Romantic Relationship Towards Marriage: An Option Approach

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Abstract

We view the decision of whether to enter a romantic relationship as an option. The decision depends on the value of the option and potential gain from searching for a new partner. We base our model on Gary Becker's marriage model, and study the impact of different factors on the probability of a romantic relationship as well as a marriage. We also extend our model to explain several different scenarios, including urgent marriage, breaking up, and different dating cultures. This paper introduces a method from financial engineering to the study of a social and economic issue. (JEL: C6, D1, G0)

"The exchange of thy love's faithful vow for mine."

Romeo and Juliet, Shakespeare

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I. Introduction

"First comes love, then comes marriage." These words from children's song tell the importance of a romantic relationship to marriage. While abundant literature has studied the marriage problem, few has paid attention to the role of a romantic relationship, which is the inception of, and also an indispensable path to a marriage, especially a love marriage. In Becker (1973)'s classic model of marriage, his arguments start with "...considers two persons, M and F, who must decide whether to marry each other or remain single." In reality, however, before making this decision, the young man and young woman in general have to decide if they would start a romantic relationship so that they will have opportunity to get acquainted with each other and to figure out whether they are suitable. Research from sociology and psychology provides evidence that this process of mutual interaction has important effects on the formation of a marriage. Berger and Calabrese's (1975) proposes the uncertainty reduction theory in which they assume that the two strangers will have the strong desire to reduce uncertainty in their relationship formation. The theory is applied to the study of romantic relationship by Parks and Adelman (1983); and Sunnafrank (1986) further extends the theory with two complementary goals: reducing uncertainty and maximizing relational outcome. Bougheas and Georgellis (1999) finds out that perfect information about the quality of the marriage precludes the impact of divorce costs. In this paper, we address the issue of reducing uncertainty in a romantic relationship towards marriage.

The economic perspective of marriage focuses on the production and distribution of family goods. According to Becker's model, only when the wife's share of joint production of family goods exceeds her single production will she have incentive to get married, so does the husband. Similar to the production of any other goods, there are costs of producing family goods. One of these costs is the search cost. People need to search in the marriage market for a suitable spouse as they search in a job market for a suitable position. Due to this similarity, literature has applied job search theory to the marriage problem, see Keeley (1977), Oppenheimer (1988), Smith (2006). The basic idea is that a single person will continue to search for the most suitable mate subject to the fact that the search is costly. The search cost can either be direct cost or opportunity cost, as addressed by early debate in North (1968) and Kafolis (1970). If one, say a young woman, encounters a potential spouse, she can choose to either continue searching for a better partner or to begin a romantic relationship with the current available mate (if this potential spouse also agrees to start a relationship). Since this romantic relationship might or might not result in a marriage, the opportunity cost of the further search would be the potential gain from the romantic relationship. Conversely, if this young woman decides to enter a romantic relationship, the opportunity cost of this relationship would be a potential higher-quality partner from further search. Her decision of whether to enter a relationship depends on the comparison between the two potential gains.

However, neither the gain from search nor from a romantic relationship is certain. In a search model, one searches for the desirable partner with desirable attributes over a certain distribution. Since search is costly, there would be a minimum acceptance level (Oppenheimer, 1988). The gain from search is not certain because after one round of search, one may either successfully find a better mate or fail to do so and remain single. We will adapt Courant (1978)'s search model in housing market to model the search in marriage market. On the other hand, the potential gain from entering a romantic relationship is ambiguous. While the relationship itself can bring positive experiences like romantic dating or sexual satisfaction, to be consistent with Becker's model, we consider the major gain from a romantic relationship as their potential share of joint family goods production if they get married in the end. As addressed in the uncertainty reduction theory, young couples face uncertainties in a romantic relationship either due to unsuitable matching or external factors. They will need to reduce the uncertainty of their own potential share of jointly produced family good if they get married. It is reasonable to assume that as they get more intimate, the level of uncertainty would decrease (Solomon and Knobloch, 2001). At the end of the dating period, if they find their own share of joint production, known as marital income, exceeds their single production, they will choose to get married and otherwise remain single. Hence the potential gain from entering a romantic relationship depends on the difference between the martial income and single productivity. This form of gain, in finance, is actually an option. In this paper, we will consider the gain from a romantic relationship as the value of such an option.

The most famous model for valuing option is Black-Scholes-Merton model, proposed in Black and Scholes (1973). The uncertainty of the BSM model lies in the price of underlying asset; while in our model, uncertainty exists in their suitability and also from external factors. Thus, the

BSM model can be a good reference for valuing the option of a romantic relationship, but it cannot be applied directly to our problem. Using risk-neutral pricing, the BSM model does not rely on investor's utility function. But since the option of the romantic relationship is obviously not a standardized contract, its risk cannot be arbitraged so the risk-neural pricing method is no longer feasible. This is the major difference between the standard BSM model and the option model in our discussion. Finally, although the BSM formula is widely applied to value different financial contracts and even real assets, to the best of our knowledge, we are the first to apply the option model in the study of the marriage problem. The remaining part of the paper is organized as follows: section II introduces the model setup and discusses the properties of option's value. Based on the basic model, we solve the decision problem between starting a romantic relationship and continuing to search. Section III extends the model to real life situations including urgent marriage, breaking up and hanging out. Section IV concludes the paper.

II. The Model

A. Basic Setup

We consider a young man whose name is Romeo and his beloved woman whose name is Juliet. Romeo wants to be together with Juliet and he expresses his affection to Juliet, asks if Juliet could be his girlfriend. Now Juliet faces two choices: she can either accept Romeo's love and take a trial to see if Romeo is the right person to marry or reject Romeo and wait for the next potential Mr. Right. Romeo here, indeed, offers Juliet an option: once Juliet agrees to be his girlfriend, then after some dating time T, if Romeo proves that he is the right person, Juliet can choose to marry him; otherwise Juliet would break up with him. We call this option as a romantic option. Meanwhile, if Juliet rejects Romeo, she can use this time T to search for the Mr. Right. She has to make a choice depending on the value of this option and expected gain from search.

We start from modeling the romantic option offered by Romeo following Gary Becker's marriage model. Suppose there is only one single aggregate commodity, denoted as Z, which can be either produced by a single person or jointly produced within a family. By the time $t = T$, when Juliet has to decide whether to marry, if she chooses to get married, she will share a proportion of jointly output $f_T \equiv \alpha Z_{mf}$, where Z_{mf} is the total output of the marriage and α is Juliet's share; if she prefers to remain single, her single output would be Z_{0f}. A necessary condition for Juliet to marry is $f_T > Z_{0f}$. In other words, if Juliet decides to marry, her expected gain from marriage is $f_T - Z_{of}$; if Juliet breaks up with Romeo, her expected gain is zero. Suppose Juliet uses a discounting factor β, at time $t = 0$, her expected gain from accepting Romeo's offer is

$$V_0 = e^{-\beta T} E\left[\max\left\{f_T - Z_{of}, 0\right\}\right]. \tag{1}$$

However, at time $t = 0$, when Juliet has to decide whether to accept Romeo's offer, she does not know the value of f_T but only knows the value of f_0, which is the joint output of current Romeo and Juliet. Meanwhile, she knows her personal traits A_f as well as Romeo's personal traits A_m, so she would expect either a positive or negative 'chemical reaction' with Romeo depending on whether a couple with A_f and A_m is suitable. We characterize this chemical reaction as these traits' effect on joint output:

$$\mu \equiv \frac{\partial^2 Z_{mf}}{\partial A_m \partial A_f}. \tag{2}$$

We assume μ to be a constant. If Romeo is a suitable partner, Juliet would expect a positive reaction between them, i.e., $\mu > 0$; otherwise she would expect $\mu < 0$. A positive reaction tends to increase their joint output, and vise versa. But this can be a process with uncertainty, since their relationship will be affected by external factors as well. Formally, we use a geometric Brownian motion to describe this process:

$$\frac{df_t}{f_t} = \mu\, dt + \sigma\, dz, \tag{3}$$

where $z_t = \varepsilon\sqrt{\mathrm{t}}$ is a standard Weiner process; σ is the volatility that characterizes the stability of their relationship. In order to be consistent with Becker's model, in which the benefit from marriage is viewed as a one-time gain, f_t here should be interpreted as Juliet's expected share of joint output based on her current judge of the relationship, in present value. Given these settings, Juliet can calculate her expected gain V_0 from accepting Romeo's as her boyfriend, which, as we will show later, is the value of Romeo's romantic option.

On the other hand, Juliet can choose to reject Romeo if she does not think Romeo is the right person. If she does so, she can use the dating time T to search for the next Mr. Right who she believes is better than the current Romeo. Suppose that, within time T, Juliet only has one search chance and at time $t = T$ she will find a man who can bring her a share of f_T^* if she marries the man. Again, she will marry the man only if $f_T^* > Z_{0f}$. If $f_T^* < Z_{0f}$, she has to remain single. Suppose the distribution of f_t is $G(f_t)$, where we assume the distribution are the same over time. Then her expected gain from a further search for Mr. Right is

$$W_0 = e^{-\beta T}\left(1 - G(Z_{0f})\right) \cdot E\left(f_T - Z_{0f} \middle| f_T > Z_{0f}\right). \tag{4}$$

Juliet's utility function is $u_J = \max\{V_0, W_0\}$, so she will choose to accept Romeo's romantic option if $V_0 > W_0$ and $V_0 \geq Z_{0f}$; and will reject Romeo if $V_0 < W_0$. And we assume if Juliet is indifferent between accept and reject, she will accept Romeo.

B. The Value of Romeo's Option

From the expression of V_0 we can see that it is indeed a call option. So far we characterize this option as a European option, which means that Juliet makes her decision only when $t = T$. We will use this as the base case and discuss the case where we allow Juliet to get married to Romeo at any time she believes appropriate before $t = T$, i.e., when the option is an American option. To value this European option, however, we cannot directly apply the well-known Black-Scholes-Merton model, because the BSM model relies on the strict assumption of no-arbitrage in the financial market. In deriving BSM partial differential equation, we require the option to be duplicated from underlying asset and riskless asset. Clearly this condition cannot be satisfied in the context of our model. It would be weird to assume that Romeo's love can be replicated. Therefore, we need to directly compute the expectation gain of Juliet.

Fortunately, our model does not deviate from BSM too much. First, since f_t follows a geometric Brownian motion, we will have that f_T follows the log-normal distribution:

$$\ln f_T \sim N\left(\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right). \tag{5}$$

where f_0 is the Juliet's share of joint output if she marries Romeo at time $t = 0$. Since $E\left[\max\{f_T - Z_{of}, 0\}\right]$ is merely the expectation of a function of f_T, and from above we already know the distribution of f_T, we can directly calculate this expectation to get the formula. The calculation is a little bit complicated. However, we can get the result quickly by paying attention to the difference between our model and BSM. In BSM, where we use risk-neutral pricing, despite the value of μ, f_T follows a slightly different log-normal distribution:

$$\ln f_T \sim N\left(\ln f_0 + \left(r - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right), \tag{6}$$

where r is the riskless interest rate. Therefore, in calculating the expectation, the only difference is that we still hold μ in our model instead of r. Thus we can get the result by slightly modifying the BSM formula:

$$V_0 = f_0 e^{(\mu-\beta)T} N(d_1) - Z_{0f} e^{-\beta T} N(d_2),$$

where (7)

$$d_1 = \frac{\log\frac{f_0}{Z_{0f}} + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\log\frac{f_0}{Z_{0f}} + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

and $N(\cdot)$ represents the cumulative distribution function of standard normal distribution.

C. Properties of the Option

a. Incentive to get married early

We would not regard the romantic relationship as a rigid contract, so we should allow Juliet to marry Romeo even at time before $t = T$ so long as she believes it is the proper time. This means the option can be an American option. In financial market, luckily we know that a European call option and an American call option have the same value, holding their parameters the same. This is because it is not profitable to exercise the option before expiry date (see [1]). If this is still true in our model, then we need not to worry about the case in which Juliet and Romeo get married before $t = T$. However, for the same reasons discussed in previous section, things are different in our model compared to financial market. So here we need to argue whether it may be a wise choice for Juliet to get married with Romeo before $t = T$.

We start from the explanation of dating time T. Why do couples usually choose to date for a period time before marriage rather than get married directly? The key reason is that couples need time get acquainted with each other. In our model, this means Juliet needs time to determine the true value of f_T. We consider the dating time T as the necessary and sufficient time for Juliet to be certain of f_T. Sufficient means at $t = T$, Juliet will have enough confidence to make the decision of whether to marry Romeo, depending on the value of f_T. Necessary means Juliet needs to spend this long time to fully know Romeo. Even though she gets married with Romeo before $t = T$, she would observe f_T to be volatile until $t = T$. This assumption seems plausible due to the need for reducing uncertainty. If they get married too early, there is higher probability of mismatch and marital instability (Oppenheimer, 1988).

Given these assumptions, it is not a wise choice for Julie to marry Romeo before $t = T$ even if at some $t = t_0$, she observes f_{t_0} to be far greater than Z_{0f}. First, if Juliet would expect $f_T > Z_{0f}$, she has no reason to get married to Romeo early, because there is no difference between marrying at $t = t_0$ and $t = T$. Second, it is possible that at $t = T$, f_T is smaller than Z_{0f}. Since $\varepsilon \sim N\ (0,1)$, with small probability it can be deeply negative. This can be the case if some rare but devastating events happen. For example, if Romeo cannot control himself and beats Juliet after he gets drunk, Juliet's expectation of their future would crack, *i.e.,* her expect of f_t would plummet. In this case, Juliet would be worse off if she gets married. If she had made the decision before $t = T$, she would have broken up with Romeo and the gain would have been zero rather than negative.

Since there is no reason for Juliet would to marry Romeo before $t = T$, we will again have that the value of an American option is equal to a European option with same conditions. The intuition is simple: it is better to get married to a person when you fully know him. We conclude the discussion in Theorem 1 and Corollary 1.

Theorem 1 If Juliet accepts Romeo's proposition, she has no incentive to get marriage with Romeo before $t = T$.

Corollary 1 Even if Juliet has the choice to marry Romeo before $t = T$, the value of Romeo's option is the same as if she must make the decision at $t = T$.

The proofs of Theorem 1 and Corollary 1 are in discussion above.

b. Greeks

We have derived the formula of the option value V_0. We need to discuss how different factors influence the value of V_0. In finance, the derivatives of an asset's value with respect to different parameters are represented by Greeks, so we title this section as Greeks. Before analyzing the impact of different parameters, we need to point out a well-known fact.

Lemma 1 The probability of the option being exercised is $N(d_2)$. i.e., $P\left(f_T > Z_{0f}\right) = N(d_2)$.

Proof. The proof is simple. We already know that $\ln f_T \sim N\left(\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right)$, then $P\left(f_T > Z_{0f}\right) = P\left(\ln f_T > \ln Z_{0f}\right) = P\left(\frac{\ln f_T - \left[\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T\right]}{\sigma\sqrt{T}} > \frac{\ln Z_{0f} - \left[\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T\right]}{\sigma\sqrt{T}}\right) = 1 - N(-d_2) = N(d_2)$.

(1) Delta and Rho

Obviously there are two major factors that influence Juliet's decision: f_0, which represents current Romeo's attractiveness in terms of this productivity of family goods Z; and μ, which represents their suitability. First we study the impact of f_0. In our model, f_0 is the counterpart of asset price; the derivative with respect to asset price is denoted as Δ. Here

$$\Delta = \frac{\partial V_0}{\partial f_0} = e^{(\mu-\beta)T} N(d_1). \tag{8}$$

This a well-known trick in Greeks, that despite f_0 exists in both d_1 and d_2 and of complicated form, the derivative takes a simple form as if f_0 was not in d_1 and d_2. We can see that $\Delta > 0$ always holds. This implies that the value of Romeo's option is more valuable as Romeo is more attractive. This result is important in our next proof of when Juliet would accept Romeo. It is very intuitive that Juliet would tend to accept a Romeo who is more productive and thus can bring her a larger amount of family goods.

Next we consider the impact of μ, the suitableness of Romeo and Juliet. μ in our model is the counterpart of r in BSM; and the corresponding derivative is denoted as P (rho):

$$P = \frac{\partial V_0}{\partial \mu} = f_0 T e^{(\mu-\beta)T} N(d_1). \tag{9}$$

Still this derivative is constantly greater than zero. But it has to be noticed that the expression of P differs from that of a standard call option due to our setting. $P > 0$ means the option would be more valuable if Romeo and Juliet are more suitable. This is also very intuitive, that Juliet is more likely to accept Romeo if they are more assortative.

Meanwhile, f_0 and μ also have another impact. We know that $N(d_2)$ is the probability Juliet will marry Romeo at $t = T$. It is easy to see d_2 and thus $N(d_2)$ is increasing in f_0 and μ. Therefore, an attractive and suitable Romeo not only makes it easier for Juliet to accept his romantic option but also increases his chance marry Juliet.

(2) Vega

We then discuss the impact of σ, the volatility on the value of the option and the probability of Juliet getting married to Romeo. Its influence on the option, denoted as V (Vega), can be calculated through the derivative

$$V = \frac{\partial V_0}{\partial \sigma} = f_0 e^{(\mu-\beta)T}\sqrt{T}N'(d_1). \quad (10)$$

This derivative is always positive, which means the value of Romeo's option is higher with larger volatility. This is because with larger volatility, Romeo is more likely to produce surprise in both directions. If the surprise has positive impact on f_t, it will increase the value of the option; otherwise, if the negative impact is so significant that it drives f_T below Z_{0f}, Juliet would choose not to exercise this option. Thus, the impact of higher volatility is asymmetric in different directions and is blocked in the negative side, resulting in a net positive influence to the value of the option.

But its influence on the probability with which Romeo and Juliet get married is ambiguous, because d_2 is not a monotone function in σ. The derivative of d_2 with respect to σ is

$$\frac{\partial d_2}{\partial \sigma} = -\frac{\log\frac{f_0}{Z_{0f}} + \mu T}{\sqrt{T}\sigma^2} - \frac{1}{2}\sqrt{T}. \quad (11)$$

When $\log\frac{f_0}{Z_{0f}} + \mu T > 0$, this derivative is strictly negative. However, if $\log\frac{f_0}{Z_{0f}} + \mu T < 0$, the sign of $\frac{\partial d_2}{\partial \sigma}$ can be either positive or negative. In this case we have

$$\frac{\partial d_2}{\partial \sigma}\begin{cases} < 0, & if\ \sigma > \sqrt{-2(\log f_0 - \log Z_{0f} + \mu T)/T} \\ > 0, & if\ \sigma < \sqrt{-2(\log f_0 - \log Z_{0f} + \mu T)/T} \end{cases}. \quad (12)$$

This result is interesting. Notice that $\log\frac{f_0}{Z_{0f}} + \mu T > 0$ is equivalent to $f_0 > e^{-\mu T}Z_{0f}$, i.e., at the beginning Romeo is attractive enough. The intuition behind the result can be explained as follows: if Romeo is already qualified enough at the time he expresses his love to Juliet, Juliet would wish their relationship to be stable, therefore large volatility could be harmful; if at the beginning Romeo is not that good, and there is almost no uncertainty, with higher probability Juliet would not marry Romeo in the end. In this case, Juliet would hope there could be some external events that can motivate Romeo to behave better than he had previously. Thus a large volatility would be helpful.

(3) Theta

We have discussed the meaning of the dating time T. Now we consider its impact on the value of the option. The derivative is denoted as Θ:

$$\Theta = \frac{\partial V_0}{\partial T} = e^{(\mu-\beta)T} f_0 \left[(\mu - \beta)N(d_1) + N'(d_1)\frac{\sigma}{2\sqrt{T}}\right] + \beta Z_{0f} N(d_2). \tag{13}$$

The sign of this derivative is vague. It can be either positive or negative when $\mu < 0$, implying a negative prospective. Figure 1 shows the different possibilities of how value of the option changes with T.

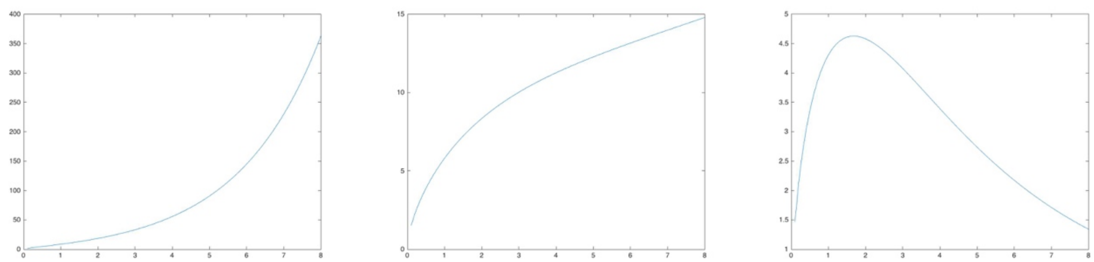


Figure 1 Option Value changing with *T*

In most cases, when T is not too short and $\mu - \beta > 0$, this derivative is positive. It has to be mentioned that the definition of Θ is different from its definition in finance, where the derivative is not with respect to T but with respect to the remaining time before expiry date. We will discuss that case in later section. The positive value of Θ implies that with longer time knowing each other, Juliet would be more confident to accept Romeo's romantic option. On the other hand, similar to σ, the impact of T on $N(d_2)$ is vague, depending on the value of $\log\frac{f_0}{Z_{0f}}$ and μ:

$$\frac{\partial d_2}{\partial\sqrt{T}} = -\frac{\log\frac{f_0}{Z_{0f}}}{\sigma T} + \left(\frac{\mu - \frac{1}{2}\sigma^2}{\sigma}\right). \tag{14}$$

There are four possibilities. First, if $\log\frac{f_0}{Z_{0f}} > 0$ and $\mu - \frac{1}{2}\sigma^2 > 0$, *i.e.*, if at the beginning Romeo is good enough and there is a strong positive reaction between them, in two scenarios will they be likely to marry: either Juliet can settle down in a very fast pace (small T), or she needs to spend a long time on knowing Romeo (large T). Second, if $\log\frac{f_0}{Z_{0f}} > 0$ and $\mu - \frac{1}{2}\sigma^2 < 0$, namely, if Romeo looks attractive but their interaction is not that positive, the derivative is strictly negative, in this case, longer dating time T can be harmful. Third, if $\log\frac{f_0}{Z_{0f}} < 0$ and $\mu - \frac{1}{2}\sigma^2 > 0$, *i.e.*, if Romeo is not that good at the beginning but they have a positive interaction, longer dating time would help to build their relationship. Finally, if $\log\frac{f_0}{Z_{0f}} < 0$ and $\mu - \frac{1}{2}\sigma^2 < 0$, although the derivative can be positive when T is small, the probability of marriage $N(d_2)$ is indeed very small, because Romeo is neither good nor suitable. To conclude, a longer dating time would increase chance of Juliet accepting Romeo, but depending on whether Romeo is attractive and suitable, longer dating time can either increase or decrease their probability of getting married.

(4) The impact of Z_{0f}

There is one more factor left to be analyzed: Z_{0f}, which is Juliet's single productivity. Unfortunately, there is no Greek representing this derivative. Intuitively, holding other factors constant, a higher Z_{0f} represents a more productive and independent Juliet. Such an independent Juliet tends to have a higher requirement for her spouse, *i.e.*, given the characteristics of Romeo, she is less likely to accept him if her life quality when single is nice enough. We can verify this argument by calculating the derivative

$$\frac{\partial V_0}{\partial Z_{0f}} = -e^{-\beta T} N(d_2) < 0, \tag{15}$$

and

$$\frac{\partial d_2}{\partial Z_{0f}} = -\frac{1}{\sigma\sqrt{T} Z_{0f}} < 0. \tag{16}$$

We finish discussing the impact of different factors on both the value of Romeo's option and the probability of them getting married. We conclude our discussion in the following Theorem:

Theorem 2 The value of Romeo's option is $V_0 = f_0 e^{(\mu-\beta)T} N(d_1) - Z_{0f} e^{-\beta T} N(d_2)$, where;

(1) Larger f_0 and μ and smaller Z_{0f} will increase both the probability of Juliet accepting Romeo and the probability of marriage.
(2) Larger σ will increase the probability of Juliet accepting Romeo, but its impact on the probability of marriage depends on f_0, μ and Z_{0f}.
(3) Longer dating time T can be either helpful or harmful, depending on the value of μ and β.

Proof. See discussion above.

D. The Gain from Search

If Juliet rejects Romeo, she will have an alternative choice of spending the dating time T on searching for the next Mr. Right. Clearly, she wants to find a new partner who can bring her a share of joint output larger than her single output. Therefore, her expected gain from further search is

$$W_0 = e^{-\beta T}\left[\left(1 - G(Z_{0f})\right) \cdot E\left(f_T - Z_{0f} \middle| f_T > Z_{0f}\right) + G(Z_{0f}) \cdot 0\right]. \tag{17}$$

If we denote the best possible share of joint output that can be obtained by Juliet from existing population as $\overline{f_T}$, then we will have

$$\begin{aligned} W_0 &= e^{-\beta T}\left[1 - G(Z_{0f})\right] \int_{Z_{0f}}^{\overline{f_T}} \left(f_T - Z_{0f}\right) \frac{g(f_T)}{1 - G(Z_{0f})} df_T \\ &= e^{-\beta T} \int_{Z_{0f}}^{\overline{f_T}} \left(f_T - Z_{0f}\right) dG(f_T). \end{aligned} \tag{18}$$

It is easy to see that W_0 is decreasing in Z_{0f}:

$$\frac{dW_0}{dZ_{0f}} = -e^{-\beta T} \int_{Z_{0f}}^{\overline{f_T}} dG(f_T) = -\mathrm{e}^{-\beta T}(1 - \mathrm{G}(\mathrm{Z}_{0f})) < 0. \tag{19}$$

This tells us that it is less likely for Juliet to find a better partner if she herself is productive enough. We already know that the value of option is decreasing in Z_{0f}. Therefore, it is not easy to tell under what conditions Juliet should accept Romeo and under what conditions should not. Meanwhile, there are a lot of factors influencing the value of the option so there can be a lot of situations. We will discuss these different situations in the next part.

E. Accept or Reject

We turn to consider the decision faced by Juliet at $t = 0$ of whether to accept Romeo's declaration of love. She has two choices: either accept Romeo or reject him and search for the next partner. Juliet maximizes her utility $u_J = \max\{V_0, W_0\}$; therefore, her decision depends on the expected value of Romeo's option and the expected gain from search. To simplify our discussion, while without loss of generality, we assume the f_T follows the uniform distribution:

$$G(f_T) = \frac{f_T - \underline{f_T}}{\overline{f_T} - \underline{f_T}}. \tag{20}$$

And we define the difference between the value of option and the expected gain from search, net of discounting, as

$$D = e^{\beta T}(V_0 - W_0) = f_0 e^{\mu T} N(d_1) - Z_{0f} N(d_2) - \int_{Z_{0f}}^{\overline{f_T}} \left(f_T - Z_{0f}\right) dG(f_T). \tag{21}$$

When f_T follows the uniform distribution, D can be reduced to

$$D = f_0 e^{\mu T} N(d_1) - Z_{0f} N(d_2) - \frac{\overline{f_T} - Z_{0f}}{2}. \tag{22}$$

Meanwhile, if under any condition Juliet will choose one strategy (either accept or reject) under any condition, we say this choice *dominates* the other. Using the expression of D, we will prove the following theorem.

Theorem 3 In general,

(1) Neither accept nor reject can dominate the other;
(2) There can be one cutoff point, multiple cutoff points, or no cutoff point in which Juliet is indifferent from accepting or rejecting Romeo.

Proof. First, notice that, so long as f_0 and Z_{0f} are greater than zero, the value of the option is strictly positive no matter what Z_{0f} is. Although according to Theorem 2, we know that both V_0 and $N(d_2)$ is decreasing in Z_{0f}; that is, as Z_{0f} gets large enough, the probability of the option to be exercised goes to zero, but is strictly larger than zero. The is due to the property of Brownian motion: although with almost zero probability, it can reach any large number within given time.

On the other hand, when $Z_{0f} = \overline{f_T}$, $W_0 = 0$ with certainty. Therefore, we know that

$$D\left(Z_{0f} = \overline{f_T}\right) > 0. \tag{23}$$

Next, take the first and second derivative of D with respect to Z_{0f}:

$$\frac{\partial D}{\partial Z_{0f}} = -N(d_2) + \frac{1}{2};\ \frac{\partial^2 D}{\partial Z_{0f}^2} = N'(d_2)\frac{1}{\sigma\sqrt{T}Z_{0f}} > 0. \tag{24}$$

The second derivative is greater than zero implies that D is a convex function in Z_{0f}. Let the first derivative equal zero:

$$\frac{\partial D}{\partial Z_{0f}} = -N(d_2) + \frac{1}{2} = 0\ \rightarrow d_2 = 0 \rightarrow Z_{0f} = f_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T}. \tag{25}$$

Since the second derivative is positive, the function will reach its minimum value at $Z_{0f} = f_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T}$:

$$D_{min} = D\left(f_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T}\right) = f_0 e^{\mu T} N(d_1) - \frac{1}{2}\overline{f_T}. \tag{26}$$

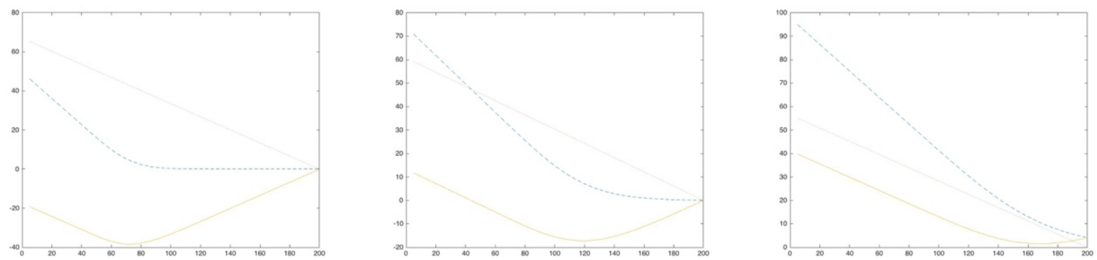
This can be either positive or negative. Then, taking into account the fact $D\left(Z_{0f} = \overline{f_T}\right) > 0$, by using Mean Value Theorem we will have the following result:

(1) If $D\left(Z_{0f} = \underline{f_T}\right) < 0$ and $D_{min} < 0$, then there is only one cutoff point;

(2) If $D\left(Z_{0f} = \underline{f_T}\right) > 0$ and $D_{min} < 0$, then there are two cutoff points;

(3) If $D\left(Z_{0f} = \underline{f_T}\right) > 0$ and $D_{min} > 0$, then there is no cutoff point.

This finishes the proof. The result is shown in the Figure 2, where the vertical axis represents the value of the option or from the search and the horizontal axis represents the Z_{0f}.



(a) One cutoff point (b) Two cutoff points (c) No cutoff point

Figure 2 Different Numbers of Cutoff Points

In Figure 2, the dashed blue line is the value of the option; the dotted straight line is the value of search; the yellow full line is the value of D. We can see directly from this figure that all three situations are possible.

It is worth a discussion of this result. From the perspective of Juliet, given Z_{0f}, she is more likely to accept Romeo if μ and f_0 is higher enough. For given Z_{0f}, higher μ and f_0 will move the line representing the option value upward so that its value will exceed the value of search. This means Juliet will accept Romeo if he is productive and suitable. From the perspective of Romeo, given f_0, finding a Juliet who is suitable for him will move the option line upward so that makes it more likely for Juliet to accept him. Meanwhile, given f_0 and μ, it would be more difficult for him to pursue a Juliet who is productive and independent, because in all cases, a higher Z_{0f} means lower value of the option. One extreme situation is interesting, which is rightmost point representing $Z_{0f} = \overline{f_T}$. In this case, Juliet's single output is so high that marrying anyone among the existing population would not make her better off. This makes it almost impossible for Juliet to accept anyone's romantic option; in this case, the value of D is almost zero. But this also implies that search among the existing population will definitely not be helpful at all. If she gets married to someone, however, it is possible that with positive chemical reaction between them they can achieve a higher joint output than they are both single. Love in the dating could have the magic to make Romeo a better person and exceed the limit.

III. Extensions

In this section, we extend the model to explain additional situations in a romantic relationship. The basic idea is still the same: to view the choice of starting a relationship with someone as an option. In our discussion above, we view the choice as an American call option. In the financial market, there also exist different kinds of exotic options. We will use the idea of pricing an exotic option to study some interesting scenarios in a romantic relationship. Meanwhile, we will discuss Romeo's problem of whether or not to express his love to Juliet which we regard as a completed, exogenous event in the previous section.

A. Romeo's Problem

We have analyzed Juliet's problem of whether or not to accept Romeo's love. In the previous section, we assume that Romeo has already made up his mind to express his love to Juliet; now we turn to consider Romeo's problem: if he knows that Juliet will consider his proposal as an option, under what situation should he bravely (or strategically) express his affection? First, it is natural that our previous analysis also holds for Romeo as well. In other words, only when Romeo believes that the expected utility from successfully being accepted as Juliet's boyfriend exceeds the potential gain from searching for another girl will he express his affection to Juliet. But this is not a sufficient condition. There are other concerns for Romeo. First since Romeo is the first mover (by assumption: one can change gender movement ordering or have same gender approaches without loss of generality) by declaring his love to Juliet, he incurs the risk of being rejected. In that case, he will suffer a psychological cost and may need time to recover from the frustration. We denote this mental trauma if he is rejected as c_r. Second, although he knows that

Juliet will make her decision based his current status f_0 and their matching degree μ, his own perception of f_0 and μ, denoted as $\widehat{f_0}$ and $\hat{\mu}$, may differ from what Juliet perceives. It is likely that, Romeo is at least somewhat arrogant (which may be more likely to be a male trait; more importantly, note that we could also analyze the behavior of insecure suitors) so he tends to overestimate his own charm and productivity, i.e., $\widehat{f_0} > f_0$; or, even more likely, overwhelmed by love, Romeo tends to persuade himself to consolidate his belief that they are a heaven-made match and therefore is willing to take more risk in expressing his love. This phenomenon is called cognitive dissonance, as studied by Akerlof and Dickens (1982). Romeo is not sure if his own perception accords with Juliet's, so there is a probability P_r with which he will be rejected. If from Romeo's perspective he does believe they are suited for each other, he needs to convince Juliet of his own perception of $\widehat{f_0}$ and $\hat{\mu}$ before he formally declares his love. This could incur a cost c_k, which in reality is often the cost spent during their informal hook-up period on creating opportunities for interaction. Given these costs, we adopt the discrete choice model in Hitsch, Hortaçsu and Ariely (2010) to depict Romeo's expected utility if he decides to express his love:

$$EU_R = (1 - P_r)V_0^R - P_r c_r - c_k, \tag{27}$$

where V_0^R is the Romeo's valuation of his expected gain from their romantic relationship, similar to Juliet's valuation of the option. Romeo will express his love to Juliet only if his expected utility, taking into account the probability of being rejected, exceeds the potential gain from search, that is

$$EU_R = (1 - P_r)V_0^R - P_r c_r - c_k > \int_{Z_{m0}}^{\overline{m_T}} (m_T - Z_{m0})dG(m_T). \tag{28}$$

The definitions of m_T and $G(m_T)$ are similar to Juliet's f_T and $G(f_T)$ that have been defined in (20); and $Z_{m0} = (1 - \alpha)Z_{mf}$, by Becker's convention. Compared with Juliet's decision problem in (22), due to the risk of being rejected, the corresponding mental trauma and necessary hook-up cost, Romeo will be more deliberate in pushing forward their relationship conditional on their beliefs of the spouse's quality and their matching level accord. In the best case, if Romeo can convince Juliet without any cost, i.e., $c_k = 0$ and $P_r = 1$, and if he does not care about the feeling of being rejected, i.e., $c_r = 0$, Romeo's problem will be reduced to Juliet's problem.

Another interesting problem is what Romeo should do if he knows he is likely to be rejected by Juliet. From Juliet's perspective, she will reject Romeo if he does not live up to her expectations (f_0 is low) or she believes that they are not suitable (μ is low). Thus, Romeo has two directions to put efforts: he can endeavor to improve his competitiveness and/or their suitability. We have derived in (8) and (9) that

$$\frac{\partial V_0}{\partial \mu} = f_0 T e^{(\mu-\beta)T} N(d_1) = f_0 T \frac{\partial V_0}{\partial f_0}. \tag{29}$$

Therefore, so long as they have a fairly long dating period, the marginal effect of suitability would be greater than the marginal effect of initial competitiveness. The initial quality of Romeo will only have a linear multiplier effect on V_0, while their matching degree has an exponential effect during the relationship development due to the geometry Brownian motion. In this sense, appropriateness is the more important factor in finding a spouse and it is the better choice for Romeo to increase their suitability, or more precisely, to increase their suitability as perceived by Juliet. While adapting him to Juliet is the more efficient way, it is hard to argue whether increasing his competitiveness and changing his personality is more difficult. Competitiveness, or Romeo's ability to provide Juliet with household goods, has a relatively quantifiable measure but his personality and their suitability do not have such measure. This feature of suitability makes increasing suitability a paradoxical option: on the one hand, effort in changing personality may have undiscernible effect and is not guaranteed to be recognized by Juliet; on the other hand, and always the case in real life, Romeo can attempt to cajole Juliet into believing that they are suitable by blandishments with almost no substantial cost. Since Juliet's decision of accepting Romeo is instantaneous in the model, she will not realize the fact that they are not as suitable as she has anticipated until they date for a period. Romeo can take advantage of this period to improve himself or adapt himself. Note that this optimization problem gives Romeo an incentive to exaggerate and lie (about everything, but especially about less verifiable suitability). Thus, women are correcting in believing that men tend to be liars and in discounting their claims, but Romeo and his fellow first movers may not be inherently deceitful, but rather are forced to be duplicitous in order to solve their optimization problems.

Having both Juliet's and Romeo's problem, our analysis using an option approach differs from the financial market in payoff mode for both parties. Although Romeo provides an option to

Juliet, he bears no risk of compensating Juliet's gain if Juliet decides to marry him at the end; in contrast, this is a win-win game for the lovers if they get married. The cost of offering the option to and accepting the option from the other side is primarily the opportunity cost that they give up the chance of dating other potential spouses, rather than the option premium as in financial market. Our option approach resembles the financial market in the sense the relational contract can be created, but in our problem two option contracts will be created simultaneously if Juliet accepts Romeo's affection.

B. Urgent Marriage

We start to rethink of the meaning of the dating time T. In previous section, we explain T as the necessary and sufficient time for Romeo and Juliet to fully know each other. Implicitly we have assumed that Juliet and Romeo would have enough time to date. However, in reality, this may not be always practical. In many cultures, especially in Confucius's world, it is an urgent task for young man and woman to get married when they reach a certain age. If they are not married by that age, their family and the society will put pressure on them and persuade them to marry someone as soon as possible. In these cases, there may not be enough time for the young couples to fully know each other before getting married. In this case, where Juliet suffers pressure from her family for an urgent marriage, is it a wise choice for her start a relationship with Romeo whose uncertainty cannot be fully eliminated, or to find someone with certainty? We will study this problem by redefining Θ.

Suppose the necessary and sufficient time for Romeo and Juliet to know each other is T, if they begin the relationship from $t = 0$, they will have enough time to reduce the uncertainty. However, in case of an urgent marriage, Romeo comes late and expresses his love to Juliet at $t = t_0$ $(0 < t_0 < T)$. Hence they will have only $T - t_0$ to know each other. The value of Romeo's option changes to

$$V_0 = f_0 e^{(\mu-\beta)(T-t_0)} N(d_1^*) - Z_{0f} e^{-\beta(T-t_0)} N(d_2^*), \tag{30}$$

where

$$d_1^* = \frac{\log\frac{f_0}{Z_{0f}} + \left(\mu + \frac{1}{2}\sigma^2\right)(T - t_0)}{\sigma\sqrt{T - t_0}}, \qquad d_2^* = \frac{\log\frac{f_0}{Z_{0f}} + (\mu - \frac{1}{2}\sigma^2)(T - t_0)}{\sigma\sqrt{T - t_0}}.$$

The derivative with respect to t_0, denoted as Θ^*, is

$$\Theta^* = \frac{\partial V_0}{\partial t_0} = -e^{(\mu-\beta)(T-t_0)} f_0 \left[(\mu - \beta)N(d_1^*) + N'(d_1^*)\frac{\sigma}{2\sqrt{T - t_0}}\right] - \beta Z_{0f} N(d_2^*), \qquad (31)$$

which is negative. That is, an urgent marriage will reduce the value of the option, and thus lower the probability of Juliet accepting Romeo. In this case, Juliet would not have enough time to develop their relationship even if they would have positive interactions. In reality, if Juliet refuses to enter the relationship with Romeo, usually the family will help her to find someone they know well. Juliet faces the dilemma that she either dates or marries someone she could not fully know, or accept someone she could know for sure but without building their relationship together. It shows that if the marriage is so urgent that she could hardly know Romeo, she will choose to accept her family's recommendation.

C. Break up

Although we allow Juliet to marry Romeo early in previous discussion, Juliet has no right to suspend the relationship before $t = T$. This is not realistic because in real life, if at some stage in their relationship Romeo behaves intolerably, Juliet will break up with him rather than keep waiting until $t = T$. In our model, this means if at some point f_t hits a certain barrier from above, the option will be nullified. In finance, this is called down-and-out barrier option. The barrier in our model can be viewed as Juliet's mental tolerance level of Romeo's revealed character. In this case where we allow breakup, if Romeo did something unbearable, Juliet would not give him any further chance. This corresponds to stories of many couples in reality. Here, we denote Juliet's barrier as c. Obviously that c makes sense only when $c < f_0$.

Luckily, in finance the value of down-and-out American call option has explicit expression. Before giving the formula, which is somewhat complicated, it is useful to think intuitively of its value. First, notice that a standard American call option can be replicated by one down-and-out call option and one down-and-in option with the same barrier. In this case, when one option is activated, the other must be nullified at the same time. Second, even with barrier, the value of an option is still positive. Thus, we can conclude that the value of a down-and-out call option must be smaller or equal to a standard call option. This is also intuitive: a standard option means Juliet will forgive Romeo's faults and give him chance until $t = T$; therefore, a standard option should be more valuable. Meanwhile, this separation also provides a way of valuing a down-and-out call option: its value is difference between a standard call option and a down-and-in call option. This is useful because a down-and-in call option is easier to value. According to [1] and [2], the value of a down-and-in American call option, in our model, is

$$C_{di} = \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1}\left[e^{(\mu-\beta)T}\frac{c^2}{f_0}N(y_1) - e^{-\beta T}Z_{0f}N(y_2)\right],$$

where (32)

$$y_1 = \frac{\log\frac{c^2}{f_0 Z_{0f}} + (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \qquad y_2 = \frac{\log\frac{c^2}{f_0 Z_{0f}} + (\mu - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}.$$

If we denote the value of a standard call option with f_0 and Z_{0f} as $V_0(f_0, Z_{0f})$, the value of this down-and-in option is indeed

$$c_{di} = \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right). \tag{33}$$

Therefore, the value of the down-and-out call option that we are interested in is

$$c_{do} = V_0 - c_{di} = V_0(f_0, Z_{0f}) - \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right). \tag{34}$$

The value of this option also varies with different parameters. We can still calculate those Greeks with respect to these parameters. We conclude the major results as the following theorem, where for simplicity, we only consider the case when Juliet and Romeo have a strong interaction, *i.e.*, when $\frac{2\mu}{\sigma^2} - 1 > 0$.

Theorem 4 The value of Romeo's option allowing breakup is $c_{do} = V_0 - c_{di} = V_0\left(f_0, Z_{0f}\right) - \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right)$, where the value of this option increases in f_0, μ, and decreases in c, Z_{0f}, when $\frac{2\mu}{\sigma^2} - 1 > 0$.

Proof. See Appendix.

Therefore, same analysis will still hold when we allow breakup during dating. From Theorem 4, we can tell that, if Juliet tends to tolerate more mistakes, she will be more likely to accept Romeo and less likely to break up with him during dating. This inspires Romeo to seek a Juliet who can tolerate his trivial shortcomings. Meanwhile, under same conditions, the value of the down-and-out option is lower than the value of the standard option, which implies that a Juliet who would refrain herself from saying breakup is more likely to accept Romeo's proposal.

It is also interesting to see what is the probability in which they would break up. That is, the probability with which the minimum of f_t is smaller than c, where $t \in [0, T]$. We have the following theorem.

Theorem 5 The probability of Juliet and Romeo breaking up during dating is

$$P\left(\min_{0\le t\le T} f_t < c\right) = 2\mathrm{N}\left(\frac{\ln\frac{c}{f_0} + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right). \tag{35}$$

Proof. This requires the distribution of the minimum of a Brownian motion. Actually, this is equivalent to the probability in which the time of f_t first hitting c is smaller than T. Denote this time as T_c. Using Lemma 3.3.2 in [2], we know that if $\{W_t\}_{t\ge 0}$ is a Brownian motion and starts from $W_0 = 0$, then the probability of W_t first hitting $a > 0$ within $[0, T]$ is $P(T_a < T) = 2\mathrm{P}(W_T > a)$. By symmetry, the probability of W_t first hitting $a < 0$ within $[0, T]$ is $P(T_a < T) = 2\mathrm{P}(W_T > -a)$.

Since

$$\ln f_t \sim N\left(\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right), \tag{36}$$

then $\ln f_t - \left[\ln f_0 + \left(\mu - \frac{1}{2}\sigma^2\right) t\right]$ is a Brownian motion that starts from 0, so that

$$
\begin{aligned}
P\left(\min_{0\le t\le T} f_t < c\right) &= P\left(\min_{0\le t\le T} \ln f_t - \left[\ln f_0 - \left(\mu - \frac{1}{2}\sigma^2\right) t\right] < \ln c - \left[\ln f_0 - \left(\mu - \frac{1}{2}\sigma^2\right) t\right]\right) \\
&= \mathrm{P}\left(T_{\ln c - \left[\ln f_0 - \left(\mu - \frac{1}{2}\sigma^2\right) t\right]} < T\right) = 2P\left(W_T > -\ln c + \left[\ln f_0 - \left(\mu - \frac{1}{2}\sigma^2\right) T\right]\right) \\
&= 2\left[1 - \mathrm{N}\left(\frac{-\ln c + \left[\ln f_0 - \left(\mu - \frac{1}{2}\sigma^2\right) T\right]}{\sigma\sqrt{T}}\right)\right] \\
&= 2\mathrm{N}\left(\frac{\ln \frac{c}{f_0} + \left(\mu - \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}\right)
\end{aligned}
$$

This result shows that the probability of breakup is increasing in Juliet’s level of tolerance for Romeo’s mistake (higher c) and would be lower if Romeo is more attractive (higher f_0) and if they are more suitable (higher μ).

D. Hang out and Dating Cultures

Dating cultures differs in eastern and western cultures. In eastern cultures, usually a romantic relationship begins with a declaration of love (usually from the man). And if it succeeds, then until breakup they are not supposed to date others simultaneously, otherwise the one who has an affair will be morally criticized by the society. Meanwhile, if they do not enter a romantic relationship formally, they are not supposed to have intimate interactions, ranging from kissing to having sexual affairs. Their serious acquaintanceship that might lead to a marriage starts from the time of establishing the relationship. However, in western cultures, especially in the United States college campus, college man and woman may hang out or ‘hook up’ with multiple partners without entering in a serious relationship with commitment. According an 18-month national survey in the United States, forty percent of college women have experienced at least one hook-up (Glenn and Marquardt, 2001). Hooking up can serve for pure ‘sex-without-commitment’ purposes, but also may result in a serious romantic relationship, if the couple acknowledge their relationship at some stage. In the paper, we regard hanging out as the beginning stage of a romantic relationship, focusing on its function of enlarging the range of selection among pursuers. Juliet may have multiple pursuers at the beginning and she could hang out with all of them to see who deserves further dating. After she decides the lucky Romeo, she would date him and make the marriage decision at the end of the dating period, as in previous section.

Formally, suppose from $t = 0$ to $t = t_1$ is the hang-out stage and from $t = t_1$ to $t = T$ is the dating stage. At $t = 0$, Juliet receives n options from her pursuers; these options specify that at $t = t_1$ Juliet has the right to choose him as her boyfriend. At $t = t_1$, when Juliet observes the value of $f_1^1, f_1^2 \dots, f_1^n$, the value of each option is

$$V_1^i = f_1^i e^{(\mu-\beta)(T-t_1)} N(d_1^*) - Z_{0f} e^{-\beta(T-t_1)} N(d_2^*),$$

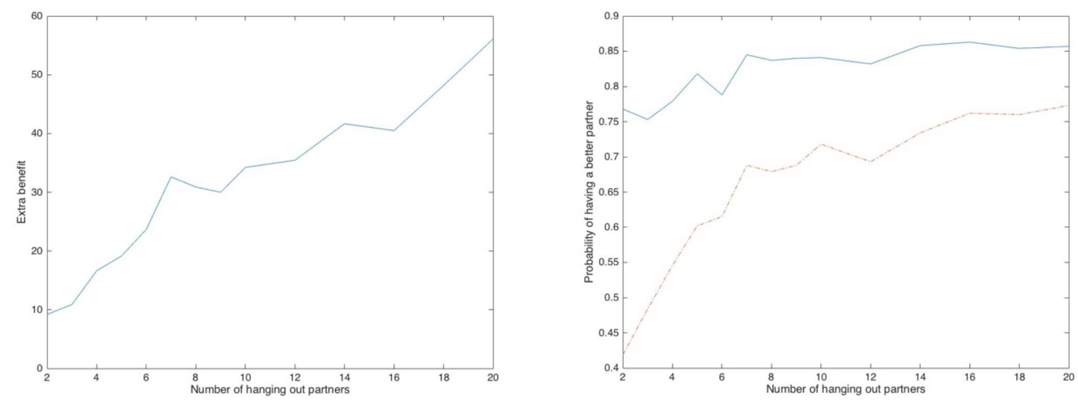
where (37)

$$d_1^* = \frac{\log \frac{f_1^i}{Z_{0f}} + \left(\mu + \frac{1}{2}\sigma^2\right)(T - t_1)}{\sigma\sqrt{T - t_1}}, \qquad d_2^* = \frac{\log \frac{f_1^i}{Z_{0f}} + (\mu - \frac{1}{2}\sigma^2)(T - t_1)}{\sigma\sqrt{T - t_1}}.$$

Therefore, the value of these options for Juliet at $t = t_1$ is $V_1 = \max_i V_1^i$, so its value at $t = 0$ should be

$$V_0^* = \mathrm{e}^{-\beta t_1} E\left[\max_i\{V_1^i, 0\}\right]. \tag{38}$$

This is indeed an option on options. (This option is different from the option on option in finance where the underlying asset is an option and the buyer has the rights to buy the underlying option at or before the expiry date.) Juliet could benefit from this option because she could have more choices over different pursuers. What is of interest in that, suppose at $t = 0$ Juliet has several pursurers who are quite similar, *i.e.*, they have almost the same f_0 and μ, and assuming the level of uncertainty σ are the same. At this point, Juliet cannot decide whom to date, because values of their options are the same, but Juliet can choose to hang out with her pursuers and decide the appropriate partner to begin a serious relationship until $t = t_1$. We know that while other factors are the same, the value of the options at $t = t_1$ is increasing in f_1. Therefore, Juliet would choose the pursuer with highest f_1 at $t = t_1$. It is not easy to calculate the value of this option at $t = 0$. But we can use simulation to see if Juliet would benefit from this option. We compare two situations in the simulation: in the first case, Juliet dates only one pursuer throughout a period with length T; in the second case, Juliet hangs out with a group of pursuers from $t = 0$ to $t = \frac{T}{2}$, and begins to date the one with highest f_1 from $t = \frac{T}{2}$. We do not assume the single pursuer in the first case to be one among the group of pursuers in the second case, implying that at $t = \frac{T}{2}$, that single pursuer may outcompete the best of the group. Nevertheless, as we will show, on average, hanging out will bring extra benefit because selection on a group of pursuers guarantees the robustness. The results are shown in Figure 3.



(a) Extra value (b) Probability of having a better partner

Figure 3 Benefit from Option on Options

The Monte-Carlo simulation calculates the extra value of the option on options and the probability of a better spouse from having a hang-out stage. For each number of partners, we simulate the result for 1000 times. Figure 3(a) shows the difference between values of an option on options and a standard option with same parameters except for the number of options. Hanging out brings extra value for any given number of partners greater than one, and this extra value increases in the number of hang-out partners. In Figure 3(b), the blue line shows the probability of $V_0^* \geq V_0$, *i.e.*, the chance of a no-worse partner, and the red dashed line represents the chance of a strict better spouse. These two probabilities have an upward trend as the number of partners increases, and the probability of a no-worse spouse is far greater than a half.

Although according to the survey, only one in ten of the women have hanged out for more than six times. The simulation tells that even having one extra partner would significantly increase the probability of a no-worse spouse. These results together prove that hanging out benefits Juliet in a sense that it reduces the randomness of her pursuers' quality and improves the robustness of the result.

IV. Conclusions

We use the idea of option and analyze the choice of Juliet that she needs to either accept or reject Romeo's romantic option. We show that Romeo is more likely to be accepted if he is more outstanding and if he is more suitable for Juliet. In contrast, a more independent Juliet will be less willing to accept Romeo under the same conditions. This prediction by the model matches well with the reality. Meanwhile, we also introduced several common scenarios in a romantic relationship including urgent marriage, breaking up, and hanging out, based on the idea of exotic option. Certainly, there are more different situations that we can analyze using different exotic options. For example, it would be more reasonable to assume that Juliet's share of joint production in their marriage is the average over the final stage of their dating rather the final value in order to eliminate the randomness. This kind of option is called Asian option. Unfortunately, there is no explicit formula for the value of an Asian option that allows us to analyze algebraically. It is also interesting to study another situation in which there is no clear limit of dating time T, so that Juliet will make the decision at any time she thinks proper. We can regard this kind of relationship as a perpetual option. In this case, we must allow breakup. Because the value of a perpetual American call option is infinity, while the value of a perpetual down-and-out American call option is bounded and has explicit solution. (see [2][15]).

Although our model is very similar to the classic BSM model, we need to emphasize the difference. In BSM model, basically the discount rate and the drift are driven by same factors, thus their influences are offset by each other. This is the reason why we can use risk-neutral pricing to get the value of the option. However, in our model, the discount rate β and the drift μ are driven by different factors: the discount rate relies on Juliet while the drift depends on both

Juliet and Romeo. Due to this reason, we keep the drift in our model while it is replaced by the riskless interest rate in BSM model.

We have made a key assumption in our paper that the f_T would not change once Romeo and Juliet get married. This is a reasonable simplification from Juliet's perspective when she needs to decide whether to accept Romeo as her boyfriend. In our model, f_T is explained as Juliet's expectation of her share of joint output after getting married. So she has to have an expectation of f_T that helps her make the decision. However, if we want to extend our model to study marriage, this assumption may not be valid in reality because marriage is such a life-long affair. If we use our model to study the marriage, there are several things that need to be modified. First, the drift term can be time-varied in a life-long relationship because not only will the relationship be shaped by them but also the relationship itself will shape them as well. Second, in a long period, the probability of rare events will be large. Thus, it is no longer suitable to use a Brownian motion to describe the motion of f_t, but it is better to introduce Poisson jump to characterize the rare events as in finance. Third, if we consider the counterpart of breakup, namely, the divorce of a marriage, according to Niko and Rasul (2008), there will be a cost of divorce. Therefore, it will be harder to exit the marriage than a romantic relationship. Finally, if we use this model to study the marriage, it is better to view the gain from marriage as an integral of the stochastic process.

There are also spaces for the improvement of the search model. If the dating time T is long enough, we would assume that Juliet will have multiple chances of searching for Mr. Right. In this case, the gain from search will increase and she is more likely to reject Romeo. To conclude, it is of our interests to apply the option model from financial engineering to study the classic marriage problem. Nevertheless, option in daily life and option in finance both are options.

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Appendix

Proof of Theorem 4

The value of the down-and-out American call option is

$$c_{do} = V_0 - c_{di} = V_0\left(f_0, Z_{0f}\right) - \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right). \tag{A1}$$

(1) Δ: From Theorem 2, we know that the value of a standard call option is increasing in f_0, thus the firm term $V_0\left(f_0, Z_{0f}\right)$ is increasing in f_0. The second term is not that obvious, since $\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1}$ can either increase or decrease in f_0, depending on the value of $\frac{2\mu}{\sigma^2} - 1$. In our simple case, when $\frac{2\mu}{\sigma^2} - 1 > 0$, since $\frac{c}{f_0} < 1$, both $\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1}$ and $V_0\left(\frac{c^2}{f_0}, Z_{0f}\right)$ are decreasing with f_0, hence with a negative sign c_{do} is increasing with f_0. When $\frac{2\mu}{\sigma^2} - 1 < 0$, we need to calculate the derivative of c_{do} with respect to f_0:

$$\frac{\partial c_{do}}{\partial f_0} = e^{-\beta T}\left[e^{\mu T} N(d_1) - \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma}-1} N(d_1^*)\right] - \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma}-1}\left(1 - \frac{2\mu}{\sigma^2}\right)\frac{1}{f_0} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right) \tag{A2}$$

(2) P: The derivative of c_{do} with respect to μ is

$$\frac{\partial c_{do}}{\partial \mu} = f_0 T e^{-(\mu-\beta)T} N(d_1) - \left[\frac{2}{\sigma^2}\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} \ln\frac{c}{f_0} V_0\left(\frac{c^2}{f_0}, Z_{0f}\right) + \left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} f_0 T e^{-(\mu-\beta)T} N(d_1^*)\right], \tag{A3}$$

where we have $\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} < 1$, $\ln\frac{c}{f_0} < 0$ and $N(d_1) > N(d_1^*)$, given $c > f_0$ and $\frac{2\mu}{\sigma^2} - 1 > 0$. Therefore, it is easy to this derivative is greater than zero. Thus the value of the option is increasing in μ.

(3) Z_{0f}: We know that the derivative of the first term with respect to Z_{0f} is $-\mathrm{e}^{\beta T} N(d_2)$. Meanwhile, the derivative of the second term with respect to Z_{0f} is

$$\frac{\partial c_{di}}{\partial Z_{0f}} = -e^{-\beta T}\left[\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} N(d_2^*)\right], \tag{A4}$$

for the same reason, when $\frac{2\mu}{\sigma^2} - 1 > 0$ we have $\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1} < 1$, therefore, $\frac{\partial c_{di}}{\partial Z_{0f}} < -\mathrm{e}^{\beta T} N(d_2)$, the total derivative will be negative.

(4) c: This is simple because the barrier level c only influences the value the second term. Given $\frac{2\mu}{\sigma^2} - 1 > 0$, both both $\left(\frac{c}{f_0}\right)^{\frac{2\mu}{\sigma^2}-1}$ and $V_0\left(\frac{c^2}{f_0}, Z_{0f}\right)$ are increasing in c, so the total value is decreasing in c.