Why Master's Programs in Top Schools Thrive

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Abstract

This paper studies the choice of education by considering education as a signal of productivity in the job market. Based on the basic setting of Spence's paper, we adopt Alós-Ferrer and Prat's model and extend the education signal into two dimensions. The first dimension reveals average productivity and the second one reveals the variance of productivity. Our study shows that multiple pooling equilibria exist in our model. Moreover, under Type II Strong Learning, the two components are substitutes, so job candidates can optimize the combination of the two components of the signal upon each equilibrium. Furthermore, compared with Alós-Ferrer and Prat's model, our model suggests a more concave education return for the high-productivity people and a more convex return for low-productivity people. This suggests that high-productivity people may generally prefer a high education signal, but the incentive of increasing one unit of education is weakened. Our paper may help to explain why many students choose master's programs in top schools rather than choosing to attend PhD programs.

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1 Introduction

Education plays an important role in the job market. As a signaling tool, it reveals job candidates' attributes that cannot be directly observed by employers. Given their own productivity and knowledge about wages in the job market, job candidates optimize their education plans and deliver signals to employers. Most of the time, employers believe that people with higher education are more likely to perform well and thus deserve higher wages. Conversely, lower education may indicate lower productivity and thus the corresponding offers will be relatively low. Given such basic knowledge, job candidates have an incentive to invest in a high level of education. Meanwhile, they also face the high costs of additional education. Hence, the choice of education level is connected with such an optimization problem, beneath which is a trade-off between wages and the relative costs of education.

An interesting observation is that many students, especially Chinese students, choose to attend US top schools. Such an education plan will definitely deliver a good signal and facilitate their job seeking in the future. However, an even more interesting phenomenon is that a considerable amount of them choose the master's programs instead of PhD programs. From their preference, we can see that their education decisions consist of two things: schools and programs. A diploma from a top school is a guarantee of an average high ability, since most of the admitted students are outstanding. However, the choices of master's programs reflect a high variance of ability, because even a bad student will not easily get eliminated in a master's program. Based on this logic, a natural idea is to split an education signal into two dimensions: mean and variance. Mean refers to the average ability represented by the signal, while variance indicates the expected deviation of students' ability from the mean delivered by the signal. This idea is also supported by Wolpin (1977), who states that schooling can be considered as an estimator of skills and thus can be characterized by mean and variance. Under such a framework, the study of education signaling is to look at how students choose the combination of the mean and variance as responses to the beliefs of employers, who also decide wages based upon the two components.

The education signaling problem in the job market was first studied by Spence (1973). In his model, job candidates make decisions about education as feedbacks to the employers' wage schedule. At the same time, employers update their beliefs and then design the wage schedule. This model gives two types of equilibrium: separating and pooling equilibrium. In the separating equilibrium, people with high productivity deliver a high education signal and people with low productivity send a low one. Employers accurately recognize the productivity beneath the signals. As a result, people will be given wages exactly equal to their productivity. In the pooling equilibrium, people with different productivity send an identical signal so that the wages for all people are the same. Although Spence builds a wonderful framework for the following research, he only considers the pre-learning process; i.e., the employers' signal learning and belief updating happen during the recruitment and then stops. In other words, on-the-job learning of signal is missing in his paper. Furthermore, education is only treated as a one-dimensional variable in his paper.

As shown in some empirical study, addition of on-the-job learning process is important. In Lange's study (2007), employers with on-the-job learning behavior decrease the expectation errors about schooling by 50% within only three years. Pierret and Altonji (1998) build a model where companies update beliefs by learning unobservable characteristics. They argue that the signaling will bring limited value because the wages will gradually become less associated with observable factors such as education. Mansour (2012) also reveals the existence of employer learning by exploring the connection between the variance of wages and growth of unobservable variables.

Addition of on-the-job learning gives some interesting theoretical findings as well. By adopting the basic framework of Spence's model, Alós-Ferrer and Prat (2012) study the effect of on-the-job learning by introducing the Reports Model. They argue that multiple pooling equilibria exist under some conditions. Specifically, in multiple pooling equilibria, candidates who randomly choose two education levels are not separated and thus receive the same wage. This finding indicates that the on-the-job learning decreases the value of the education signal. Moreover, a high education cannot directly help distinguish a person with high productivity from his competitors, because revealing of productivity no longer solely depends on the education signal and also can be achieved by employers' on-the-job learning.

However, the findings given by Alós-Ferrer and Prat's paper cannot fully explain students' preference for master's programs in top schools. Although the weakening of education signal in their paper may simply imply a fading fad of PhD degrees, it cannot explain why students still want to be admitted into top schools. However, Alós-Ferrer and Prat's model is very useful for our extension of signal dimensions. By extending the signal into two dimensions: mean and variance, employers can learn not only the average productivity represented by a signal, but also its underlying variance. As we will later show, our model suggests that mean and variance delivered by a signal are substitutes, which means that a student's potential losses from choosing a master's program can be compensated for by choosing a high-ranking school.

The remaining of the paper is organized as follows. Section 2 introduces several important concepts in our model. Section 3 gives the layout and defines the learning process of our model. Section 4 derives the condition for the existence of multiple pooling equilibria. Section 5 discusses the people's choice of signal assembling. Section 6 focuses on the overall education return. Proofs and some calculation details are provided in Appendix.

2 Signaling Model

In this paper, we still follow the basic settings of Spence's model. All job candidates are divided into two groups — one with high productivity and one with low productivity. To make their productivity observable to employers, they can choose different levels of education and use them as signals. Recruitment is considered as a feedback system. First, employers generate conditional probabilistic beliefs based on the previous knowledge regarding the relation between signal and productivity. Second, they derive a wage schedule based on these beliefs. Third, both low- and high-type job candidates observe the wage schedule, optimizing their signal (education) to maximize their payoff. Finally, employers finish hiring and further adjust their conditional probabilistic beliefs for the second round. The equilibrium in Spence's model is reached when the employers' beliefs reach a steady state. This is called a "self-confirming" belief in Spence's paper.

However, as we mentioned above, on-the-job learning is missing in Spence's model. Signal learning in Spence's model only happens during recruitment. In the real world, learning will keep going afterwards in the form of seasonal personnel assessment. Therefore, the updating of probabilistic beliefs will continue during on-the-job learning. In this paper, we include on-the-job learning process by adopting the Reports Model introduced by Alós-Ferrer and Prat (2012), where on-the-job learning is conducted in the form of "report" immediately after recruitment. The only one report in Alós-Ferrer and Prat's model basically answers the question "*Does this signal represents good performance on job?*". However, since we are considering the signal in two dimensions —- mean and variance, we will include two reports in our model.

1. Report 1: *Does this signal **on average** represent good on-the-job performance?*
2. Report 2: *Is **this person** with such a signal likely to perform well on the job?*

The first report examines the average productivity represented by the education signal, while the second examines the possible variance. The idea of the two reports can be best illustrated by the following example. Suppose there are two candidates, A and B. A got his master's degree from University Top while B got a PhD degree from University Mediocre. From the perspective of average productivity, an employer may prefer A to B because University Top ranks above University Mediocre. Meanwhile, as for variance, the employer may prefer B to A, because B's PhD degree provides a stronger guarantee for a low variance in performance than A's master's degree.

Like Alós-Ferrer and Prat, we only include two possible results for each report: good reports (G) and bad reports (B). Both types of candidates could receive good or bad reports at a certain probability d. Given this, employers further update their beliefs and decide a new wage schedule. By adding the dimension of variance, we will show that the multiple pooling equilibria will be more "multiple" in the sense that people can have not only multiple education equilibria, but also multiple combinations of mean and variance upon each equilibrium level of education.

3 The Model

3.1 Basic Set Up

We assume that there are two groups of candidates in the job market: low-type group, denoted as l and high-type group, denoted as h. Their corresponding productivities are p_l and p_h with $p_h > p_l$. The share of high-type candidates is μ_0 and $0 < \mu_0 < 1$. We denote their education level as e and the cost of education as $c(p, e)$. As set in Spence's and Alós-Ferrer and Prat's models, our model has

$$c_e(p, e) > 0, \quad c_p(p, e) < 0$$

The first condition indicates that the cost is increasing with respect to education level. The second condition indicates a higher learning ability of high-type candidates. Furthermore, we have

$$c_{ee}(p,e) \geq 0, \quad c_{pe}(p,e) < 0$$

, where the second inequality, according to Alós-Ferrer and Prat, satisfies *Single-crossing Property*, i.e., low-type people have a steeper indifference curve.

The initial belief of the employer, μ, is a function of the education signal and thus $\mu = \mu(e)$. The belief here refers to the probability of candidates being high-type. We denote the initial wage as $w(\mu) = \mu p_h + (1-\mu)p_l$ [1]. When $\mu = 1$, the employer believes that the candidate must be high-type and pays him exactly at his productivity p_h. Also, when $\mu = 0$, the employer believes that the candidate must be low-type and pay him exactly at p_l.

The employer's learning level, i.e., a measure of on-the-job signal learning ability, is denoted as $\mathbf{s} = (s_1, s_2)'$. Different from Alós-Ferrer and Prat's model, the learning level here is a vector composed by two elements. s_1 denotes the learning level about the average productivity in Report 1. s_2 denotes the learning level about productivity variance in Report 2. If s_1 is high, the employer will learn the average ability represented by candidates' signals in a very fast way. Similarly, if s_2 is high, the employer will soon become aware of how likely a candidate would be to have performance worse than the average level of the signal. In this model, we define the range of s_1, s_2 as $(0, +\infty)$.

Candidates consider their payoff from the job as a value function. They have a discount factor $\frac{1}{r} \in (0, 1]$[2]. The value function is defined conditional on the level of r and $\mathbf{s}$ with respect to p and $\mu(e)$. We denote the value function as $v(p, \mu | r, \mathbf{s})$.

[1]Note that here the wage function will be applied again in the following updating. For example, if the belief is updated to be μ' in the second step, new wage will be $\mu' p_h + (1-\mu')p_l$

[2]To be consistent with the notations of Alós-Ferrer and Prat's paper, we denote the discount factor as an inverse.

Given what we have set up above, we then introduce several properties of the value function. Note that we mark with "AP" those properties directly adopted from Alós-Ferrer and Prat's paper, and mark with "Modified AP" those properties we slightly modify based on their paper. The rest are newly defined ones in our paper.

- P0 (AP)

$$\forall \mu \in (0,1), \frac{p_h}{r} > v(p_h, \mu) \geq \frac{w(\mu)}{r} \geq v(p_l, \mu) > \frac{p_l}{r}$$

$$\mu \in \{0,1\}, v(p_h, \mu) = v(p_l, \mu) = \frac{w(\mu)}{r}$$

- P1 (AP)

$$\forall \mu \in (0,1), \frac{p_h}{r} > v(p_h, \mu) > \frac{w(\mu)}{r} > v(p_l, \mu) > \frac{p_l}{r}$$

$$\mu \in \{0,1\}, v(p_h, \mu) = v(p_l, \mu) = \frac{w(\mu)}{r}$$

We directly adopt the first two properties from Alós-Ferrer and Prat's paper. Intuitively, it indicates that when employers are not fully informed of the types of each candidate, i.e., $\mu \in (0,1)$, they will not categorize anyone into the high or low-type group. That is to say, everyone's type is undetermined. As a result, an actual low-type person will get $v(p_l, u)$ greater than or equal to $\frac{p_l}{r}$ and an actual high-type person will get $v(p_h, \mu)$ less than or equal to $\frac{p_h}{r}$. Besides, when $\mu \in (0,1)$, employers will update beliefs and gradually become informed. As a result, the high-type's value function will be greater than or equal to the initial level $\frac{w(\mu)}{r}$ and the low-type's will be less than or equal to the initial one. The strict inequality in P1 for $\mu \in (0,1)$ indicates a stronger learning level of the employers: when they are uncertain about candidates' types, they always update beliefs and change the wage schedule from initial wage level. The equalities in both P0 and P1 indicate that updating will stop once employers confirms the beliefs, i.e., $\mu = 0$ or 1 and the initial value becomes a steady state. Generally speaking, we can see that employers' informedness built in on-the-job learning will benefit the high-type and hurt the low-type. Based on this idea, we have the P2 and

P2'.

- P2 (Modified AP)

$$\forall \mu \in (0,1), \frac{\partial v(p_h, \mu | r, \mathbf{s})}{\partial s_1} > 0, \frac{\partial v(p_l, \mu | r, \mathbf{s})}{\partial s_1} < 0$$

- P2'

$$\forall \mu \in (0,1), \frac{\partial v(p_h, \mu | r, \mathbf{s})}{\partial s_2} > 0, \frac{\partial v(p_l, \mu | r, \mathbf{s})}{\partial s_2} < 0$$

Next, we have several properties concerning two learning abilities.

- P3 (Modified AP)

$$\forall \mu \in (0,1), \lim_{s_1 \to 0, s_2 \to 0} v(p, \mu | r, s) = \frac{w(\mu)}{r}, \lim_{s_1 \to \infty, s_2 \to \infty} v(p, \mu | r, s) = \frac{p}{r}$$

- P3'

$$\forall \mu \in (0,1), \lim_{s_1 \to \infty, s_2 \to 0} v(p, \mu | r, s) = \lim_{s_1 \to 0, s_2 \to \infty} v(p, \mu | r, s) = \frac{p}{r}$$

P3 suggests that when both s_1 and s_2 go to infinity, the employer can confirm the category of each candidate and assign them with corresponding wages. When both s_1 and s_2 are zero, the employer cannot do on-the-job learning and hence only give the initial wage. P3' shows a substitution effect between two learning abilities — even when one learning reaches zero, the employer would recognize the types of candidates based on the other ability if it is infinitely high.

- P4 Given $s_1' > s_1$, there exists $s_2' < s_2$ such that

$$v(p, \mu | r, (s_1, s_2)) = v(p, \mu | r, (s_1', s_2'))$$

P4 indicates the substitutability between two learnings in a finite case. It implies that an employer's low learning level in one dimension can always be compensated for by a high level in the other dimension.

Definition 1. *Let the value function $v(p, \mu) : \{p_l, p_h\} \times [0, 1] \to [0, \infty)$. It is strictly increasing with respect of μ and satisfies P0. It is a* ***Weak Learning*** *function if and only if it satisfies P1. It is a* ***Type I Strong Learning*** *function if and only if it satisfies P1, P2, P2' and P3. It is a* ***Type II Strong Learning*** *function if and only if it satisfies P1-P4, P2' and P3'.*

The difference between weak and strong learning, as mentioned in Alós-Ferrer and Prat's paper, is that strong learning requires employers to reach the "truth" by on-the-job learning. The innovation here is the introduction of Type I and Type II strong learning. Type II requires a complete substitutability between s_1 and s_2 while Type I does not have such a requirement.

From the properties above, we can see that our model is a more general framework for Spence's and Alós-Ferrer and Prat's model. Spence's model is a specific case where $\mathbf{s} = 0$. Alós-Ferrer and Prat's model is also a specific case with $s_2 = 0$.

3.2 Bayesian Updating Process

In our model, we assume that the employer follows the Bayesian updating process to make inferences from on-the-job learning. As mentioned before, there are two reports conducted immediately after the candidates begin working. Both reports have only two results: good (G) or bad (B). Report 1 examines the average quality by answering "*Does this signal* ***on average*** *represent good on-the-job performance?*". Under a positive learning level, the probability for a high-type person to get G in Report 1 is denoted as $d_1(s_1)$ with $d_1(s_1) > \frac{1}{2}$. The probability of a low-type person to get G in

Report 1 is $1 - d_1(s_1) < \frac{1}{2}$. Report 2 examines the productivity variance by answering ”*Is **this person** with such a signal likely to perform well on the job?*”. The probability of a high-type person to get G in Report 2 is denoted as $d_2(s_2)$ with $d_2(s_2) > \frac{1}{2}$. Also, the probability of a low-type person to get G in Report 2 is $1 - d_2(s_2) < \frac{1}{2}$. For each type of candidates, the probability of getting B in each report is the complementary part of the probability of getting G. The probabilities of getting G and B are showed in Table 1 and 2 respectively.

Table 1: Probability of Good Results in Reports

	High-type	Low-type
P(get G in Report 1)	$d_1 > \frac{1}{2}$	$1 - d_1 < \frac{1}{2}$
P(get G in Report 2)	$d_2 > \frac{1}{2}$	$1 - d_2 < \frac{1}{2}$

Table 2: Probability of Bad Results in Reports

	High-type	Low-type
P(get B in Report 1)	$1 - d_1 < \frac{1}{2}$	$d_1 > \frac{1}{2}$
P(get B in Report 2)	$1 - d_2 < \frac{1}{2}$	$d_2 > \frac{1}{2}$

Both d_1 and d_2 are increasing in s_1 and s_2 respectively. We define

$$\lim_{s_1 \to \infty} d_1(s_1) = 1, \lim_{s_1 \to 0} d_1(s_1) = \frac{1}{2}$$

$$\lim_{s_2 \to \infty} d_2(s_2) = 1, \lim_{s_2 \to 0} d_2(s_2) = \frac{1}{2}$$

Following Alós-Ferrer and Prat, we define the value function as

$$v(p, \mu | r, \mathbf{s}) = \frac{1}{r} E[w(\mu') | p, \mu, \mathbf{s}]$$

, where μ' is the updated belief based on μ.

Given the two dimensions of signal in our model, there are four possible results of two reports: good&good (GG), good&bad (GB), bad&good (BG) and bad&bad (BB). Then, the updated belief μ' has four possible forms[3]:

$$\mu'(\mu, GG) = \frac{\mu d_1 d_2}{\mu d_1 d_2 + (1-\mu)(1-d_1)(1-d_2)}$$
$$\mu'(\mu, GB) = \frac{\mu d_1 (1-d_2)}{\mu d_1 (1-d_2) + (1-\mu)(1-d_1)d_2}$$
$$\mu'(\mu, BG) = \frac{\mu (1-d_1) d_2}{\mu (1-d_1) d_2 + (1-\mu)d_1(1-d_2)}$$
$$\mu'(\mu, BB) = \frac{\mu (1-d_1)(1-d_2)}{\mu (1-d_1)(1-d_2) + (1-\mu)d_1 d_2}$$

Consequently, the corresponding wage schedule $w(\mu, \cdot)$ has the following four forms[4]:

$$W(\mu, GG) = \frac{\mu d_1 d_2 p_h + (1-\mu)(1-d_1)(1-d_2)p_l}{\mu d_1 d_2 + (1-\mu)(1-d_1)(1-d_2)}$$

$$W(\mu, GB) = \frac{\mu d_1 (1-d_2) p_h + (1-\mu)(1-d_1)d_2 p_l}{\mu d_1 (1-d_2) + (1-\mu)(1-d_1)d_2}$$

$$W(\mu, BG) = \frac{\mu (1-d_1) d_2 p_h + (1-\mu)d_1(1-d_2) p_l}{\mu (1-d_1) d_2 + (1-\mu)d_1(1-d_2)}$$

$$W(\mu, BB) = \frac{\mu (1-d_1)(1-d_2) p_h + (1-\mu)d_1 d_2 p_l}{\mu (1-d_1)(1-d_2) + (1-\mu)d_1 d_2}$$

Hence, the expected wage $E[w(\mu')|p, \mu, \mathbf{s}]$ can be written as

$$E[w(\mu')|p_l, \mu] = (1-d_1)(1-d_2)W(\mu, GG) + (1-d_1)d_2 W(\mu, GB) + d_1(1-d_2)W(\mu, BG) + d_1 d_2 W(\mu, BB)$$

$$E[w(\mu')|p_h, \mu] = d_1 d_2 W(\mu, GG) + d_1(1-d_2)W(\mu, GB) + (1-d_1)d_2 W(\mu, BG) + (1-d_1)(1-d_2)W(\mu, BB)$$

We can easily prove that the value functions show all of the properties of *Type II Strong Learning* listed in Definition 1.

[3]Here we use Baye's rule. For example, $\mu'(\mu, GG) = \frac{P(\text{be a high-type who gets GG})}{P(GG)}$, where $P(GG) = P(\text{low-type})P(\text{low-type gets GG}) + P(\text{high-type})P(\text{high-type gets GG}) = \mu d_1 d_2 + (1-\mu)(1-d_1)(1-d_2)$.

[4]Note that $W(\mu, \cdot) = \mu'(\mu, \cdot)p_h + [1-\mu'(\mu, \cdot)]p_l$.

4 Multiple Pooling Equilibria

Before showing the existence of multiple pooling equilibria, we have to first show the settings of utility in our model. We define the **pecuniary** utility of candidates to be their net payoff, $u(e, \mu|p) = v(p, \mu) - c(p, e)$. Second, like Alós-Ferrer and Prat's model, the cost of education, $c(p, e)$, is also linear in our model, namely

$$c(p_i, e_j) = g(p_i)e_j \tag{1}$$

, where $i = \{h, l\}$ and $j = 1, 2, 3, \cdots$ is the index of education level.

Next, we will show the existence of multiple pooling equilibria. Here, we follow Alós-Ferrer and Prat. Suppose there are two pooling equilibria, e_1, e_2 for both low- and high-type people, i.e., both of the two type can randomize between the two education levels and will not be seperated. Then, we have

$$v(p_i, \mu(e_1)) - c(p_i, e_1) = v(p_i, \mu(e_2)) - c(p_i, e_2)$$

By simple rearrangement, we can derive the necessary condition for the existence of multiple pooling equilibria, which is

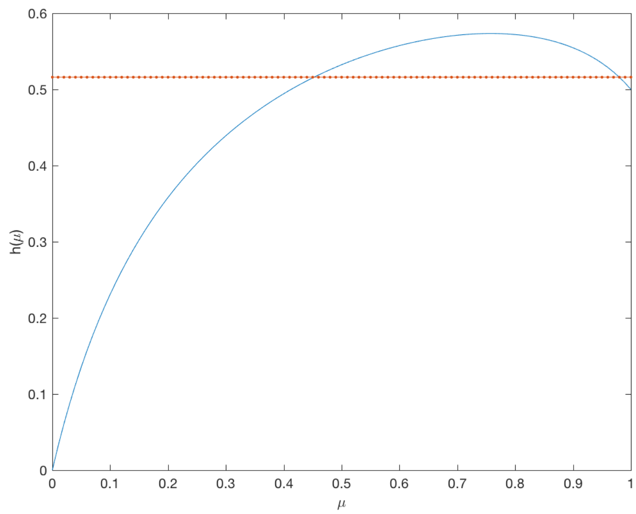
$$v(p_h, \mu_1) - gv(p_l, \mu_1) = v(p_h, \mu_2) - gv(p_l, \mu_2) \tag{2}$$

, where $g = \frac{g(p_h)}{g(p_l)} < 1, \mu_j = \mu(e_j)$[5]. We define the function $h(\mu) = v(p_h, \mu) - gv(p_l, \mu)$. Hence, equation 2 can be rewritten as

$$h(\mu_1) = h(\mu_2) \tag{3}$$

[5]The reason why $g = \frac{g(p_h)}{g(p_l)} < 1$ is that the education cost of the high-type is lower than that of the low-type given the same education e

Figure 1: Multiple Pooling Equilibria



Parameters: $p_l = 0, p_h = 1, d_1 = 0.8, d_2 = 0.6, g = 0.5$

As shown in Figure 1, h is concave with respect to μ so that multiple pooling equilibria are possible. To see the intuition of this result, we consider h as a wage gap between the high and low type candidates under μ [6]. The concavity of h implies that although a higher μ (i.e., a higher education) may lead to a higher wage gap, such a gap gradually shrinks as μ (education) grows. Consequently, a low education may be equivalent to a high education when we include employers' on-the-job learning.

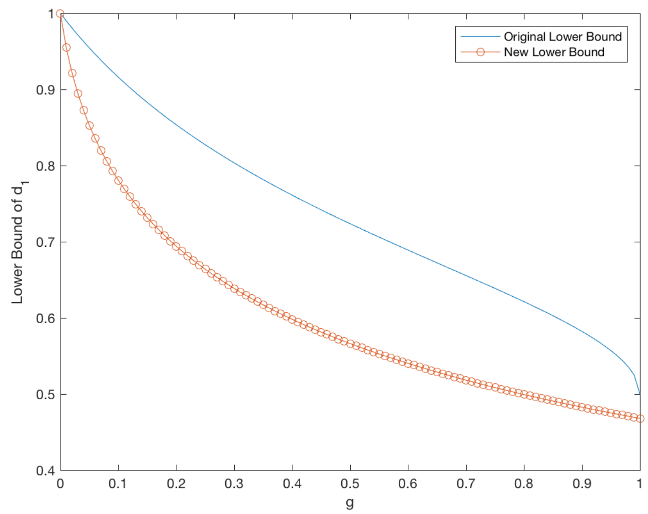
What we are interested in here is that on which level of d_1 and d_2 will there be multiple equilibria. The Proposition 4 of Alós-Ferrer and Prat's paper shows an necessary and sufficient condition for its existence, which is

$$d_1 > \frac{1}{2}(1 + \sqrt{\frac{1-g}{1+3g}}) \tag{4}$$

Nevertheless, since the property of *Type II Strong Learning* suggests the substi-

[6]Considering that g measures how relatively weak the low-type's learning ability is, it plays a role of further "discounting" the low-type value function in the formula of h.

Figure 2: Lower Bound of d_1



Parameters: $p_l = 0, p_h = 1, d_2 = 0.7$

tutability between Report 1 and 2, the lower bound of d_1 in our model will be lower than that of (4) under some conditions. To get the lower bound, we have to solve a cubic inequality, which unfortunately does not have an analytical solution. To simplify our proposition, we only give a sufficient condition of d_1. However, it turns out that this is enough to show that our second dimension lowers the bound.

Proposition 1. *With a linear cost equation* $c(p_i, e_j) = g(p_i)e_j$ *and* $\frac{g(p_h)}{g(p_l)} \in (0, 1)$*, multiple pooling equilibria exist if*

$$d_1 > \frac{1}{2}\frac{\sqrt{(1-d_2)4{d_2}^2 g + (1-d_2)^3} + {d_2}^2 - 1}{{d_2}^2 g + ({d_2}^2 - d_2)} \tag{5}$$

Figure 2 shows the the lower bound in (4) and the new lower bound given by (5). Clearly, Proposition 1 suggests that the second dimension in on-the-job learning can lower the lower bound of d_1, i.e., including Report 2 lowers the requirement of learning level in Report 1 under some conditions.

5 Optimal Assembling of Signals

In this section, we will discuss how candidates choose the combination of two components in signaling. When we only consider a one-dimensional signal, candidates simply randomize among multiple equilibria. However, when we look at the two dimensions: mean and variance, not only can they randomize among the two signals, but also can choose to "assemble" each equilibrium signal with different combinations of mean and variance.

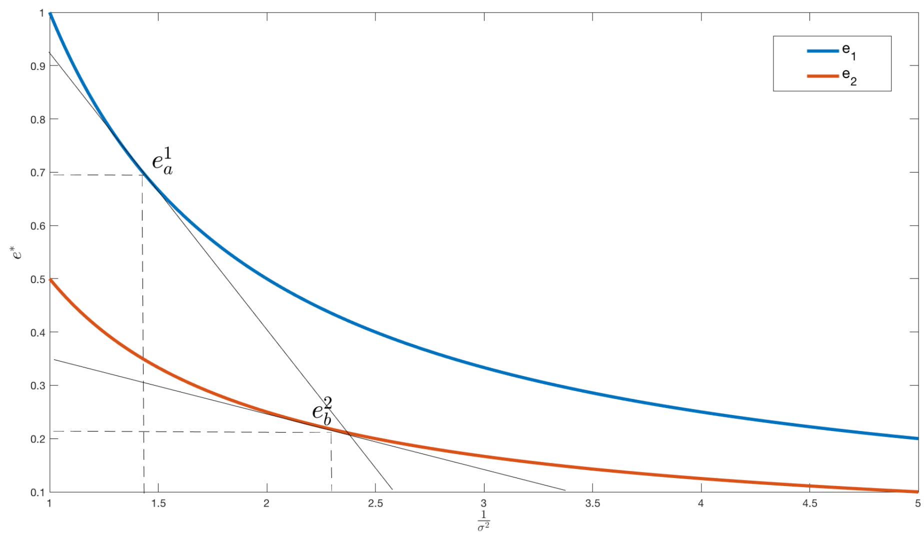
Now, let us see how candidates choose the combinations. Given the two equilibrium education levels e_1 and e_2, we have two curves in Figure 3. They look the same as indifference curves, except that a conventional indifference curve reflects the indifference only within one curve, while the multiple equilibria setting captures the indifference within more than one curves. Candidates weigh the two components to construct a "budget line" to optimize their assembling. To better understand the idea of "budget line", we can consider that both components have some opportunity cost (or benefit). For example, an ordinary student may prefer a low-average (mediocre) school since the intelligence needed to gain entry to a high-average (top) school is too "costly" to him. Also, a strong preference for high-variance (master) program may be caused by a large opportunity cost of choosing low-variance (PhD) program, including a longer period of study, though that is not formally modeled in our paper. In a word, people trade off between high mean and low variance. They can choose to invest more in the component that costs them less.

We denote the mean as e_j^* and variance as σ_j^2 . We denote the former cost as c^e and latter as c^s. Then, the budget line can be constructed as

$$B = c^e e^* + \frac{c^s}{\sigma^2}$$ [7]

[7] Note that since a low variance implies good quality, we take an inverse of variance in the budget

Figure 3: Choice of Signal Assembling



Assume there are two budget lines for candidates A and B, denoted as B_a and B_b respectively. With budget line and indifferent curves, we can then see the optimal assembling of signal e_a^1 and e_b^2 in Figure 3. In this figure, we assume that the A makes academic efforts at a higher cost than B so that the B_a is steeper than B_b. It turns out that e_a^1 has a higher e^* and higher σ^2 than e_b^2, indicating that A prefers programs in high ranking schools, while B prefers programs with higher academic standard. This possible result in our model somehow gives us a flavor of signal mimicking — students who study reluctantly may attend a school with higher ranking than those with higher academic enthusiasm[8].

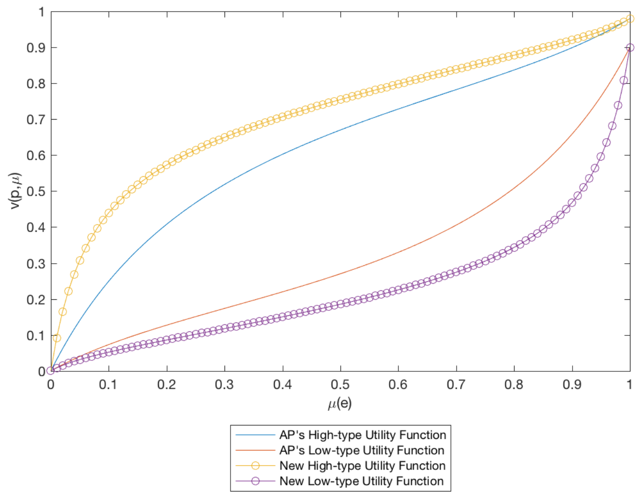
line.

[8]However, this is not a typical signal mimicking. Signal mimicking happens when a low-type person chooses a high education to pretend to be high-type. This is a concern only in a separating equilibrium. In this paper, we only focus on pooling equilibrium, where mimicking is not a concern.

6 Returns to Education

In this section, we analyze the education returns for both groups of candidates. One way to show this is by graphing utility under different education levels, which is demonstrated in Figure 4. Clearly, the addition of Report 2 increases the gap between the two groups. Moreover, it also increases the concavity of the high-type utility function and the convexity of low-type function. Combining the two changes, we can see that although high-type people can get higher payoff by sending a high education signal, they are less encouraged to make a marginal increase upon the current level. For low-type people, they are more encouraged to increase one unit of education although their average return is lower than before. If we regard most of the students as high-type candidates, this result gives us a possible explanation for their preference for master's programs in top schools — they still have an incentive to send high education signals, so they choose top schools to finish master's degrees. However, their incentive of studying as PhD students in top schools is weakened because of the low marginal return indicated by the high concavity.

Figure 4: Utility Function



Parameter: $r = 1, p_h = 1, p_l = 0, g_h = 0.2, g_l = 1, d_1 = 0.8, d_2 = 0.8$

Furthermore, we can also compute the average return to education. Following Alós-Ferrer and Prat, we define it as

$$AR_j = \frac{E(v|e_j) - E(v|0)}{e_j} \tag{6}$$

The equation above can be rearranged as

$$AR_j = g(p_l) + \mu(e_j)\frac{v(p_h, \mu(e_j)) - v(p_l, \mu(e_j))}{e_j} \tag{7}$$

When e increases, the consequent increase of $\mu(e)$ enlarges the education premium. Meanwhile, it also leads to an increase of the denominator of the second term, implying a decrease of education return. Such an offset is called a **composition effect** of education in Alós-Ferrer and Prat. The second negative effect dominates most of the time so that investing in high education seems to be unattractive in their model.

By including one more dimension, our model gives some new findings. We also use the similar way to measure education returns. The only thing modified here is that we split the signal e into two dimensions in cost function, namely

$$c(e_j, p_i) = g(p_i)e_j^* f(\frac{1}{\sigma_j^2})$$

Then, the average return becomes

$$AR_j = g(p_l) + \frac{\mu(e)}{e_j^* f(\frac{1}{\sigma_j^2})}[v(p_h, \mu(e)) - v(p_l, \mu(e))] \tag{8}$$

For further explanation, we here make a definition about education quality.

Definition 2. *An education choice e_j is called* ***Quality-undiscounted Education*** *if $f(\frac{1}{\sigma_j^2}) \geq 1$. An education choice e_j is called* ***Quality-discounted Education*** *if $0 < f(\frac{1}{\sigma_j^2}) < 1$. The set of quality-undiscounted education choices is denoted as E_u and the set of quality-discounted education choices is denoted as E_d.*

Then, for all $e_j \in E_u$, the negative effect of e_j^* increases since $e_j^* f(\frac{1}{\sigma_j^2}) > e_j^*$; for all $e_j \in E_d$, the negative effect of e_j^* shrinks since $e_j^* f(\frac{1}{\sigma_j^2}) < e_j^*$. This simply means that people who choose a high-average education have low motivation to choose a quality-undiscounted education, while people who choose a low-average education have low motivation to choose a quality-discounted education. This implies that people tend to prefer some master's programs (quality-discounted) to PhD programs (quality-undiscounted) if they are intrinsically relatively smart and have chance to be admitted into top schools.

7 Conclusion

In this paper, we have studied a model of job market. We include on-the-job learning in our model by adopting Alós-Ferrer and Prat's framework, in which the employers update beliefs with a Bayesian learning method based on the results of on-the-job performance reports. The innovation of our paper is extending education signals into two dimensions by including two reports. The first report evaluates the average productivity represented by the signal, while the second one evaluates the variance of performance delivered by the signal. We have proved the existence of multiple pooling equilibria, where both groups of people will not be separated by randomizing over multiple education choices. Furthermore, our results show that people can optimize their assembling of signals by choosing different combinations of mean and variance upon the equilibria set.

Our study gives some explanations for students' choice of education. The results show that the mean and variance of education are substitutes. Therefore, students' losses from attending high-variance academic programs can be compensated for by their choice of high-ranking schools. In terms of education returns, we can observe that for high-type students in top schools, marginal return of education is smaller in our model, which discourages them from choosing low-variance or lengthy programs. Generally, our model implies that students in master's programs of high-ranking schools may not have much incentive to attend PhD programs.

Our model also gives some insights into the master's programs in top schools. To raise students' motivation in study, they need to invest in master's program and screen the admitted heavily to control the variance of programs. As a student graduated from the master's program at the Department of Economics in Duke University, I am glad to see that our program always has a high requirement for master's students and thus has so far built a great reputation in the United States. Moreover, it is the rigorous academic training in this program that has evoked the desire of many master's students, including me, to become serious scholars and continue our study in ”low-variance” PhD programs.

There are also some questions unsolved in our model. First, we only focus on pooling equilibria and give no discussion about separating equilibrium. Hence, the effect of the second dimension on separating equilibrium is still unclear. Second, we have not defined any specific form for each report or built any connection between report results and signals. Third, like Spence and Alós-Ferrer and Prat, we only consider education as signals without any augmentation of people's productivity. However, education does improve people's ability and will probably turn a low-type person into a high-type one. Future study is expected to develop in these aspects.

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8 Appendix

Proof of Proposition 1

Basically, we follow the proof in Alós-Ferrer and Prat's paper.

Define

$$K_d = d_1 d_2 - g(1-d_1)(1-d_2)$$

$$K_1 = d_1 - g(1-d_1)$$

$$K_2 = d_2 - g(1-d_2)$$

$$K_3 = d_1 d_2 + (1-d_1)(1-d_2)$$

We denote $W(\mu, SS)$ as W_{SS} for simplification, where $S = \{G, B\}$. Then, we can rewrite $rh(\mu)$ as

$$rh(\mu) = K_d W_{GG} + (K_1 - K_d)W_{GB} + (K_2 - K_d)W_{BG} + [(1-g)K_3 - K_d]W_{BB}$$

To derive the first derivative of $rh(\mu)$, we need to first derive the first derivative of each W_{SS}. We denote the denominator of W_{SS} as D_{SS}. Also, some notations are listed below

1. $d^1 \triangleq 1 - d_1, \quad d^2 \triangleq 1 - d_2$
2. $d_1^2 \triangleq d_1(1-d_2), \quad d_2^1 \triangleq (1-d_1)d_2$

3. $d_{12} \triangleq d_1 d_2, \quad d^{12} \triangleq (1-d_1)(1-d_2)$

Then, we get

$$W'_{GG} = \frac{(p_h - p_l)d_{12}d^{12}}{D^2_{GG}}$$
$$W'_{GB} = \frac{(p_h - p_l)d_{12}d^{12}}{D^2_{GB}}$$
$$W'_{BG} = \frac{(p_h - p_l)d_{12}d^{12}}{D^2_{BG}}$$
$$W'_{BB} = \frac{(p_h - p_l)d_{12}d^{12}}{D^2_{BB}}$$

Then, we have the first and second derivative of $rh(\mu)$

$$\begin{aligned} &rh(\mu)' \\ &= (p_h - p_l)d_{12}d^{12}[\frac{K_d}{D^2_{GG}} + \frac{K_1 - K_d}{D^2_{GB}} + \frac{K_2 - K_d}{D^2_{BG}} + \frac{(1-g)K_3 - K_d}{D^2_{BB}}] \\ &= (p_h - p_l)d_{12}d^{12}[\frac{d_{12} - gd^{12}}{D^2_{GG}} + \frac{d_1^2 - gd_2^1}{D^2_{GB}} + \frac{d_2^1 - gd_1^2}{D^2_{BG}} + \frac{d^{12} - gd_{12}}{D^2_{BB}}] \end{aligned}$$

$$\begin{aligned} &rh(\mu)'' \\ &= -2(p_h - p_l)d_{12}d^{12}[(d_{12} - gd^{12})\frac{d_{12} - d^{12}}{D^3_{GG}} + (d_1^2 - gd_2^1)\frac{d_1^2 - d_2^1}{D^3_{GB}} \\ &+ (d_2^1 - gd_1^2)\frac{d_2^1 - d_1^2}{D^3_{BG}} + (d^{12} - gd_{12})\frac{d^{12} - d_{12}}{D^3_{BB}}] \end{aligned}$$

Since $g < 1$, we can prove that the second derivative is non-positive for all $\mu \in [0, 1]$ by

$$\begin{aligned}
& rh(\mu)'' \\
& \leq -2(p_h - p_l)d_{12}d^{12}[\frac{(d_{12} - d^{12})^2}{D_{GG}^3} + \frac{(d_1^2 - d_2^1)^2}{D_{GB}^3} + \frac{(d_2^1 - d_1^2)^2}{D_{BG}^3} + \frac{(d^{12} - d_{12})^2}{D_{BB}^3}] \leq 0
\end{aligned}$$

The equality holds when $d_1 = d_2 = \frac{1}{2}$. Thus, $rh(\mu)$ is concave with respect to μ. Then, we consider $rh(\frac{1}{2})$, which is

$$\begin{aligned}
& rh(1/2)' \\
& = (p_h - p_l)d_{12}d^{12}[\frac{d_{12} - gd^{12}}{(d_{12} + d^{12})^2} + \frac{d_1^2 - gd_2^1}{(d_1^2 + d_2^1)^2} + \frac{d_2^1 - gd_1^2}{(d_1^2 + d_2^1)^2} + \frac{d^{12} - gd_{12}}{(d^{12} + d_{12})^2}] \\
& = (p_h - p_l)d_{12}d^{12}[\frac{1 - g}{d_{12} + d^{12}} + \frac{1 - g}{d_1^2 + d_2^1}] \\
& > 0
\end{aligned}$$

Therefore, given $rh(\mu)'' \leq 0$, the multiple equilibria exist if and only if $rh(1)' < 0$. That is,

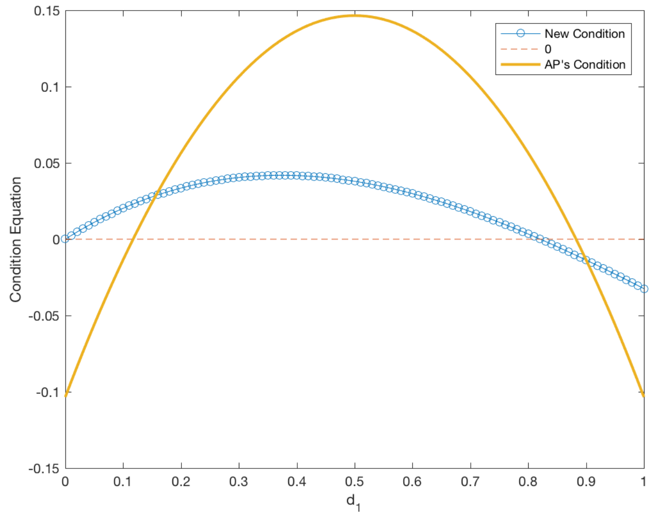
$$\begin{aligned}
& rh'(1) \\
& = (p_h - p_l)d_{12}d^{12}[\frac{d_{12} - gd^{12}}{(d_{12})^2} + \frac{d_1^2 - gd_2^1}{(d_1^2)^2} + \frac{d_2^1 - gd_1^2}{(d_2^1)^2} + \frac{d^{12} - gd_{12}}{(d^{12})^2}] \\
& = (p_h - p_l)\frac{d_{12}d^{12}[d^{12} + d_2^1 + d_1^2] - g[(d^{12})^3 + (d_2^1)^3 + (d_{12})^3]}{d_{12}d^{12}} \\
& < 0
\end{aligned}$$

Since $p_h > p_l$, this condition is equivalent with

$$d_{12}d^{12}[d^{12} + d_2^1 + d_1^2] - g[(d^{12})^3 + (d_2^1)^3 + (d_{12})^3] < 0 \tag{9}$$

Unfortunately, we cannot give an analytic solution to this cubic inequality. Hence, we decide to derive a sufficient condition to show that the lower bound of d_1 given by (9) is lower than that given by AP's Proposition 4 under some conditions. We know that

Figure 5: Comparison of Condition for Multiple Pooling Equilibria



Parameter: $r = 1, p_h = 1, p_l = 0, g = 0.15, d_2 = 0.6$

$$d_{12}d^{12}[d^{12} + d_2^1 + d_1^2] - g[(d^{12})^3 + (d_2^1)^3 + (d_{12})^3] < d_{12}d^{12}[d^{12} + d_2^1 + d_1^2] - g(d_{12})^3$$

Therefore, a sufficient condition here is then

$$d_{12}d^{12}[d^{12} + d_2^1 + d_1^2] - g(d_{12})^3 < 0 \tag{10}$$

Figure 5 shows the value of condition inequality of our and AP's model. We can see that the our second intersection with horizontal axis is in the left side of AP's, suggesting that our lower bound of d_1 can be lower than AP's under some values of g and d_2. Also, there exists an analytical solution for (10), which is

$$d_1 > \frac{1}{2}\frac{\sqrt{(1-d_2)4{d_2}^2 g + (1-d_2)^3} + {d_2}^2 - 1}{{d_2}^2 g + ({d_2}^2 - d_2)}$$

Then, we complete the proof of Proposition 1.

Calculation of Average Education Return

In this part, we are going to show how to derive (7) and (8) . AP shows that under some conditions, there will be an exclusive education level 0 for low-type people in multiple pooling equilibria[9]. This means that high-type candidates randomize among $\{e_1, e_2\}$ and low-type people randomize among $\{0, e_1, e_2\}$. Suppose $e_j \in \{e_1, e_2\}$, then low-type people are indifferent between e_j and 0, which is

$$v(p_l, \mu(e_j)) - g(p_l)e_j = v(p_l, \mu(0))$$

Because 0 is an exclusive signal sent by the low-type, then the low-type who send 0 will be believed to be low-type with 100% confidence. Hence, $\mu(0) = 0$. Besides, the low-type people who send 0 will be given p_l and thus $v(p_l, \mu(0)) = \frac{p_l}{r}$[10]. Besides, high-type people who send signal 0 will also be categorized as the low-type and are given p_l, so $v(p_h, \mu(0)) = \frac{p_l}{r}$. Plug all this into (6) and we can get the results.

[9] Please check Proposition 4(b) in AP's model

[10] Note that although the low-type receive p_l, their net payoff still equals what they get from sending e_j.