 1$ we bound $P (P \in [0, 1])$.

To see when adverse selection is a problem, we first solve the model assuming that it is not; i.e., that high types are willing to accept the same contract as low types. If this is the case, then competition in the finance market implies

$$P = \alpha \mathbb{E}[\mu] \implies \alpha = \frac{P}{\mathbb{E}[\mu]}$$

The certainty equivalent of a high type who finances with such an ISA is

$$(1-\alpha)\mu_H - 0.5\rho\sigma^2(1-\alpha)^2,$$

and the certainty equivalent if he finances with debt is

$$\mu_H - P - 0.5\rho\sigma^2$$

Therefore adverse selection will occur if

$$(1-\alpha)\mu_H - 0.5\rho\sigma^2(1-\alpha)^2 < \mu_H - P - 0.5\rho\sigma^2$$

Substituting for α and rearranging gives

$$\frac{\mu_H}{\mathbb{E}[\mu]}P - P > 0.5\rho\sigma^2\left(1 - \left(1 - \frac{P}{\mathbb{E}[\mu]}\right)^2\right)$$

For simplification purposes and to better understand the underlying intuition behind this expression, it can be rewritten as the following

$$\alpha\mu_H - P > .5\rho\sigma^2\alpha\left(2-\alpha\right)$$

Both sides are positive. The left side is the loss in expected income from pooling with the low wage earners; i.e., the benefit from switching to debt. This can be seen as it is represented as the product of α and μ_H (i.e. the expected amount owed under an ISA for a high income borrower) minus P (the amount to be repaid under debt). If this difference is positive, then it clearly shows that the high income earner is paying more under the ISA than under the debt instrument, thus this can be thought of as the loss in expected income from pooling with the low wage earners. However, this difference must still be greater than the right side for the inequality to hold and for the individual to elect to choose debt. This is because the right side represents the increase in the risk premium from switching to debt; i.e., the cost. This is clearly shown by the presence of ρ and σ^2 which define the risk premium. Returning to the original derived form

$$\frac{\mu_H}{\mathbb{E}[\mu]}P - P > 0.5\rho\sigma^2\left(1 - \left(1 - \frac{P}{\mathbb{E}[\mu]}\right)^2\right)$$

Further algebra renders the following expression

$$(1-\phi)(\mu_H - \mu_L) > 0.5\rho\sigma^2\left(2 - \frac{P}{\mu_H - (1-\phi)(\mu_H - \mu_L)}\right) \tag{7}$$

Similar to the previous form, the left side of the expression is the product of the proportion of low income earners and the wage gap between high type and low type individuals. The right side represents the risk premium.

Proposition 6. *As the proportion of high income earners tends to zero ($\phi \to 0$), the propensity for adverse selection increases.*

Proof. Consider the following equivalent expressions

$$\begin{aligned}
& \frac{\partial}{\partial(1-\phi)}\left[CE_D\right] > \frac{\partial}{\partial(1-\phi)}\left[CE_I\right] \\
\iff \quad & \frac{\partial}{\partial(1-\phi)}\left[(1-\phi)(\mu_H - \mu_L)\right] > \frac{\partial}{\partial(1-\phi)}\left[0.5\rho\sigma^2\left(2 - \frac{P}{\mu_H - (1-\phi)(\mu_H - \mu_L)}\right)\right] \\
\iff \quad & (\mu_H - \mu_L) > \frac{-0.5\rho\sigma^2 P\left(\mu_H - \mu_L\right)}{\left[\mu_H - (1-\phi)(\mu_H - \mu_L)\right]^2} \\
\iff \quad & 1 > \frac{-0.5\rho\sigma^2 P}{\left[\mu_H - (1-\phi)(\mu_H - \mu_L)\right]^2}
\end{aligned}$$

The right side of the inequality is clearly negative, therefore, the claim is established. □

This intuition holds as $\phi \to 0$ implies that there will be a higher proportion of low-income earners in the wage pool. Because of this, ISA financing terms will become less favorable across the population (α will increase). Due to the presence of asymmetric information, high-earning individuals will be disproportionately penalized, thereby incentivizing them to adversely select out of the ISA pool in order to obtain better financing terms.

Proposition 7. *As the wage gap increases, the propensity for adverse selection increases.*

Proof. Consider the following equivalent expressions

$$
\begin{aligned}
& & \frac{\partial}{\partial(\mu_H - \mu_L)}\left[CE_D\right] &> \frac{\partial}{\partial(\mu_H - \mu_L)}\left[CE_I\right] \\
\iff & & \frac{\partial}{\partial(\mu_H - \mu_L)}\left[(1-\phi)(\mu_H - \mu_L)\right] &> \frac{\partial}{\partial(\mu_H - \mu_L)}\left[0.5\rho\sigma^2\left(2 - \frac{P}{\mu_H - (1-\phi)(\mu_H - \mu_L)}\right)\right] \\
\iff & & (1-\phi) &> \frac{-0.5\rho\sigma^2 P\,(1-\phi)}{\left[\mu_H - (1-\phi)(\mu_H - \mu_L)\right]^2} \\
\iff & & 1 &> \frac{-0.5\rho\sigma^2 P}{\left[\mu_H - (1-\phi)(\mu_H - \mu_L)\right]^2}
\end{aligned}
$$

The right side of the inequality is clearly negative, therefore, the claim is established. □

Similar to the intuition of Proposition 6, due to the presence of asymmetric information, as the wage gap increases, the financing terms for a high-earning individual become disproportionately worse as they are pooled with low-earning individuals under the ISA.

Proposition 8. *As the risk premium ($\rho\sigma^2$) increases, the propensity for adverse selection decreases.*

Proof. Consider the following equivalent expressions

$$
\begin{aligned}
& & \frac{\partial}{\partial(\rho\sigma^2)}\left[CE_D\right] &< \frac{\partial}{\partial(\rho\sigma^2)}\left[CE_I\right] \\
\iff & & 0 &< 0.5\left(2 - \frac{P}{\mu_H - (1-\phi)(\mu_H - \mu_L)}\right)
\end{aligned}
$$

The right side of the inequality is clearly positive, therefore, the claim is established. □

As the risk premium increases, the propensity for adverse selection decreases as the student puts increased weight on the value of the risk-sharing structure under the ISA.

Proposition 9. *As the student borrows more (P increases), the propensity for adverse selection increases.*

Proof. Consider the following equivalent expressions

$$
\begin{aligned}
& & \frac{\partial}{\partial P}\left[CE_D\right] &< \frac{\partial}{\partial P}\left[CE_I\right] \\
\iff & & 0 &> -\frac{.5\rho\sigma^2}{\mu_H - (1-\phi)\left(\mu_H - \mu_L\right)}
\end{aligned}
$$

The right side of the inequality is clearly negative, therefore, the claim is established. □

The intuition here is less straightforward. This relationship establishes that as the student borrows more, they are more likely to adversely select out of the ISA contract. This goes back to the relationship between α and P. α is a function of P, and thus, as P increases, by nature of ISA contracts requiring a proportion of the student's income as repayment, the take-home wage of the borrower decreases more than one-to-one (as is the case under a debt contract); i.e. they now owe a higher percentage of their income. Thus, while not initially intuitive, as P increases, so too does the propensity for adverse selection as high-earners begin to face relatively less favorable income-share terms.

Recall (7). Note that when the inequality holds so that adverse selection occurs, then the finance market anticipates that only low types use ISAs, therefore $\alpha = \frac{P}{\mu_L}$. In the absence of adverse selection, low-earners were benefiting from pooling because of cross subsidization by high-earners. This produces an efficiency loss, for if there was not asymmetric information, then high-earners would be able to secure separate (and better) ISA financing terms and be better off than from opting for debt.

7 Combining Moral Hazard and Adverse Selection

Finally, return to the moral hazard setting in which the cost of obtaining expected income μ requires effort cost by the graduate of $0.5\theta\mu^2$. Suppose, however, that there are two types of students in the population. A fraction ϕ have a low effort cost $0.5\theta_1\mu^2$ and the complementary fraction $1-\phi$ have high effort cost $0.5\theta_2\mu^2$, where $\theta_2 > \theta_1 > 0$. As in the previous section we first solve the model in the case adverse selection does not occur. Under an ISA the type i graduate solves

$$\max_{\mu}(1-\alpha)\mu - 0.5\theta_i\mu^2 - 0.5\rho(1-\alpha)^2\sigma^2$$

The first-order condition reveals

$$\mu_i = \frac{1-\alpha}{\theta_i} \quad i = 1, 2$$

Note that the low cost students have higher expected earnings. Substituting the first-order condition into the student's certainty equivalent yields

$$(1-\alpha)\mu_i - 0.5\theta_i\mu_i^2 - 0.5\rho(1-\alpha)^2\sigma^2 = 0.5(1-\alpha)^2(1/\theta_i - \rho\sigma^2), \quad i = 1, 2$$

Note that the low cost students' (θ_1) certainty equivalent is higher due to their higher expected earnings. Define $\mathbb{E}[\mu] = \phi\mu_1 + (1-\phi)\mu_2$. If adverse selection does not occur, then

$$P = \alpha\mathbb{E}[\mu] \implies \alpha = \frac{P}{\mathbb{E}[\mu]}$$

Therefore, the low-cost student's certainty equivalent if adverse selection does not occur is

$$0.5\left(1-\frac{P}{\mathbb{E}[\mu]}\right)^2\left(\frac{1}{\theta_1}-\rho\sigma^2\right)$$

If the low-cost students do not pool with the high-cost ones, then there are two effects. They benefit from better financing terms (as in the previous section) and they also benefit from lack of moral hazard. Specifically, a low-cost student who finances with debt solves

$$\max_{\mu}\mu-0.5\theta_1\mu^2-P-0.5\rho\sigma^2$$

The first-order condition reveals $\hat{\mu}_1=1/\theta_1$. This yields (indirect) certainty equivalent of

$$\frac{1}{2\theta_1}-P-0.5\rho\sigma^2$$

Therefore, the low-cost students will not finance with an ISA if

$$\frac{1}{2\theta_1}-P-0.5\rho\sigma^2>0.5\left(1-\frac{P}{\mathbb{E}[\mu]}\right)^2\left(\frac{1}{\theta_1}-\rho\sigma^2\right)$$

Going forward, we will seek to maximize α with respect to the exogenous variables. This maximization differs from Section 3 (Moral Hazard) in that it is dependent on two individual types, low-cost and high-cost. In the meantime, we can examine the propensity for adverse selection depending upon movements in ρ, σ^2, and $E[\mu]$. Previous Propositions regarding changes in risk coefficients hold for both ρ and σ^2 in the above relationship, following the same underlying intuition that risk-sharing decreases the propensity for adverse selection. Additionally, the right side can be shown to be increasing in $\mathbb{E}[\mu]$, thereby decreasing the propensity for adverse selection, albeit the intricacies of this relationship are ultimately dependent on a number of exogenous parameters, including, but not limited to θ_1, θ_2, and ϕ which will be explored in subsequent research.

Conclusion

As stated, the current theoretical understanding of Income Share Agreements is largely unexplored. While undertaking this paper, we hoped to add a theoretical foundation to the nascent literature in the space, so as to hopefully promote further research and understanding of such securities in the future. Some key takeaways and areas of additional research and interest for us include developing a more robust understanding of θ – particularly in how it relates to the pricing and preference for ISA securities. θ is a parameter that represents a student's ability level and subsequent marginal cost of financing. A student with a low θ will incur lower marginal cost to achieve a given wage relative to a student with a high θ. Therefore, from a financing prospective, low θ students will have access to cheaper financing options under an ISA. Proxies for θ could be, IQ, undergraduate GPA, SAT/ACT. We ultimately feel this parameter will be extremely valuable in the evolution of the ISA underwriting process and hope to develop a more robust understanding of how it can impact a student's ability to earn a given wage. Additionally, we hope to look further into quantifying risk parameters when evaluating ISA contracts. Further understanding in these two dimensions will improve upon our current research, and hopefully expand the opportunity for future theoretical and empirical applications.

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