An Analysis of Upstream and Downstream Interaction, From a View of Principal-Agent Relationship*

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Abstract: This paper discusses pricing strategies and production of upstream and downstream firms from the view of a principal-agent relationship. As an agent, the upstream firm offers intermediate products and allocates them between downstream firms according to their incentives. As principals, downstream firms determine the quantities of final products they separately supply based on the competition or cooperation in the final market, which further determine the quantities of intermediate products they need and the incentives they are willing to offer. The research on the dual principals-agent setting indicates a price dispersion of intermediate product because of the difference in productivity of downstream firms. This paper also discusses the possibility of a unified price of intermediate product, a kind of price collusion between downstream firms, and its effect on profits of upstream agent. It is shown that the firm with higher productivity can always benefit from the collusion, and the collusion is usually supported when downstream firms have close technology levels. Furthermore, the analysis on the monopolistic downstream indicates that whether the upstream agent can benefit from the monopoly structure depends on the features of the final market and the difference in productivity of downstream firms. In short, this paper offers some new findings of price and allocation of intermediate products and the supply of final goods, as well as the effects of price commitment and monopoly structure in downstream on them.

Key words: Upstream, Downstream, Dual-principals, Productivity, Price dispersion, Collusion, Monopoly Structure.
JEL Numbers: L11, L12, L14.

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1 Introduction

The principal-agent theory is frequently discussed to figure out an optimal contract for the principal to incentivize the agent, with asymmetric information and their inconsistent utilities. It is applied in determining commissions paid to employees, job allocation among multiple agents, limits on outside activities and asset ownership. This theory refers to the relationship of firm owner and managers, electorate and officers, doctors and patients, brand owner and franchisee. Most cases have one principal-one agent or one principal-multi agents structures, which have been traditionally discussed. However, it is possible to have multiple principals in reality, when the upstream and downstream industries are jointly considered. For example, manufacturers or brands may choose a platform such as a retailing store to sell their products. Here the multiple upstream firms are the principals who gain profits via the selling of the store or platform that acts as an agent. The comparative roles of principal and agent played by upstream and downstream firms may not be fixed. Considering an upstream firm producing accessories as intermediate products, and downstream firms are electronic or machinery manufacturers who offer final products to consumers, the upstream firm acts as the agent obtaining the commission from the downstream firms on its supply of intermediate products, and the downstream firms gain their profits by producing final products based on intermediate products purchased from the agent as an input. Thus, this paper mainly discusses the later case, especially when competition or cooperation happens in the downstream.

This paper will research on the pricing strategies and production of upstream and downstream firms with a background of principal-agent relationship. As an agent, the upstream firm provides intermediate products to the downstream principals for reproduction. Downstream firms are responsible to offer final products to consumers. According to the competition or cooperation in the final market, downstream firms determine their optimal quantities of final goods to maximize profits. And the market prices of final products are determined by aggregate consumer demand. The optimal supply of each downstream firm also determines the quantity of intermediate products they need for the reproduction, which consequently decides incentives that they are willing to pay to the agent. That is the process of pricing the intermediate product. Based on the Cournot competition in the downstream, this study figures out the equilibrium prices of intermediate product offered by two downstream firms and the allocation of intermediate products between them, considering that downstream firms have different levels of productivity. This paper further discusses how collusion on pricing the intermediate product between downstream firms affects the supply of final product and its market price, as well as profits of upstream and downstream firms. Finally, the study focuses on the effect of a monopolistic downstream on the agent's profits.

This paper consists of seven parts. Section 1 and 2 give an introduction of this problem and related literature. Section 3 introduces a basic upstream-downstream model, and derives optimal prices and allocation of intermediate products. Section 4 discusses price collusion between downstream firms and figures out the victim and beneficiary of this commitment. In section 5, the monopoly structure in downstream is considered, with the attempt to find its influence on intermediate product's price, supply and the upstream agent's profits, as well as the price and supply of final goods. Section 6 and 7 summarize the conclusions of this paper and show some extensions.

2 Relevant Literature

In terms of principal-agent theory, Holmstrom and Milgrom (1987) present a linear model in which they specify that the agent's cost only depends on total efforts devoted to all his tasks, which indicates that an increase of any one task leads to a reallocation of efforts devoted to other tasks. They (1991) consider a random observation error in the information signal to represent the measurability of multitasks. Their model implies that in most cases commissions are optimal for measurable tasks, while employers should offer fixed wages for non-measurable tasks. Additionally, a complementary relationship among multitasks in the cost function leads to a higher commission while the substitution decreases it. Their research also reveals that for a higher degree of agent's risk aversion and non-measurability of performance, it is optimal to adopt employment instead of independent contracting. Holmstrom and Milgrom also indicate that an agent should have a higher freedom to pursue personal business if his marginal reward for performance is higher. As for the job allocation between two agents, a division of labor should be adopted and the agent expected to provide more efforts should be responsible for the measurable task while the agent devoting less attention should work on the non-measurable task. This conclusion is based on the setting of a substitution between agents' work.

Considering the discussion on upstream and downstream firms, Tirole (2015) shows that a monopoly downstream is beneficial for an oligopoly upstream, since its profits are capped by the downstream profits, which is equal to the Cournot industry profits and these will be destroyed by downstream competition. The upstream firm has incentive to maintain its market power by blocking the access of new firms entering downstream. It leads to a vertical integration with downstream subsidiary. Regulators can observe this high price and less competition in downstream; they can then attempt to promote competition by mandating the upstream firm to offer the downstream industry fair accesses. However, Tirole indicates that the policy of transparent contracts cannot avoid the monopoly profits obtained by the upstream firm and only affects the rate of return on inputs. Therefore, he suggests that regulators should check the undeserved market power instead of monopoly in general. This problem is from the lack of regulatory information, including market condition and firm's operating structure. Together with Patrick Rey, Tirole (2013) also focuses on demand functions ranging from perfect substitutes and perfect complements. They find that the tacit collusion (coordinated increase in price) is feasible in a substitutional case while the tacit cooperation (coordinated decrease in price) is preferred by complementary firms, with the assumption that firms are patient.

In recent years, the vertical structure between the upstream and downstream frequently has been discussed from the view of manufacturers and retailers, a hot focus of distribution channel in marketing research. Geylani et al (2007) establish a model in which a dominant retailer masters the pricing power relative to the manufacturer, while the manufacturer sets the wholesale price for the weaker retailer. They indicate that the upstream manufacturer can intervene in the downstream competition by adjusting demand between retailers with joint promotions and advertising with the weaker one, although the manufacturer sets a higher wholesale price for the weak retailer at the beginning. This model implies a source of dominant retailers' low prices. Draganska et al (2010) derive a Nash bargaining solution of the wholesale prices and split of margins in the channel, which also shows that the bargaining power is determined by exogenous retail and manufacturer features. They study the role of firm size increase, store brand introduction and service level differentiation as determinants of the power shift between retailer and manufacturer. Their empirical work on the coffee market in Germany presents various degrees of bargaining power among different pairs of manufacturers and retailers and the possibility of decreasing share of profits with increasing overall pie in the channel. In the discussion of multilateral bargaining and downstream competition, Guo and Iyer (2013) compare the outcomes of simultaneous and sequential bargaining, with the conclusion that the former is beneficial for the manufacturer when the retail prices are similar while the later is preferred with a large dispersion of retail prices. Sofia Berto (2007) uses the data on yogurt sold in a large urban area of the United States to illustrate the pricing power of retailers in the vertical chain, since the wholesale prices are close to the marginal cost.

3 Basic Upstream-Downstream Model

3.1 Description of the basic model

As shown in Figure 1, an upstream firm U offers intermediate products for downstream firms D_1 and D_2 to produce final products. Consumers then purchase these products in the final market. This setting is commonly observed in the real business world. For example, in the food industry, farms as upstream firms are responsible for supplying raw materials such as fruits and meat and consumers purchase the processed food products offered by downstream firms. Similarly, with the development of industrial specification and professionalization, upstream firms supply accessories to downstream firms to produce final products, in terms of electronic and machinery industries. Here, the problem is studied from a perspective of principal-agent relationship. Intermediate products offered by the upstream firm are regarded as key components of producing final goods. Thus, downstream firms acting as principals gain profits by selling final products of which the production is based on the work offered by the upstream firm acting as the agent. The quality and quantity of intermediate products from the agent influence principals' revenues, while quantities sold to principals at price levels that principals are willing to pay decide the agent's profits. To simplify the problem, only the quantity of intermediate products is considered since it has a great measurability. Meanwhile, considering that principals are in the same industry, they are assumed to produce homogeneous products. The competition between them in the final market is from the aspect of quantity. Following the corresponding notations used by Holmstrom and Milgrom (1991), we denote that firm U has a vector of efforts $t = (t_1, t_2)$ allocated to D_1 and D_2, with production $P(t)$ and cost $C(t)$. The price level at which D_1 and D_2 purchase the intermediate products is the incentive vector $\alpha = (\alpha_1, \alpha_2)$. Therefore, the optimization problem faced by the agent is to maximize its payoff $\pi_U = \alpha P(t) - C(t)$.

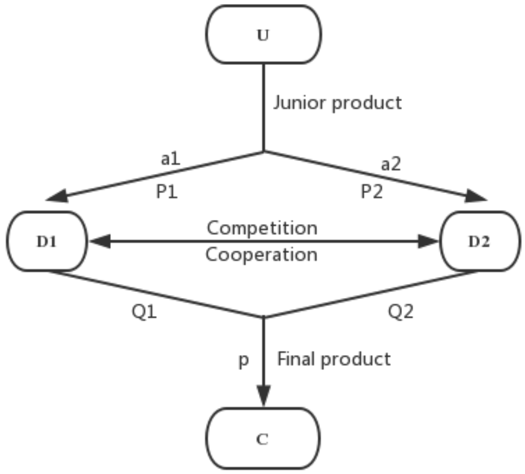

Figure 1: The system of upstream and downstream firms

Considering the downstream side, denote that $q_i(\cdot)$ is the production function of D_i, which maps intermediate products $P_i(t)$ received by i to the quantity of final products it supplies. The price of final product is decided by the assumed linear relationship $p = a - b(q_1 + q_2)$. Thus principal i faces the problem to maximize its profits $\pi_i = (p - \alpha_i)q_i$, for $i = 1, 2$. This basic model is to figure out the allocation of intermediate products between dual downstream firms and corresponding incentive levels with a principal-agent background, when optimality is achieved.

3.2 The optimal price and allocation of intermediate products

As for these settings above, assume that production functions $P(t), q_i(\cdot)$ are increasing and concave ($P' > 0, P'' \leq 0, q' > 0, q'' \leq 0$) and cost function $C(t)$ is increasing and strictly convex ($C' > 0, C'' > 0$), where t is defined as effort as mentioned above. In the relationship $p = a - b(q_1 + q_2)$, assume that $a, b > 0$. Because of the same intermediate products offered to D_1 and D_2 to produce homogenous final products, it is reasonable to assume a perfect substitution between t_1 and t_2 in $C(t)$. For mathematical convenience, the optimality problem faced by the agent is:

$$\max_{t,\alpha} \quad \sum_{i=1}^{2} \alpha_i P(t_i) - \sum_{i=1}^{2} C(t_i) \tag{1}$$

Consider FOC and check SOC:

$$\frac{\partial \pi_U}{\partial t_i} = \alpha_i P_i' - C_i = 0, \quad \frac{\partial^2 \pi_U}{\partial t_i^2} = \alpha_i P_{ii} - C_{ii} < 0 \tag{2}$$

Based on FOC and the assumption $P_i' \neq 0$, for $i = 1, 2$, we derive $\alpha_i = \frac{C_i}{P_i'} \overset{\triangle}{=} g(t_i)$, of which the monotony is easy to be proved as $\frac{\partial g}{\partial t_i} = \frac{\partial \alpha_i}{\partial t_i} = \frac{C_{ii}P_i' - P_{ii}C_i}{P_i'^2} > 0$. Thus, t_i can be denoted as $t_i = g^{-1}(\alpha_i)$. To simplify notation, denote $Q_i(\alpha_i) = q_i[P(g^{-1}(\alpha_i))]$. In this model, the asymmetric productivities of downstream firms are induced and reflected by different technology levels, which is an important contribution of this study. Considering different productivities demonstrated by heterogenous $q_i(\cdot)$, simply assume $Q_i(\alpha_i) = \mu_i \alpha_i$[1], for $i = 1, 2$, where μ_i specifically indicates the technology level of firm i. The reasonability of this setting is proved in appendix, which shows that such $P(\cdot), C(\cdot), q(\cdot)$ generally exist and the level of productivity shown by $q(\cdot)$ can be completely indicated by μ here. Therefore, the optimality problem of principal i is

$$\max_{\alpha_i} \quad [a - b(\mu_1\alpha_1 + \mu_2\alpha_2) - \alpha_i]\mu_i\alpha_i \tag{3}$$

Derive from FOC,

$$\alpha_i^* = \frac{b\mu_{-i} + 2}{3\mu_1\mu_2 b^2 + 4(\mu_1 + \mu_2)b + 4} a, \quad i = 1, 2 \tag{4}$$

It can be trivially proved that the sign of $\alpha_i^* - \alpha_{-i}^*$ is consistent with that of $\mu_{-i} - \mu_i$. Furthermore, the increasing property of function $g(\cdot)$ provides that $\frac{\partial t}{\partial \alpha} > 0$. Without losing generality, set $\mu_1 = 1, \mu_2 = \lambda \in (0, 1]$, which indicates that with a higher level of technology, downstream firm D_1 is more productive than D_2.

Proposition 1: The downstream firm with a higher level of technology is willing to pay less for intermediate products, and correspondingly, agent allocates less intermediate products to the more productive downstream firm, with less efforts spared, i.e. $\alpha_1 \leq \alpha_2, t_1 \leq t_2, P_1 \leq P_2$, for $\lambda \in (0, 1]$. Equalities hold if and only if $\lambda = 1$.

Then consider market share $Q_i^* = \mu_i \alpha_i^*$, specifically

[1] The implication of this setting is to extract the effect of technology level on the relationship between production and incentive for intermediate goods. Although this effect of technology level cannot be summarily represented by a constant μ for most cases when formats of $q(\cdot), P(\cdot), g(\cdot)$ are specifically given, this simplification is for the further theoretical analysis. The abstract way is to reinforce our focus on the effect of different technology levels.

$$Q_1^* = \frac{\lambda b + 2}{3\lambda b^2 + 4(\lambda+1)b + 4} a, Q_2^* = \frac{\lambda b + 2\lambda}{3\lambda b^2 + 4(\lambda+1)b + 4} a \tag{5}$$

Proposition 2: $Q_1^* \geq Q_2^*$ for $\lambda \in (0, 1]$, and the equality holds if and only if $\lambda = 1$. The downstream firm with a higher technology level has a larger market share.

Proposition 1 and 2 indicate that the downstream firm with a higher productivity has cost advantage and ends up with a higher market share in the competition of quantity. On the contrary, the less productive firm needs to input more to compete with his opponent in the final market. Although it is obviously that the productivity level decides the input level in terms of quantity competition, with the consideration of agent's optimality, cost advantage of the firm with a higher technology level is from two aspects, not only the number of intermediate products needed but also the marginal cost on them. It implies a dual power of technology, leading to the relatively costless input and less amount needed. Despite that the less productive firm inputs more with a higher marginal cost, it is also beaten by the more productive opponent, in terms of market share.

4 Who will benefit from an unified price?

The analysis above indicates a price dispersion in the intermediate product market. D_1 and D_2 separately decide their equilibrium prices to purchase from agent, based on their quantity competition in the final market. The agent's allocation of intermediate products, as the best response to incentives separately offered by principals, inversely affects the downstream firms' Cournot optimal quantities. The source of price dispersion is the difference in productivity of downstream firms. The process implies that difference in technology level is commonly known for downstream firms. However, it is more realistic that downstream firms only know about their own productivities; thus, they cannot decide equilibrium incentives on intermediate products and optimal quantities of final products to supply, corresponding to the opponent's productivity. It is likely for them to have a commitment to making a price offer to intermediate supplies, setting a unified price which sounds fair for both of them. What would be results of this price collusion? Here is another important contribution of this study: we will discuss what the unified price of intermediate products would be and how the pricing collusion between principals affects the welfare of the upstream agent, downstream firms, and consumers.

In this scenario, it is reasonable to assume that both D_1 and D_2 suppose that their opponent has the same productivity as they do. Thus, the optimal problem for them is to

$$\max_{\alpha} \quad (a - 2b\mu_i\alpha - \alpha)\mu_i\alpha \tag{6}$$

Then they bring their preferred price $\alpha_i = \frac{a}{2(2b\mu_i+1)}$ to the negotiation meeting. Assume that both D_1 and D_2 know that D_i sets price by supposing they both have the same technology level μ_i. Assume that they simultaneously show their preferred α_i to the other, which therefore reveals the information of their productivity level μ_i. Assume that D_1 and D_2 are both faithful to the price commitment and still decide to offer the same price to agent. Then, based on their knowledge of μ_1 and μ_2, D_1 and D_2 derive the agreed optimal $\hat{\alpha}$ in this negotiation meeting. In this case, the decision problem of agent is to figure out the optimal quantity of intermediate products to supply. Since price offered by the principals is identified, intermediate products are equally allocated between them according to the strict monotonicity of t in α, in section 3. Therefore, the optimality problem is to

$$\max_{t,\alpha} \quad [a - b(\mu_1 + \mu_2)\alpha - \alpha]\mu_i\alpha, \qquad s.t \ \max_{t} \quad \alpha P(t) - C(t) \tag{7}$$

And then derive the solution $\hat{\alpha} = \frac{a}{2(b\mu_1+b\mu_2+1)}$. With the simplification above, $\hat{\alpha} = \frac{a}{2(b+b\lambda+1)}$. Denote $i's$ equilibrium quantity of final products $\hat{Q}_i$ and the total quantity $\hat{Q}$. Correspondingly, the equilibrium price of final product is $\hat{p}$ and that in the previous setting is p^*

Proposition 3: The collusion on pricing intermediate products leads to a less total quantity of final products and correspondingly a higher market price, which is bad for consumers, i.e. $\hat{Q} < Q^*, \hat{p} > p^*$.

Proof: Based on the maximization problem, $Q^* = Q_1^* + Q_2^* = \frac{2\lambda b+2(\lambda+1)}{3\lambda b^2+4(\lambda+1)b+4}a$, $\hat{Q} = \frac{1+\lambda}{2(b+b\lambda+1)}a$. And thus $Q^* - \hat{Q} = \frac{\lambda(\lambda+1)b^2+4\lambda b}{2[3\lambda b^2+4(\lambda+1)b+4](b+b\lambda+1)}a > 0$

Proposition 4: The commitment always ends up with a lower input price for the less productive firm D_2, i.e. $\hat{\alpha} < \alpha_2^*$, and correspondingly $\hat{Q}_2 < Q_2^*$. As for D_1, $\hat{\alpha} < \alpha_1^*, \hat{Q}_1 < Q_1^*$ when $\lambda \geq \frac{1}{2}$, for $\forall b > 0$.

Proposition 5: The more productive firm D_1 can always benefit from the commitment, i.e. $\hat{\pi}_1 > \pi_1^*$. As for D_2, $\exists \ \underline{\lambda} \in (0.5, 1)$, when $\lambda \geq \underline{\lambda}$, for $\forall b > 0$, $\hat{\pi}_2 > \pi_2^*$.

Proof of these two propositions can be found in the appendix[2]. The intuition behind these three propositions is that the collusion on pricing intermediate products leads to a decrease of final product supply and consequently increases its price in the final market. The collusion hurts the welfare of consumers. The decrease of supply is mainly from the less productive firm D_2. The commitment limits the incentive that D_2 can offer to the agent; thus, he cannot obtain enough input for production. Even if D_1 loses his marginal cost advantage with holding the commitment, his higher productivity still guarantees an even larger market share compared to D_2, implied by $\hat{Q}_1/\hat{Q}_2 = 1/\lambda > Q_1^*/Q_2^*$. The decrease of supply from D_2 promotes market price, and then the stronger market advantage benefits D_1 from this increase of price. Additionally, the consistent sign of $\hat{\alpha}_i - \alpha_i^*$ and $\hat{Q}_i - Q_i^*$ is based on increasing monotonicities among α, t, q. The proposition 4 tells that if the difference of technology level is high, i.e small $\lambda < 0.5$, the profitable strategy for D_1 with the commitment is to show a higher incentive to agent to obtain more input for expanding production. Even so, the increase of Q_1 is still less than the decrease of Q_2, and thus D_1 can benefits from expanding production with a higher market price. The commitment offers D_1 the power to limit the agent's supply of intermediate products to D_2, in order to control the total quantity in final market which consequently promotes a higher final good price. The agreed equilibrium price $\hat{\alpha}$ makes D_2 to gain the amount of input far below than that he needs for the quantity competition in the final market, when D_2 is far less productive with a small λ. It is also the reason that D_2 can only benefit from this commitment when his productivity is close to D_1's (a relatively large λ), implied in proposition 5. Aware of it, D_1 would be willing to share some technologies with D_2 to decrease the difference in their productivities which can motivate D_2 to cooperate on such a commitment.

The analysis above indicates that commitment on pricing intermediate products is always profitable for the more productive downstream firm. When two downstream firms have close levels of technology, they both support this price collusion and benefit from it. However, the collusion results in a decreasing supply of final products that hurts consumers' welfare with a higher market price. The former discussion is from the perspective of consumer and principal, and then the following analysis will focus on the influence of collusion on agent. To compare the agent's profits with and

[2]Specific results of $Q^*, \alpha^*, p^*, \hat{Q}, \hat{\alpha}, \hat{p}$ are driven by the linearity assumption $p = a - b(q_1 + q_2)$, which is also the foundation of propositions 3-5. This assumption is commonly used in economic studies to reduce the complexity of calculation, which is sometimes reasonable in reality. Since it is likely to attain conclusions when other formats of the relationship between price and demand are used, we state that all results in this paper are responsible for cases with an approximately linear relationship.

without commitment, we make a specification of the agent's production and cost functions which satisfy the assumption for basic setting in section 3, without losing generality. For mathematical convenience, assume that $P(t) = t, C(t) = \frac{1}{2}t^2$. According to $\pi_U = \alpha P(t) - C(t)$,

$$\pi_U^* = \frac{1}{2}(\alpha_1^{*2} + \alpha_2^{*2}), \quad \hat{\pi}_U = \frac{1}{2}\hat{\alpha}^2 \tag{8}$$

Considering the difference in π_U^* and $\hat{\pi}_U$,

Proposition 6: In most cases (b is not too small or too large), $\pi_U^* > \hat{\pi}_U$, that is to say the price collusion decreases the agent's equilibrium profits, especially for a relatively large $\lambda \in (0, 1]$.

Proof can be found in appendix.

This proposition intuitively means that, in terms of a general market of final products, of which the demand is not extremely sensitive or insensitive to price, indicated by a very small and large b respectively, the equilibrium profits that upstream firm U can obtain decreases when downstream firms have a collusion on pricing intermediate products. And the decreasing property of the agent's profits because of this commitment is more obvious for a λ closer to 1, which indicates a closer level of technology between D_1 and D_2. This finding is consistent with results shown in proposition 4 and 5 that this collusion leads to a lower price of intermediate products for both D_1 and D_2 when $\lambda \geq \frac{1}{2}$. The decrease of intermediate product's price, together with its lower demand from downstream firms essentially result in the decrease of upstream's profits.

In short, propositions 3-6 give answers to the question "who will benefit from the unified price of intermediate products?" The commitment on the incentive to agent is always profitable for the downstream firm with a higher technology level; thus, it will support such a collusion. With this commitment, the more productive firm has a higher comparative market share, reflected by a higher Q_1/Q_2. For this firm, the unified incentive to agent is higher than that in the case without commitment when the opponent's technology level is far less. Even if its marginal cost of input increases, the firm can still benefit from the increasing supply of final products with a higher market price. The close productivities of downstream firms make the agreed price lower, also leads to a lower supply of final products from both firms, while the consequently increasing price of final goods results in a higher profits for the more productive firm. If the less productive firm is behind far from its opponent, the lowered agreed price of intermediate products, which leads to the less input available for final production, decreases the firm's profits. Only when the difference in productivity is small can the less productive firm benefit from this collusion, despite that both its production and comparative market share decrease. As for the quantity of final products, this collusion decreases the total supply from downstream firms even if the more productive firm tends to produce more when its technology level is much higher than its opponent's, which implies that the less productive firm produces much less than the other increases. Therefore, consumers in the final market are always victims of the collusion, facing a higher price of final goods. So does the upstream firm as an agent, gaining a lower equilibrium profits with the commitment, given a general market of final products.

5 A Monopolistic Downstream

In this part, the influence of merging downstream firms on the price of intermediate and final products will be discussed, which also reflects the effect of downstream competition. The most basic Cournot model has told that the duopoly competition increases the quantity of products compared to the oligopoly structure, and consequently leads to the decrease of market price. Similarly, this conclusion does not change when the principal-agent relationship is induced. In this case, it is reasonable to assume that the new firm after merging has the highest technology level of original firms. Thus, the optimality problem for principal is

$$\max_{\alpha} \quad (a - b\mu_{max}\alpha - \alpha)\mu_{max}\alpha \tag{9}$$

Apply the setting of λ, here $\mu_{max} = 1$, so derive $\widetilde{\alpha} = \frac{a}{2(b+1)} = \widetilde{Q}$. Compare it with former results,

Proposition 7: The monopolistic downstream leads to the lowest quantity of final products and correspondingly the highest market price, i.e.$\widetilde{Q} < \hat{Q} < Q^*, p^* < \hat{p} < \widetilde{p}$.

To consider the change of agent's equilibrium profits and price of intermediate products, we still assume that $P(t) = t, C(t) = \frac{1}{2}t^2$,

Proposition 8: $\exists\ \underline{b}$, when $b \in (0, \underline{b}]$, for $\forall \lambda \in (0,1), \widetilde{\pi}_U < \pi_U^*$. If $b > \underline{b}, \widetilde{\pi}_U < \pi_U^*$ as $\lambda \to 0$ and $\widetilde{\pi}_U > \pi_U^*$ as $\lambda \to 1$.

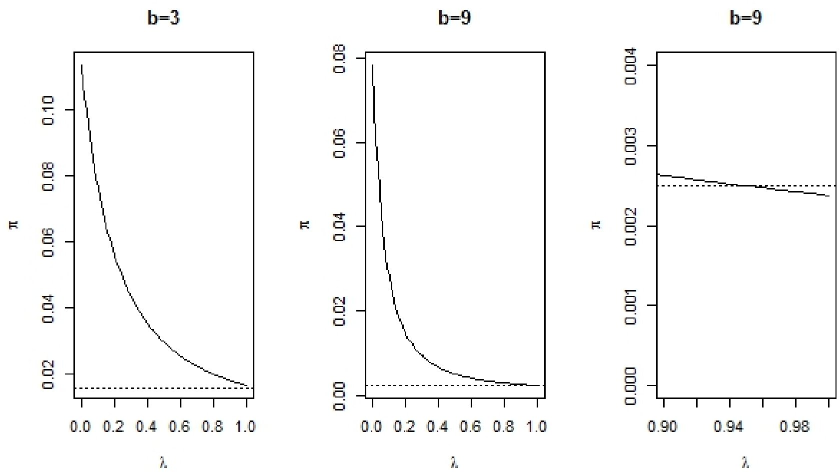

Figure 2: The agent's profits before and after merging

Detailed proofs can be found in the appendix. Proposition 7 is consistent with the analysis on oligopoly and duopoly structures, which is easy to be understood that monopoly hurts consumers' welfare by decreasing supply, thus increasing market price. Here, it is interesting to have proposition 8 which implies that when the demand of final goods is sensitive to its price, reflected by a large $\frac{1}{b}$ (i.e. $b \in (0, \underline{b}]$), the monopolistic downstream always hurts the welfare of upstream agent. Actually the fixed value $\underline{b}$ is about 4.8. As shown in the first graph of Figure 2, when $b = 3$ agent's profits before merging (the solid curve) is always beyond profits after merging (the dotted line) for any value of $\lambda \in [0,1]$. However, if the final goods has a rigid demand which is insensitive to its price, the upstream firm can benefit from merging when original downstream firms have close productivities. When their difference in technology level is huge, the acquisition that happens in downstream market would be still bad for the upstream firm. This result is also reflected by the case of $b = 9$ in Figure 2. The third graph reinforces the point that the agent's profits after merging begin to exceed its profits before merging with a λ larger than 0.94. It is realized that a larger b (a more rigid demand) may lead to a smaller threshold λ for the agent to be better off. Intuitively, compared to the case of rigid demand, the decrease of demand because of a rise of price due to the merging is more drastic when the demand of final goods is sensitive to its price. Thus, in this case the decrease of total demand and incentive on intermediate goods is also more drastic, which is more likely for upstream agent to be worse off. The conclusion of rigid demand in proposition

8 is driven by the interaction between increasing incentive and decreasing demand of intermediate products which can be explained by the following proposition. Compare the price of intermediate products before and after merging,

Proposition 9: It is always the case that $\widetilde{\alpha} > \alpha_1^*$. If $\lambda \leq \frac{1}{2}, \widetilde{\alpha} < \alpha_2^*$ for all $b > 0$; if $\lambda \geq \frac{2}{3}, \widetilde{\alpha} > \alpha_2^*$ for all $b > 0$.

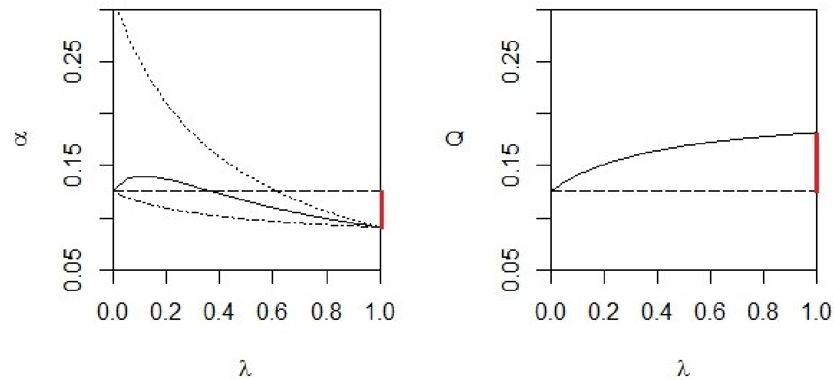

Figure 3: Incentives on intermediate goods and demand of final goods, with $b = 3$.

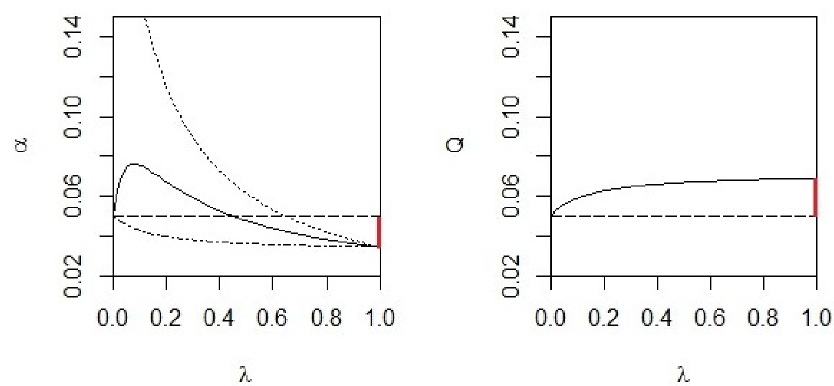

Figure 4: Incentives on intermediate goods and demand of final goods, with $b = 9$.

Detailed proofs can be found in appendix. This proposition tells us that the incentive offered by monopoly the firm in downstream is always larger than the minimum incentive when duopoly firms separately price the intermediate product. Additionally, as $\lambda \to 1$, which means that productivities of downstream firms are close, the incentive on intermediate products offered by the single downstream principal is higher than both α_1^* and α_2^*. As seen in Figure 3 and 4, the dotted line representing α_1^* is always below the line of $\widetilde{\alpha}$ while the curve of α_2^* begins to be under the line of $\widetilde{\alpha}$

with a λ larger than 0.6. Additionally, the average incentive on intermediate goods before merging which is computed by $\bar{\alpha}^* = \frac{Q_1^*}{Q_1^*+Q_2^*}\alpha_1^* + \frac{Q_2^*}{Q_1^*+Q_2^*}\alpha_2^*$ firstly increases and then decreases in λ. As shown in Figure 3 and 4, this curve begins to be under the line of $\widetilde{\alpha}$ with a λ larger than 0.4. Considering the implication of proposition 8, for the market in which demand of final goods is insensitive to its price, the upstream agent now benefits from the monopoly structure in downstream as $\lambda \to 1$. Intuitively, even if the demand of intermediate products decreases as implied in proposition 7 which implies that the supply of final products decreases because of the monopoly structure, the price of intermediate products is larger than both α_1^* and α_2^*. Thus, the positive effect of merging on agent's profits is mainly due to the powerfully increasing price of intermediate products, which is beyond the negative effect of decreasing demand. As illustrated in Figure 3 and 4, it is easier for this positive effect dominates the negative one in the case with a larger b, due to the fact that the length of red part in $\alpha - \lambda$ graph and that of red part in $Q - \lambda$ graph decrease in b, which implies that the positive effect of increasing price intensifies and negative effect of decreasing demand fades in b. Nevertheless, if original downstream firms have a large difference in productivities, it is hard for upstream agent to benefit from this monopoly structure in downstream, as jointly reflected in proposition 8 and 9. In this case, the price of intermediate goods is not always higher than the average price before merging since $\widetilde{\alpha} < \alpha_2^*$ as λ is small, however, the demand of intermediate goods must decrease, which makes it harder for upstream agent to be better off.

This result is different from that described by Tirole (2015). In that paper, the upstream firm owns essential facilities or key patents and it prefers the monopoly in downstream to maintain its market power, since competition destroys downstream profits which caps upstream profits. Therefore, the upstream has incentive to restrain the entry of downstream. However, the upstream firm acts as principal and the downstream firms are agents in that problem, which is opposite to the scenario in this paper. And the key point is that the cost of downstream firms is profits of upstream firm here, given that the former are principals while the later is the agent.

6 Conclusions

In the view of principal-agent relationship, the analysis above shows optimal incentives offered by downstream firms to upstream firm for intermediate products, considering Cournot competition happens between two firms with different productivities. In the case of freely pricing intermediate products, as described by the basic model, the principal with a higher productivity tends to offer a lower incentive to the agent, with a correspondingly smaller demand of intermediate products, but it can still have a larger share in the final market, and vice versa. The collusion on pricing intermediate products between downstream firms will decrease the quantity of final products and consequently increase its price. The unified price of intermediate products is always lower than that offered by the less productive firm without collusion. Thus, the less productive firm will provide less final products because of the decreasing supply of intermediate products from upstream. The more productive firm uses this collusion to restrain its opponent's input and consequently gains a higher market share. It always benefits from this collusion, because of a higher price of final products and a stronger market dominance from the technology power. Only when their technology levels are close, the less productive firm can benefit from this collusion, which will destroy its profits otherwise. This result implies that such a collusion can only be simultaneously supported by two downstream firms with close productivity levels. In most cases, the upstream agent is the victim of this collusion, with a relatively lower demand of intermediate products reflected by lower incentives. When merging happens in downstream, the monopoly structure leads to the smallest quantity of final products, compared to the basic case and that of collusion. Thus the monopolistic downstream hurts the welfare of consumers. However, whether upstream agent will benefit from the monopoly depends on the relationship of price and quantity in the market of final products and the difference in downstream firms' productivity. When the sensitivity of price to quantity is lower than a threshold, the upstream agent is always hurt by the monopoly structure. When it is beyond the threshold, conclusion depends on the difference in downstream firms' productivities. The upstream agent benefits from the monopoly structure only when original downstream firms are close to each other in productivity.

7 Discussion and Extension

Analysis of this paper is based on following assumptions:

- The quality of intermediate products is not considered to affect the production of final products.
- Quantity competition happens in downstream.
- Final products offered by downstream firms are homogeneous.

However, these assumptions seem too ideal to fit the real business world. In fact, besides the quantity of intermediate product, its quality also influences downstream firm's profits. The quality of agent's work is an aspect of its total efforts, and this problem has been considered by Holmstrom and Milgrom (1991) in their discussion on multitask scenario. Additionally, in contrast to the quantity competition, it is more realistic to observe the price competition in the final market, which is described by the Bertrand model. Moreover, as for the same category, an individual always has different preferences on different brands, which indicates that products offered by different downstream firms in the final market are not homogenous. This heterogeneity would affect the perfect substitution between products offered by different downstream firms. Therefore, a future study should relax these assumptions to be closer to reality. There are some specific considerations about extending research from these aspects:

The quality of intermediate products: Since the measurability of quantity and quality is different,which is shown by the fact that counting quantity is always easier than observing quality, a vector of random errors could be induced to reflect their various measurabilities. The risk types of upstream agent and downstream principals should be further assumed to compute their expected utilities and certainty equivalent payoffs. This analysis is similar with the linear principal agent model discussed by Holmstrom and Milgrom (1991), while the big difference is the setting of dual principals for whom final profits $B(t)$ are correlated via the competition or cooperation in downstream in this paper. The existence of profits-correlated principals may affect the incentive paid by principals on the quantity and quality of intermediate products, which consequently affects the agent's allocation of efforts on quantity and quality of intermediate products for these downstream firms. And actually, the difference in allocating quality of intermediate products may be one of sources of product heterogeneity in the final market.

Price competition in downstream: Game theory learning tells us that price competition is more realistic than quantity competition while Cournot outcomes seem to be more realistic than Bertrand outcomes which indicate a zero-profits result. The basic analysis of the Bertrand model assumes that the marginal cost c_i of each competitor is an exogenously given constant. In the scenario of this paper, the marginal cost is the source of upstream agent's profits, and backward induction implies that the supply of intermediate products as an input of principals' production is decided by this cost. In this case, price competition should consider the endogeneity of cost which then decides the market share of each downstream firm. Thus, it is likely to have some new findings of price competition under this endogenous setting.

Differentiated final products: In reality, the competition among brands within a category is not only related to price but also their heterogeneity for consumers. Game theory indicates that inducing differentiated products to the Bertrand model results in higher prices and higher profits, thus the product differentiation is supported by downstream firms. This result is obviously more realistic, compared to the zero-profits implied by basic Bertrand model. Therefore, it is reasonable to consider product differentiation when the downstream competition is from the aspect of price. With the principal-agent relationship, the heterogeneity of final products may be from heterogenous intermediate products offered by upstream agent, and consumers' individual preferences. The heterogeneity influences the demand of final products from each downstream firm, which determines their market shares. It is reasonable to have a conjecture that the effect of heterogeneity on downstream demands will be delivered to the price of intermediate product and its allocation between downstream firms via the backward induction that connects agent and principals.

These extensions on quality of intermediate products, price competition in downstream and heterogeneity of final products can possibly lead to new findings. Since they may change profits of downstream principals and demands of final products, they influence incentives that principals are willing to pay for the agent, which consequently affects the allocation of intermediate products.

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Appendix

0. Considering the reasonability of setting $Q_i(\alpha_i) = q_i[P(g^{-1}(\alpha_i))] = \mu_i\alpha_i$, the goal is to prove that there always exists an increasing production function $q_i(\cdot)$, corresponding to $P(\cdot), C(\cdot)$, for which the technology level can be reflected by the item μ_i. First, we check the monotonicity of $q_i(\cdot)$. Since it has been illustrated that $t_i = g^{-1}(\alpha_i) \Rightarrow \frac{\partial g^{-1}}{\partial \alpha_i} = \frac{\partial t_i}{\partial \alpha_i} = \frac{P_i'^2}{C_{ii}P_i' - P_{ii}C_i} > 0$. According to the increasing monotonicity of function $P(\cdot), Q(\cdot)$, it is obviously that $q_i(\cdot)$ satisfies the increasing property that a production function has. Considering the concavity of $q_i(\cdot)$, $Q_i(\alpha_i) = q_i[P(g^{-1}(\alpha_i))] = \mu_i\alpha_i \Rightarrow q''P'\frac{\partial g^{-1}}{\partial \alpha_i} + P''q'\frac{\partial g^{-1}}{\partial \alpha_i} + q'P'\frac{\partial^2 g^{-1}}{\partial \alpha_i^2} = Q'' = 0$. Thus, it is consistent with the basic assumption that $q_i(\cdot)$ is concave ($q' > 0, q'' \leq 0$). Then we verify that the technology level included in $q_i(\cdot)$ can be abstractly reflected by the item μ_i. Trivially, $\mu_i = q_i[P(g^{-1}(\alpha_i))]/\alpha_i$ when set $\alpha_i = \alpha_j$, the difference of $q_{i,j}(\cdot)$ can be reflected by $\mu_{i,j}$.

1. Proof of proposition 4:

$$\alpha_2^* - \hat{\alpha} = [\frac{b+2}{3\lambda b^2 + 4(1+\lambda)b + 4} - \frac{1}{2(b+b\lambda+1)}]a = \frac{(2-\lambda)b^2 + 2b}{2[3\lambda b^2 + 4(1+\lambda)b + 4](2(b+b\lambda+1))}a > 0 \quad (A.1)$$

Thus,

$$Q_2^* - \hat{Q}_2 = \lambda(\alpha_2^* - \hat{\alpha}) > 0 \qquad (A.2)$$

Additionally,

$$\alpha_1^* - \hat{\alpha} = [\frac{b\lambda+2}{3\lambda b^2 + 4(1+\lambda)b + 4} - \frac{1}{2(b+b\lambda+1)}]a = \frac{\lambda(2\lambda-1)b^2 + 2\lambda b}{2[3\lambda b^2 + 4(1+\lambda)b + 4](b+b\lambda+1)}a \qquad (A.3)$$

and

$$Q_1^* - \hat{Q}_1 = \alpha_1^* - \hat{\alpha} \qquad (A.4)$$

Thus, when $\lambda \geq \frac{1}{2}$, for $\forall b > 0$, $\alpha_1^* - \hat{\alpha} = Q_1^* - \hat{Q}_1 > 0$.

2. Proof of proposition 5:

$$\pi_1^* = \frac{(\lambda b+2)^2(b+1)}{[3\lambda b^2 + 4(1+\lambda)b + 4]^2}a^2, \quad \pi_2^* = \frac{\lambda(\lambda b+1)(b+2)^2}{[3\lambda b^2 + 4(1+\lambda)b + 4]^2}a^2 \qquad (A.5)$$

$$\hat{\pi}_1 = \frac{a^2}{4(b+b\lambda+1)}, \quad \hat{\pi}_2 = \frac{\lambda a^2}{4(b+b\lambda+1)}$$

Therefore, for $\lambda \in (0, 1]$

$$\hat{\pi}_1 - \pi_1^* = \frac{\lambda^2(5-4\lambda)b^4 + 4\lambda(2-\lambda^2)b^3 + 4\lambda(2-\lambda)b^2}{4(b+b\lambda+1)[3\lambda b^2 + 4(1+\lambda)b + 4]^2}a^2 > 0 \qquad (A.6)$$

Consider D_2,

$$\hat{\pi}_2 - \pi_2^* = \frac{\lambda(5\lambda-4)b^4 + 4(2\lambda^2-1)b^3 + 4(2\lambda-1)b^2}{4(b+b\lambda+1)[3\lambda b^2 + 4(1+\lambda)b + 4]^2}\lambda a^2 \qquad (A.7)$$

thus, when $\lambda \geq 0.8$, for $\forall b > 0$, $\hat{\pi}_2 - \pi_2^* > 0$.

3. Proof of proposition 6:

$$\pi_U^* - \hat{\pi}_U = \frac{1}{2}(\alpha_1^{*2} + \alpha_2^{*2} - \hat{\alpha}^2) = \frac{1}{2}\frac{A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E}{4(b+b\lambda+1)[3\lambda b^2 + 4(1+\lambda)b + 4]^2} \qquad (A.8)$$

where,

$$A = 4b^4, \quad B = 8b^3(b+3), \quad C = b^2(-b^2+32b+52), \quad D = 8b(b^3+4b^2+9b+6), \quad E = (b+1)^2[(b+2)^2-12]$$

It is only possible for C, E to be negative, while for most cases that b is not too small or too large, $\pi_U^* - \hat{\pi}_U > 0$, especially for a relatively large λ.

4. Proof of proposition 7:

$$\hat{Q} - \widetilde{Q} = \frac{1+\lambda}{2(b+b\lambda+1)}a - \frac{a}{2(b+1)} = \frac{\lambda a}{2(b+b\lambda+1)(b+1)} > 0 \tag{A.9}$$

Since it has been proved that $Q^* > \hat{Q}$, now we have $Q^* > \hat{Q} > \widetilde{Q}$. Consequently, $p^* < \hat{p} < \widetilde{p}$.

5. Proof of proposition 8:

$$\pi_U^* - \widetilde{\pi}_U = \frac{1}{2}(\alpha_1^{*2} + \alpha_2^{*2} - \widetilde{\alpha}^2) = K[-b^2(5b+6)(b+2)\lambda^2 - 8b(b+1)(b+2)\lambda + 4(b+1)^2(b+2)^2] \tag{A.10}$$

where K is an item always positive. Consider $y = -b^2(5b+6)(b+2)\lambda^2 - 8b(b+1)(b+2)\lambda + 4(b+1)^2(b+2)^2$, it is a quadratic function of λ with a negative coefficient of the quadratic term and its axis of symmetry locates at a negative λ. When $\lambda = 0, y > 0$, and for $\lambda > 0$, y decreases in λ. Thus consider $\lambda = 1$, $y = -b^4 + 16b^2 + 32b + 16$. The following figures indicate that $\exists\ \underline{b} \approx 4.8$, when $b \in (0, \underline{b}]$, for $\forall \lambda \in (0, 1)$, $\pi_U^* - \widetilde{\pi}_U > 0$.

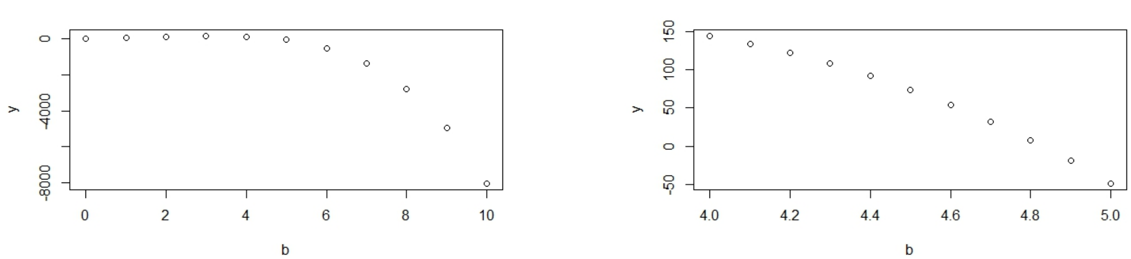

Figure 5: Locating the critical value $\underline{b}$

6. Proof of proposition 9:

$$\alpha_1^* - \widetilde{\alpha} = \frac{-\lambda b^2 - 2\lambda b}{2(b+1)[3\lambda b^2 + 4(\lambda+1)b + 4]}a < 0 \tag{A.11}$$

$$\alpha_2^* - \widetilde{\alpha} = \frac{(2-3\lambda)b^2 + (2-4\lambda)b}{2(b+1)[3\lambda b^2 + 4(\lambda+1)b + 4]}a$$

Thus, if $\lambda \leq \frac{1}{2}, \widetilde{\alpha} < \alpha_2^*$ for all $b > 0$; if $\lambda \geq \frac{2}{3}, \widetilde{\alpha} > \alpha_2^*$ for all $b > 0$.