A Theory of Ambiguous Property Rights: Choices, Efficiencies, and Transition Paths

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A group of countries has transited from centrally planned economies into market economies in the past several decades. Although the transition process of these transitional economies diverges to a large extent, ambiguous property rights often prevail during this process. This paper proposes a micro-economic theory to study the choice between ambiguous and unambiguous property rights, the resulting social efficiency, and the transition process. I find the two types of property rights possess the same efficiency in both short-run and long-run equilibrium. Moreover, the convergence paths toward the long-run equilibrium deriving from my model reconciles those observed divergent transition processes. This study implies that in order to improve long-run market environments and social efficiencies, policymakers in transitional economies should prioritize legal and institutional reforms instead of solely focusing on enterprise reforms. (*JEL* D23, O12, P21)

I. Introduction

A group of countries has transited from centrally planned economies into market economies in the past several decades. The transition process from centrally planned economies into market economies is worthwhile for economic research, since a smooth transition process can foster a significant and sustainable improvement on the economic welfare of a society. An interesting phenomenon is that several distinct transition paths appear in the economy history. To be specific, some governments become stronger in this process, while others weaken. Some economies become more efficient during their transitions, while others turn out to be less efficient. Some countries keep evolving, while others stagnate. Another interesting phenomenon is that ambiguous property rights often prevail during this process.

This paper proposes a micro-economic theory to study the choice between ambiguous and unambiguous property rights, the resulting social efficiency, and the transition process. In my paper, I extend the conventional theories at two aspects.

First, Li (1996) proposes a theory of ambiguous property rights with exogenous

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market environment and government bargaining power. However, local governments may promulgate rules and regulations to influence the market environment and encroach private firms. This is not uncommon in reality, since there is no commitment based on law and institutions designed to prevent the local governments from encroaching on private enterprises (Che and Qian, 1998). So, in my model, the market environment is endogenously determined by the government.

Second, I also introduce a top-level government to determine the bargaining power of its local agencies endogenously. Intuitively, changing market environment takes much less time than modifying government's bargaining power. So there is some stickiness in government's bargaining power but such stickiness does not exist in market environment.

Lai and Lorne (2013) summarize several prominent studies on ambiguous property rights. The comparison between ambiguous and unambiguous property rights constitutes a major part of previous research. Li (1996) and Le (2004) argue that ambiguous property rights may prove to be more efficient than the private holding arrangement. I find that the ambiguous property rights arrangement can be favorable over the unambiguous one only in off-equilibrium situations. Indeed, the two types of property rights have to possess the same efficiency in both short-run and long-run equilibrium. Several empirical studies, such as Svejnar (1990) and Weitzman and Xu (1994) reports similar findings that the productive efficiency does not vary systematically among several types of ownership after differences in inputs and other variables are controlled.

In an empirical study about the emerging real estate market in China, Zhu (2002) argues that the problem of ambiguous property rights derived from gradualism in the reforms does not lead to an efficient allocation of development resources. The long-run equilibrium derived from my model conform this result. I also point out the reason why an efficient allocation with clearly defined property rights is hard to achieve.

In the transition process of a transitional economy, the government plays a crucial role in the reforms. A broad class of previous studies examines functions, structures, and interactions of the government sector. Shleifer and Vishny (1993) develop a model to analyze corruption. In my model, to simplify the analysis, I ignore the conflict of interests between different levels of a government. This simplification not only makes the model tractable, but also elucidates the influences of government behaviors on transitional process. Hart, Shleifer and Vishny (1997) discuss the optimal scope of the government. They compare the choice between in-house provision and out-sourcing when providing public goods in the government's perspective. However, in my model, both of the entrepreneur and the local government involve in the decision-making process regarding the choice of property rights. Moreover, the discussion is not about providing public goods but pertaining to produce commercial goods. (Le, 2004) proposes a sim-

ilar three-agent model with an entrepreneur, a local government, and a central government. However, instead of studying the interactions between the local and central government, I adopt this similar setting to analyze the transition process of transitional economies.

II. A Motivating Story

This section offers a motivating story in order to illuminate the intuition. The inspiration of this story comes from Gang and Tower (1988).

A. The Story

Consider an economy made up of a Giant, a Midget, some golden eggs and shovels. Both of the Giant and the Midget have strictly increasing utilities in golden eggs. Suppose 100 golden eggs appear in this economy everyday at a random point of time. These eggs are perfectly perishable, which means they will disappear at the end of the day if they remain uncollected. To collect these eggs, the Giant needs to prepare some shovels in advance as the initial investment. In fact, the Giant is the only one who is trained to use shovels. So only the Giant is capable to pick up the golden eggs.

A day is divided into two parts, namely day and night. Suppose daytime and nighttime during a day are fixed at 4/5 day and 1/5 day respectively. An interesting fact is that the golden eggs are discernible only at daytime. Although being incompetent for picking up eggs, the Midget possesses the ability to use certain magics. By casting a spell over these eggs, 1/2 of the total number of golden eggs start to flash and glitter in the dark.

There are only two feasible ways for these two agents to accomplish the task of picking up eggs. Firstly, the Giant may work alone. To be specific, if the eggs appears during the daytime, the Giant will get all of the eggs. If, unfortunately, the eggs appear during nighttime, the Giant will seek helps from the Midget. As a return for the help, 1/2 of the harvest goes to the pocket of the Midget. Alternatively, the Giant may work together with the Midget as a team. In this case, the two agents accomplish the task at first, and then divide the total harvest by a fight. The Giant, of course, has an advantage when fighting with the Midget. As a result, the Giant will receive 4/5 of the total harvest in the end.

The two agents meet at the very beginning of each day to negotiate the form in which they cooperate. The one inducing the larger amount of initial investment on shovels is chosen, since it is considered as more efficient. Assume that the initial investment is commensurate with the expected number of golden eggs being distributed to the Giant. Thus, the proposal from which the Giant gains more is adopted.

B. Outcomes

The story above completes the start-up setting for my further analysis. Now, with this start-up setting, one may have the following question in mind.

QUESTION 1: *What is the outcome according to this setting?*

Table 1 summarizes the main results.

Table 1—The Start-up Setting

(1) Form	(2) Time	(3) Probability	(4) Total	(5) Share	(6) Expected	(7) Efficiency
Alone	Day	0.80	100	1.00	85	0.85
	Night	0.20	50	0.50		
Together	Day	0.80	100	0.80	72	0.72
	Night	0.20	50	0.80		

The first column shows the two feasible forms of cooperation. The second and third columns collect the probabilities of daytime and nighttime when the golden eggs appear. According to the story, the probability of daytime is 4/5 or 0.80 and the probability of nighttime is 1/5 or 0.20 at a particular point of the day. The fourth column records the total number of golden eggs picked up by the Giant. If the golden eggs show up during daytime, all of them (100 golden eggs) will be collected. However, if these eggs appear at night, only a half of them (50 golden eggs) will be discovered and gathered.

How many golden eggs will eventually go to the pocket of the Giant? Without forming a team, the Giant receives 100% or 1.00 of the eggs collected during daytime (100 golden eggs) and 50% or 0.50 of the eggs picked up at night (25 golden eggs). Working as a team, the Giant receives 4/5 or 0.80 of the eggs collected either during daytime (80 golden eggs) or at night (40 golden eggs). This information is recorded in the fifth column.

Based on the preceding information, the expected number of golden eggs the Giant received is

$$0.80 \times 100 \times 1.00 + 0.20 \times 50 \times 0.50 = 85$$

in the first case and

$$0.80 \times 100 \times 0.80 + 0.20 \times 50 \times 0.80 = 72$$

in the second case. The results from these calculations are noted down in the sixth column.

The expected number of eggs received by the Giant attains its maximum, 100 golden eggs, when the Giant works alone given the probability of daytime is 1.00. Since the initial investment is in commensurate with the expected number of eggs received, the initial investment also reaches its maximum. So the economy at the most efficient situation. Comparing to this situation, the relative efficiency is 85/100 or 0.85 in the first case and 72/100 or 0.72 in the second case. This derives the numbers in the last column.

From the last column of Table 1, the efficiency of the first case is higher than the second case. Therefore, the form of cooperation with the the start-up setting is that the Giant works without forming a team with the Midget.

In this story, the Midget uses the magics to discern eggs in darkness. However, what if the magic is so powerful that it can change the length of day and night? If so, is it possible for the second form of cooperation dominates the first one? So one may encounter the following question.

QUESTION 2: *What if nighttime becomes longer?*

Suppose the Midget casts a spell to reverse day and night. So the probability of daytime is now 1/5 or 0.20 and the probability of nighttime is now 4/5 or 0.80 at a particular point of the day. These changes are recorded at the third column of Table 2 with bold characters.

Table 2—A Setting with Longer Nighttime

(1) Form	(2) Time	(3) Probability	(4) Total	(5) Share	(6) Expected	(7) Efficiency
Alone	Day	**0.20**	100	1.00	40	0.40
	Night	**0.80**	50	0.50		
Together	Day	**0.20**	100	0.80	48	0.48
	Night	**0.80**	50	0.80		

As a result, the expected number of golden eggs the Giant received decreases from 85 to 40 in the first case and drops down from 72 to 48 in the second case. These results show up in the sixth column. Correspondingly, in the last column of Table 2, the efficiency declines to 0.40 and 0.48 respectively.

The last column of Table 2 shows that the efficiency of the first case is now lower than the second case. Therefore, the form of cooperation with longer nighttime is that the Giant teams up with the Midget.

The enhanced magics are capable to change the length of day and night. Can you imagine the magics become even more powerful? Can you imagine that the magics is powerful enough to turn a Midget into a Giant? If the Midget in our

story becomes a giant who is well-trained in using magics instead of shovels, will our conclusion change? So a third question is encountered.

QUESTION 3: *What if the Midget becomes a giant?*

Suppose, magically, the Midget becomes a giant. So when working as a team, the Midget has less disadvantage fighting with the Giant for a larger share of the harvest. So the Giant can only receive 1/2 or 0.50 of the total golden eggs collected now. These changes are showed at the fifth column of Table 3 with bold characters.

Table 3—A Setting with a "Midget" giant

(1) Form	(2) Time	(3) Probability	(4) Total	(5) Share	(6) Expected	(7) Efficiency
Alone	Day	0.20	100	1.00	40	0.40
	Night	0.80	50	0.50		
Together	Day	0.20	100	**0.50**	30	0.30
	Night	0.80	50	**0.50**		

As a result, the expected number of golden eggs the Giant received in the first case remains 40, while decreases from 48 to 30 in the second case. Correspondingly, in the last column of Table 3, the efficiency now becomes 0.40 and 0.30 respectively.

The last column of Table 3 shows that the efficiency of the first case is now higher than the second case. Therefore, the form of cooperation with longer nighttime is that the Giant works without forming a team with the "Midget" giant. The result with the previous setting is reversed, since a stronger Midget attenuates the share of harvest of the Giant when forming a team.

C. Remarks

The story above serves as an analogy to a typical scenario in transitional economies. The key elements of the motivating story may find their correspondences in the transitional economies. Table 4 clarifies this analogy.

Actually, the two agents of this economy are a Giant and a Magician. The Giant represents the entrepreneur. The Magician stands for the government, who has power (magic) to change the rules, regulations, and its own organizational structure. In the motivating story, the Giant picks up golden eggs with shovels. In the real world, the entrepreneur sets up firms to transform labor and capital inputs into outputs. Day and night in the story stand for the two states of the market, which is going to be defined in the coming section. The two forms of cooperation represent the two types of ownership arrangement, called the unambiguous property rights arrangement and the ambiguous property rights arrangement.

Table 4—The Analogy

The Motivating Story	The Transitional Economies
Giant	Entrepreneur
Magician	Government
Magics	Power of the Government
Golden Eggs	Outputs
Shovels	Capital Inputs
Day	White Market State
Night	Black Market State
Alone	Unambiguous Property Rights Arrangement
Together	Ambiguous Property Rights Arrangement

Previous studies, such as Li (1996), prove that ambiguous property rights arrangement dominates the unambiguous one under certain circumstance. I replicate this finding in Table 2. Li (1996) also argues that a market will be less than perfectly efficient when there is a chance of the black state. The motivating story I proposed stays in line with this finding. The last columns of Table 1, 2, and 3 all records relative efficiency less than 1.00.

However, Li (1996) set the probability of white and black state as exogenous variable. So, one of the major extensions of my paper is to endogenize this probability. I argue that the local government is often capable to influence the market environment by setting up rules and regulations.

In fact, the economy proposed in this story is an over-simplified world for illustration purposes. It does not contain the notion of optimization, the core of economic analysis. Instead, everything is based on some predetermined rules. In the following sections, I take steps to build up profit functions and utility functions for the agents. Moreover, the golden eggs are endowed to the economy in the motivating story. However, I introduce a production function into my model to examine the significance of input factors. The difference in abundance of labor and capital and the discrepancy of their significance in production plays a crucial role in resolving the paradox of the transition process of transitional economies.

Recall the story, the magic is capable to change the length of daytime and nighttime as well as the physical strength of the Magician. In my paper, I assume that the length of daytime and nighttime can be set freely, while there is some stickiness when setting the physical strength of the Magician. In other words, the market environment can be changed in the very short run, while the power of the government can only be modified gradually. For this reason, I examine the optimization problem of the market environment first, before optimizing the power of the government.

The major contributions of my paper is to endogenize two key variables, say the

market environment and the power of the government. This enables me to trace and explain the transition process of the property right arrangements and the efficiency of transitional economies.

III. Assumptions

In this model, there are three sub-period in each period. In my model, building on Li (1996), I have three agents: an entrepreneur, a local government, and the top-level government.

At time 1, the entrepreneur and the local government fully negotiate on the structure of ownership based on the anticipated market environment. There are two potential outcomes of the market environment: white and black. I define them as follows.

DEFINITION 1: *The white state stands for a market environment favorable to business.*

DEFINITION 2: *The black state stands for a market environment unfavorable to business.*

If the white state occurs, the firm will operate normally. In this case, no involvement of the government is needed. However, if the black state occurs, the firm will not be productive at all, except that it receives some helps from the local government. In this situation, the entrepreneur has to pay the local government some tax for its involvement. The possibilities of a white state and a black state are p and $1-p$, respectively, where $p \in [0,1]$. The local government is able to modify p freely *before* its negotiation with the entrepreneur.

The tax comes in different formats under distinct structures of ownership. In my model, I consider two possible ownership structures, including an unambiguous property rights arrangement and an ambiguous property rights arrangement. The unambiguous property rights arrangement allows the entrepreneur to be the sole owner of the firm. So, in the black state, the local government, as an outsider, taxes the firm in a monopolistic manner to maximize its own tax revenue. However, the ambiguous property rights arrangement includes the local government as an ambiguous owner in this firm, which means the entrepreneur has to pay the government a portion of the total revenue (*not* the total profit), $1-\delta_E$ in this case, regardless of the states of the market.

After the completion of the negotiation process, the entrepreneur decides the enterprize's capital investment according to the ownership structure.

Similar to Li (1996), the market environment, either black or white, is revealed at time two. To better present the influence of the market environment, I incorporate a premium factor and a discount factor in correspondence to each outcome. To be specific, if the white state prevails, the firm will enjoy a premium $q > 1$ on its

payoff. If the black state prevails, the firm will suffer a discount $0 < d < 1$ on its payoff.

At time three, the top-level government conducts an adjustment to the bargaining power of the government, $1 - \delta_E$, according to its predetermined strategy. In my model, the top-level government chooses one of the three strategies: efficiency-oriented, revenue-oriented, and balanced. I make two assumptions regarding the choice of the top-level government. First, the top-level government applies the *same* strategy forever. Second, the top-level government is *always* able to modify the bargaining power, $1 - \delta_E$, corresponding to its strategy. In addition, the modification follows a *smooth* manner. In other words, the top-level government adjusts the bargaining power only once in each period with a restricted degree.

After these three periods, another round of the game starts until the system reaches its equilibrium.

In general, the key assumptions of this model are as follows.

ASSUMPTION 1: *The profitability of firm θ follows a uniform distribution on [0,1], that is $\theta \sim$ Uniform $[0, 1]$. The entrepreneur and the government shares this same set of information regarding profitability.*

ASSUMPTION 2: *The premium $q > 1$ is a magnifying factor to emphasize on the additional return of a firm in the white state.*

ASSUMPTION 3: *The discount $0 < d < 1$ is a discounting factor to emphasize on the adverse effect of a firm's operation in the black state.*

ASSUMPTION 4: *The labor input into the production $a > 0$ is given and fixed.*

ASSUMPTION 5: *The output elasticity of labor α and the output elasticity of capital β satisfies $0 < \alpha < 1$, $0 < \beta < 1$.*

ASSUMPTION 6: *Cost function $C(k) = k^{\beta+1}$ is convex in the initial capital investment. Indeed, $C(0) = 0$, $C'(k) > 0$, and $C"(k) > 0$.*

ASSUMPTION 7: *Tax rate t measures the additional portion of tax required by the government for the help in the black state under the unambiguous property rights arrangement. This rate is determined in monopolistic manner by the government.*

ASSUMPTION 8: *By the Coase Theorem proposed by Coase (1960), the outcome of the negotiation is the most efficient ownership arrangement, the one with most capital investment k.*

ASSUMPTION 9: *The factor $0 \leq \delta_E \leq 1$ measures the bargaining power of the entrepreneur relative to the government. Conversely, the factor $\delta_G \equiv 1 - \delta_E$ measures the bargaining power of the government relative to the entrepreneur.*

ASSUMPTION 10: *The government faces no cost when providing help to the firm.*

IV. The Model

A. Profit Functions

In this part, I focus on the first and second period and the interaction between the entrepreneur and the local government. I develop the profit functions for the entrepreneur and the local government ("the government") for the two types of property rights arrangements – unambiguous property rights and ambiguous property rights, respectively.

Unambiguous Property Rights

In the white state, the expected payoff to the entrepreneur is as follows.

$$E(\theta)qa^{\alpha}k^{\beta}.$$

In the black state, the involvement of the government is indispensable to make the business productive. However, the entrepreneur will seek the help of the government, if and only if the profit is greater than the cost of the help, that is $\theta > t$.

So the government who acts as a monopolist is facing the following maximization problem.

$$\max_t Pr(\theta > t)tda^{\alpha}k^{\beta}.$$

Since, $\theta \sim$ Uniform $[0, 1]$, $Pr(\theta > t) = 1 - t$. Then, I have

$$\max_t (1 - t)tda^{\alpha}k^{\beta}.$$

Solving this maximization problem, I find the optimal tax rate for the government, $t^* = \frac{1}{2}$.

The expected payoff to the entrepreneur then equals to

$$Pr(\theta > t^*)[E(\theta|\theta > t^*) - t^*]da^{\alpha}k^{\beta}.$$

Substitute $t^* = \frac{1}{2}$ into it, I get

$$(1 - \frac{1}{2})[E(\theta|\theta > \frac{1}{2}) - \frac{1}{2}]da^{\alpha}k^{\beta}.$$

Therefore, under the arrangement of unambiguous property rights, the expected

profit function for the entrepreneur is

$$\pi_{E,U} = p\frac{1}{2}qa^{\alpha}k^{\beta} + (1-p)\frac{1}{8}da^{\alpha}k^{\beta} - k^{\beta+1}, \tag{1}$$

and the expected profit function for the government is

$$\pi_{G,U} = (1-p)\frac{1}{4}da^{\alpha}k^{\beta}. \tag{2}$$

Ambiguous Property Rights

Similarly, in the white state, the expected payoff to the firm is

$$E(\theta)qa^{\alpha}k^{\beta}.$$

In the black state, the expected payoff to the firm is

$$E(\theta)da^{\alpha}k^{\beta}.$$

In the context of ambiguous property rights arrangements, the entrepreneur can procure a portion of δ_E of the total payoff and incur the full cost of production. However, the government can receive a slice of $1-\delta_E$ of the total payoff and bear no cost of production.

Therefore, with the ambiguous property rights arrangement, the expected profit function for the entrepreneur is

$$\pi_{E,A} = p\delta_E\frac{1}{2}qa^{\alpha}k^{\beta} + (1-p)\delta_E\frac{1}{2}da^{\alpha}k^{\beta} - k^{\beta+1}, \tag{3}$$

and the expected profit function for the government is

$$\pi_{G,A} = p(1-\delta_E)\frac{1}{2}qa^{\alpha}k^{\beta} + (1-p)(1-\delta_E)\frac{1}{2}da^{\alpha}k^{\beta}. \tag{4}$$

B. Optimal Initial Capital Investment

In this part, I determine the optimal capital investment that maximize the profit functions.

To maximize the equation (1) respect to k, I have

$$\frac{\partial \pi_{E,U}}{\partial k} = p\frac{1}{2}qa^{\alpha}\beta k^{\beta-1} + (1-p)\frac{1}{8}da^{\alpha}\beta k^{\beta-1} - (\beta+1)k^{\beta} = 0.$$

Rearranging terms I obtain the optimal capital investment for the unambiguous

property rights arrangement.

$$k_U = \frac{p\frac{1}{2}qa^{\alpha}\beta + (1-p)\frac{1}{8}da^{\alpha}\beta}{\beta+1}. \tag{5}$$

Taking the first-order condition respect to p, I have

$$\frac{dk_U}{dp} > 0. \tag{6}$$

To maximize the equation (3) respect to k, I have

$$\frac{\partial \pi_{E,A}}{\partial k} = p\delta_E \frac{1}{2} qa^{\alpha}\beta k^{\beta-1} + (1-p)\delta_E \frac{1}{2} da^{\alpha}\beta k^{\beta-1} - (\beta+1)k^{\beta} = 0.$$

Rearranging terms, I obtain the optimal capital investment for the ambiguous property rights arrangement.

$$k_A = \frac{p\delta_E\frac{1}{2}qa^{\alpha}\beta + (1-p)\delta_E\frac{1}{2}da^{\alpha}\beta}{\beta+1}. \tag{7}$$

By taking the first-order condition respect to p, I have

$$\frac{dk_A}{dp} > 0. \tag{8}$$

By comparing Equation (5) and (7), I find that when $\delta_E < \frac{1}{4}$, the capital investment for the ambiguous property rights arrangement is always below that for the unambiguous property rights arrangement. Conversely, if $\frac{1}{4} < \delta_E < 1$, the capital investment for the ambiguous property rights arrangement may grow above that for the unambiguous property rights arrangement. These findings imply that the ambiguous property rights arrangement can be more efficient than the unambiguous property rights arrangement. This implication is in line with the one proposed by Li (1996).

C. The Equivalent Efficiency Line

From Equation (5) and (7), I can find the combination, (p, δ_E), which leads the two ownership arrangements to have the same efficiency. In this part, I derive such combinations, (p, δ_E).

According to Assumption 8, by the Coase Theorem, given that the entrepreneur and the government can negotiate on the arrangement at time 1, the final outcome must be the ex ante most socially efficient ownership arrangement. In other

words, the arrangement leading to more initial capital investment, k, is chosen. Consequently, in the case that the two ownership arrangements are equivalent, k_U from Equation (5) and k_A from Equation (7) must be equal. Then, by equating Equation (5) with (7), I can find the combinations, (p, δ_E), that equate the efficiency of the two ownership arrangements. To be specific, denoting such kinds of p as $\tilde{p}$, I have

$$\tilde{p} = \frac{(\delta_E - \frac{1}{4})d}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}. \tag{9}$$

So I have the following lemma.

LEMMA 1: *The relationship between p and $\tilde{p}$ determines the relative efficiency of ownership arrangements.*
1) The unambiguous property rights arrangement is more efficient, if and only if $p > \tilde{p}$;
2) The ambiguous property rights arrangement is more efficient, if and only if $p < \tilde{p}$;
3) The two ownership arrangements are equally efficient, if and only if $p = \tilde{p}$.

By taking the first-order and second-order conditions of $\tilde{p}$ with respect to δ_E, I get

$$\frac{d\tilde{p}}{d\delta_E} > 0, \tag{10}$$

$$\frac{d^2\tilde{p}}{d{\delta_E}^2} > 0. \tag{11}$$

The first-order and second-order conditions show that the function $\tilde{p}(\delta_E)$ is strictly monotonic increasing and strictly convex on $0 \leq \delta_E \leq 1$. Intuitively, the more powerful the government the more profitable for it to become an ambiguous owner of a firm. So the government increases the probability of black state.

Figure 1 summarizes my findings in this part. Since $\tilde{p}(\frac{1}{4}) = 0$, by Equation (10), I know that $\tilde{p}(\delta_E) < 0$ for $0 \leq \delta_E < \frac{1}{4}$. The broad line in Figure 1 denotes $\tilde{p}(\delta_E)$ on $\frac{1}{4} \leq \delta_E \leq 1$, where $0 \leq \tilde{p} \leq 1$. I define this line as the *Equivalent Efficiency Line*.

Lemma 1 implies that the final outcome will be one of unambiguous property rights if $p > \tilde{p}$, while the final outcome will be the ambiguous property rights if $p < \tilde{p}$, because the most socially efficient ownership arrangement will be chosen.

In fact, the set $S \equiv [0,1] \times [0,1]$ contains all possible combinations (p, δ_E). The *Equivalent Efficiency Line* proposed in this subsection divides the set S into two parts, S_U (the unshaded area in Figure 1) and S_A (the shaded area in Figure 1).

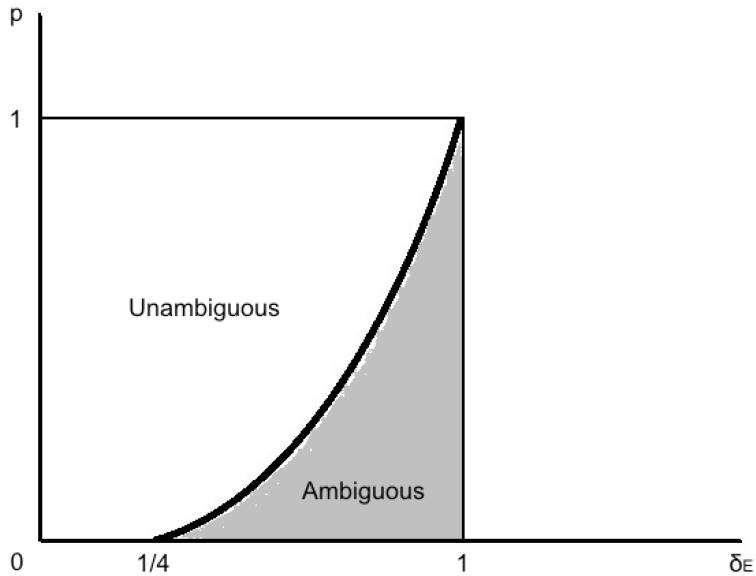

Figure 1. The Equivalent Efficiency Line.

On one hand, the set S_U represents the situations when I have the unambiguous property rights arrangement as the final outcome. On the other hand, the set S_A represents the situations when I have the ambiguous property rights arrangement as the final outcome.

However, the division is not perfect, given the ambiguity of the final outcomes along the *Equivalent Efficiency Line*. Since the entrepreneur and the local government can fully negotiate the ownership arrangement at the negotiation stage, the preference of the government determines the final outcome when the efficiency (initial capital investment) of both arrangements is equal.

In the following subsection, I examine the preference of the government, and find the ownership arrangement along the *Equivalent Efficiency Line*.

D. The Equivalent Revenue Line

In this part, I find the preference of the government regarding the two ownership arrangements with the same efficiency, that is, $k_U = k_A$. To achieve this goal, I first derive the combination (p, δ_E) that makes the government indifferent between the ownership arrangements with the same efficiency. So I equate Equation (2)

and (4), that is $\pi_(G, U) = \pi_(G, A)$, and denote p as $\hat{p}$. Then, I have

$$\hat{p} = \frac{(\delta_E - \frac{1}{2})d}{(1-\delta_E)q + (\delta_E - \frac{1}{2})d}. \tag{12}$$

So I have the following lemma.

LEMMA 2: *The relationship between p and $\hat{p}$ determines the relative amount of government revenue between ownership arrangements with equivalent efficiency.*
1) The government collects more revenue from firms with ambiguous property rights arrangement, if and only if $p > \hat{p}$;
2) The government collects more revenue from firms with unambiguous property rights arrangement, if and only if $p < \hat{p}$;
3) The government collects the same amount of revenue from firms with either arrangement, if and only if $p = \hat{p}$.

By taking the first-order and second-order conditions of $\hat{p}$ with respect to δ_E, I get

$$\frac{d\hat{p}}{d\delta_E} > 0 \tag{13}$$

$$\frac{d^2\hat{p}}{d{\delta_E}^2} > 0 \tag{14}$$

The first-order and second-order conditions show that the function $\hat{p}(\delta_E)$ is strictly monotonic increasing and strictly convex. Since $\hat{p}(\frac{1}{2}) = 0$, $\hat{p}(1) = 1$, and $0 \leq \hat{p} \leq 1$, I find that $\hat{p}(\delta_E) < 0$ for $0 \leq \delta_E < \frac{1}{2}$.

Figure 2 summarizes my findings of this part. The new broad line in Figure 2 denotes $\hat{p}(\delta_E)$ on $\frac{1}{2} \leq \delta_E \leq 1$. I define this line as the *Equivalent Revenue Line* for the government. In fact, each point on the *Equivalent Revenue Line* describes a situation where the government revenue is equal between the two ownership arrangements given that they have the same efficiency.

Lemma 2 implies that the final outcome along the *Equivalent Efficiency Line* is the ambiguous property rights except the point $(1, 1)$. Since $p > \tilde{p}$ for all $\frac{1}{4} \leq \delta_E < 1$, the government can collect more revenue from a particular firm with an ambiguous property rights arrangement than with an unambiguous property rights arrangement. So the final outcome is an ambiguous property rights arrangement. However, when $\delta_E = 1$, I have $p = \tilde{p}$. This implies the government collects the same amount of revenues from firms with either arrangement. As a result, either arrangement can be the outcome at the particular point $(1, 1)$. This is equivalent to say that $(1, 1) \in S_U \cap S_A$. In fact, I have $S_U \cup S_A = S$ and $S_U \cap S_A = (1, 1)$.

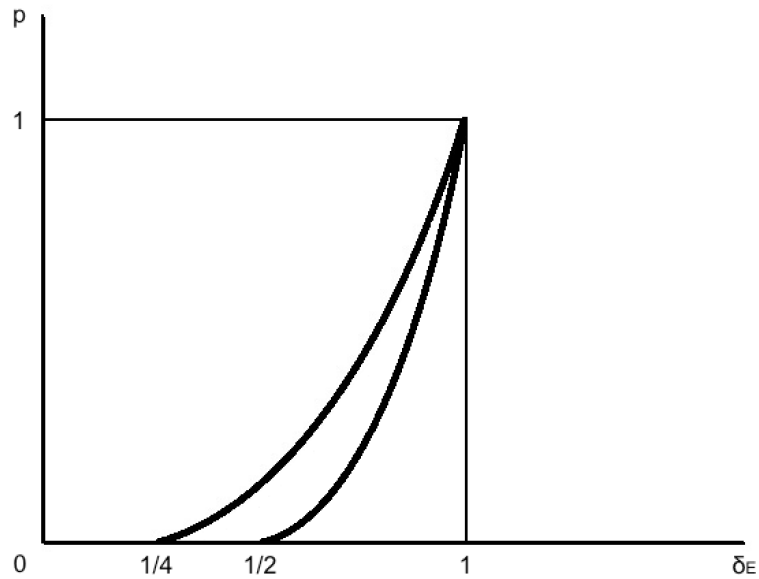

Figure 2. The Equivalent Revenue Line

Since the government can implement various policies to manipulate the market environment p, I am interested in the p the government will set. In the following part, I determine such a p for each δ_E.

E. The Equilibrium of the Market Environment

In this part, I determine the equilibrium of the market environment p by solving the profit maximization problem of the government. Recall Lemma 2, it seems that the ambiguous property rights arrangement may dominate the unambiguous property rights arrangement, since the Equivalent Efficiency Line lies above the Equivalent Revenue Line, that is $p > \hat{p}$. However, according to Lemma 1 and Figure 1, one may observe that the ambiguous property rights arrangement is not feasible for $0 < \delta_E < \frac{1}{4}$. Consequently, instead focusing solely on the ambiguous property rights arrangement, I need to focus on both arrangements.

Recall that Equations (2) and (4) represent the revenue to the government under the arrangement of unambiguous property rights and the arrangement of ambiguous property rights respectively.

The government faces the following maximization problem for the unambiguous

property rights arrangement.

$$\max_p (1-p)\frac{1}{4}da^{\alpha}{k_U}^{\beta}.$$

By solving this maximization problem, I have the following finding.

For $0 < \beta \leq \frac{d}{4q-d}$ and for all $0 \leq p \leq 1$,

$$\frac{\partial \pi_{G,U}}{\partial p} \begin{cases} = 0, & p = 0; \\ < 0, & 0 < p \leq 1. \end{cases} \tag{15}$$

For $\frac{d}{4q-d} < \beta < 1$, there exists a $p_0 \in (0,1)$, such that

$$\frac{\partial \pi_{G,U}}{\partial p} \begin{cases} > 0, & 0 \leq p < p_0; \\ = 0, & p = p_0; \\ < 0, & p_0 < p \leq 1. \end{cases} \tag{16}$$

Similarly, the government faces the following maximization problem for the ambiguous property rights arrangement.

$$\max_p p(1-\delta_E)\frac{1}{2}qa^{\alpha}{k_A}^{\beta} + (1-p)(1-\delta_E)\frac{1}{2}da^{\alpha}{k_A}^{\beta}.$$

By solving this maximization problem, I find that for all $0 < \beta < 1$ and for all $0 \leq p \leq 1$,

$$\frac{\partial \pi_{G,A}}{\partial p} > 0. \tag{17}$$

Therefore, I have the following lemma.

LEMMA 3: *The bargaining power of the entrepreneur and the output elasticity of capital, that is the combination (δ_E, β), determine the optimal choice of the market environment p^* for the government.*
1) For labor-intensive industries where $0 < \beta \leq \frac{d}{4q-d}$, the optimal market environment is

$$p^* = \begin{cases} 0, & 0 \leq \delta_E < \frac{1}{4}; \\ \tilde{p}, & \frac{1}{4} \leq \delta_E \leq 1. \end{cases} \tag{18}$$

2) For capital-intensive industries where $\frac{d}{4q-d} < \beta < 1$, the optimal market envi-

ronment is

$$p^* = \begin{cases} p_0, & 0 \leq \delta_E < \tilde{\delta}_E; \\ \tilde{p}, & \tilde{\delta}_E \leq \delta_E \leq 1; \end{cases} \tag{19}$$

for some $\tilde{\delta}_E \in (\frac{1}{4}, \tilde{p}^{-1}(p_0))$.

Note, in Lemma 3, the classification of labor-intensive and capital-intensive is not in absolute, but in relative terms. To be specific, since the threshold $\frac{d}{4q-d} \in (0, \frac{1}{3})$, industries with $0 < \beta \leq \frac{d}{4q-d}$ are labor-intensive in absolute terms. However, industries with $\frac{d}{4q-d} < \beta < 1$ are not necessarily capital-intensive in absolute term, since it is uncertain whether $\beta > \frac{1}{2}$. Rather, in Lemma 3, industries are divided into two groups by the threshold, $\frac{d}{4q-d}$. In fact, I define industries with $0 < \beta \leq \frac{d}{4q-d}$ as labor-intensive industries, and industries with $\frac{d}{4q-d} < \beta < 1$ as capital-intensive industries.

F. The Equilibrium

In this subsection, I point out the equilibrium and figure out the ownership arrangement at each equilibrium point.

First, for the equilibrium, I have the following lemma.

LEMMA 4: *For each δ_E, there exists an unique and stable equilibrium point. Indeed, equilibrium points locate on $p^*(\delta_E)$.*

So, I define $p^*(\delta_E)$ as the *Equilibrium Line*. Then I determine the equilibrium ownership arrangement along it.

Since $S_U \cup S_A = S$ and $S_U \cap S_A = (1,1)$, the only point where I cannot determine the ultimate ownership arrangement is $(1,1)$. So I have $S_U \backslash \{(1,1)\} \cup S_A \backslash \{(1,1)\} = S \backslash \{(1,1)\}$ and $S_U \backslash \{(1,1)\} \cap S_A \backslash \{(1,1)\} = \Phi$.

Therefore, in the equilibrium, one of the two types of ownership arrangements is *always* superior than the other, *except* in the free market situation where $\delta_E = 1$. In other words, each equilibrium point except $(1,1)$ has a unique equilibrium ownership arrangement.

So far, I have proven four lemmas. Lemma 1 shows the outcome of the negotiation given an market environment and government power, (p, δ_G). Lemma 2 takes account of the behavior of the government. From Lemma 1 and 2, I divide all possible outcomes of the ownership arrangements into 3 categories according to (p, δ_G). Then in Lemma 3, I determine the equilibrium of the system for every (p, δ_G). In Lemma 4, I prove that such equilibrium is stable and unique for each (p, δ_G). In the following lemma, I determine the ultimate outcomes of the ownership arrangements. In fact, the ultimate outcome for each predetermined (p, δ_G) (or (p, δ_E)) is essentially the equilibrium ownership arrangement.

LEMMA 5: *The bargaining power of the government, $\delta_G \equiv 1 - \delta_E$, determines the equilibrium of property rights arrangement.*
1) The unambiguous property rights arrangement is the equilibrium, if and only if the government has strong bargaining power;
2) The ambiguous property rights arrangement is the equilibrium, if and only if the government has weak bargaining power;
3) A mix of both types of arrangements is the equilibrium, if and only if the government has no bargaining power.

Figure 3 and 4 summarizes my findings. The blue line in Figure 3 represents the *Equilibrium line* for labor-intensive industries. The blue line in Figure 4 represents the *Equilibrium line* for capital-intensive industries. In general, the *Equilibrium line* in Figure 3 and 4 can be divided into three parts according to the three parts of Lemma 5.

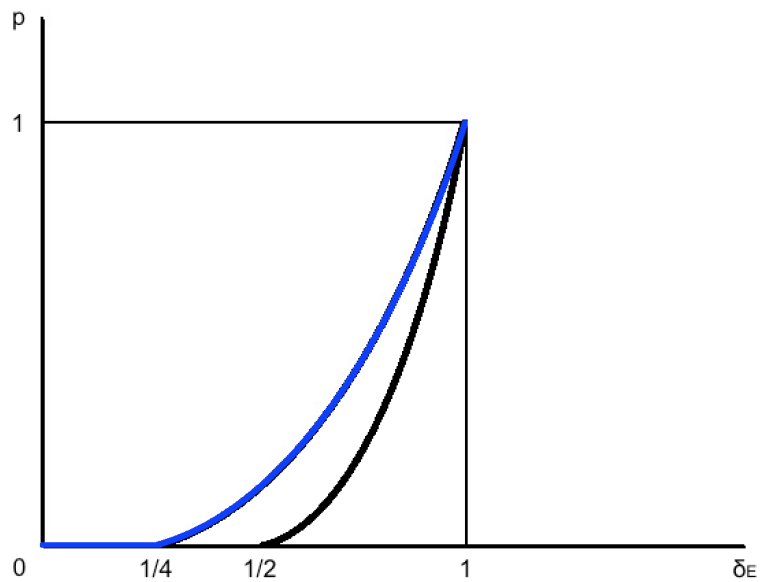

Figure 3. The Equilibrium Line for Labor-intensive Industries

The finding in Lemma 5 is not a coincidence. Intuitively, a government with strong bargaining power leads the entrepreneur to under-invest in the ambiguous property rights arrangement, since the entrepreneur can only receive a small portion of the total revenue of the firm. In this situation, the unambiguous property rights arrangement is more efficient, since it leads more initial capital investment from the entrepreneur. Conversely, the ambiguous property rights arrangement may becomes more efficient than the unambiguous property rights arrangement,

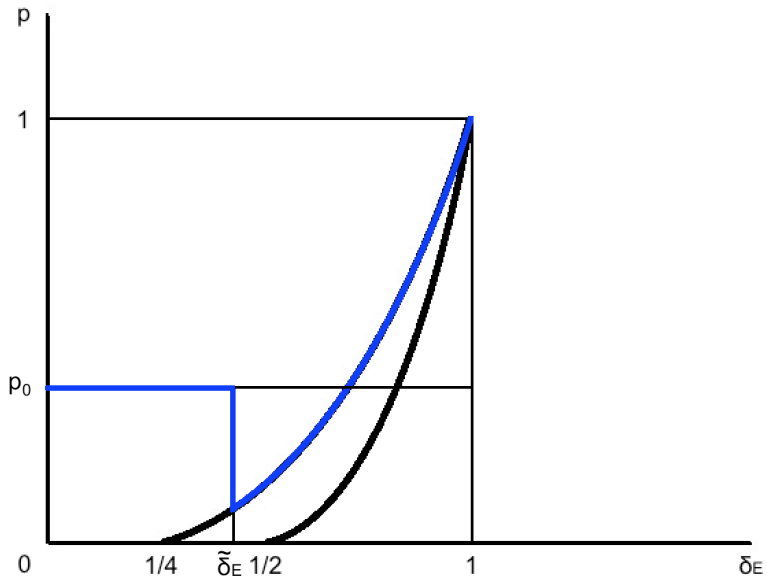

Figure 4. The Equilibrium Line for Capital-intensive Industries

if the government leaves most of the revenue to the entrepreneur. Finally, in a free market, the government has no bargaining power. So it is not reasonable for the government to manipulate some black states, since it can neither collect more revenue nor improve the efficiency of the economy by doing so. As a result, the market is perfectly white, and these two ownership arrangements are equivalent.

V. Implication

In this section, I apply my model to resolve the paradox in a variety of transition of developing economy settings. I mainly conduct the analysis for an economy where labor-intensive industries with $0 < \beta \leq \frac{d}{4q-d}$ prevail. There are two reasons for this treatment. First, the well-behaved continuous equilibrium function $p^*(\delta_E)$ simplifies the analysis. Second, this treatment can lead to a generalized result, which can be applied to an economy where capital-intensive industries with $0 < \beta \leq \frac{d}{4q-d}$ prevail in most cases. At the end of this section, I point out a special case in the latter type of economy. In general, the explanation I proposed is consistent with economic history.

In section 3, I set the market environment p as an endogenous variable while leaving the bargaining power of the entrepreneur δ_E exogenous. To be specific, the profit-maximizing government takes charge of setting the market environment p. In this section, similar to (Le, 2004), I introduce a top-level government,

denoted as T, which can modify the bargaining power of the government $\delta_G \equiv 1 - \delta_E$. According to Fischer (1993), the creation of a new legal and institutional system takes years to implement. So in contrast to the market environment, the bargaining power of the government can only be changed gradually.

Generally, I assume there are three types of top-level government, including efficiency-oriented, revenue-oriented, and balanced. In this manner, I set the bargaining power of the entrepreneur δ_E endogenous as well to analyze the transition process from a central planned economy into a free market economy.

A. Efficiency-oriented Top-level Government

A efficiency-oriented top-level government only cares about the efficiency of the economy. In other words, it prefers to set the bargaining power of the government δ_G to a level that maximizes the economy's capital investment. This means the government has a utility function

$$U_{T,e} = u_e(\delta_G), \tag{20}$$

where $u_e'(\delta_G) > 0$. This leads to the economy to evolve in a process from low capital investment to high capital investment. So I have the following proposition.

PROPOSITION 1: *An economy with the efficiency-oriented top-level government transits along the Equilibrium Line toward free market.*

Figure 5 summarizes this result. In the transition process, the business structure develops in four steps.
1) A central planned economy with state-owned firms and without private-owned business;
2) A transitional economy with private-owned firms;
3) A transitional economy with partially local government controlled firms;
4) A free market economy with efficient firms.

B. Revenue-oriented Top-level Government

A revenue-oriented top-level government only cares the tax revenue of the government. In other words, it prefers to set the bargaining power of the government δ_G to a level that maximize the capital investment of the economy. This means the government has a utility function

$$U_{T,r} = u_r(\delta_G), \tag{21}$$

where $u_r'(\delta_G) > 0$. This leads to the economy to evolve in a process from two ends of the *Equilibrium Line* to a point in the middle. So I have the following proposition.

PROPOSITION 2: *An economy with the revenue-oriented top-level government*

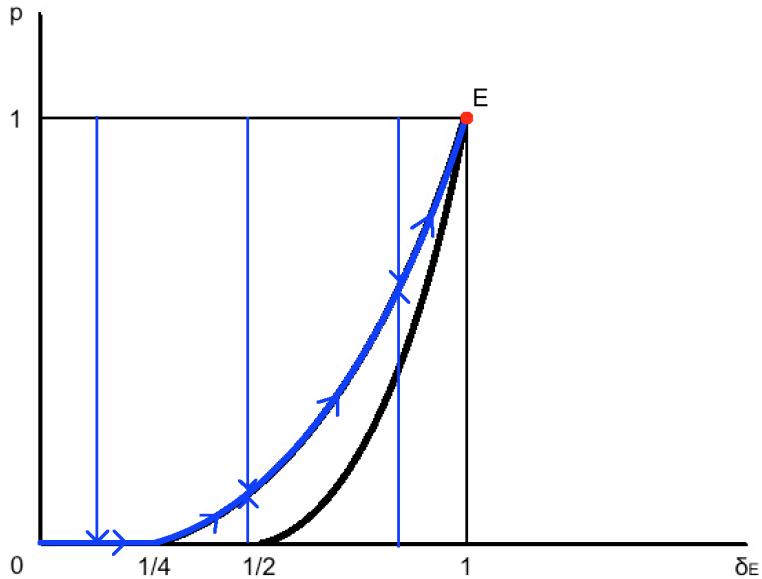

Figure 5. The Convergence Path for Efficiency-oriented Top-level Government

transits along the Equilibrium Line toward to a point on the Equivalent Efficiency Line.

Figure 6 summarizes this result. In the transition process, the business structure develops in two ways, either
1) A central planned economy with state-owned firms and without private-owned business;
2) A transitional economy with private-owned firms;
3) A transitional economy with partially local government controlled firms;
or
1) A central planned economy with state-owned firms and without private-owned business;
2) A transitional economy with a transitional economy with partially local government controlled firms.

C. Balanced Top-level Government

A balanced-oriented top-level government cares both the efficiency of the economy and the tax revenue of the government. In this case, the top-level government is a combination of the previous two extreme cases. This means the government

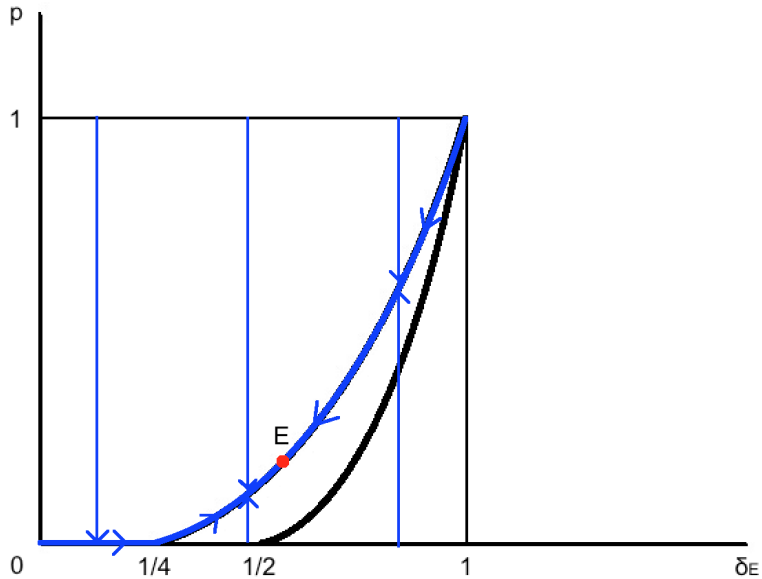

Figure 6. The Convergence Track for Revenue-oriented Top-level Government

has the following utility function.

$$U_{T,b} = w_e u_e(\delta_G) + w_r u_r(\delta_G), \tag{22}$$

subject to

$$w_e + w_r = 1, 0 \leq w_e, w_r \leq 1,$$

where w_e and w_r are weightings for the utility from efficiency and the utility from tax revenue, respectively. This leads the economy to evolve in a process similar to the revenue-oriented top-level government case, except that a smaller bargaining power of the government is assigned. So I have the following corollary.

COROLLARY 1: *An economy with the balanced top-level government transits along the Equilibrium Line toward to a point on the Equivalent Efficiency Line, which lies between the two point specified in Proposition 1 and Proposition 2.*

Figure 7 summarizes this result. The transition process, in this case, the business structure develops in a similar manner as the revenue-oriented top-level government case.

This corollary together with Proposition 1 and 2 resolves the paradox of two typical transition process. On one hand, if the power of the government is weak initially, the government will become stronger along the process of transition. According to Figure 7, the economy moves from point A to the equilibrium point

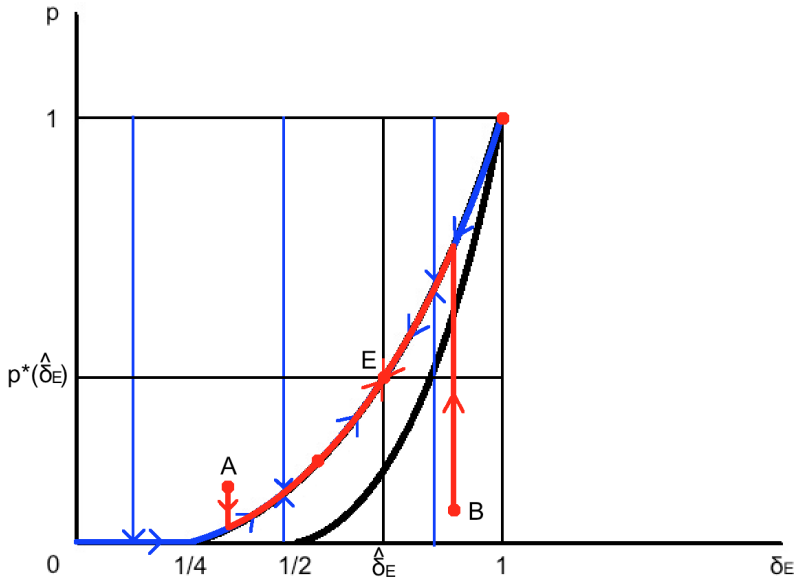

Figure 7. The Convergence Track for Balanced Top-level Government

E. On the other hand, if the power of the government is strong initially, the government will become weaker along the process of transition. According to Figure 7, the economy moves from point B to the equilibrium point E.

Furthermore, Proposition 1 and 2 and Corollary 1 imply that the more the top-level government cares about efficiency, the weaker its power will be at the steady state and vice-versa.

Finally, based on my analysis in section 4, I have the following corollary.

COROLLARY 2: *An economy can evolve to a free market economy if and only if it has a efficiency-oriented top-level government.*

This corollary explains the reason why the a perfectly free market economy is extremely rare. Since the top-government always values tax revenue to some extent, a perfectly efficiency-oriented top-level government rarely exists in reality.

However, why do some economies, like Singapore, have a nearly perfect free market economy? It does not mean the government does not care about tax revenue at all. The reason is that it violates my assumption that top-level government is capable of modifying its the bargaining power. Such governments, like the one in Singapore, are small, so they are clearly not able to do so. Therefore, although it is rare, a nearly perfect free market is possible in reality.

VI. Extension

In the previous section, I explain the transition process of transitional economies. In general, I find two types of converging tracks. However, in reality, several transitional economies stagnate in the process of transition. What is even worse, they stay at their status quo and never make any progress.

In this section, I extend my model to explain these paradoxical phenomena in the real world. In general, I cover two types of stagnating economy, including the trapped economy and the failed state. To be specific, a trapped economy refers to a stagnating economy with a powerful government and fairly clear market environment. On the contrary, a failed state refers the one stagnated with a weak government and fairly dark market environment.

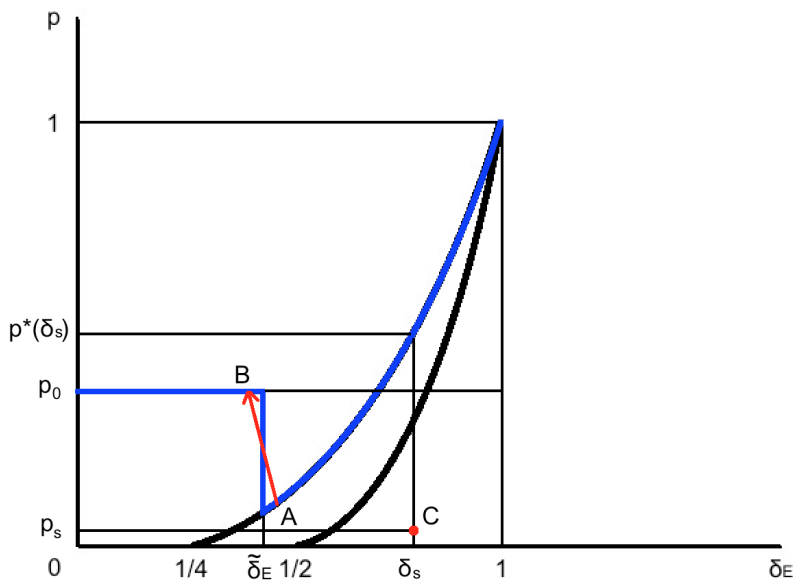

Figure 8. The Trapped Economy and The Failed State

A. *The Trapped Economy*

In an economy where capital-intensive industries prevail, the top-level government may increase efficiency by increasing the bargaining power of the government δ_E and increasing the whiteness of the market environment p. I describe this phenomenon in the following proposition.

PROPOSITION 3: *An economy where capital-intensive industries prevail may*

stagnate at a steady state with a strong government and fairly clear market environment, therefore becoming a trapped economy.

This extension introduces the concept of discounting over time. In fact, this is quite consistent with the reality. In general, an effective project that can be completed quickly is better than a perfect proposal that requires continuous effort and long-term commission. In this case, the top-level government has the incentive to increase the bargaining power δ_G to stimulate the capital investment of the entrepreneur, even though decreasing the bargaining power can lead to more capital investment in the long run.

B. The Failed State

The case of a failed state describes a converse situation comparing to the situation of a trapped economy. I describe this case in the following proposition.

PROPOSITION 4: *An economy may stagnate at a steady state with a weak government and fairly dark market environment, therefore becoming a failed state.*

This extension introduces monopolistic power and bribes into my model, which are ruled out in my basic model. As we can see, monopolistic power and bribe introduces discontinuities into my model, therefore distort the general result.

Figure 8 summarizes my findings in this section. In figure 8, the trapped economy moves from point A to point B and stagnates at there, while the failed state sticks at point C.

VII. Discussion

In this discussion section, I introduce several directions for further research.

First, Lipton and Sachs (1990) and Fischer and Gelb (1991) point out six major categories of reforms in transitional economies, including macroeconomic stabilization, price liberalization and market reform, enterprise reform, trade liberalization and account convertibility, creation of a social safety net, and creation of the legal and institutional framework for a market economy. I focus my analysis on the legal and institutional reforms in the transition process from centrally planned economies into market economies. Further studies can focus on the remaining issues to analyze the transition process.

Second, my analysis is based on a specific profit function. The production function of the firm follows a Cobb-Douglas structure. The revenue function of the firm follows a structure of homogeneous degree of β. The cost function, $k^(\beta + 1)$, is a specific convex function. Further research can refine the profit function into more generalized forms.

Third, my model rules out a conflict of interest between two levels of the government by assigning different functions to them. In reality, as Le (2004) puts

it, the interactions between a local government and the central government can influence the outcomes. In fact, different levels of governments may collude in the decision making process. Furthermore, the influence of the relative bargaining power between different levels of government is also worthwhile to study.

Fourth, similar to Li (1996), I define the efficiency as the productive efficiency. To be specific, I assume that the more the capital investment from the entrepreneurs the more productive efficient the firm. However, one may measure the surplus efficiency by comparing the total revenue resulting from various types of property rights.

VIII. Conclusion

In this paper, I proposed a refined theory of ambiguous property rights with endogenous market environment and bargaining power. From this refined theory, I find that in off-equilibrium situations, ambiguous property rights can be more efficient than unambiguous ones. This finding conforms the conclusions of Li (1996) and Le (2004). Moreover, I also find that ambiguous and unambiguous property rights have to possess the same efficiency in both short-run and long-run equilibrium. This conclusion conforms the empirical findings by Svejnar (1990) and Weitzman and Xu (1994).

I also find that the transition process diverges not by coincidence. It is the differences in fundamentals that cause the divergence in transition paths. First, the differences in the relative abundance of the production factors and the time preference of the top-level government determine whether a transitional economy evolves or stagnates. To be specific, it is more likely for an economy with abundant capital or natural resources and a shortsighted top-level government to stagnate, vice-versa. Second, the differences in the initial bargaining power of the local government agencies and the long-run orientation of the top-level government determine whether the government strengthen or weaken. In other words, it is more likely for a country with strong local government agencies and a efficiency-oriented top-level government to weaken, vice-versa.

This study implies that the long-run success of a transitional economy is not stems from the property rights because property rights possess the same efficiency in equilibrium. Instead, it is that the success of legal and institutional reforms, which reduce the bargaining power of local governments via increased competitions. Therefore, in order to improve long-run market environments and social efficiencies, policymakers in transitional economies should prioritize legal and institutional reforms instead of solely focusing on enterprise reforms. This conclusion conforms one of the main arguments of Fischer and Gelb (1991) that legal and institutional reforms should start at a very early stage preceding enterprise reforms. In fact, my model provides more theoretical supports to Fischer and Gelb (1991).

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Appendix A: Derivations

Derivation of Equation (6)

By taking the first-order condition of k_U respect to p, I get

$$\begin{aligned}\frac{dk_U}{dp} &= \frac{\frac{1}{2}qa^\alpha\beta - \frac{1}{8}da^\alpha\beta}{\beta+1} \\ &= \frac{a^\alpha\beta}{\beta+1}(\frac{1}{2}q - \frac{1}{8}d) \\ &> 0.\end{aligned}$$

Derivation of Equation (8)

By taking the first-order condition of k_A respect to p, I get

$$\begin{aligned}\frac{dk_A}{dp} &= \frac{\frac{1}{2}\delta_E qa^\alpha\beta - \frac{1}{2}\delta_E da^\alpha\beta}{\beta+1} \\ &= \frac{\delta_E a^\alpha\beta}{2(\beta+1)}(q-d) \\ &> 0.\end{aligned}$$

Derivation of Equation (9)

To derive the Equation (9), I equate k_U with k_A. Then, I have

$$\begin{aligned}\frac{p\frac{1}{2}qa^\alpha\beta + (1-p)\frac{1}{8}a^\alpha\beta d}{\beta+1} &= \frac{p\delta_E\frac{1}{2}qa^\alpha\beta + (1-p)\delta_E\frac{1}{2}a^\alpha\beta d}{\beta+1} \\ p\frac{1}{2}q + (1-p)\frac{1}{8}d &= p\delta_E\frac{1}{2}q + (1-p)\delta_E\frac{1}{2}d \\ p(\frac{1}{2}q - \frac{1}{8}d - \delta_E\frac{1}{2}q + \delta_E d) &= -\frac{1}{4}d + \delta_E d \\ p[(1-\delta_E)q + (\delta_E - \frac{1}{4})d] &= (\delta_E - \frac{1}{4})d \\ p &= \frac{(\delta_E - \frac{1}{4})d}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}.\end{aligned}$$

Denote the p as $\tilde{p}$, I have the equation (9).

$$\tilde{p} = \frac{(\delta_E - \frac{1}{4})d}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}.$$

Derivation of Equation (10)

By taking the first-order condition of $\tilde{p}$ respect to δ_E, I get

$$\begin{aligned}
\frac{d\tilde{p}}{d\delta_E} &= \frac{d[(1-\delta_E)q + (\delta_E - \frac{1}{4})d] - (\delta_E - \frac{1}{4})d(-q+d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^2} \\
&= \frac{(1-\delta_E)dq + (\delta_E - \frac{1}{4})d^2 + (\delta_E - \frac{1}{4})dq - (\delta_E - \frac{1}{4})d^2}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^2} \\
&= \frac{(1-\delta_E)dq + (\delta_E - \frac{1}{4})dq}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^2} \\
&= \frac{\frac{3}{4}dq}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^2} \\
&> 0.
\end{aligned}$$

Derivation of Equation (11)

By taking the second-order condition of $\tilde{p}$ respect to δ_E, I get

$$\begin{aligned}
\frac{d^2\tilde{p}}{d{\delta_E}^2} &= \frac{-\frac{3}{4}dq2[(1-\delta_E)q + (\delta_E - \frac{1}{4})d](-q+d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^4} \\
&= \frac{\frac{3}{2}dq(q-d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^3} \\
&> 0.
\end{aligned}$$

Derivation of Equation (12)

To derive the Equation (12), I equate the right-hand-side of Equation (2) with

the right-hand-side of Equation (4). I have the following derivation.

$$
\begin{aligned}
(1-p)\frac{1}{4}a^{\alpha}k^{\beta}d &= p(1-\delta_E)\frac{1}{2}qa^{\alpha}k^{\beta} + (1-p)(1-\delta_E)\frac{1}{2}a^{\alpha}k^{\beta}d \\
(1-p)\frac{1}{2}d &= p(1-\delta_E)q + (1-p)(1-\delta_E)d \\
p(1-\delta_E)q &= (1-p)(\delta_E - \frac{1}{2})d \\
p[(1-\delta_E)q + (\delta_E - \frac{1}{2})d] &= (\delta_E - \frac{1}{2})d \\
p &= \frac{(\delta_E - \frac{1}{2})d}{(1-\delta_E)q + (\delta_E - \frac{1}{2})d}.
\end{aligned}
$$

Denote the p as $\hat{p}$, I have the Equation (12).

$$
\hat{p} = \frac{(\delta_E - \frac{1}{2})d}{(1-\delta_E)q + (\delta_E - \frac{1}{2})d}.
$$

Derivation of Equation (13)

By taking the first-order condition of $\hat{p}$ with respect to δ_E, I have

$$
\begin{aligned}
\frac{d\hat{p}}{d\delta_E} &= \frac{d[(1-\delta_E)q + (\delta_E - \frac{1}{2})d] - (\delta_E - \frac{1}{2})d(-q+d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{2})d]^2} \\
&= \frac{(1-\delta_E)dq + (\delta_E - \frac{1}{2})d^2 + (\delta_E - \frac{1}{2})dq - (\delta_E - \frac{1}{2})d^2}{[(1-\delta_E)q + (\delta_E - \frac{1}{2})d]^2} \\
&= \frac{\frac{1}{2}dq}{[(1-\delta_E)q + (\delta_E - \frac{1}{2})d]^2} \\
&> 0.
\end{aligned}
$$

Derivation of Equation (14)

By taking the second-order condition of $\hat{p}$ with respect to δ_E, I have

$$
\begin{aligned}
\frac{d^2\hat{p}}{d{\delta_E}^2} &= \frac{-\frac{1}{2}dq2[(1-\delta_E)q + (\delta_E - \frac{1}{2})d](-q+d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{2})d]^4} \\
&= \frac{dq(q-d)}{[(1-\delta_E)q + (\delta_E - \frac{1}{2})d]^3} \\
&> 0.
\end{aligned}
$$

Derivation of Equation (15)

By taking the first-order condition of $\pi_{G,U}$ from Equation (2) respect to p, I have

$$\begin{aligned}
\frac{\partial \pi_{G,U}}{\partial p} &= -\frac{1}{4} d a^{\alpha} {k_U}^{\beta} + (1-p)\frac{1}{4} d a^{\alpha} \beta {k_U}^{\beta-1} \frac{dk_U}{dp} \\
&= -\frac{1}{4} d a^{\alpha} {k_U}^{\beta} + (1-p)\frac{1}{4} d a^{\alpha} \beta {k_U}^{\beta-1} \frac{a^{\alpha}\beta}{\beta+1}(\frac{1}{2}q - \frac{1}{8}d) \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + (1-p)\frac{1}{k_U}\frac{a^{\alpha}\beta^2}{\beta+1}(\frac{1}{2}q - \frac{1}{8}d)] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + (1-p)\frac{\beta+1}{p\frac{1}{2}q a^{\alpha}\beta + (1-p)\frac{1}{8}a^{\alpha}\beta d}\frac{a^{\alpha}\beta^2}{\beta+1}(\frac{1}{2}q - \frac{1}{8}d)] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + (1-p)\beta\frac{\frac{1}{2}q - \frac{1}{8}d}{p\frac{1}{2}q + (1-p)\frac{1}{8}d}].
\end{aligned}$$

By taking the second-order condition of $\pi_{G,U}$ from Equation (2) respect to p, I have

$$\begin{aligned}
\frac{\partial^2 \pi_{G,U}}{\partial p^2} &= -\frac{1}{4} d a^{\alpha} \beta k^{\beta-1} \frac{dk_U}{dp} \\
&= -\frac{1}{4} d a^{\alpha} \beta k^{\beta-1} \frac{a^{\alpha}\beta}{\beta+1}(\frac{1}{2}q - \frac{1}{8}d) \\
&< 0.
\end{aligned}$$

So, $\frac{\partial \pi_{G,U}}{\partial p}$ is strictly monotone decreasing in p. Since $0 \leq p \leq 1$, I know that $\frac{\partial \pi_{G,U}}{\partial p}$ attains its maximum at $p = 0$ and its minimum at $p = 1$.

The minimum of $\frac{\partial \pi_{G,U}}{\partial p}$ is $\frac{\partial \pi_{G,U}}{\partial p}\big|_{p=1} = -\frac{1}{4} d a^{\alpha} {k_U}^{\beta} < 0$. Since $0 < \beta \leq \frac{d}{4q-d}$, then the maximum of $\frac{\partial \pi_{G,U}}{\partial p}$ is

$$\begin{aligned}
\frac{\partial \pi_{G,U}}{\partial p}\Big|_{p=0} &= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + \beta\frac{\frac{1}{2}q - \frac{1}{8}d}{\frac{1}{8}d}] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + \beta\frac{4q-d}{d}] \\
&\leq \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + \frac{d}{4q-d}\frac{4q-d}{d}] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta}[-1 + 1] \\
&= 0.
\end{aligned}$$

Then, I have the Equation (15).

Derivation of Equation (16)

This derivation follows the previous one.

Recall that $\frac{\partial \pi_{G,U}}{\partial p}$ is strictly monotone decreasing in p. The minimum of $\frac{\partial \pi_{G,U}}{\partial p}$ is $\frac{\partial \pi_{G,U}}{\partial p}\Big|_{p=1} = -\frac{1}{4} d a^{\alpha} {k_U}^{\beta} < 0$. However, if $\frac{d}{4q-d} < \beta \leq 1$ instead, then the maximum of $\frac{\partial \pi_{G,U}}{\partial p}$ is

$$\begin{aligned}
\frac{\partial \pi_{G,U}}{\partial p}\Big|_{p=0} &= \frac{1}{4} d a^{\alpha} {k_U}^{\beta} [-1 + \beta \frac{\frac{1}{2}q - \frac{1}{8}d}{\frac{1}{8}d}] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta} [-1 + \beta \frac{4q-d}{d}] \\
&> \frac{1}{4} d a^{\alpha} {k_U}^{\beta} [-1 + \frac{d}{4q-d} \frac{4q-d}{d}] \\
&= \frac{1}{4} d a^{\alpha} {k_U}^{\beta} [-1 + 1] \\
&= 0.
\end{aligned}$$

Then, since $\frac{\partial \pi_{G,U}}{\partial p}$ is a continuous function on the closed interval $p \in [0, 1]$, by the Mean Value Theorem, there exists some $p_0 \in (0, 1)$ such that $\frac{\partial \pi_{G,U}}{\partial p}\Big|_{p=p_0} = 0$.

Since $\frac{\partial^2 \pi_{G,U}}{\partial p^2} < 0$ for $p_0 \in (0, 1)$, then $\frac{\partial \pi_{G,U}}{\partial p} < 0$ for $p_0 < p \leq 1$ $\frac{\partial \pi_{G,U}}{\partial p} > 0$ for $0 \leq p < p_0$. Therefore, I have the Equation (13).

Derivation of Equation (17)

This derivation follows a similar manner with those of previous two derivations.

By taking the first-order condition of $\pi_{G,A}$ from Equation (4) respect to p, for all

$0 < \beta < 1$ and $0 \leq p \leq 1$, I have

$$\begin{aligned}
\frac{\partial \pi_{G,A}}{\partial p} &= (1-\delta_E)\frac{1}{2}qa^{\alpha}{k_A}^{\beta} + p(1-\delta_E)\frac{1}{2}qa^{\alpha}\beta{k_A}^{\beta-1}\frac{dk_A}{dp} \\
&- (1-\delta_E)\frac{1}{2}da^{\alpha}{k_A}^{\beta} + (1-p)(1-\delta_E)\frac{1}{2}da^{\alpha}\beta{k_A}^{\beta-1}\frac{dk_A}{dp} \\
&= p(1-\delta_E)\frac{1}{2}qa^{\alpha}\beta{k_A}^{\beta-1}\frac{dk_A}{dp} + (1-p)(1-\delta_E)\frac{1}{2}da^{\alpha}\beta{k_A}^{\beta-1}\frac{dk_A}{dp} \\
&= \frac{1}{2}(1-\delta_E)a^{\alpha}\beta{k_A}^{\beta-1}\frac{dk_A}{dp}[pq + (1-p)d] \\
&> 0.
\end{aligned}$$

Appendix B: Proofs

Proof of Lemma 1

PROOF:

The proof follows from the comparison between k_U and k_A from Equation (5) and (7). I define

$$\begin{aligned}
f(p) &\equiv k_U - k_A \\
&= \frac{p\frac{1}{2}qa^{\alpha}\beta + (1-p)\frac{1}{8}da^{\alpha}\beta}{\beta+1} - \frac{p\delta_E\frac{1}{2}qa^{\alpha}\beta + (1-p)\delta_E\frac{1}{2}da^{\alpha}\beta}{\beta+1} \\
&= \frac{a^{\alpha}\beta}{2(\beta+1)}[p(1-\delta_E)q + (1-p)(\frac{1}{4}-\delta_E)d].
\end{aligned}$$

Taking the first-order condition of $f(p)$, I have

$$\begin{aligned}
f'(p) &= \frac{a^{\alpha}\beta}{2(\beta+1)}[(1-\delta_E)q - (\frac{1}{4}-\delta_E)d] \\
&= \frac{a^{\alpha}\beta}{2(\beta+1)}[(q-\frac{1}{4}d) - (q-d)\delta_E] \\
&\geq \frac{a^{\alpha}\beta}{2(\beta+1)}[(q-\frac{1}{4}d) - (q-d)] \\
&= \frac{a^{\alpha}\beta}{2(\beta+1)}\frac{3}{4}d \\
&> 0.
\end{aligned}$$

According to my setting, $f(\tilde{p}) = 0$. So, I have

$$f(p) \equiv k_U - k_A \begin{cases} > 0, & p > \tilde{p}; \\ = 0, & p = \tilde{p}; \\ < 0, & p < \tilde{p}; \end{cases}$$

Since the ownership arrangement leads to more initial capital investment is more efficient, this proves Lemma 1. ■

Proof of Lemma 2

PROOF:

The proof follows from the comparison between $\pi_{G,U}$ and $\pi_{G,A}$ from Equation (2) and (4). Since the two ownership arrangements have the same efficiency, I have $k_U = k_A$. Then I define $k \equiv k_U = k_A$ and

$$\begin{aligned} g(p) &\equiv \pi_{G,U} - \pi_{G,A} \\ &= (1-p)\frac{1}{4}da^\alpha k^\beta - p(1-\delta_E)\frac{1}{2}qa^\alpha k^\beta - (1-p)(1-\delta_E)\frac{1}{2}da^\alpha k^\beta \\ &= \frac{1}{4}a^\alpha k^\beta[(1-p)d - 2p(1-\delta_E)q - 2(1-p)(1-\delta_E)d] \\ &= \frac{1}{4}a^\alpha k^\beta[(1-p)(2\delta_E - 1)d - 2p(1-\delta_E)q]. \end{aligned}$$

Taking the first-order condition of $g(p)$, I have

$$\begin{aligned} g'(p) &= \frac{1}{4}a^\alpha k^\beta[-(2\delta_E - 1)d - 2(1-\delta_E)q] \\ &= \frac{1}{4}a^\alpha k^\beta 2(q-d)(\delta_E - 1) \\ &\leq 0. \end{aligned}$$

According to my setting, $f(\tilde{p}) = 0$. So, I have

$$g(p) \equiv \pi_{G,U} - \pi_{G,A} \begin{cases} < 0, & p > \tilde{p}; \\ = 0, & p = \tilde{p}; \\ > 0, & p < \tilde{p}; \end{cases}$$

Since the government always prefers the ownership arrangement with higher tax revenue given that the two ownership arrangements have the same efficiency, this proves Lemma 2. ■

Proof of Lemma 3

PROOF:

For the first part of Lemma 3, I have $0 < \beta \leq \frac{d}{4q-d}$. Recall Equation (15) and (17), I have

$$\frac{\partial \pi_{G,U}}{\partial p} \begin{cases} = 0, & p = 0; \\ < 0, & 0 < p \leq 1; \end{cases}$$

and

$$\frac{\partial \pi_{G,A}}{\partial p} > 0.$$

If $0 \leq \delta_E < \frac{1}{4}$, Lemma 1 suggests that the unambiguous property rights arrangement is chosen regardless of the market environment. So by Equation (15), to maximize profit, the government needs to set the market environment as black as possible. So in this case, $p^* = 0$.

Then, when $\frac{1}{4} \leq \delta_E \leq 1$, there are two options.

First, if $p > \tilde{p}$, by Lemma 1, the unambiguous property rights arrangement is chosen. In this case, by Equation (15), the government sets the market environment as black as possible to maximize profits. So $p^* = \tilde{p}$.

Second, if $p < \tilde{p}$, by Lemma 1, the ambiguous property rights arrangement is chosen. In this case, by Equation (17), the government sets the market environment as white as possible to maximize profits. So $p^* = \tilde{p}$.

This proves the first part of Lemma 3.

Similarly, for the second part of Lemma 3, I have $\frac{d}{4q-d} < \beta < 1$. Recall Equation (16) and (17), I have

$$\frac{\partial \pi_{G,U}}{\partial p} \begin{cases} > 0, & 0 \leq p < p_0; \\ = 0, & p = p_0; \\ < 0, & p_0 < p \leq 1; \end{cases}$$

and

$$\frac{\partial \pi_{G,A}}{\partial p} > 0.$$

Since $\pi_{G,A}$ can be transformed into a single variate function of δ_E, $\pi_{G,A}(\delta_E)$, and $\pi_{G,U}$ can be transformed into a single variate function of p, $\pi_{G,U}(p)$.

Now, if I can prove there exists a unique $\tilde{\delta}_E \in [\frac{1}{4}, \tilde{p}^{-1}(p_0))$, such that $\pi_{G,A}(\delta_E) < \pi_{G,U}(p_0)$ for all $\delta_E \in [\frac{1}{4}, \tilde{\delta}_E)$ and $\pi_{G,A}(\delta_E) > \pi_{G,U}(p_0)$ for all $(\tilde{\delta}_E, \tilde{p}^{-1}(p_0)]$, then I can prove the remaining second part of Lemma 2 in the same manner as the proof for the first part.

Define $h(\delta_E) \equiv \pi_{G,A}(\tilde{\delta}_E) - \pi_{G,U}(p_0)$. Since $\pi_{G,A}$ is a continuous function on $[\frac{1}{4}, \tilde{p}^{-1}(p_0)]$ and $\pi_{G,U}(p_0)$ is a constant, $h(\delta_E)$ is a continuous function on $[\frac{1}{4}, \tilde{p}^{-1}(p_0)]$.

Taking the first-order condition of $h(\delta_E)$, I have

$$\begin{aligned}\frac{dh(\delta_E)}{d\delta_E} &= \frac{d\pi_{G,A}(\delta_E)}{d\delta_E} - 0 \\ &= \frac{d\pi_{G,A}(\delta_E)}{d\tilde{p}} \frac{d\tilde{p}}{d\delta_E} \\ &> 0.\end{aligned}$$

By Lemma 1 and Lemma 2, at $(\tilde{p}^{-1}(p_0), p_0)$ of the set S, I have $\pi_{G,A}(\tilde{p}^{-1}(p_0)) > \pi_{G,U}(p_0)$, that is $h(\delta_E) > 0$. Since $h(\delta_E)$ is a strictly monotonic increasing continuous function on the closed interval $[\frac{1}{4}, \tilde{p}^{-1}(p_0)]$, I have 2 options.

First, $\pi_{G,A}(\frac{1}{4}) < \pi_{G,U}(p_0)$, that is $h(\delta_E) < 0$. So, by the Intermediate Value Theorem, there exists a unique $\tilde{\delta}_E$ such that $h(\delta_E) = 0$. Since $h(\delta_E) < 0$ for $\delta_E < \tilde{\delta}_E$ and $h(\delta_E) > 0$ for $\delta_E > \tilde{\delta}_E$, such a $\tilde{\delta}_E$ is exactly the one I am looking for.

Second, $\pi_{G,A}(\frac{1}{4}) \geq \pi_{G,U}(p_0)$, that is that is $h(\delta_E) \geq 0$. So, by the Intermediate Value Theorem, $h(\delta_E) > 0$ for all $\delta_E \in (\frac{1}{4}, \tilde{\delta}_E]$. In this case, I choose $\tilde{\delta}_E = \frac{1}{4}$.

So I find such a $\tilde{\delta}_E$. This completes the proof. ■

Proof of Lemma 4

PROOF:

To prove this lemma, I fix δ_E at some $\delta_{E0} \in [0, 1]$.

Given δ_{E0}, the government always choose p accordingly to maximize profit. According to Lemma 3, p^* is the optimal market environment where the government can maximize profit. Therefore, the point on $(\delta_{E0}, p^*(\delta_{E0}))$ is the equilibrium point.

Since δ_{E0} is an arbitrary point chosen from $[0, 1]$, the equilibrium points locate on the curve, $p^*(\delta_E)$, for $\delta_E \in [0, 1]$. So I have proved the existence and the uniqueness of the equilibrium.

Now, I only need to show the stability of the equilibrium. In fact, there are 2 options that one can deviate from the equilibrium at every given δ_E.

First, if $p > p^*$, then I define $\epsilon_1 > 0$ such that $\epsilon_1 \equiv p - p^*$. According to Lemma 1, the entrepreneur will set the property rights of the firm unambiguous. By Equation (15) and (16), the government will reset the market environment as

black as possible to increase the profit. As a result, ϵ_1 converges to 0, and p converges to p^* from above.

Second, if $p < p^*$, then I define $\epsilon_2 > 0$ such that $\epsilon_2 \equiv p^* - p$. According to Lemma 1, the entrepreneur will set the property rights of the firm unambiguous. By Equation (17), the government will reset the market environment as white as possible to increase the profit. As a result, ϵ_2 converges to 0, and p converges to p^* from below.

Therefore, I prove that for any disturbance ϵ that deviates the market environment from the equilibrium, the market environment will bounce back to the equilibrium. This shows that the equilibrium is stable. ■

Proof of Lemma 5

PROOF:

According to Lemma 4, the equilibrium points locate on the curve p^*. Therefore, to show the equilibrium property rights arrangement is equivalent to show the property rights arrangement at each point of the curve p^*. There are two options.

First, for $0 < \beta \leq \frac{d}{4q-d}$, recall that I have

$$p^* = \begin{cases} 0, & 0 \leq \delta_E < \frac{1}{4}; \\ \tilde{p}, & \frac{1}{4} \leq \delta_E \leq 1. \end{cases}$$

So I have the following result, which is consistent with Lemma 5.
1) For $0 \leq \delta_E < \frac{1}{4}$, the unambiguous property rights arrangement is the equilibrium;
2) For $\frac{1}{4} \leq \delta_E < 1$, the ambiguous property rights arrangement is the equilibrium;
3) For $\delta_E = 1$, A mix of both types of arrangements is the equilibrium.

Second, for $\frac{d}{4q-d} < \beta < 1$, recall that I have

$$p^* = \begin{cases} p_0, & 0 \leq \delta_E < \tilde{\delta}_E; \\ \tilde{p}, & \tilde{\delta}_E \leq \delta_E \leq 1; \end{cases}$$

So I have the following result, which is also consistent with Lemma 5.
1) For $0 \leq \delta_E < \tilde{\delta}_E$, the unambiguous property rights arrangement is the equilibrium;
2) For $\tilde{\delta}_E \leq \delta_E < 1$, the ambiguous property rights arrangement is the equilibrium;
3) For $\delta_E = 1$, A mix of both types of arrangements is the equilibrium.

So this proves Lemma 5. ■

Proof of Proposition 1

PROOF:

From Equation (20), the top-level government prefer to modify the δ_E to increase the capital investment from the entrepreneurs. According to Lemma 4, the equilibrium points lie on the *Equilibrium Line*. So I need to compare the capital investment at each point along the *Equilibrium Line*. From Equation (18), the *Equilibrium Line* can be divided into two parts, the situation with $0 \leq \delta_E < \frac{1}{4}$ and the situation with $\frac{1}{4} \leq_E \leq 1$. By Lemma 1 and Lemma 5, I should analyze k_U for the first part and k_A for the second part.

From Equation (5) I find that k_U is a single variate function of p and it is independent with δ_E. So the capital investment k_U at the first part is the same, that is $k_U(0)$.

Substituting Equation (9) into Equation (7), I find that k_A is a single variate function of δ_E, that is

$$\begin{aligned}
k_A(\delta_E) &= \frac{\frac{(\delta_E-\frac{1}{4})d}{(1-\delta_E)q+(\delta_E-\frac{1}{4})d}\delta_E\frac{1}{2}qa^\alpha\beta + [1-\frac{(\delta_E-\frac{1}{4})d}{(1-\delta_E)q+(\delta_E-\frac{1}{4})d}]\delta_E\frac{1}{2}da^\alpha\beta}{\beta+1} \\
&= \frac{(\delta_E-\frac{1}{4})d\delta_E\frac{1}{2}qa^\alpha\beta + (1-\delta_E)q\delta_E\frac{1}{2}da^\alpha\beta}{(\beta+1)[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]} \\
&= \frac{q\delta_E da^\alpha\beta[(\delta_E-\frac{1}{4})+(1-\delta_E)]}{2(\beta+1)[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]} \\
&= \frac{\frac{3}{8}q\delta_E da^\alpha\beta}{(\beta+1)[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]}.
\end{aligned}$$

By taking the first-order condition of k_A with respect to δ_E, I have

$$\begin{aligned}
\frac{dk_A(\delta_E)}{d\delta_E} &= \frac{\frac{3}{8}qda^\alpha\beta(\beta+1)[(1-\delta_E)q+(\delta_E-\frac{1}{4})d] - \frac{3}{8}q\delta_E da^\alpha\beta(\beta+1)(d-q)}{(\beta+1)^2[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]^2} \\
&= \frac{\frac{3}{8}qda^\alpha\beta(\beta+1)[(1-\delta_E)q+(\delta_E-\frac{1}{4})d-\delta_E(d+q)]}{(\beta+1)^2[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]^2} \\
&= \frac{\frac{3}{8}qda^\alpha\beta(\beta+1)(q-\frac{1}{4}d)}{(\beta+1)^2[(1-\delta_E)q+(\delta_E-\frac{1}{4})d]^2} \\
&> 0.
\end{aligned}$$

We find that k_A is an increasing function in $\delta_E \in [\frac{1}{4}, 1]$. So k_A reaches its maximum at $\delta_E = 1$.

Since by Lemma 1, the point $(\frac{1}{4}, 1)$ lies on the *Equivalent Efficiency Line*, I have $k_A = k_U$ at $\delta_E = \frac{1}{4}$. So $k_A(\delta_E) > k_U(0)$ for all $\delta_E \in [\frac{1}{4}, 1]$. So the capital investment reaches the global maximum at $\delta_E = 1$. Therefore, the economy converges to the point $(1, 1)$ along the *Equilibrium Line*. ■

Proof of Proposition 2

PROOF:

From Equation (21), the top-level government prefer to modify the δ_E to increase the tax revenue of the government. According to Lemma 4, the equilibrium points lie on the *Equilibrium Line*. So I need to compare the tax revenue at each point along the *Equilibrium Line*. From Equation (18), the *Equilibrium Line* can be divided into two parts, the situation with $0 \leq \delta_E < \frac{1}{4}$ and the situation with $\frac{1}{4} \leq_E \leq 1$. By Lemma 2 and Lemma 5, I should analyze $\pi_{G,U}$ for the first part and $\pi_{G,U}$ for the second part.

From Equation (2) and (5) I can transform $\pi_{G,U}$ to a single variate function of p and it is independent with δ_E. So the tax revenue $\pi_{G,U}(p)$ at the first part is the same, that is $\pi_{G,U}(0)$.

For the second part, I can transform $\pi_{G,A}$ to a single variate function of δ_E. From Equation (4), I have

$$\begin{aligned}
\pi_{G,A} &= p(1-\delta_E)\frac{1}{2}qa^\alpha k^\beta + (1-p)(1-\delta_E)\frac{1}{2}da^\alpha k^\beta \\
&= \frac{1}{2}(1-\delta_E)a^\alpha k^\beta[pq + (1-p)d] \\
&= \frac{1}{2}(1-\delta_E)a^\alpha k^\beta[p(q-d) + d].
\end{aligned}$$

From Equation (7), I have

$$\begin{aligned}
k &\equiv k_A \\
&= \frac{p\delta_E\frac{1}{2}qa^\alpha\beta + (1-p)\delta_E\frac{1}{2}da^\alpha\beta}{\beta+1} \\
&= \frac{a^\alpha\beta\delta_E}{2(\beta+1)}[pq + (1-p)d] \\
&= \frac{a^\alpha\beta\delta_E}{2(\beta+1)}[p(q-d) + d].
\end{aligned}$$

From Equation (9), I have

$$\begin{aligned} p &\equiv \tilde{p} \\ &= \frac{(\delta_E - \frac{1}{4})d}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}. \end{aligned}$$

Then, substituting Equation (24) and (25) into (23), I have

$$\begin{aligned} \pi_{G,A}(\delta_E) &= (\frac{a^\alpha}{2})^{1+\beta}(\frac{\beta}{\beta+1})^\beta {\delta_E}^\beta (1-\delta_E)[p(q-d)+d]^{\beta+1} \\ &= (\frac{a^\alpha}{2})^{1+\beta}(\frac{\beta}{\beta+1})^\beta {\delta_E}^\beta (1-\delta_E)[\frac{(\delta_E - \frac{1}{4})d}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}(q-d)+d]^{\beta+1} \\ &= (\frac{a^\alpha}{2})^{1+\beta}(\frac{\beta}{\beta+1})^\beta ({\delta_E}^\beta - {\delta_E}^{\beta+1})[\frac{\frac{3}{4}qd}{(1-\delta_E)q + (\delta_E - \frac{1}{4})d}]^{\beta+1} \\ &= (\frac{3a^\alpha qd}{8})^{1+\beta}(\frac{\beta}{\beta+1})^\beta \frac{{\delta_E}^\beta - {\delta_E}^{\beta+1}}{[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^{\beta+1}} \end{aligned}$$

We observe that $\pi_{G,A}(0) = 0$ and $\pi_{G,A}(1) = 0$. From the last term of the last line of Equation (26), $[(1-\delta_E)q + (\delta_E - \frac{1}{4})d]^{\beta+1}$ is a monotone decreasing function on $\delta_E \in [0,1]$ and ${\delta_E}^\beta - {\delta_E}^{\beta+1}$ increase monotonically first and then decrease monotonically on $\delta_E \in [0,1]$. So there exists $\delta_E \in [0,1]$ such that $\pi_{G,A}$ is maximized.

Since by Lemma 2, at $\delta_E = \frac{1}{4}$, I have $\pi_{G,A} > \pi_{G,U}$. So, there exists a $\hat{\delta}_E \in [\frac{1}{4}, 1)$, such that $\pi_{G,A}(\hat{\delta}_E)$ is the global maximum. Therefore, the economy converges to the point $(\hat{\delta}_E, p^*(\hat{\delta}_E))$ along the *Equilibrium Line.* ■

Proof of Corollary 1

PROOF:

From Equation (22), the utility function of the top-government in this case is simply an linear combination of the utility functions from Proposition 1 and Proposition 2. So there exists a point $\bar{\delta}_E \in (\hat{\delta}_E, 1)$ such that the utility function in Equation (22) is maximized.

By Lemma 4, the equilibrium must lie on the *Equilibrium Line* $p^*(\delta_E)$. Therefore, the economy converges to the point $(\bar{\delta}_E, p^*(\bar{\delta}_E))$ along the *Equilibrium Line.* ■

Proof of Corollary 2

PROOF:

In general, from Equation (22), the utility function of a top-government can be given by

$$U_{T,b} = w_e u_e(\delta_G) + w_r u_r(\delta_G),$$

subject to

$$w_e + w_r = 1, 0 \leq w_e, w_r \leq 1,$$

where w_e and w_r are weightings for the utility from efficiency and the utility from tax revenue respectively.

By Proposition 1 and 2 and Corollary 1, I find that the the economy converges to the point $(1, 1)$, if and only if $w_e = 1$ and $w_r = 0$. This proves Corollary 2. ■

Proof of Proposition 3

PROOF:

To prove this proposition, I fix a small positive number $\epsilon_3 > 0$. I assume the economy starts at $\delta_E = \tilde{\delta}_E + \epsilon_3$. Then, I consider the case $\delta_E = \tilde{\delta}_E - \epsilon_3$.

According to Lemma 1,

$$k_U(p^*(\tilde{\delta}_E + \epsilon_3)) = k_A(\tilde{\delta}_E + \epsilon_3).$$

From Equation (19),

$$p^*(\tilde{\delta}_E + \epsilon_3) < p_0.$$

From Equation (6),

$$k_U(p^*(\tilde{\delta}_E - \epsilon_3)) > k_U(p^*(\tilde{\delta}_E + \epsilon_3)).$$

So I have

$$k_U(p^*(\tilde{\delta}_E - \epsilon_3)) > k_A(\tilde{\delta}_E + \epsilon_3).$$

According to the proof of Lemma 3,

$$\pi_{G,U}(p^*(\tilde{\delta}_E - \epsilon_3)) = \pi_{G,U}(p_0) \simeq \pi_{G,A}(\tilde{\delta}_E - \epsilon_3).$$

From Equation (22), I have

$$U_{T,b}\big|_{\delta_E = \tilde{\delta}_E - \epsilon_3} > U_{T,b}\big|_{\delta_E = \tilde{\delta}_E + \epsilon_3}.$$

So the economy will transit from $(\tilde{\delta}_E + \epsilon_3, p^*(\tilde{\delta}_E + \epsilon_3))$ to $(\tilde{\delta}_E - \epsilon_3, p_0)$. ■

Proof of Proposition 4

PROOF:

To prove this proposition, I fix a point $s \in S$, such that $s = (\delta_{Es}, p_s)$ where

$p_s < p^*(\delta_{Es})$. So, according to Equation (17), I have

$$\pi_{G,A}\big|_{p=p^*(\delta_{Es})} > \pi_{G,A}\big|_{p=p_s}.$$

Define $B_0 > 0$, such that $B_0 \equiv \pi_{G,A}\big|_{p=p^*(\delta_{Es})} - \pi_{G,A}\big|_{p=p_s}$. Suppose the benefit from gaining the monopolistic power $B \gg B_0$. Although I have

$$k_A\big|_{p=p^*(\delta_{Es})} < k_A\big|_{p=p_s},$$

by Equation (22),

$$U_{T,b}(\delta_{Es}, p_s) > U_{T,b}(\delta_{Es}, p^*(\delta_{Es})).$$

Therefore, the government and the firm can be better-off by staying at the off-equilibrium point (δ_{Es}, p_s). This proves Proposition 2. ■

Appendix C: Notations

The key notations used in this study is summarized as follows.

E	Agent E (the enterprise)
G	Agent G (the local government)
T	Agent T (the top-level government)
k_U	Agent E's initial investment with unambiguous property rights arrangement
k_A	Agent E's initial investment with ambiguous property rights arrangement
a	Agent E's labor input (exogenous and fixed)
θ	Profitability of the firm
t	Agent G's charge rate (set by G)
$\pi_{E,U}$	Agent E's profit with unambiguous property rights arrangement
$\pi_{E,A}$	Agent E's profit with ambiguous property rights arrangement
$\pi_{G,U}$	Agent G's profit with unambiguous property rights arrangement
$\pi_{G,A}$	Agent G's profit with ambiguous property rights arrangement
δ_E	Agent E's relative bargaining power
δ_G	Agent G's relative bargaining power
p	Probability of the white state
$1-p$	Probability of the black state
q	Premium to the firm's revenue in the white state
d	Discount to the firm's revenue in the black state
$U_{T,e}$	The utility function of a efficiency-oriented Agent T
$U_{T,r}$	The utility function of a revenue-oriented Agent T
$U_{T,b}$	The utility function of a balanced Agent T