The Role of Conflict Diamond Sanctions in Civil War Resolution

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Abstract

This paper models post-civil war conflict recurrence in Africa as a two-player sequential game. I treat the two "players" in my model, an incumbent government and a rebel group, as profit-maximizing firms who must each allocate a fixed supply of labor between diamond production and armed warfare. I then analyze the impact of conflict diamond sanctions on the players' optimal labor allocations and on the likelihood that the Rebel will choose to demobilize after a civil war rather than return to armed conflict. I find that the minimum level of sanction needed to achieve demobilization is larger when the world price of diamonds is higher, and when the Rebel controls a smaller proportion of the country's labor resources. The results of this study could inform policymakers about the value of diamond sanctions as a preventive tool against post-war conflict recurrence, as well as the most cost effective sanction that a mediator could impose given a certain set of circumstances.

I: Introduction

In the past three decades, several regions in west and central sub-Saharan Africa have suffered from an epidemic of costly civil wars. In most cases, these conflicts began when one or more armed rebel factions attempted to seize political power from an incumbent one-party government. Power struggle wars tended to last longer and have especially devastating consequences in countries naturally endowed with abundant deposits of rough diamonds. Perpetually in high demand from retailers and industrialists alike, diamonds were valuable wartime assets because rebel groups could easily extract, transport and sell them at high prices on the international market.

The sale of African “conflict diamonds” not only bankrolled the military activities of many rebel groups and governments, but also generated considerable profits for both groups, particularly for rebel factions who did not have access to legitimate sources of revenue. In the 1990s, conflict diamonds were an especially important source of income for rebels in Angola, Sierra Leone, and the Democratic Republic of the Congo. In 1999, the National Union for the Total Independence of Angola (UNITA) rebel group earned an estimated $150 million of Angola’s total $600 million worth of diamond production (Tamm 2002).

In many cases, a profitable trade in conflict diamonds prolonged warfare by giving rebel groups an economic incentive to renege on peace agreements and return to war. In 1994, UNITA and Angolan government leaders signed the Lusaka Protocol calling for a reconciliation between UNITA and the Government of Unity and National Reconciliation (GURN), the internationally recognized government of

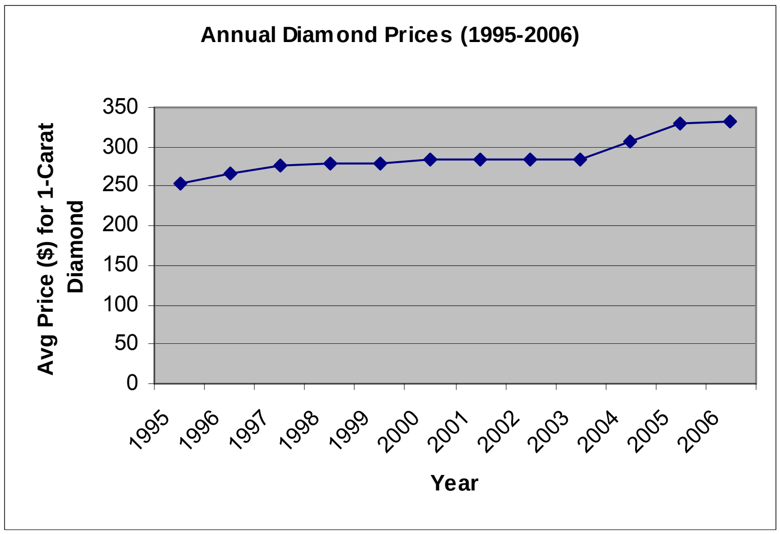
Angola, and for the demobilization of all rebel troops (United Nations [UN], 1994). However, UNITA, who by 1999 had captured the great majority of the country's diamond mines, failed to demobilize in compliance with the terms of the Protocol and returned to war within a few months of signing the agreement (Tamm 2002).

In 1999, Revolutionary United Front (RUF) leaders and the government of Sierra Leone reached a similar agreement under the Lomé Peace Accords requiring the rebels, who at that point controlled most of Sierra Leone's diamond territories, to end hostilities, demobilize and become inaugurated as a legitimate political party. The Lomé Accords also proposed a generous peace settlement that promised RUF leaders prominent positions in the postwar government and a share in the country's legitimate diamond industry. Despite these concessions, RUF failed to demobilize in accordance with the terms of the settlement and returned to armed conflict within a year of the agreement (Tamm 2002).

In recent years, the United Nations Security Council (UNSC) and the international diamond trading community have made a cooperative effort to stem ongoing conflict in Africa by issuing sanctions against the sale of conflict diamonds in the international market. UNSC diamond sanctions prohibited all UN member states from importing African diamonds that did not carry a special certificate of origin issued by a government that the UN recognized as "legitimate." The objective of these sanctions was to keep diamonds originating from rebel-controlled territories from entering the mainstream international market (Tamm 2002).

Diamond sanctions first emerged as a policy instrument in African civil conflict resolution in 1998 when the UNSC passed Resolution 1173 banning the import of diamonds that had been mined in territories controlled by the UNITA rebel group. UNSC Resolution 1173 prohibited member states from importing diamonds that were not certified by GURN, the legitimate postwar government established in 1997 under a previous UNSC Resolution (United Nations Security Council [UNSC], 1998). In 2000, the UNSC issued Resolution 1306, which imposed similar sanctions against diamonds mined by the RUF rebel group in Sierra Leone. This Resolution called for the Government of Sierra Leone to cooperate with the Diamond High Council in Western diamond importing countries to certify and electronically confirm all diamond exports leaving Sierra Leone and entering these countries (UNSC 2000).

Initially, these and other UNSC sanctions had little effect in mitigating the conflict diamond trade because of non-compliance problems on the part of diamond importing member states. After much pressure from NGOs such as Global Witness, however, the international diamond monopoly DeBeers agreed to cooperate with the United Nations and with African government leaders to devise a more effective diamond monitoring system. In December 2000, the United Nations General Assembly passed a resolution calling for the establishment of an international certification scheme to enforce UNSC diamond sanctions (Kimberley Process Background, 2003, para. 2). In 2003, the Kimberley Process Certification Scheme (KPCS) was created to serve this purpose. The KPCS required officials in participating diamond exporting countries to attach a forgery-proof certificate of origin to each “legitimate” shipment of diamonds leaving the country, and for officials in importing countries to verify the shipment upon arrival (Kimberley Process Background, 2003, para. 6). According to KPCS estimates, the percentage of conflict diamonds in the world market has dropped from 15% in the 1990s to less than 1% in 2006 (Kimberley Process Background, 2003, para. 6). As shown in the chart below, world diamond prices have been steadily increasing since 2003, suggesting that the supply of illicit diamonds entering the world market has been shrinking.



(*Data taken from the Diamond High Council of Antwerp's 2005-2006 Diamond Price Index, available at www.diamondregistry.com)

Although the Kimberley Process has been successful in reducing the supply of rebel-mined conflict diamonds in the world market, the KPCS did not come into being until after 2003, when most of the recent diamond-fueled civil wars in Africa had already come to a permanent end; the civil wars in Angola and Sierra Leone were officially resolved in 2002. It is therefore unclear whether these KPCS-enforced sanctions would have played a significant role in conflict resolution had they taken effect immediately following the signing of peace agreements in the 1990s. More specifically, it is not clear whether these sanctions, if effectively enforced, would have given rebels enough of an economic incentive to demobilize and commit to the peace process at that time.

In this paper, I use a game theoretical model to determine the impact of diamond sanctions on a Rebel's decision to demobilize or to return to war following a major civil conflict, had these sanctions been imposed as a punishment against non-demobilization. I also analyze the effects of various exogenous parameters, such as the size of the country's available labor supply, the size of the rebel group, and the world price of diamonds on the Rebel's optimal allocation of labor towards conflict and the minimum level of sanctions needed to achieve demobilization. I also define a parameter range on which sanctions have an impact on the Rebel's demobilization decision. The conclusions drawn from this study could inform future policymakers about the relevance of diamond sanctions as a preventive tool in avoiding post-war conflict recurrence, and the cost effectiveness of different levels of sanctions given a certain set of circumstances. If UNSC diamond sanctions could successfully motivate rebel groups to demobilize in compliance with the terms of peace agreements, they would greatly reduce the duration, and thus the economic and humanitarian costs, of diamond-related civil wars in Africa.

There is a wealth of existing literature on the causal relationship between natural resources and civil war (e.g. Collier and Hoeffler, 2004, Ross, 2004), although a much smaller number of studies address specifically the role of wartime sanctions on natural resource commodities. Studies that do focus on sanctions use game theoretical or empirical models to determine their impact on civil war resolution. Strandow (2006) did an empirical study on the effectiveness of different types of economic sanctions, including commodity sanctions, on the likelihood that warring parties would reach a peace agreement. Addison and Murshed (2002) used game theoretical microeconomic models to measure the impact of sanctions on rebel and government "welfare" and their decisions to cooperate with or to renege on peace agreements. Although Addison and Murshed used utility functions to determine the players' economic gains and losses in the presence of sanctions, they did not explain how sanctions change each group's optimal allocation of labor towards the war effort.

Olsson (2007) used a Stackelberg model to measure the effect of diamond resource abundance on the likelihood of civil war initiation. In Olsson's model, the government owned an exogenously given quantity of diamond rents that the rebel group could capture through its war efforts, and the size of this diamond supply was the main parameter of interest that determined players' optimal allocations of labor between formal sector (agricultural) production and armed warfare. The model I use in this paper is similar to Olsson's model. However, rather than fixing a country's diamond supply as an exogenous parameter, I examine how diamond sanctions change the optimal quantities of labor that players allocate between diamond production and armed warfare. In my model, the amount of diamond revenue that players can generate during war is an endogenous parameter determined by the size of sanctions and of the quantity of labor that they allocate to diamond production. I also focus on the effect of diamond sanctions on civil war resolution, rather than initiation; therefore my model also takes into account the impact of sanctions and of labor allocations on the probabilities of different military outcomes should the rebel group choose to return to war.

The paper is organized into four subsequent sections. In the first section, I discuss in more detail the relevant literature surrounding my topic and the contributions that this paper makes to the existing literature. In the second section, I describe the theoretical framework and mathematical methodology I use, and perform the relevant optimization and comparative statics analyses. In the third section, I summarize my results from the previous section. In the fourth and final section, I discuss the conclusions drawn from my study and their implications for sanction policy.

II: Literature Review

The link between natural resource abundance and the incidence of civil war in the developing world has been widely researched by economists at international development organizations such as the World Bank and the International Monetary Fund. In an empirical study conducted in 2004, World Bank researchers Paul Collier and Anke Hoeffler found that the incidence and duration of civil war in developing countries are more highly correlated with the economic characteristics of the region, such as national per capita income, population and natural resource abundance, than with non-economic measures such as ethno-linguistic fractionalization.

Using the Small-Singer (1982) data set of civil conflicts that occurred over the period 1816-1992, Collier and Hoeffler ran a probit regression of all economic and non-economic independent variables on the likelihood of civil war occurrence, and a tobit regression of these same independent variables on civil war duration. They concluded from this study that natural resource abundance initially increased with both civil war incidence and duration, but that both of these war measures decreased as diamond abundance increased beyond a certain point. However, the overall correlation between resource abundance and the probability of civil war was positive and strongly significant, with the exception of countries with extremely large resource endowments. The results of Collier and Hoeffler's study emphasized the importance of economic incentives and of the availability of economic resources in civil war occurrence and duration.

The literature survey conducted by UCLA political scientist Michael Ross (2004) discussed Collier and Hoeffler's findings in the context of other cross-country econometric studies that have been done on the relationship between natural resources and civil war. According to Ross's survey findings, studies have collectively shown that "lootable" natural resource commodities, such as diamonds, gemstones, and drugs, have no significant correlation with civil war initiation but do have a significant positive correlation with civil war duration. Ross compared this result to the collective findings for legal agricultural commodities, which in most studies had no significant correlation with any aspect of civil war. Ross's findings suggested that, although civil wars were initiated for a variety of reasons, the presence of highly valued commodities with low transportation costs, such as diamonds, gave rebels a strong profit incentive to prolong conflict.

Tony Addison and Mansoob Murshed (2002) used a game theoretical model to specifically analyze the role of the profit incentive in civil war duration and to study the impact of conflict commodity sanctions on parties' incentives to renege on postwar peace agreements. Addison and Murshed's study focused on the negative effect of sanctions on the attractiveness of conflict for the parties involved, namely, the profits that a warring government or rebel group could potentially gain from warfare. Addison and Murshed used utility functions to represent a party's preferences for capturing rents through warfare as opposed to other sources of income, and found that the equilibrium level of conflict chosen by a warring party is a direct increasing function of the size of capturable rents and of the party's "greed", its innate preference for wartime rent seeking as opposed to peaceful sources of income. Addison and Murshed conclude from this analysis that sanctions reduce a party's incentive to return to war by reducing its greed, as denoted in their model by the single variable "c". This sanctions parameter "c" determines the slope of the party's indifference curve between wartime revenue and other income. When the size of sanctions increases, the party's preference for capturing rents through warfare decreases, and it is more willing to substitute towards other sources of income. Although Addison and Murshed provide a thorough analysis of the impact of sanctions on a party's preferences for warfare, they do not analyze the effect of these sanctions on the party's allocation of labor or other resources towards conflict.

Daniel Strandow (2006) supports Addison and Murshed's theoretical findings in his empirical study of the impact of different types of economic sanctions on civil war resolution. Using time series cross-sectional data on the recent civil conflicts in Liberia and Ivory Coast up until March 2006, Strandow ran a binary logit regression to determine the impact on conflict resolution of three different types of economic sanctions imposed in this time period (arms embargoes, sanctions on commodities such as diamonds and oil, and sanctions on individual finances).

Strandow's independent variable was a dummy coded "0" or "1" to indicate the presence or absence of each type of sanction in a specific timeframe, and his dependent variables were proxies for conflict resolution such as military expenditures, casualties, and incidences of armed conflict that occurred in this timeframe. Strandow found that in at least one regression, commodity sanctions had a small, but statistically significant, positive correlation with conflict resolution. However, the correlation between arms embargoes and conflict resolution was much stronger and appeared more consistently in all regressions.

Ola Olsson (2007) used a Stackelberg predator-prey model to measure the effect of diamond abundance on how two players, a potential Rebel and an incumbent Government, chose to allocate labor between formal sector activities and armed warfare. In the first time period, both parties are at peace and devote all their resources to formal sector activities, and the Government is the sole owner of the country's entire exogenously fixed diamond supply. In the second time period, the Rebel must decide whether to keep producing formal sector output peacefully or to initiate a war against the government in order to capture its diamond resources. When the Rebel's optimal profit-maximizing allocation of labor towards warfare is larger than 0, the Rebel initiates a civil war. Olsson concluded from her study that the likelihood of the Rebel initiating conflict in order to capture the Government's diamond resources increased proportionately with the amount of labor the Government put into defending their diamond mines up to a certain threshold. When the Government's defense increased above this threshold, the Rebel's efforts to capture the diamonds decreased and eventually became 0. She also found that the likelihood of civil war initiation is a direct increasing function of the size of a country's diamond resources, since larger available resources give the rebel a greater incentive to initiate conflict.

Although Olsson included in her model a variable "δ" that discounts the value of the country's diamond resources in the case where sanctions are imposed, she did not analyze the effects of this parameter on the Rebel's labor allocation and war initiation decisions. In this paper, I build on Olsson's theoretical framework by analyzing in particular the sanctions variable "δ" and its effect on the total intertemporal utility that a rebel group earns from returning to war. I also study the impact of sanctions on a rebel's optimal labor allocation between warfare and diamond production, rather than formal sector production. While Olsson treats diamond resources as a fixed exogenous quantity to be captured through warfare, diamond revenue in my model is an endogenous, variable quantity that is determined by the amount of labor that players choose to allocate to its production. In my model, sanctions not only discount the value of the rebel group's diamond production, but also reduce the quantity of diamonds produced by raising the opportunity costs of diamond production. Since I focus on the role of sanctions in conflict resolution rather than initiation, I also extend Olsson's model to include three time periods, so as to take into account the probabilities of different military outcomes should the rebel group choose to return to war. I hope to contribute to the existing economic literature on conflict commodity sanctions by examining the specific causal mechanisms through which sanctions affect a rebel group's optimal resource allocation between commodity production and military activities, and, consequently, their impact on the group's decision to demobilize or to return to war following a major civil conflict.

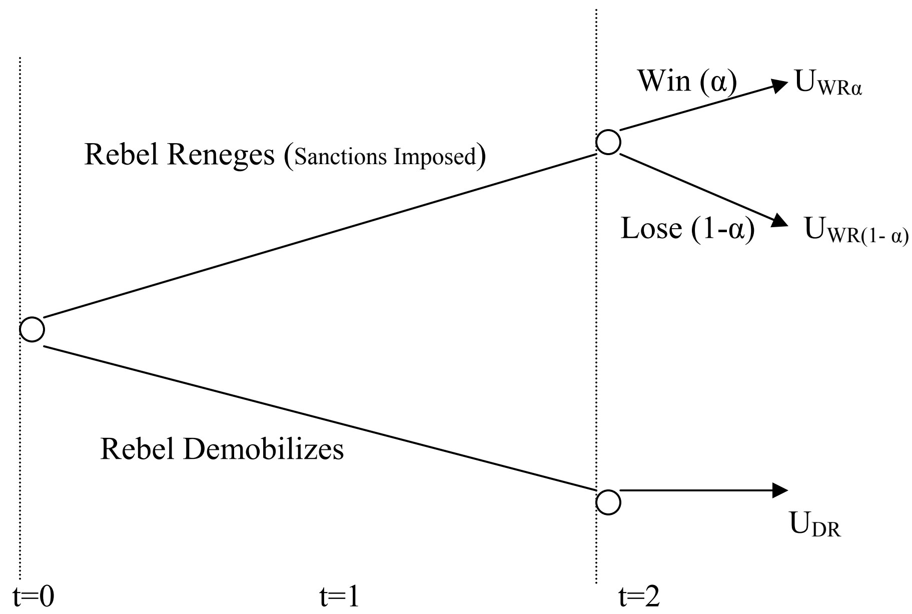
III: Theoretical Framework

Overview

I model the impact of sanctions as a two-player sequential move game between a rebel group and an incumbent government. This game takes place over three time periods, which I denote in my model as $t = 0, 1, 2$. At $t = 0$, the country is in a temporary period of peace following a major civil conflict. At this time, a third party mediator, such as the UN, announces a peace settlement, which includes the threat to impose sanctions on the Rebel's diamond production at time $t = 1$ should he choose to renege on the settlement and return to war; no sanctions are imposed if he chooses to demobilize. At time $t = 1$, the Rebel chooses to either renege and be subject to sanctions, or to demobilize, become part of the legitimate postwar government and share control of the country's diamond resources with the incumbent party.

If the rebel chooses to renege, then its efforts from time $t = 1$ will result in one of two outcomes at time $t = 2$: a military victory, in which case the Rebel becomes the country's legitimate one-party government, sanctions against him are lifted and he gains absolute control of the country's entire diamond resources, or a military defeat, in which the Rebel group is simply disbanded and gains zero utility from returning to war in the previous time period. Rebel's possible choices at time $t = 1$, and the possible outcomes of those choices at time $t = 2$, are outlined as follows:

Figure 1: Rebel's Possible Choices and Outcomes in Time Periods t = 0, 1, 2



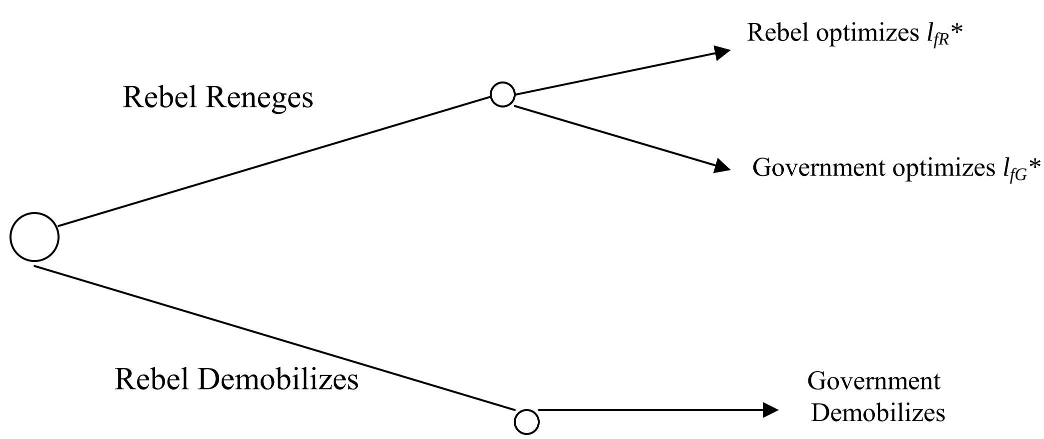
The parameter α denotes the probability, a value between 0 and 1, that the Rebel will win a military victory if he chooses to renege and return to war, and (α-1) expresses the probability he will lose. $U_{WR\alpha}$ is the total intertemporal utility that the Rebel receives from returning to war if he wins, $U_{WR(1-\alpha)}$ is his total intertemporal utility if he loses, and U_{DR} is his intertemporal utility if he chooses to demobilize.

The two "players" in the post-war period, the Rebel and the Government, each control a proportion of the country's total labor supply, $\bar{L}$, at the time of the peace settlement. At time t = 1, the Rebel and the Government each have a fixed labor endowment L_R and L_G, respectively, such that $L_R + L_G = \bar{L}$. In this time period, each player must decide how to allocate its labor between diamond production (l_d) and armed warfare (l_f), and in any situation each player always chooses the labor allocation that maximizes his total intertemporal utility.

In my model, the Rebel first decides whether to renege on the settlement and return to war, or to accept the settlement and demobilize. If the Rebel chooses to demobilize, then the Government, whom I assume always prefers peace to war, responds to the Rebel's choice by also demobilizing. If the Rebel decides to renege, however, the Rebel and the Government simultaneously choose the utility-maximizing quantities of labor that each allocates towards warfare in a Cournot duopoly game. Because the likelihood of a Rebel victory is a function of the quantity of labor that each player devotes to warfare, the Rebel's optimization problem depends on the optimal amount of labor that the Government allocates towards military activities (l_{fG}*) as well as his own optimal warfare allocation (l_{fR}*).

The subgame perfect outcome can be determined through backward induction. Given the equilibrium allocations l_{fG}* and l_{fR}*, I determine the maximum intertemporal utility that the Rebel earns from returning to war. If the Rebel's total expected utility from warfare exceeds his utility from demobilizing, he chooses to return to war. The two-stage Cournot game can be illustrated as follows:

Figure 2: Two-Stage Cournot Game



The Rebel's utility-maximizing allocation of labor towards warfare depends not only on the size of the Government's and his own war efforts, but also on a variety of exogenous factors such as the price of rough diamonds on the legitimate world market, the size of the rebel group (i.e. the amount of labor it has access to in the first time period), the size of the country's total labor supply to be won in the second time period and the costs of warfare. Since I assume that labor is the only input needed for both diamond production and warfare, the resource to be captured in this game is actually human labor. By winning a victory in time $t = 2$, the Rebel would have access to the country's entire labor supply $\bar{L}$, which could all be used to produce diamonds. Thus, labor to be captured in time $t = 2$ is what gives the Rebel an incentive to return to war. In my model, I assume for the sake of simplicity that the size of the country's total labor force stays constant over time and is unaffected by changes in the country's population.

When international sanctions are imposed on diamonds originating from rebel-controlled territory, the main effect of these sanctions is to raise the transaction costs of trading diamonds for the rebel group. In order to sell his diamonds, the Rebel must use costly methods to smuggle them into the legitimate world market, such as bribing officials, buying fake certificates and exporting the diamonds to be sold in neighboring countries (as was the case with RUF, who exported most of their illicit diamonds to be sold in Liberia during the civil war in Sierra Leone). As a result, the rebels must sell their diamonds at a discounted price below the world market price in order to maintain their supply.

By discounting the value of the Rebel's diamond production, sanctions have a dual effect on his total expected intertemporal utility from returning to war. The immediate effect of sanctions in time $t = 1$ is to reduce the total revenue that the Rebel could earn from producing diamonds, thus decreasing his total wartime utility. However, by discounting the price of the Rebel's diamonds, sanctions also raise the opportunity costs of diamond production in terms of warfare. Consequently, the imposition of sanctions also results in a substitution effect where the Rebel allocates less of his labor towards diamond production and more towards warfare. Increasing the Rebel's optimal labor allocation towards warfare also raises the probability of a Rebel victory in time $t = 2$, thus increasing the likelihood that the Rebel will win control of the country's entire diamond resources if he chooses to return to war. Therefore, sanctions could also increase Rebel's total expected intertemporal utility from returning to war. While they reduce the Rebel's immediate diamond profits in the present, they also raise the expected value of diamond production in the future. A variety of other exogenous parameters, such as the world price of diamonds and the size of the country's labor force, determine whether the present discount effect dominates the increase in the future probability of victory, and thus the overall impact of sanctions on the Rebel's total wartime utility.

Some basic assumptions of my model are that both players have perfect information about the future imposition and size of sanctions, the probability of military outcomes, and the payoffs from choosing each option, and that any level of sanction imposed by a third party mediator is effectively enforced to achieve the desired discount effect. I also assume that the rebel has no time preferences, and is indifferent between immediate consumption (as represented by diamond revenue in time $t = 1$) and future consumption (as represented by the expected utility to be earned at time $t = 2$).

Utility Functions and Production Constraints

In my model, I assume that the utility that a rebel group derives from reneging or demobilizing consists only of the total profit (total revenue-total costs) that it can obtain from either option, and that he has perfect information about military probabilities and the size of sanctions. The rebel's intertemporal utility functions can be written as such:

$$E(U_{WR}) = U_0 + U_{1W} + \alpha U_{2W\alpha} + (1-\alpha)U_{2W(1-\alpha)} \qquad (1)$$

$$U_{DR} = U_0 + U_{D1} + U_{D2} \qquad (2)$$

In Equation (1), the total expected utility that the rebel receives from returning to war is equal to the sum of the utility that he receives in the first time period (U_0), the utility he receives from returning to war in the second time period (U_{1W}), and the utility he receives from each of the two possible military outcomes in the third period multiplied by their respective probabilities of occurring ($\alpha U_{2W\alpha} + (1-\alpha)U_{2W(1-\alpha)}$). Equation (2) represents the sum of the utility that the rebel receives from each time period if he chooses to demobilize.

From here on out, I omit the far left-hand side of Equation (1) and (2) because at time $t = 0$ the rebel receives exactly the same utility whether he chooses to renege or to demobilize at time $t = 1$. Since my objective is to compare the rebel's total intertemporal utility from returning to war to his utility from demobilizing, the parts of Equations (1) and (2) that concern utility earned in time $t = 0$ are irrelevant to my analysis. Keeping this in mind, Equations (1) and (2) can be rewritten as such:

$$E(U_{WR}) = [pq_{1W} - wL_R] + [\alpha (pq_2 - w\bar{L})] + [(1-\alpha)(0)] \qquad (3)$$

$$U_{DR} = 2 (p\, q_{1D} - wL_R) \qquad (4)$$

Equation (3) expresses the Rebel's expected intertemporal utility as the total revenue he earns from diamond production (pq_t) minus the costs of production (wL_R) in each time period. The parameter p represents the world price of diamonds, which I assume remains constant over time periods; q_t represents the quantity of diamonds produced in each time period t, and w is the marginal cost of each unit of labor that the rebel group employs for either purpose (diamond production or warfare) in each time period. Costs of labor could include maintaining the rebel army, supplying workers with arms or tools and providing for their immediate consumption. The parameter w could also be thought of as a market "wage", although in reality rebel group members are not paid formal wages for their activities during war. If the Rebel chooses to return to war, then at time $t = 1$, the Rebel has access to a fixed supply of labor L_R and produces q_{1W} diamonds. If he chooses to demobilize, then at time $t = 1$, he has the same supply of labor L_R and produces q_{1D} diamonds.

In this time period, sanctions are imposed on the rebel's diamond supply and his diamond revenue is discounted as a result. The parameter δ in my model is a value between 0 and 1 that measures the size of this price discount, which reflects the effectiveness of the sanction in reducing the value of the Rebel's diamond supply. More specifically, the variable δ represents the fraction of the world diamond price that the Rebel receives in the presence of sanctions. A smaller value of δ corresponds to a "larger" sanction, while a larger value of δ is a "smaller" sanction. The parameter δ equals 0 in the case of a perfect sanction under which the Rebel receives 0% of the world diamond price; δ equals 1 in the case where no sanctions are imposed.

In time t = 2, the rebel's efforts in the previous time period could result in a military victory (α), in which case he gains access to the country's entire labor supply ($\bar{L}$), in which case his utility is the diamond revenue he earns if he uses the entire labor supply $\bar{L}$ to produce diamonds, minus the total amount of wages ($w\bar{L}$) that is paid out to the labor force. At time t = 2, the Rebel could also suffer a military loss (1-α), in which case the rebel group is disbanded and receives an utility of 0. Equation (4) represents the rebel's intertemporal utility from demobilizing. I assume that, under the terms of the peace agreement made at time t = 0, the Rebel keeps his entire first period labor supply if he chooses to demobilize. Equation (4) therefore expresses the Rebel's intertemporal utility from demobilizing as twice the diamond revenue that he earns in the first time period, using his first period labor supply L_R . Under the resource-sharing peace deal, the Rebel receives the same utility in time t =2 as he does at time t = 1 in the event that he chooses to demobilize.

In all cases, the Rebel's utility is subject to the following constraints:

$$q_{tW} = A\ l_{dtW}, \quad q_{tD} = A\ l_{dtD} \qquad (5)$$

$$l_{d1W} + l_{fR} = L_R\ , \quad l_{dtD} = L_R \qquad (6)$$

The constraints in Equation (5) are two simple linear production functions. The first expresses the rebel's diamond output q_{tW} in time period t as a function of diamond sector productivity (A) and the amount of labor allocated towards diamond production in time period t (l_{dt}) in the case that he chooses to return to war, and the second expresses his diamond output q_{tD} in time period t as a function of the amount of labor allocated towards diamond production in time period t (l_{dtD}), in the case that he chooses to demobilize. When the Rebel chooses to renege, at time t =1, his production is also constrained by the total fixed labor supply L_R. The first equation in (6) shows this constraint as a sum of the Rebel's two possible labor allocations in each time period, l_{d1W} and l_{fR}. In the case of demobilization, in both time periods the Rebel's entire labor supply L_R is allocated to diamond production ($L_R = l_{d1D}$ and $L_R = l_{d2D}$), as shown in the second equation in (6).

If the rebel chooses to renege in time t = 1, the probability of winning a military victory in time t = 2 is a direct function of the amount of labor he allocates to warfare in time t = 1. This function can be written as follows:

$$\alpha = \frac{l_{fR}}{l_{fR} + l_{fG}} \qquad (7)$$

where l_{fG} represents the amount of labor that the government will allocate towards warfare in time t = 1 to defend itself from the rebel's military actions. Inserting the constraints in Equations (5), (6), and (7) into Equation (3), I can rewrite the rebel's utility functions as follows:

$$E(U_{WR}) = [A\,\delta\, p\,(L_R - l_{fR}) - w(L_R)] + \left[\frac{l_{fR}}{l_{fR} + l_{fG}}(Ap - w)(\bar{L})\right] \qquad (8)$$

$$U_D = 2(Ap - w)(L_R) \qquad (9)$$

First Order Conditions

I now maximize the Rebel's total expected utility ($E(U_R)$) in order to derive first order conditions for l_{fR}*, his optimal allocation of labor towards warfare. Adding together the utility functions in Equations (8) and (9) results in the following:

$$E(U_R) = E(U_{WR}) + U_{DR} = [A\,\delta\, p\,(L_R - l_{fR}) - wL_R] + \left[\frac{l_{f1}}{l_{f1} + l_{f1G}}(Ap - w)\,(\bar{L})\ \right]$$

$$+\ 2(Ap - w)(L_R) \qquad (10)$$

Taking first order derivatives of (10) with respect to l_f results in the following first order conditions:

$$\frac{\partial E(U_R)}{\partial l_{fR}} = \frac{l_{fG}}{(l_{fR} + l_{fG})^2}\ (Ap - w)(\bar{L}) - A\,\delta\, p = 0$$

$$l_{fR}^* = \sqrt{\frac{l_{fG}(Ap - w)(\bar{L}))}{A\delta p}} - l_{fG} \qquad \text{if } \frac{\partial E(U_R)}{\partial l_{fR}} = 0,\ \ l_{fR}^* \,\square\, [0, L_R] \qquad (11)$$

$$l_{fR}^* = L_1 \qquad \text{if } \frac{\partial E(U_R)}{\partial l_{fR}} > 0,\ \ l_{fR}^* \,\square\, [0, L_R] \qquad (12)$$

$$l_{fR}^* = 0 \qquad \text{if } \frac{\partial E(U_R)}{\partial l_{fR}} < 0,\ \ l_{fR}^* \,\square\, [0, L_R] \qquad (13)$$

Equation (11) defines an interior local maximum on the interval $[0, L_R]$, for which the Rebel's utility-maximizing warfare l_{fR}* is expressed by the first order condition above. Equation (12) represents an upper boundary solution in which the rebel's utility does not reach a local maximum on the interval l_{fR}^* □ $[0,L_R]$ but is rather increasing everywhere on this interval. In this case, the Rebel allocates his entire first period labor supply to warfare because the opportunity cost of diamond production is too high. Equation (13) represents a lower boundary solution in which the Rebel's utility decreases everywhere on the interval l_{fR}^* □ $[0,L_R]$. In this case, the Rebel allocates no labor towards warfare and instead devotes his entire first period labor supply towards diamond production, because the opportunity cost of warfare is too high.

According to the first order condition expressed in Equation (11), l_{fR}^* is a negative convex function of δ. As δ increases, l_{fR}^* decreases at an increasing marginal rate (i.e. for smaller values of δ, an increase in δ causes a smaller decrease in l_{fR}^* than it does for larger values of δ). This relationship makes intuitive sense. As the value of δ increases (i.e. sanctions cause a smaller price discount), the opportunity cost of allocating labor towards warfare, in terms of diamond production, also increases. Therefore, as δ increases the Rebel allocates less of his first period labor endowment towards warfare. When δ is closer to 1, an increase in δ results in a larger increase in the opportunity cost of warfare in terms of diamond production, and consequently the optimal amount of labor that the rebel allocates towards warfare decreases by a larger amount.

The first order condition in (11) also indicates a positive relationship between p and l_{fR}^*. As the world price of diamonds increases, the profits to be made from diamond production in the third time period also increase, providing a greater incentive for the rebel to return to war in an effort to capture these profits. The size of the country's total labor supply $\bar{L}$ also shares a positive relationship with l_{fR}*. As $\bar{L}$ (i.e. the amount of labor that the rebel would have access to as a result of a military victory in the third time period) increases, the Rebel's incentive to return to war increases and he allocates more labor towards warfare.

Equation (11) expresses l_{fR}* as a quadratic function of l_{fG}. For small values of l_{fG}, l_{fR}* increases as l_{fG} increases. For large values of l_{fG}, l_{fR}* decreases as l_{fG} increases. When the government army is small, the Rebel's chances of achieving a military victory are large, so an increase in the amount of labor the government puts into warfare will cause the rebel to reciprocate and increase the size of his own army in order to match the strength of the government forces. When the government army is large, the chance of the rebel winning a military victory is small. Consequently, the rebel will decrease the size of its army when government strength increases, since it is not productive for the rebel to continue allocating more labor towards warfare when his probability of victory is small.

The condition expressed in (11) only holds true when $l_{fR}* \leq L_R$, the total fixed supply of labor that the Rebel has access to in the first time period when he makes his allocative choices. In the case where $l_{fR}* > L_R$, we arrive at the upper boundary solution expressed in Equation (12). Setting l_{fR}* in the first order condition in Equation (11) equal to L_R and rearranging the equation in terms of δ results in the following:

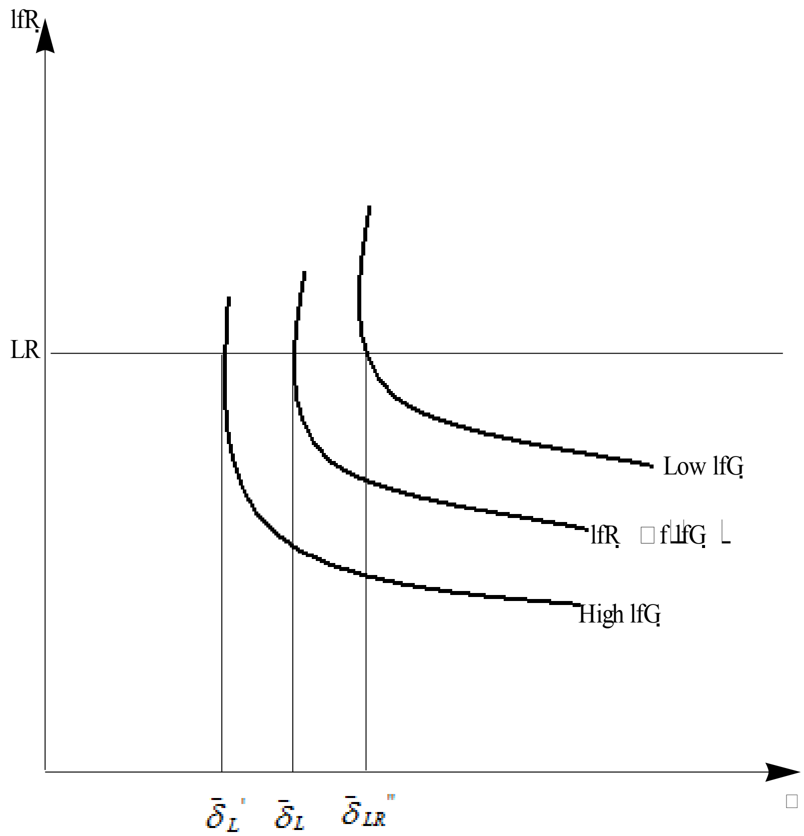
$$\bar{\delta}_{LR} = \frac{l_{fG}(Ap - w)(\bar{L})}{Ap(L_R + l_{fG})^2} \qquad (14)$$

In Equation (14), $\bar{\delta}_{LR}$ is the minimum value of δ at which the Rebel allocates any labor to diamond production. When δ falls below $\bar{\delta}_{LR}$, we arrive at the upper boundary solution in which the Rebel allocates his entire labor supply L_R towards warfare.

Equation (14) also expresses $\bar{\delta}_{LR}$ as a quadratic function of l_{fG}. For larger values of l_{fG}, $\bar{\delta}_{LR}$ increases when l_{fG} increases, whereas for larger values of l_{fG}, $\bar{\delta}_{LR}$ decreases as l_{fG} increases. This observation reflects the relationship found between l_{fG} and l_{fR} in Equation (11). Since the Rebel allocates more labor towards warfare when Government strength increases in the case where the Government army is small, the minimum threshold value of δ at which the Rebel allocates any labor towards diamond production increases in this case. In other words, when the Government army is small, sanctions do have to be as large in order for the Rebel to allocate his entire labor supply to warfare, since in this case he naturally increases his war efforts in response to an increase in Government strength.

In the case where the Government army is large, the Rebel decreases his warfare allocation when Government strength increases. In this case, the minimum value of δ at which the Rebel allocates any labor towards diamond production is smaller.

Figure 3: The Effects of an Increase in l_{fG} on $\bar{\delta}_L$



The curve in Figure (3) represents $l_{fR}*$ as a function of δ. The horizontal line labeled L_R marks the Rebel's total labor supply and intersects the curve at $\bar{\delta}_{LR}$. When l_{fG} is high, an increase in l_{fG} causes the curve to shift down and outwards so that $\bar{\delta}_{LR}$ decreases to $\bar{\delta}_L{}'$. When l_{fG} is low, an increase in l_{fG} causes the curve to shift up and inwards so that $\bar{\delta}_{LR}$ increases to $\bar{\delta}_{LR}{}''$.

When δ hits a maximum value $\hat{\delta}_{LR}$, the rebel allocates his entire second period labor supply to diamond production and $l_{fR}* = 0$. Setting $l_{fR}*$ in the first order condition in Equation (11) equal to 0 and rearranging the equation in terms of δ results in the following:

$$\hat{\delta}_{LR} = \frac{(Ap - w)(\bar{L})}{Ap(l_{fG})} \qquad (15)$$

When δ is larger than the maximum threshold value expressed in (15), the rebel automatically demobilizes since he allocates no labor towards warfare in such a case.

Inserting the first order condition in Equation (13) for l_{f1} in (12), I can write the rebel's equilibrium utility function of reneging as follows:

$$E(U_{WR})^* = A\,\delta\, p\,(L_R - \sqrt{\frac{l_{fG}(Ap - w)(\bar{L})}{A\delta p}} + l_{fG}) - wL_R + [(1 - \frac{l_{fG}}{\sqrt{\frac{l_{fG}(Ap - w)(\bar{L})}{A\delta p}}})(Ap - w)(\bar{L})] \qquad (16)$$

I can rewrite the utility function of demobilizing in Equation (4) as follows:

$$U_{DR}^* = 2(Ap - w)(L_R) \qquad (17)$$

Equation (15) expresses $E(U_{WR})^*$ as a quadratic function of δ. $E(U_{WR})^*$ decreases as δ increases for small values of δ, and increases as δ increases for large values of δ. We observe this parabolic relationship because δ has a dual effect on the rebel's intertemporal utility from returning to war. A higher value of δ results in a greater reduction in the revenue that rebels could earn from producing diamonds at t = 1. However, a higher value of δ , by reducing the productivity and thereby raising the opportunity cost of producing diamonds, also causes a substitution effect towards warfare at time $t = 1$. As the opportunity cost of diamond production increases, the rebel takes labor out of diamond production and reallocates it towards warfare. When this happens, the rebel's equilibrium warfare allocation $l_{fR}{}^*$ increases, increasing the probability of a military victory at time $t = 2$, which increases the rebel's total expected intertemporal utility. The imposition of price discounting sanctions therefore causes a tradeoff between the utility at time $t = 1$ and the expected utility in time $t = 2$. When δ is small, the diamond price discount effect dominates; when δ is large, the effect of the increasing likelihood of victory overrides the price discount. Setting $E(U_{WR})^* = U_{DR}{}^*$ results in two intercepts: $\bar{\delta}_{UR}$ and $\hat{\delta}_{UR}$. The first intercept expresses the minimum value of δ at which $E(U_{WR})^* \leq U_{DR}{}^*$, while the second expresses the maximum value of δ at which $E(U_{WR})^* \leq U_{DR}{}^*$. Therefore, the Rebel chooses to demobilize when $\delta \in [\bar{\delta}_{UR}, \hat{\delta}_{UR}]$.

A Simulation with Numerical Parameters

I will now examine the case where the parameters in Equation (15) take on the following numerical values:

$$A = 1$$

$$p = 5$$

$$L_R = 10000$$

$$\bar{L} = 30000$$

$$l_{fG} = 3500$$

$$w = 1$$

Inserting these values into Equations (15) and (16), I can rewrite the rebel's equilibrium utility functions as follows:

$$E(U_{WR})^* = -5\delta\sqrt{\frac{420000000}{5\delta+1}} + 67{,}500\,\delta + 110{,}000 - \frac{420000000}{\sqrt{\dfrac{42000000}{5\delta+1}}} \qquad (18)$$

$$U_{DR}^* = 80{,}000 \qquad (19)$$

Inserting different values of δ into Equations (18) and (19), I generate the following table of values:

δ	$E(U_{WR})^*$	U_{DR}^*
0	89506.1	80000
0.1	83283.6	80000
0.2	80025.87	80000
0.3	78404.07	80000
0.4	77839.2	80000
0.5	78023.29	80000
0.6	78771.34	80000
0.7	79962.66	80000
0.8	81513.64	80000
0.9	83363.71	80000
1	85467.4	80000

(Table 1)

Plotting these values on a chart with Utility on the y-axis and δ on the x-axis, I generate the following graphical representation:

Figure 4: Rebel’s Utility as a Function of δ

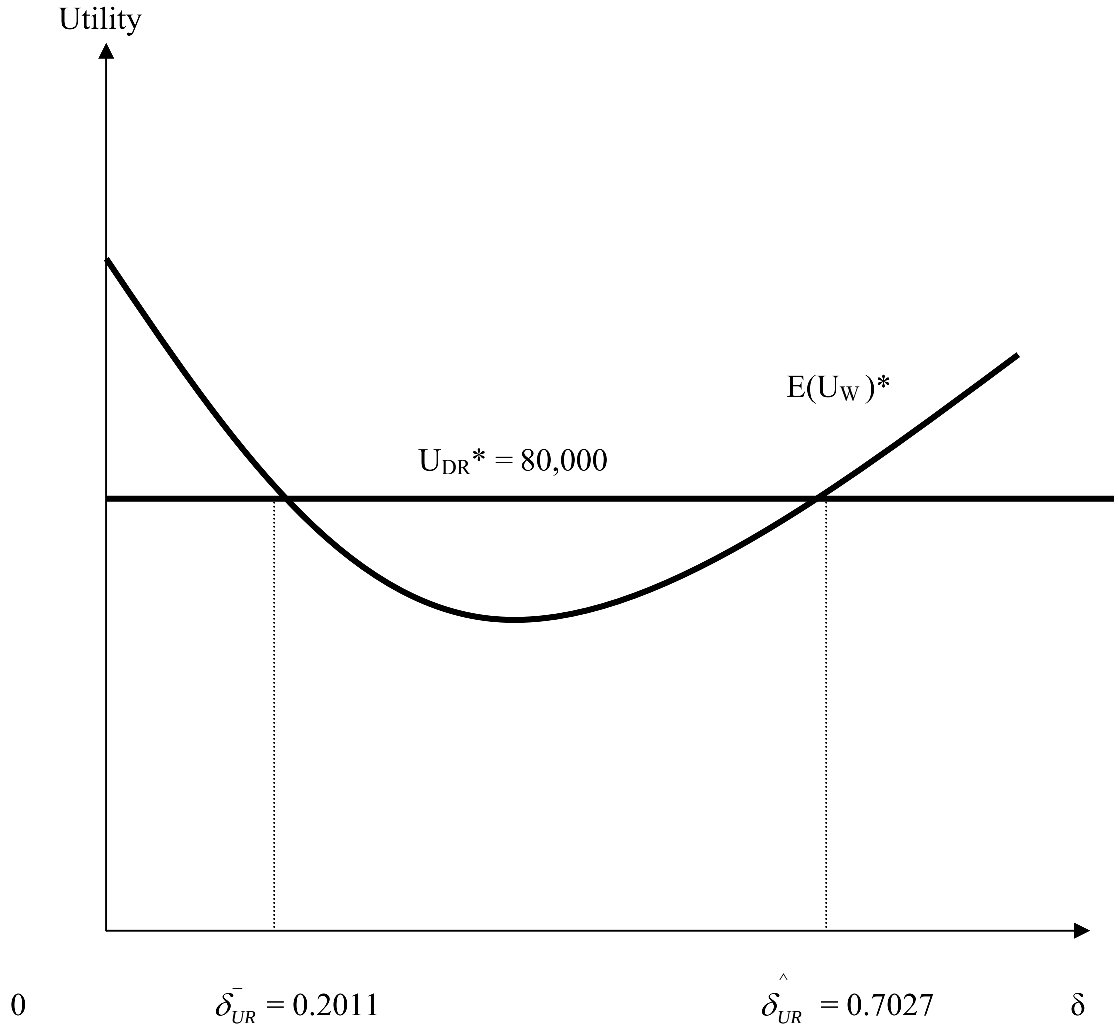


Figure (4) depicts the rebel’s utility as a function of δ[2]. Since sanctions do not affect the utility of demobilizing, U_D* is simply a constant term where U_D* = 80,000 (as calculated from Equation (18)). The utility of reneging, E(U_{WR})*, is a parabola with intercepts at (0.2011, 0.7027); these intercepts are also found by setting Equation

[2] All graphs and charts in this paper are not drawn to scale.

(17) equal to Equation (18) and solving for δ. In this case, $E(U_{WR})^* \leq U_{DR}^*$ on the interval δ □ [0.2011, 0.7027].

Inserting the numerical parameter values into Equation (14), I find that $\bar{\delta}_L =$ 0.2609. The utility function in (17) only applies when $\delta \geq 0.2609$. When δ falls below this minimum threshold, the Rebel's utility function can be written as follows:

$$E(U_{WR})^* = \frac{L_R}{L_R + l_{fG}} (Ap - w)(\bar{L}) - wL_R \qquad (20)$$

Equation (20) represents the utility that the rebel receives from reneging at the upper boundary solution $l_{fR}^* = L_R$. Since the rebel produces no diamonds in the second time period, he generates no revenue in this period but incurs the cost of warfare, wL_R. Since $l_{fR}^* = L_R$, the probability of a military victory in the third time period is now expressed as a function of L_R.

For the purpose of this analysis, I disregard the lower boundary solution found in Equation (15). Inserting the numerical parameter values into (15) results in a value of $\hat{\delta}_{LR}$ that is significantly greater than 1 (close to 6). I can therefore conclude that, given my chosen parameter values, a lower boundary solution does not occur on the interval [0,1] (i.e. the Rebel always allocates at least part of his labor supply to warfare on this interval). Since the variable δ is only meaningful on this interval, there is no need to include the lower boundary solution in this analysis.

Inserting the numerical parameter values into Equation (20) above, I find that, in the case where $\delta < 0.2609$,

$$E(U_R)^* = 78888.8889 \qquad (21)$$

The graphical representation in Figure (4) can be redrawn as follows:

Figure 5: Rebel's Utility as a Function of δ with Upper Boundary Constraint

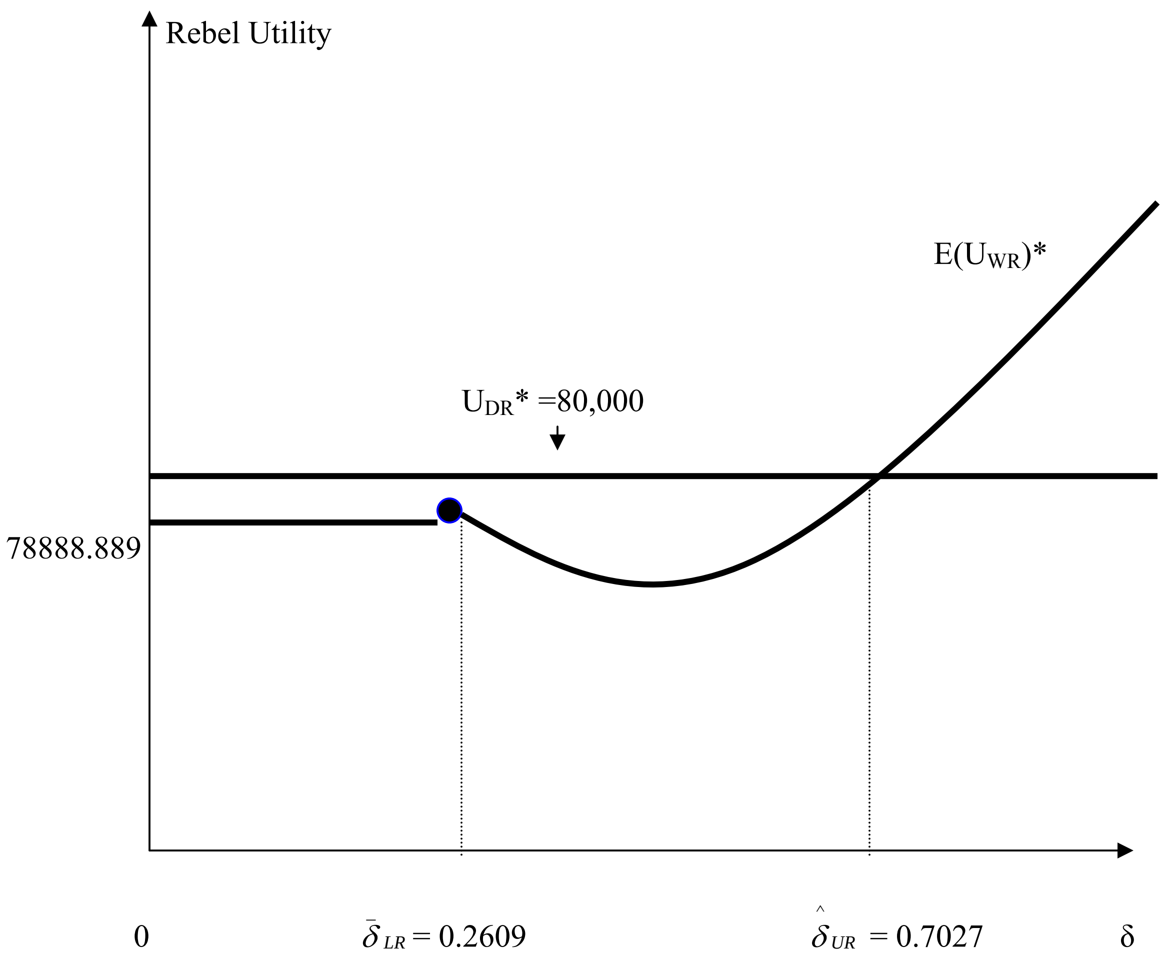


Figure (5) shows a discontinuity when δ = 0.2609. For all values of δ below this value, the rebel's total utility from reneging is 0. Since the rebel produces no diamonds when δ < 0.2609, sanctions have no effect on the interval δ = [0, 0.2609). When δ > 0.2609, the rebel's utility from reneging is represented by the function expressed in Equation (17). In the case outlined above, the rebel demobilizes if δ < 0.7027. This is the maximum value of δ at which the rebel still chooses to demobilize, and is therefore the optimal sanction that a mediator could impose given the parameter values.

Comparative Statics on Government Warfare Allocation

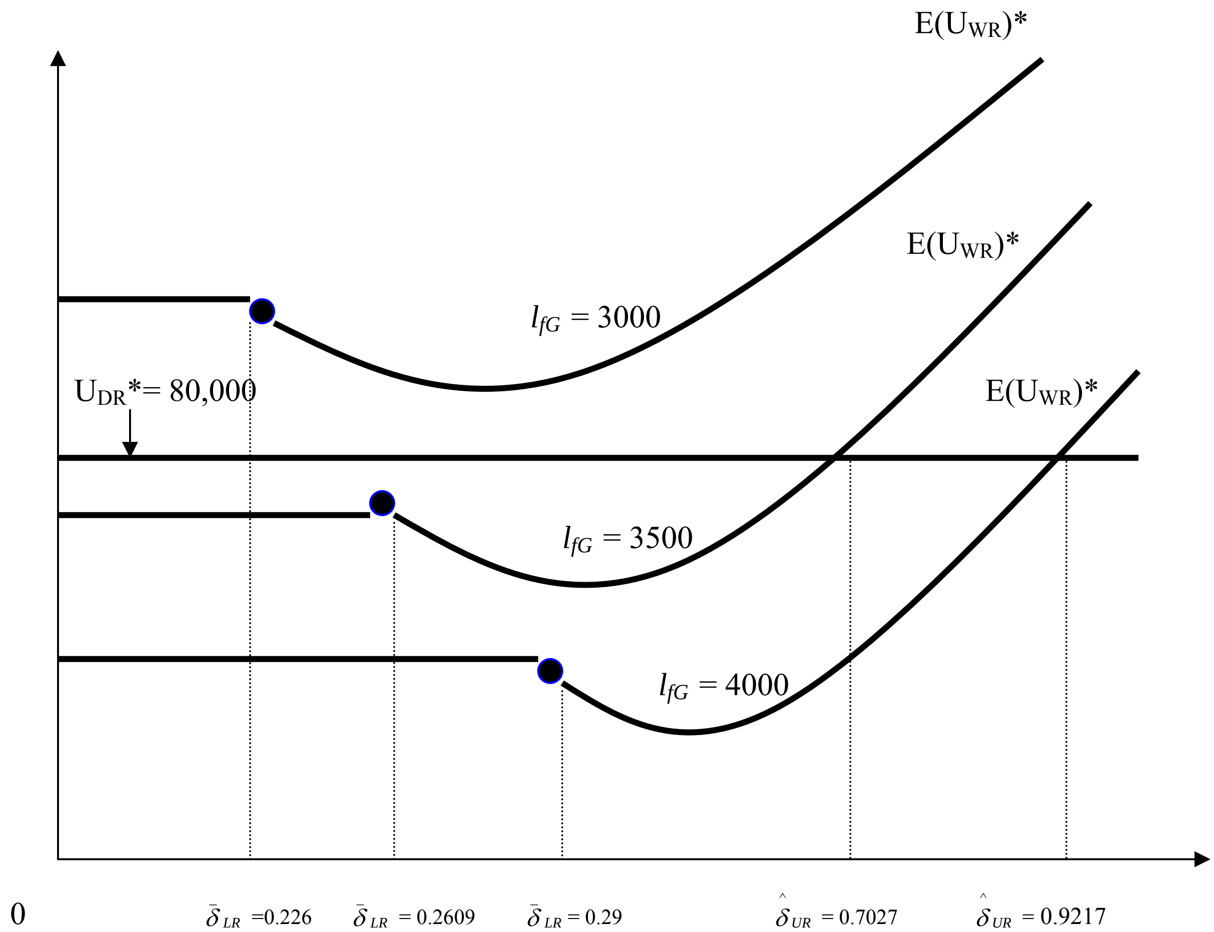
I now do a comparative statics analysis using the numerical simulation in the previous section in order to determine how $\bar{\delta}_{LR}$ and $\hat{\delta}_{UR}$ changes with changes in l_{fG} . Table (1) below shows the results for different levels of l_{fG}:

Table 1: Comparative Statics Results for Changes in l_{fG}

l_{fG}	$\bar{\delta}_{LR}$ δ ϵ[0,1]	$\hat{\delta}_{UR}$ δ ϵ[0,1]	Rebel behavior on δ ϵ[0,1]
3000	0.226	None	Renege
3500	0.2609	0.7027	Demobilize when δ <0.7027
4000	0.29	0.9217	Demobilize when δ<0.9217

These results can be represented graphically as follows:

Figure 6: The Impact of l_{fG} on Rebel Utility



When the value of l_{f1G} increases from 3500 to 4000, the utility curve of reneging shifts downwards so that $E(U_{WR})^*$ is smaller for every value of δ. $\bar{\delta}_{UR}$ increases to 0.9217, and $\bar{\delta}_{LR}$ also increases to 0.29. In this case, the optimal sanction occurs at δ = 0.9217. When the value of l_{fG} decreases from 3500 to 3000, the utility curve of reneging shifts upwards so that $E(U_{WR})^*$ is larger for every value of δ and $\bar{\delta}_{LR}$

decreases to 0.226. At this point, $E(U_{WR})^* > U_D^*$ for every value of δ, so that the rebel always chooses to renege regardless of sanctions.

As l_{fG} increases, the following occurs:

1) $E(U_{WR})^*$, the total equilibrium utility that the rebel receives from reneging, decreases for every value of δ.

2) The interval $[\hat{\delta}_{UR}, \bar{\delta}_{UR}]$ on which the rebel demobilizes increases and $\hat{\delta}_{UR}$, the optimal value of δ needed to achieve demobilization, increases.

3) $\bar{\delta}_{LR}$, the minimum value at which $E(U_{WR})^*$ is defined by the function expressed in Equation (15), also increases while $E(U_{WR})^*$ on the interval $[0, \bar{\delta}_L]$ decreases. For very large values of l_{fG}, the rebel always chooses to renege, since $E(U_{WR})^* > U_{DR}^*$ for any value of δ. In this case, sanctions do not have any effect on the Rebel's choices.

4) For very small values of l_{fG}, the optimal value of δ needed to achieve demobilization equals 1 (i.e., no sanctions are necessary to give the Rebel an incentive to demobilize).

Determining the Cournot Nash Equilibrium Warfare Allocations

Up until this point, I have treated l_{fG} as an exogenous parameter in order to determine its effect on the Rebel's wartime utility and demobilization decision. I now model both the Rebel's and the Government's labor allocation choices in a Cournot duopoly framework, so that the equilibrium l_{fG}^* chosen by the Government becomes a variable function of the other parameters in my model. I start by writing the Government's intertemporal utility functions as follows:

$$E\,(U_{WG}) = [Ap\,(\bar{L} - L_R - l_{fG}) - wL_R] + \left[\frac{l_{fG}}{l_{fR} + l_{fG}}(Ap - w)\,(\bar{L})\right] \qquad (22)$$

$$U_{DG} = 2(Ap - w)(\bar{L} - L_R) \qquad (23)$$

Equation (22) expresses the Government's expected intertemporal utility in the event that the Rebel chooses to renege and the two players return to war. Equation (23) expresses the Government's utility if the Rebel chooses to demobilize. The Government's utility functions take the same form as the Rebel's functions in Equations (8) and (9), except I replace L_R with $\bar{L}$ - L_R, since the amount of labor that the Government controls in the first time period, and in both time periods in the case of demobilization, is equal to the country's total labor supply minus the amount of labor that the Rebel controls. The probability (1-α), or the likelihood that the Government wins a military victory in the second time period, is represented in Equation (22) as the fraction $\frac{l_{fG}}{l_{fR} + l_{fG}}$.

I now derive first order conditions for the utility-maximizing amount of labor that the Government allocates to warfare. I first find the Government's total expected utility by adding together Equations (22) and (23):

$$E\,(U_G) = [Ap\,(\bar{L} - L_R - l_{fG}) - wL_R] + \left[\frac{l_{fG}}{l_{fR} + l_{fG}}(Ap - w)\,(\bar{L})\right] + 2(Ap - w)(\bar{L} - L_R)$$

(24)

Taking derivatives of Equation (24) with respect to l_{fG} results in the following first order conditions:

$$\frac{\partial E(U_G)}{\partial l_{fG}} = \frac{l_{fR}}{(l_{fR} + l_{fG})^2} \, (\mathrm{A}p - w)(\bar{L}) - \mathrm{A}p \quad = 0$$

$$l_{fG}^* = \sqrt{\frac{l_{fR}(Ap - w)(\bar{L})}{Ap}} - l_{fR} \qquad \text{if } \frac{\partial E(U_{WG})}{\partial l_{fG}} = 0,\ l_{fG}^* \ \square\ [0,\ \bar{L} - L_R] \qquad (25)$$

$$l_{fG}^* = \bar{L} - L_R \qquad \text{if } \frac{\partial E(U_{WG})}{\partial l_{fG}} > 0,\ l_{fG}^* \ \square\ [0,\ \bar{L} - L_R] \qquad (26)$$

$$l_{fG}^* = 0 \qquad \text{if } \frac{\partial E(U_{WG})}{\partial l_{fG}} < 0,\ l_{fG}^* \ \square\ [0,\ \bar{L} - L_R] \qquad (27)$$

Equation (25) defines an interior local maximum on the interval [0, $\bar{L}$ -L_R], for which the Government's utility maximizing allocation of labor towards warfare, l_{fG}*, is expressed by the first order condition in (25). Equation (26) defines an upper boundary solution at which E (U_G) is increasing everywhere on the interval [0, $\bar{L}$ -L_R]. In this case, the Government allocates her entire first period labor supply, $\bar{L}$ -$L_{R,}$ to warfare, and produces no diamonds. Equation (27) defines a lower boundary solution in which the Government allocates no labor towards warfare, and automatically demobilizes.

If the Rebel chooses to return to war in the first time period, then the Rebel and the Government both choose their optimal labor allocations towards warfare in a simultaneous Cournot game. The Rebel's optimal allocation, l_{fR}*, is expressed in Equation (11) as a function of l_{fG}. The Government's optimal allocation, l_{fG}*, is expressed in Equation (25) as a function of $l_{fR.}$ I rewrite Equation (11) as a function of $l_{fG}*$ as follows:

$$l_{fR}* = \sqrt{\frac{l_{fG}*(Ap-w)(\bar{L})}{A\delta p}} - l_{fG}* \qquad (28)$$

and Equation (25) as a function of $l_{fR}*$,

$$l_{fG}* = \sqrt{\frac{l_{fR}*(Ap-w)(\bar{L})}{Ap}} - l_{fR}* \qquad (29)$$

Equations (28) and (29) respectively represent the Rebel's and the Government's best response functions. Given any value of δ, the equilibrium allocations $l_{fR}*$ and $l_{fG}*$ can be found at the intersection of these two functions. I first rearrange Equation (28) as follows:

$$\frac{(Ap-w)(\bar{L})}{Ap} = \delta\frac{(l_{fR}*+l_{fG}*)}{l_{fG}*} \qquad (30)$$

Next, I rearrange Equation (29) as follows:

$$\frac{(Ap-w)(\bar{L})}{Ap} = \frac{(l_{fR}*+l_{fG}*)}{l_{fR}*} \qquad (31)$$

Since the left-hand side of Equation (30) cancels with the left-hand side of Equation (31), I can set (30) equal to (31) and solve for $l_{fG}*$ in terms of $l_{fR}*$:

$$\delta\frac{(l_{fR}*+l_{fG}*)}{l_{fG}*} = \frac{(l_{fR}*+l_{fG}*)}{l_{fR}*}$$

$$\delta l_{fR}* = l_{fG}* \qquad (32)$$

Substituting Equation (32) into (28) results in the following expression for the equilibrium value of l_{fR}*:

$$l_{fR}* = \frac{(Ap - w)(\bar{L})}{Ap(1+\delta)^2} \quad , \quad l_{fR}* \in (0, L_R) \qquad (33)$$

Substituting Equation (32) into (29) results in the following expression for the equilibrium value of l_{fG}*:

$$l_{fG}* = \frac{\delta(Ap - w)(\bar{L})}{Ap(1+\delta)^2} \quad , \quad l_{fG}* \in (0, \bar{L} - L_R) \qquad (34)$$

Equations (33) and (34) define the equilibrium values of l_{fR}* and l_{fG}* given that an interior solution exists.

The following graphical representation shows the location of the Nash equilibrium allocation points and how they change with a change in δ.

Figure 7: Cournot Nash Equilibrium Allocations of l_{fR}* and l_{fG}*

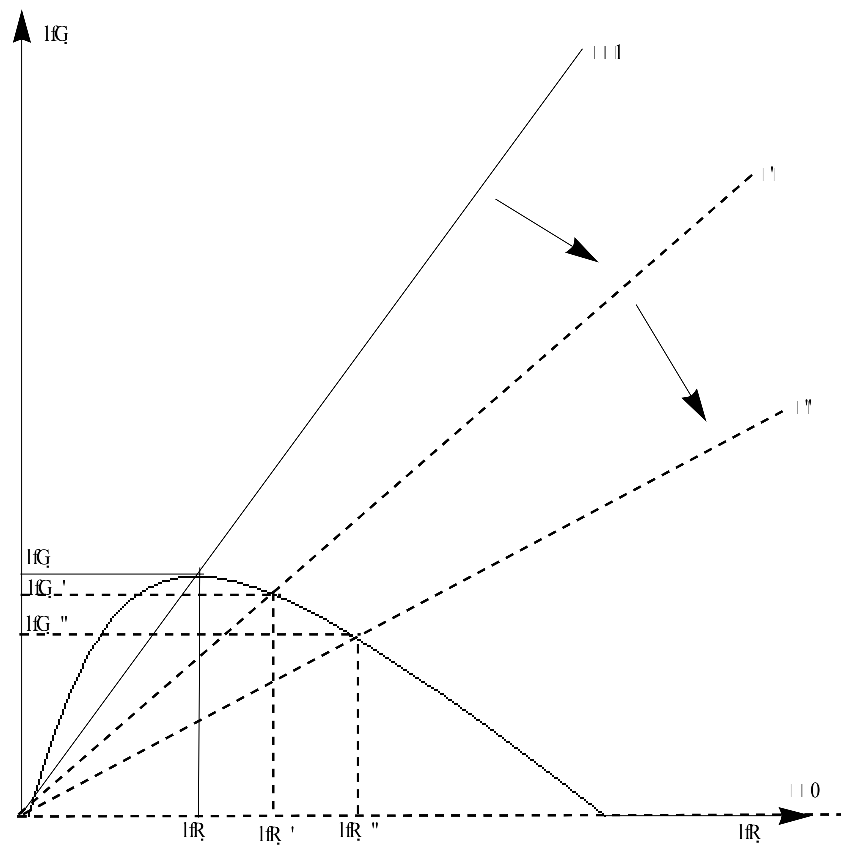


Figure (7) above shows the Government's best response function as a quadratic curve. All possible sets of Nash equilibrium labor allocations lie on this curve. For low values of l_{fR}*, l_{fG}* increases as l_{fR}* increases, and for high values of l_{fR}*, l_{fG}* decreases as l_{fR}* increases. In other words, when the Rebel army is small, an increase in the Rebel's war efforts motivates the Government to increase her war efforts as well. When the Rebel's army is large, an increase in the Rebel's war efforts causes the Government to decrease the amount of labor she devotes to warfare, because her chances of victory are small when the Rebel army is large. The lines drawn from the origin in this graph represent the linear relationship between l_{fR}* and l_{fG}* that I found in Equation (32) for different values of δ. These lines intersect the Government's best response function at two points: (0,0) and another point (l_{fR}*, l_{fG}*) such that $l_{fR}* > 0$. The second intercept is the Nash equilibrium allocation, given any value of δ.

The original solid line in Figure (7) represents the case where no sanctions are imposed and $\delta = 1$. This line intersects the Government's best response function very close to the maximum value of l_{fG}*; at this point, $l_{fG}* = l_{fR}*$ since $\delta = 1$. When sanctions are imposed, δ decreases to δ' and δ'', causing the slope of the line to decrease. The line then rotates down and outwards to the dashed lines as drawn above. When a perfect sanction is imposed such that $\delta = 0$, the line rotates downwards to lie horizontally over the x-axis, as illustrated by the dashed line in Figure (7) above.

In Figure (7), we see that the Rebel's equilibrium labor allocation towards warfare, l_{fR}*, always increases when δ decreases (i.e. when sanctions are larger). On the other hand, the Government's equilibrium level of warfare, l_{fG}*, increases when δ decreases on the increasing segment of the quadratic function, and decreases when δ decreases on the decreasing segment. Intuitively, we can explain this observation by the fact that, for small values of l_{fR}*, the Government responds to a decrease in δ, which increases l_{fR}*, by increasing her own war effort l_{fG}*. However, for large values of l_{fR}* when the Government's chances of victory are small, a decrease in δ, which increases l_{fR}*, causes the Government to allocate a smaller amount of labor to warfare.

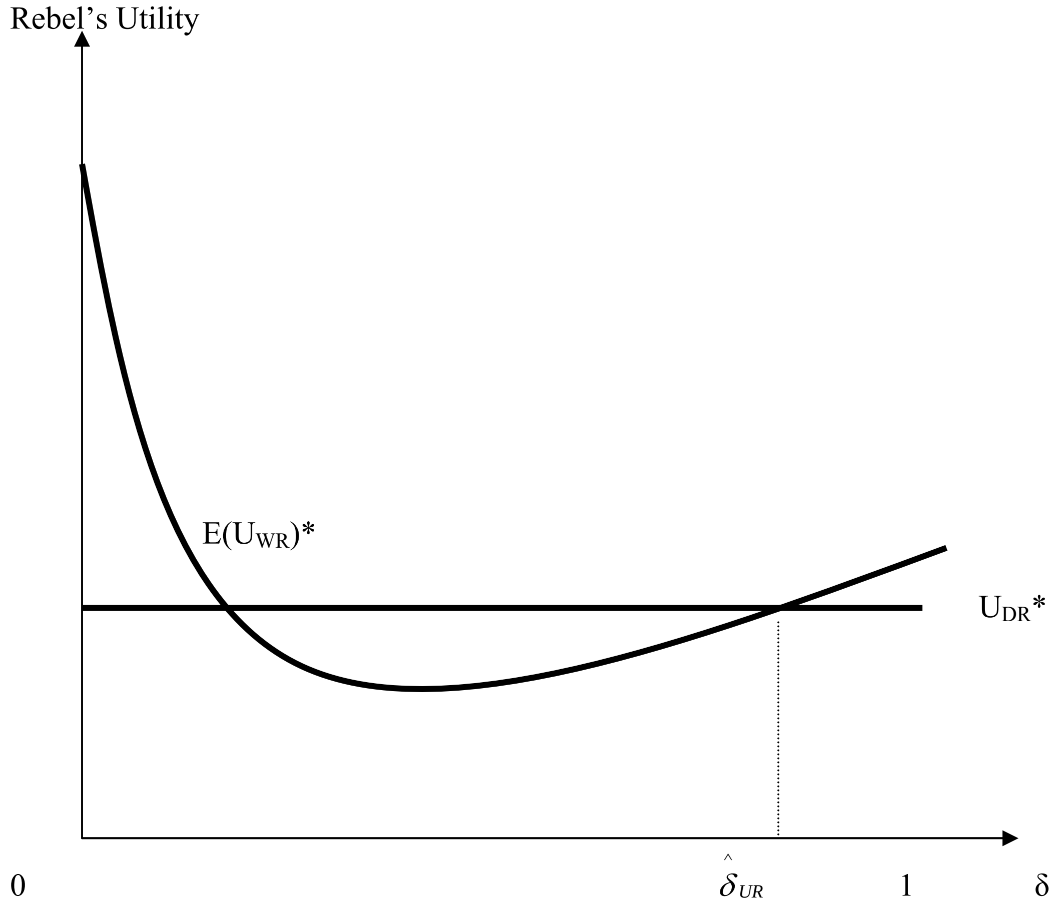
For the purpose of this analysis, I am only interested in the interval δ □ [0, 1], since the price discount caused by sanctions is a fraction that, by definition, must fall between 0 and 1. Since l_{fG}* reaches its maximum close to δ=1, it is safe to assume that l_{fG}* decreases everywhere on the interval δ [0, 1]. On this interval, the Government always decreases the amount of labor it allocates to warfare when δ decreases (i.e. sanctions are larger) and the line from the origin rotates downwards. This occurs because as δ increases, the Rebel devotes more labor to warfare in response to the increasing opportunity costs of producing diamonds. As the Rebel army gets larger, the chances of a Government victory decrease. As a result, the Government's opportunity costs of allocating labor towards warfare increase as δ increases and she devotes less labor to warfare.

When I insert Equations (33) and (34) into the Rebel's original wartime utility function in Equation (8) and simplify, the Rebel's utility function becomes:

$$\mathrm{E}(\mathrm{U}_{\mathrm{WR}})^* = Ap\delta L_R - wL_R + \frac{(Ap-w)(\bar{L})}{(1+\delta)^2} \qquad (35)$$

In (35), the probability of a Rebel victory, $\frac{l_{fG}}{l_{fR}+l_{fG}}$, becomes $\frac{1}{1+\delta}$. According to the relationship expressed in Equation (32), the equilibrium quantity of labor that the Government allocates to warfare is a proportion, δ, of the Rebel's equilibrium allocation. Consequently, the probability of a Rebel victory becomes a function of δ, which now also represents the size of the Government army relative to the Rebel's. The general graphical form of the function expressed in (35) is shown in the illustration below, as is the Rebel's utility function of demobilizing, as expressed in Equation (9).

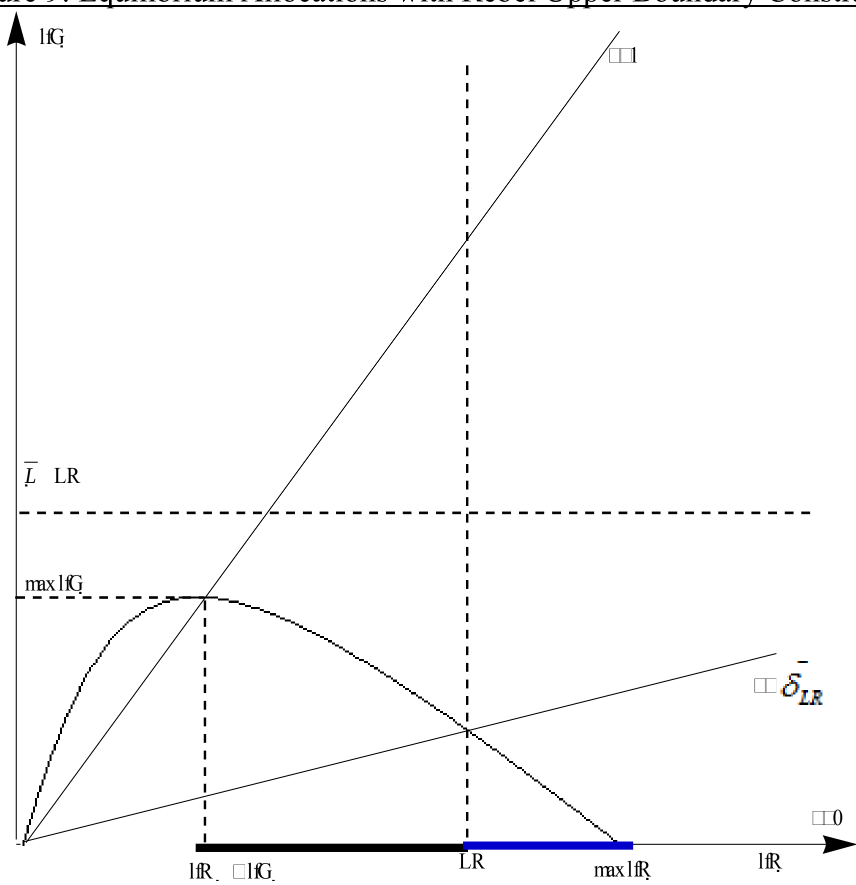
Figure 8: Rebel's Utility Function with all Interior Solutions



Checking for Upper Boundary Solutions

I draw the shape of the E (U_{WR})* function in Figure (8) with the assumption that only interior solutions for (l_{fG}*, l_{fR}*) exist on the interval $\delta \in [0, 1]$.
In reality, however, the Rebel's and the Government's warfare allocations are both constrained by the quantity of available labor that they each have access to in the first time period. The maximum amount of labor that the Rebel could possibly allocate towards warfare is L_R, while the Government could devote a maximum of $\bar{L}$-L_R labor units to warfare. Figure (9) below shows the players' Nash equilibrium labor allocations in the case where the Rebel faces an upper boundary constraint.

Figure 9: Equilibrium Allocations with Rebel Upper Boundary Constraint



The vertical dashed line in Figure (9) above marks L_R, the total quantity of labor that the Rebel controls in the first time period when he makes his allocation decisions. As illustrated in Figure (9), the Rebel faces an upper boundary solution when the maximum value of l_{fR}*, at which $\delta = 0$, is larger than the Rebel's total labor supply L_R. Substituting "0" in for δ in Equation (33), I find that the Rebel's upper boundary solution occurs given the following condition:

$$L_R \leq \frac{(Ap - w)(\bar{L})}{Ap} \qquad (36)$$

For a certain value of δ between 0 and 1, which I call $\bar{\delta}_{LR}$, the Rebel allocates his entire first period labor supply to warfare, so that $l_{fR}^* = L_R$ when $\delta = \bar{\delta}_{LR}$. When δ falls below this minimum threshold value, l_{fR}* remains fixed at the maximum possible allocation L_R, and l_{fG}* remains fixed at $\bar{\delta}_{LR} L_R$. The interval $\delta \in [\bar{\delta}_{LR}, 1]$, on which only interior solutions exist for (l_{fR}*, l_{fG}*), is highlighted in black in Figure (9) above, while the interval $\delta \in [0, \bar{\delta}_{LR}]$, on which the Nash equilibrium (l_{fR}*, l_{fG}*) is defined by the upper boundary solution ($l_{fR}^* = L_R$, $l_{fG}^* = \bar{\delta}_{LR} L_R$) is highlighted in blue.

In the case that an upper boundary solution occurs, the minimum threshold value $\bar{\delta}_{LR}$ at which the Rebel allocates any labor to diamond production is found by setting L_R equal to Equation (33) and rearranging in terms of δ. The minimum threshold value $\bar{\delta}_{LR}$ is written as follows:

$$\bar{\delta}_{LR} = \sqrt{\frac{(Ap - w)\,\bar{L}}{L_R\,Ap}}\ \text{-1} \qquad (37)$$

After I insert the upper boundary equilibrium allocations, l_{fR}* = L_R and $l_{fG}* = \bar{\delta}_{LR}\ l_{fR}*$, into Equation (8), the Rebel's wartime utility function becomes:

$$\text{E }(U_{WR})* = -\,wL_R + \frac{(Ap - w)(\bar{L})}{(1 + \bar{\delta}_{LR})} \qquad (38)$$

Equation (36) represents the Rebel's utility from reneging at the upper boundary solution ($l_{fR}* = L_R$, $l_{fG}* = \bar{\delta}_{LR}\ l_{fR}*$). This function applies on the interval δ □ [0, $\bar{\delta}_{LR}$], as illustrated below:

<u>Figure 10: Rebel's Utility with Rebel Upper Boundary Constraint</u>

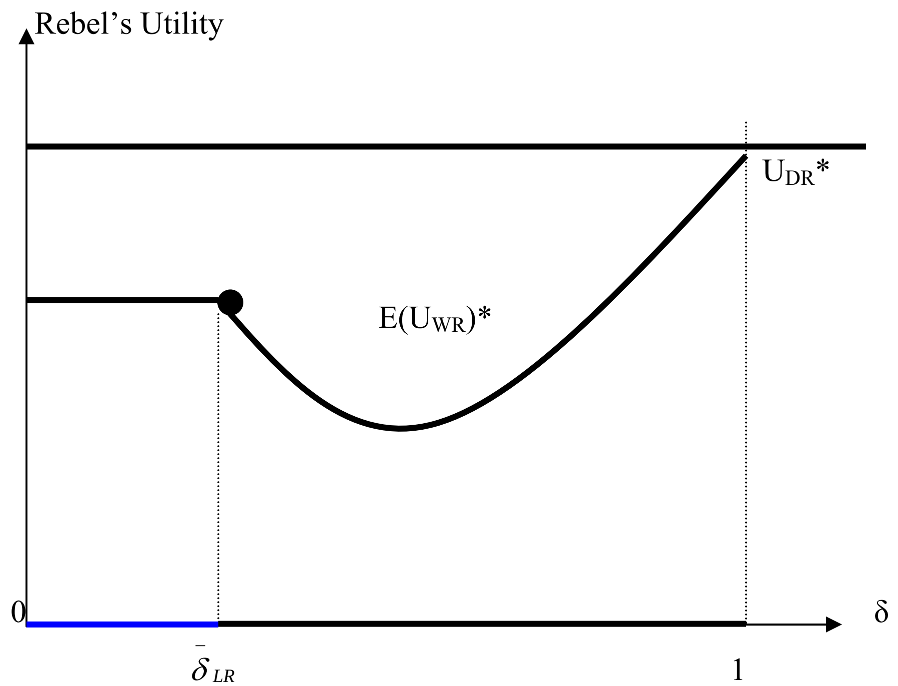
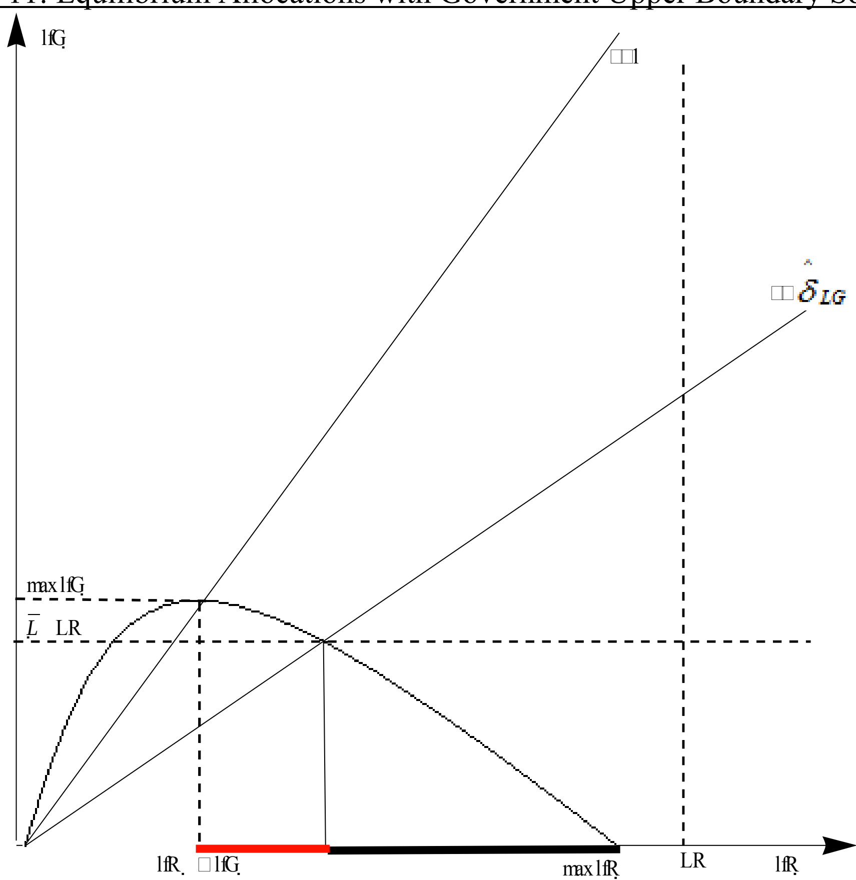


Figure (10) represents the Rebel's utility functions $E(U_{WR})^*$ and U_{DR}^* in the case that the Rebel faces an upper boundary solution at $\delta = \bar{\delta}_{LR}$. When $\delta \in [0, \bar{\delta}_{LR}]$, $E(U_{WR})^*$ is expressed by the upper boundary function in Equation (36). When $\delta \in [\bar{\delta}_{LR}, 1]$, $E(U_{WR})^*$ is expressed by the interior solution function in Equation (35). These intervals are highlighted in blue and black, respectively, and correspond to the similarly highlighted intervals in the Cournot Nash equilibrium chart in Figure (9).

In the case where the Government faces an upper boundary constraint, the maximum amount of labor that she can allocate towards warfare is $\bar{L}$ -L_R. This situation is illustrated as follows:

Figure 11: Equilibrium Allocations with Government Upper Boundary Solution



The lower horizontal dashed line in Figure (11) above marks the Government's total labor supply, $\bar{L}$ -L_R. As illustrated in the chart above, an upper boundary solution occurs when the maximum value of l_{fG}*, at which the value of δ is close to 1, is larger than the Government's total labor supply L_R.

For a certain value of δ between 0 and 1, which I call $\hat{\delta}_{LG}$, the Government allocates his entire first period labor supply to warfare, so that $l_{fG}* = \bar{L} - L_R$ when $\delta = \hat{\delta}_{LG}$. When δ rises above this maximum threshold value, l_{fG}* remains fixed at the maximum possible allocation $\bar{L}$ -L_R, and l_{fR}* remains fixed at $\frac{\bar{L} - L_R}{\hat{\delta}_{LG}}$. The portion of the horizontal axis highlighted in black in Figure (11) represents the interval δ □ $[0, \hat{\delta}_{LG}]$, on which only interior solutions exist for (l_{fR}*, l_{fG}*), while the interval δ □ $[\hat{\delta}_{LG}, 1]$, on which the Nash equilibrium (l_{fR}*, l_{fG}*) is defined by the upper boundary solution ($l_{fR}* = \frac{\bar{L} - L_R}{\hat{\delta}_{LG}}$, $l_{fG}* = \bar{L} - L_R$), is highlighted in red.

After I insert the upper boundary equilibrium allocations, $l_{fR}* = \frac{\bar{L} - L_R}{\hat{\delta}_{LG}}$ and $l_{fG}* = \bar{L} - L_R$, into Equation (8), the Rebel's wartime utility function becomes:

$$E(U_{WR})^* = A\delta p\left(L_R - \frac{\bar{L} - L_R}{\hat{\delta}_{LG}}\right) - wL_R + \left(\frac{(Ap - w)(\bar{L})}{1 + \hat{\delta}_{LG}}\right) \tag{39}$$

The Rebel's utility functions in the case of a Government upper boundary constraint are illustrated in the chart below:

<u>Figure 12: Rebel's Utility with Government Upper Boundary Solution</u>

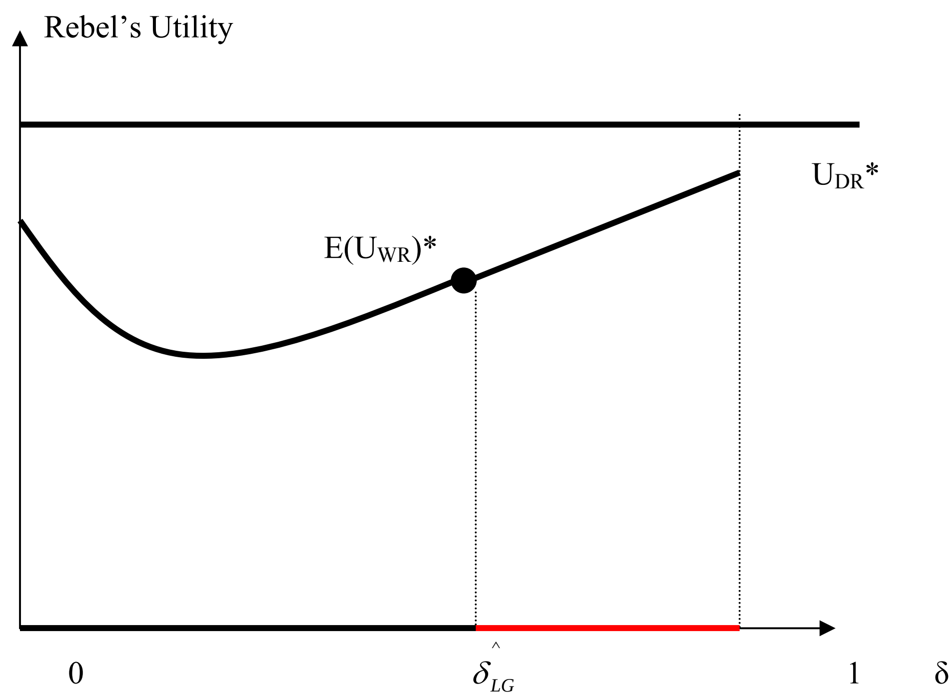


Figure (12) represents the Rebel's utility functions $E(U_{WR})^*$ and U_{DR}^* in the case that the Government faces an upper boundary solution at $\delta = \hat{\delta}_{LG}$. When $\delta \in [0, \hat{\delta}_{LG}]$, $E(U_{WR})^*$ is expressed by the interior solution function in Equation (35). When

δ □ [$\hat{\delta}_{LG}$, 1], E(U_{WR})* is expressed by the upper boundary solution in Equation (39). These intervals are highlighted in black and red, respectively, and correspond to the similarly highlighted intervals in the chart in Figure (11). Note that the right-hand side of this function, where $\delta \geq \hat{\delta}_{LG}$, is increasing as δ increases. On this interval, the quantity of labor that the Rebel allocates to warfare is fixed (according to the size of the Government's labor supply) and does not change with increasing values of δ. However, the value of the Rebel's diamond revenue increases as δ increases. Therefore, when $\delta \geq \hat{\delta}_{LG}$, the Rebel's expected wartime utility is an increasing function of δ.

<u>Defining a Range on which Sanctions are Effective</u>

Given the upper boundary scenario illustrated in Figure (10), I now define a parameter range on which sanctions have an impact on the Rebel's decision to return to war or to demobilize.

I define a "demobilization range" for the first scenario in which the Rebel faces an upper boundary constraint on the amount of labor that he can allocate towards warfare. As illustrated in Figure (9) above, E (U_{WR})* < U_{DR}* everywhere on the interval δ □ [0,1] when E (U_{WR})* < U_{DR}* at $\delta = 1$. In this case, the Rebel always chooses to demobilize, regardless of whether sanctions are imposed. In order to find the values of L_R for which the Rebel always demobilizes, I substitute "1" for δ in the Rebel's utility function, E(U_{WR})*, in Equation (35). Using Equations (35) and (9), I write the inequality as follows:

$$ApL_R - wL_R + \frac{(Ap - w)(\bar{L})}{4} \leq 2(Ap - w)(L_R)$$

$$(Ap - w)(L_R) \geq \frac{(Ap - w)(\bar{L})}{4}$$

$$L_R \geq \frac{\bar{L}}{4} \qquad (40)$$

According to the inequality in Equation (40), the Rebel always demobilizes when he already controls more than one-fourth of the country's entire labor force in the first time period. Since my model assumes that the peace agreement made in time $t = 0$ allows the Rebel to keep his entire first period labor supply if he chooses to demobilize, he chooses demobilization as an "insurance" option that guarantees him a minimum level of utility. The "demobilization" range applies in situations where, in time $t = 1$, the Rebel controls a large enough proportion of the country's labor force to give him an incentive to demobilize and to keep the labor he already owns, rather than returning to war and facing the risk of losing everything in the event of a military defeat. When the proportion of the country's labor force that the Rebel already owns is larger, he has to less to gain and more to lose from returning to war, and is more likely to choose demobilization.

Comparative Statics with Numerical Parameters

I now perform a comparative statics analysis on the parameter $\bar{L}$, which represents the country's total labor force that the Rebel could potentially win in time $t = 2$. I start with these numerical parameters:

$$A = 1$$

$$p = 5$$

$$L_R = 10000$$

$$\bar{L} = 40000$$

$$w = 1$$

I have chosen these parameters as a "baseline" case. Inserting these numerical values into the "demobilization range" defined in Equation (43), I find that the Rebel faces an upper boundary constraint and owns exactly one-fourth of the country's total labor supply in the first period. Therefore, the Rebel is at the minimum level of $\bar{L}$ at which he still always chooses to demobilize regardless of the level of sanction imposed. When $\bar{L}$ is larger than 40,000 (i.e. the proportion of the labor force that the Rebel controls is smaller than one-fourth), sanctions have an impact on whether the Rebel chooses to renege or to demobilize.

In order to find the minimum level of sanction needed to achieve demobilization when $L_R < 1/4\,\bar{L}$, I determine the Rebel's labor allocation and demobilization decisions for values of $\bar{L}$ equal to or greater than 40,000:

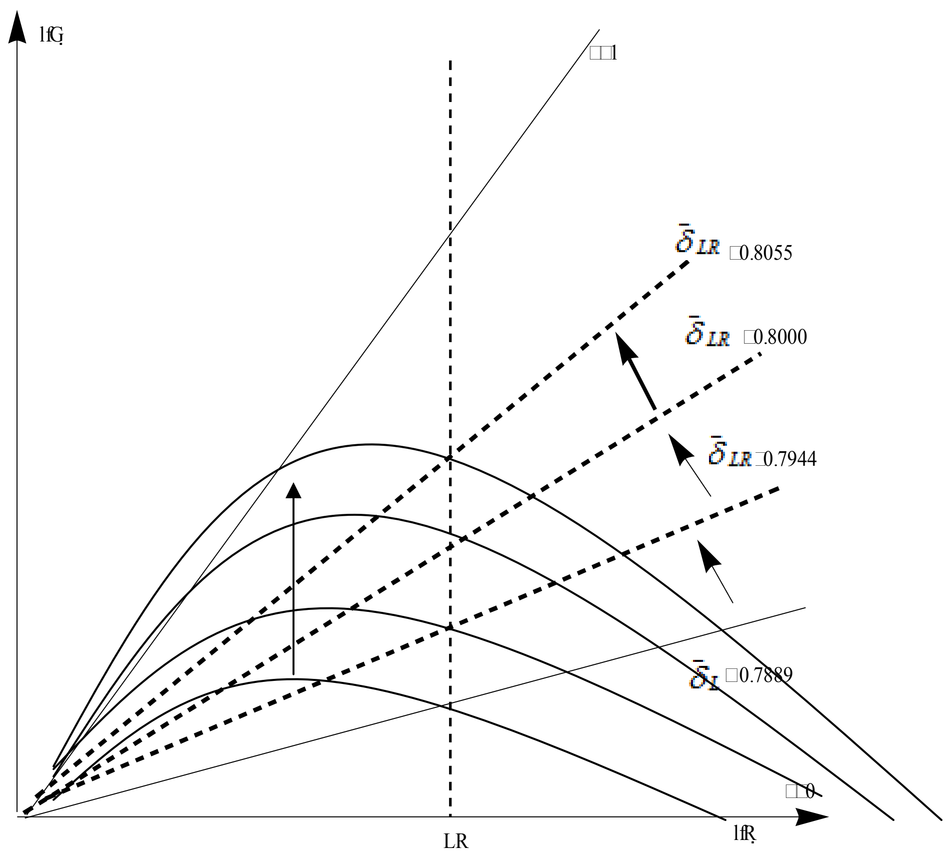
Table 2: Comparative Statics Results for Changes in $\bar{L}$

$\bar{L}$	$E(U_{WR})^*$ $\delta \leq \bar{\delta}_{LR}$	$E(U_{WR})^*$ $\delta > \bar{\delta}_{LR}$	U_{DR}^*	$\bar{\delta}_{LR}$ $\delta \epsilon [0,1]$	$\hat{\delta}_{UR}$ $\delta \epsilon [0,1]$	Rebel behavior on $\delta \epsilon [0,1]$
40,000	$\frac{160{,}000}{(1+\bar{\delta}_{LR})} - 10{,}000$	$\frac{160{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.7889	1	Demobilize
40,250	$\frac{161{,}000}{(1+\bar{\delta}_{LR})} - 10{,}000$	$\frac{161{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.7944	0.9719	Demobilize when $\delta < 0.9719$
40,500	$\frac{162{,}000}{(1+\bar{\delta}_{LR})} - 10{,}000$	$\frac{162{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.8000	0.9318	Demobilize when $\delta < 0.9318$
40,750	$\frac{163{,}000}{(1+\bar{\delta}_{LR})} - 10{,}000$	$\frac{163{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.8055	None	Renege

Table (2) above shows the Rebel's utility functions for different levels of $\bar{L}$, the value of $\bar{\delta}_{LR}$ at which he chooses to allocate his entire labor supply to warfare, and the Rebel's behavior at each level of $\hat{\delta}_{UR}$. The functions for $E(U_{WR})^*$ and U_{DR}^* in the second, third and fourth columns from the right were found by inserting the numerical parameters into Equations (38), (35), and (9), respectively. In the sixth column to the right, the parameter $\hat{\delta}_{UR}$ (recall Table 1) is the value of δ at which $E(U_{WR})^* = U_{DR}^*$. Specifically, this is the maximum value of δ (minimum sanction) at which the Rebel chooses to demobilize. When δ rises above $\hat{\delta}_{UR}$ (i.e. sanctions are smaller than the required threshold level), the Rebel chooses to renege. In Table (2) above, we see that $\hat{\delta}_{UR}$ decreases for larger values of $\bar{L}$ whereas $\bar{\delta}_{LR}$ increases for larger values of $\bar{L}$. When $\bar{L} = 40{,}000$ (i.e. the proportion of the labor force that the

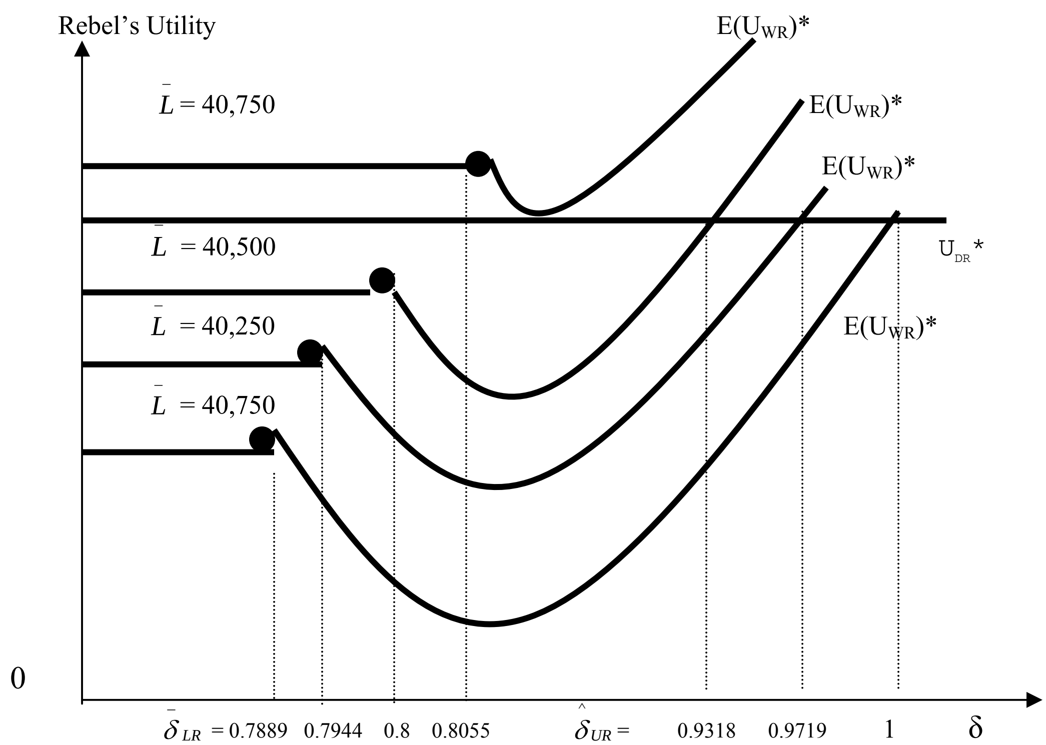
Rebel owns is exactly one-fourth), =1. In this case, no sanctions are necessary to achieve demobilization, because the Rebel is just within the range, expressed in Equation (40), on which he always demobilizes. When the value of $\bar{L}$ rises above this "demobilization range", the impact of an increase in $\bar{L}$ on $\bar{\delta}_{LR}$ is shown in the equilibrium allocation chart below:

Figure 13: The Impact of Increasing $\bar{L}$ on $\bar{\delta}_{LR}$



When the Rebel controls less than one-fourth of the country's total labor force in the first time period, an increase in $\bar{L}$, which effectively decreases the proportion $\frac{L_R}{\bar{L}}$ of the labor force that the Rebel owns, causes the allocation curve in Figure (13) above to shift upwards and to the right so that l_{fR}* increases for every level of δ. As a result, the equilibrium line $\delta l_{fR}* = l_{fG}*$ rotates upwards to the left (becomes closer to 1) and the minimum threshold value at which the Rebel still allocates any labor towards diamond production increases. In other words, when the country's total labor force is larger, the size of the sanction at which the Rebel allocates his entire labor supply to warfare becomes smaller. As δ increases, the Rebel substitutes towards warfare at a faster rate and consequently reaches his upper boundary constraint at a smaller value of δ. Figure (14) below illustrates the effect of increasing on the Rebel's expected wartime utility $E(U_{WR})$*.

Figure 14: The Impact of $\bar{L}$ Increasing in Rebel Utility



As shown in Figure (14) above, the $E(U_{WR})^*$ curve shifts upwards and to the right as $\bar{L}$ increases, causing $\hat{\delta}_{UR}$ to become smaller for larger values of $\bar{L}$. The rationale behind this observation is simple: when the size of the country's labor force that the Rebel could potentially capture at time $t = 2$ is larger, he has a greater incentive to return to war at time $t = 1$. We can also think of an increase in $\bar{L}$, holding L_R constant, as a decrease in the proportion of the country's labor force that the Rebel already owns in the first time period. When this proportion is small, the Rebel has much more to gain from returning to war. If he manages to win a military victory in the second time period, the diamond revenue that he earns at that time (using the much larger labor supply $\bar{L}$) would be much greater than the diamond revenue he could earn if he chooses to demobilize and simply keeps his current labor supply L_R. Therefore, when $\bar{L}$ is larger, the value of δ at which the Rebel demobilizes is smaller (i.e. a larger sanction is needed to achieve demobilization, since the Rebel's incentive to return to war is greater). As shown in the last row of Table (2), the Rebel always chooses to renege when $\bar{L}$ increases beyond a certain point.

I now examine the impact of decreasing L_R on the threshold values $\bar{\delta}_{LR}$ and $\hat{\delta}_{UR}$. Table (3) below shows the changes in these parameters and in the Rebel's utility for different levels of L_R :

Table 3: Comparative Statics Results for Changes in L_R

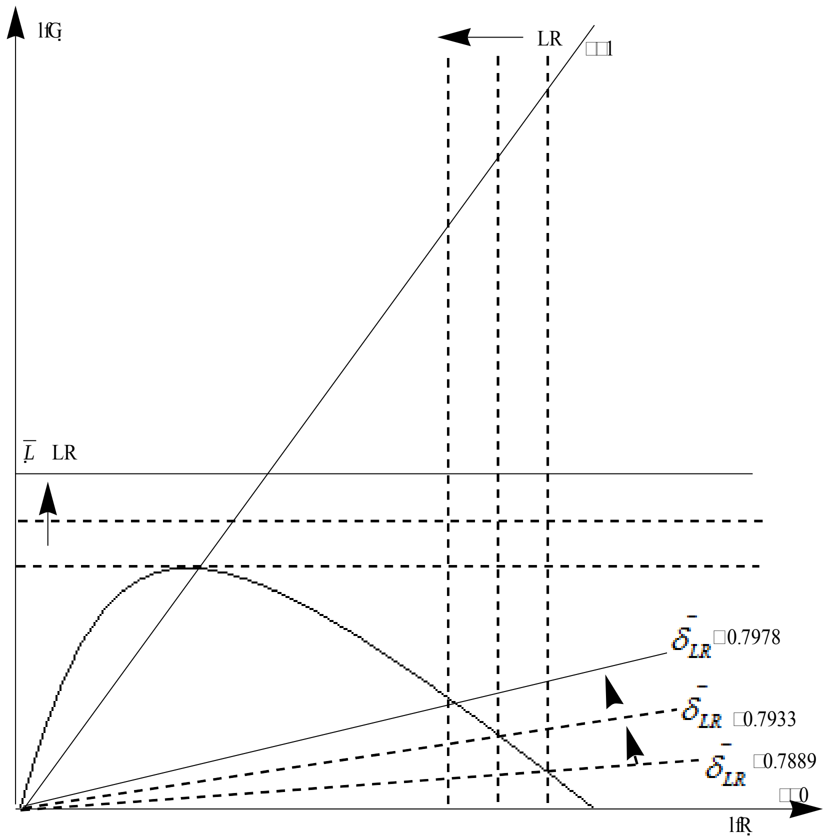
L_R	$E(U_{WR})^*$ $\delta \le \bar{\delta}_{LR}$	$E(U_{WR})^*$ $\delta > \bar{\delta}_{LR}$	U_{DR}^*	$\bar{\delta}_{LR}$ $\delta \epsilon [0,1]$	$\hat{\delta}_{UR}$ $\delta \epsilon [0,1]$	Rebel behavior on $\delta \epsilon [0,1]$
10,000	$\frac{160{,}000}{(1+\bar{\delta}_{LR})} - 10{,}000$	$\frac{160{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.7889	1	Demobilize
9,950	$\frac{160{,}000}{(1+\bar{\delta}_{LR})} - 9{,}950$	$\frac{160{,}000 + 49750\delta}{(1+\delta)^2} - 9{,}950$	79,600	0.7933	0.9780	Demobilize when $\delta < 0.9780$
9,900	$\frac{160{,}000}{(1+\bar{\delta}_{LR})} - 9{,}900$	$\frac{160{,}000 + 4975008}{(1+\delta)^2} - 9{,}900$	79,200	0.7978	0.9496	Demobilize when $\delta < 0.9496$

As shown in Table (3) above, $\bar{\delta}_{LR}$ increases and $\hat{\delta}_{UR}$ decreases as L_R decreases. When L_R decreases beyond a certain level, the Rebel always chooses to renege.

Decreasing L_R has the same effect as increasing $\bar{L}$. In both cases, the proportion of the country's total labor supply that the Rebel controls in the first time period is smaller, giving him a greater incentive to return to war in an effort to capture more resources in the second time period. As a result, the maximum value of δ at which the Rebel chooses to demobilize is smaller (i.e. the minimum level of sanction needed to achieve demobilization is larger) for smaller values of L_R. At the same time, the threshold value of δ at which the Rebel allocates his entire labor supply to warfare increases for smaller values of L_R. When L_R decreases, increasing the Rebel's incentive to return to war, the Rebel substitutes labor towards warfare at a faster rate for every level of sanction that is imposed, thus arriving at his upper boundary constraint at a larger value of δ.

Figure (15) below illustrates the impact of a decrease in L_R on the Rebel's equilibrium labor allocation towards warfare:

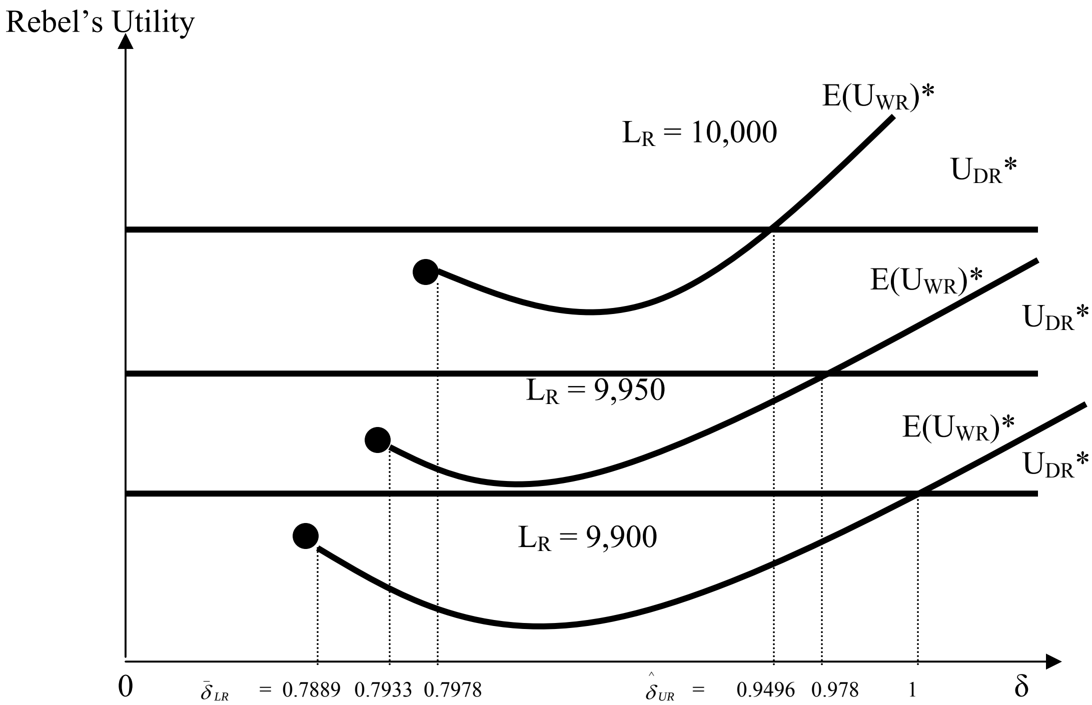
Figure 15: The Impact of Decreasing L_R on $\bar{\delta}_{LR}$



Although the allocation curve in Figure (15) does not change in response to a change in L_R, holding $\bar{L}$ constant, the Rebel's upper boundary constraint shrinks in, as shown by the movement of the vertical line marked L_R in the chart above. At the same time, the Government's upper boundary constraint, as marked by the horizontal line $\bar{L}$ -L_R, moves upwards, since a smaller Rebel labor force necessarily means a larger Government labor supply, holding $\bar{L}$ constant. The equilibrium line $\delta l_{fR}{}^* = l_{fG}{}^*$ rotates up and inwards as L_R decreases, so that $\bar{\delta}_{LR}$ increases for smaller values of L_R.

The impact of a decrease in L_R on the minimum level of sanction, needed to motivate the Rebels to choose demobilization is shown below:

Figure 16: The Impact of Decreasing L_R on Rebel Utility



In Figure (15) above, the Rebel's wartime utility curve $E(U_{WR})^*$ shifts downwards and to the right as L_R decreases. However, U_{DR}^* also shifts downwards when L_R decreases, according to the Rebel's utilty function from demobilizing expressed in Equation (9). The downwards shift of U_{DR}^* causes $\hat{\delta}_{UR}$ to increase, so that the minimum level of sanction needed to achieve demobilization is smaller when the Rebel's labor supply L_R decreases.

In the case where the world diamond price *p* increases, the Rebel's utility and threshold parameters $\bar{\delta}_{LR}$ and $\hat{\delta}_{UR}$ are as follows:

Table 4: Comparative Statics Results for Changes in P ($\bar{L}$ = 40,250)

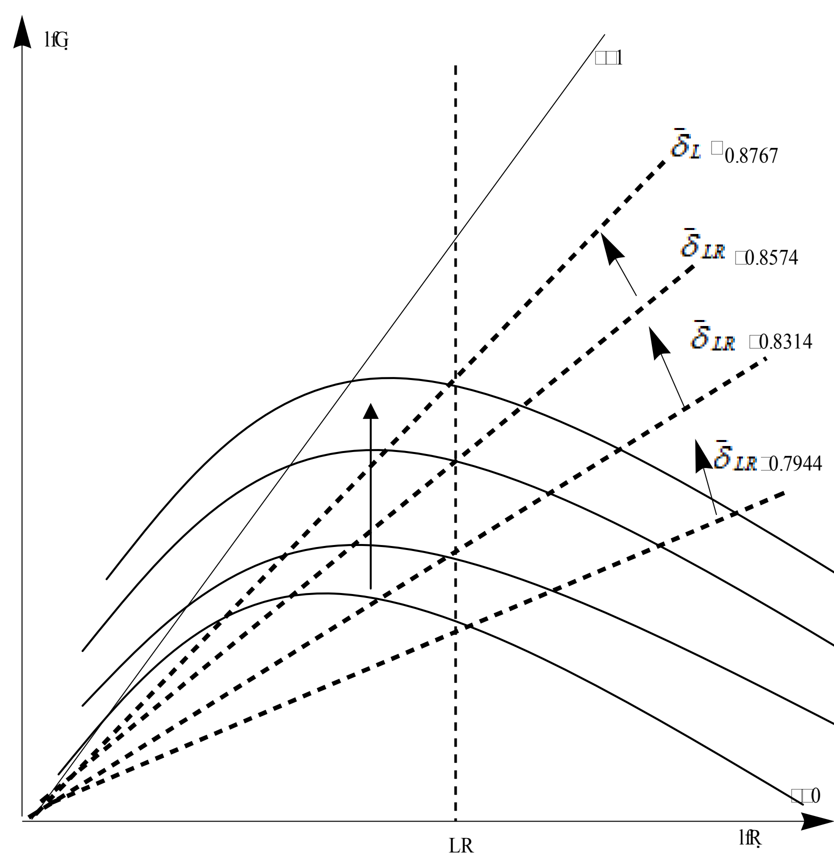
p	E(U$_{WR}$)* $\delta \leq \bar{\delta}_{LR}$	E(U$_{WR}$)* $\delta > \bar{\delta}_{LR}$	U$_{DR}$*	$\bar{\delta}_{LR}$ δϵ[0,1]	$\hat{\delta}_{UR}$ δϵ[0,1]	Rebel behavior on δϵ[0,1]
5	$\frac{161{,}000}{(1+\delta)} - 10{,}000$	$\frac{161{,}000 + 50000\delta}{(1+\delta)^2} - 10{,}000$	80,000	0.7944	0.9719	Demobilize when δ<0.9719
6	$\frac{201{,}250}{(1+\delta)} - 10{,}000$	$\frac{201{,}250 + 60000\delta}{(1+\delta)^2} - 10{,}000$	100,000	0.8314	0.9620	Demobilize when δ<0.9620
7	$\frac{241{,}500}{(1+\delta)} - 10{,}000$	$\frac{241{,}500 + 70000\delta}{(1+\delta)^2} - 10{,}000$	120,000	0.8574	0.9477	Demobilize when δ<0.9477
8	$\frac{281{,}750}{(1+\delta)} - 10{,}000$	$\frac{281{,}750 + 80000\delta}{(1+\delta)^2} - 10{,}000$	140,000	0.8767	None	Renege

To analyze the impact of the price *p* on the Rebel's utility, I use $\bar{L}$ = 40,250 rather than $\bar{L}$ = 40,000, as I have done previously. When $\bar{L}$ = 40,000, the Rebel owns one-fourth of the country's total labor supply and therefore is within the "demobilization range" on which he always demobilizes regardless of the diamond price *p*. In order to determine the effects of changes in the parameter *p*. In order to determine the impact of a change in the price *p* on the Rebel's utility, therefore, I must use a higher level of $\bar{L}$ in my analysis. As shown in Table (4) above, $\bar{\delta}_{LR}$ increases and $\hat{\delta}_{UR}$ decreases as the diamond price *p* increases. When *p* increases beyond a certain level, the Rebel always chooses to renege. An increase in the world diamond price has the same effect as a decrease in the proportion ; it gives the Rebel a greater incentive to return to war and thus increases the minimum level of sanction needed to achieve demobilization, while decreasing the level of sanction at which the

Rebel allocates all of his first period labor towards warfare. Although a higher diamond price at t =1 raises the opportunity costs of warfare in terms of diamond production, causing the Rebel to substitute labor away from warfare, a higher diamond price also increases the value of the Rebel's future diamond revenue at time t = 2, should he win a military victory at that time. This gives the Rebel a larger incentive to return to war in the first time period, despite the higher opportunity costs.

Figure (17) below shows the impact of increasing the world diamond price p on the Rebel's equilibrium labor allocation decisions.

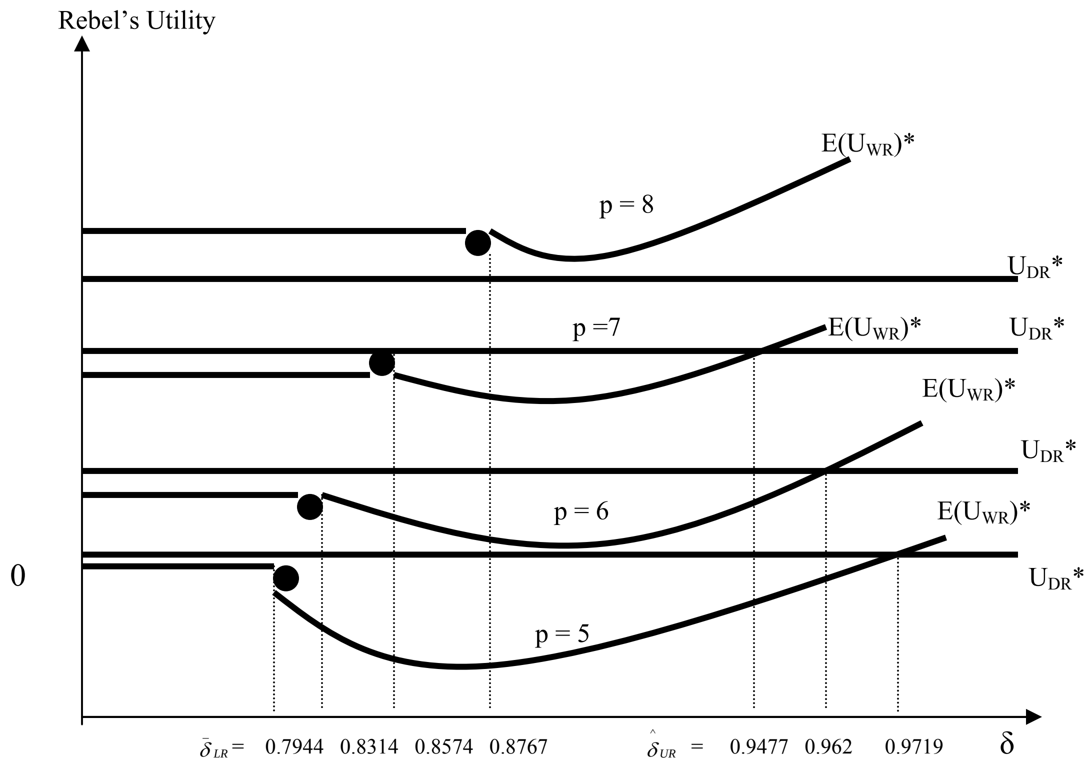
Figure 17: The Impact of Increasing p on $\bar{\delta}_{LR}$



In Figure (17) above, a decrease in the diamond price p causes the allocation curve to shift upwards and to the right, increasing l_{fR}* for every value of δ. The equilibrium line $\delta l_{fR}{*} = l_{fG}{*}$ rotates upwards to the left, causing the threshold value $\bar{\delta}_{LR}$, at which the Rebel allocates his entire first period labor supply to warfare, to increase.

Figure (18) below illustrates the impact of increasing the world diamond price *p* on the utility that the Rebel receives from reneging or demobilizing:

Figure 18: The Impact of Changes in *p* on Rebel Utility



As shown in the chart above, when the diamond price *p* increases, both the Rebel's expected wartime utility $E(U_{WR}*)$ and his utility from demobilizing ($U_{DR}*$) shift upwards. As a result, the maximum value of δ at which the Rebel chooses to demobilize decreases (i.e. the minimum sanction needed to achieve demobilization increases). Although an increase in the diamond price *p* increases the value of the diamond revenue that the Rebel could earn if he chooses to demobilize and to keep his first period labor supply, it also increases the value of the diamond revenue he could potentially earn in the second time period if he returns to war and wins control of the country's entire labor force. Since the labor resources available to the Rebel in the case of a military victory are substantially larger than the labor supply he keeps after demobilizing, an increase in the diamond price increases his incentive to return to war. When the proportion of the country's labor force that the Rebel owns in the first time period is smaller, an increase in the price results in a larger increase in the utility that the Rebel earns from returning to war (and therefore it takes a larger sanction to achieve demobilization).

IV. Summary of Results

The size of δ (i.e. the fraction of the world diamond price that the Rebel retains when sanctions are imposed on him), has a dual effect on his expected intertemporal utility from returning to war. As δ decreases, the value of the diamond revenue that the Rebel earns in the first time period decreases. However, a decrease in δ also raises the opportunity costs of producing diamonds, which causes the Rebel to substitute labor towards warfare, increasing the size of the Rebel army and thus raising the probability of a military victory in the second time period. For small values of δ, the substitution effect dominates, and the Rebel's expected wartime intertemporal utility decreases as δ increases (i.e. as sanctions become smaller). For large values of δ, the impact of the immediate price discount overrides the substitution effect and the

Rebel's utility increases as δ increase. Demobilization occurs, therefore, for moderate rather than extreme values of δ.

The size of sanctions only has an impact on the Rebel's behavior on a small interval. When the Rebel already controls more than one-fourth of the country's entire labor force in the first time period when he makes the decision to demobilize or to renege, he always chooses to demobilize regardless of the size of sanctions. The logical explanation behind this observation is that, when the Rebel already owns a substantial proportion of the country's labor supply, he will not have the incentive to return to war, which carries the risk of losing his entire labor supply in the case of a military defeat. In the case where the Rebel owns less than one-fourth of the country's labor resources, certain levels of sanctions could reduce his wartime utility and encourage him to demobilize rather than to return to war.

When the Rebel owns a smaller proportion of the country's entire labor force, the minimum level of sanction needed to achieve demobilization is larger (i.e. the maximum value of δ at which the Rebel chooses to demobilize is smaller). This occurs when the size of the country's labor force is large or when the Rebel's first period labor supply is small. When the Rebel owns less than one-fourth of the country's entire labor force in the first time period, the minimum sanction needed to achieve demobilization is larger when the world diamond price is higher. An increase in the world diamond price increases the value of the diamond revenue that the Rebel could potentially earn in the future, giving him a greater incentive to return to war. The smaller the proportion of the country's entire labor resources that the Rebel owns, the greater the impact of the world diamond price on raising the size of the sanction necessary to achieve demobilization.

V. Conclusions and Policy Implications

The first objective of this paper was to identify the conditions under which diamond sanctions, or the threat to impose diamond sanctions on a rebel group if it chooses to renege on a peace settlement, are an effective policy tool in facilitating demobilization. My finding that the Rebel automatically demobilizes when he owns a small share of the country's labor supply is counterintuitive. Historically, large rebel groups have usually been more successful in prolonging conflict than small groups; the more members of the group there are, the more labor the group has at its disposal to allocate to either diamond production or warfare.

However, my findings make sense given that my model assumes that the Rebel is allowed to keep his entire first period labor supply if he complies with demobilization. The interval on which the Rebel automatically demobilizes is large because of this "insurance" factor; if the Rebel is guaranteed a minimal level of utility in the case of demobilization, he chooses to demobilize when he already owns substantial labor reosurces in the first time period. These findings suggest that the inclusion of a resource-sharing deal in the terms of a peace settlement that gives the rebel group an incentive to demobilize may be more effective in conflict resolution than placing sanctions on its resources.

The second objective of this paper was to determine the minimum level of sanction needed to achieve demobilization given a set of exogenous conditions, on the interval where sanctions do have an effect on the Rebel's behavior. Since larger sanctions are more costly for the mediator to enforce and for the international diamond-importing community to comply with, in any scenario the most cost effective sanction is the minimum sanction that still gives the rebel group sufficient incentive to demobilize; this is the sanction that allows the rebel to retain the largest fraction of the world diamond price. The minimum sanction needed to achieve demobilization is larger when the size of the country's labor force is larger, or when the size of the rebel group is smaller. Since labor is a resource to be captured through warfare, a rebel group's incentive to return to war increases greatly when he owns a smaller proportion of the country's labor resources in the first time period, and therefore has more to gain and less to lose from warfare. Similarly, a higher world diamond price raises the value of the diamond revenue to be earned through warfare, giving the rebel group a greater incentive to return to war and raising the minimum level of sanction needed to achieve demobilization. When the world price of diamonds is higher than a certain level, or when the proportion of the country's labor supply that the Rebel owns is smaller than a certain level, the Rebel will always choose to renege and sanctions would be ineffective in this situation.

Although sanctions do play a role in reducing rebels' incentives and encouraging demobilization, the interval on which they have an impact is very small. The size of sanctions matters only when the size of the rebel group relative to the size of the country's entire labor force is moderately low, but not so low that the Rebel always chooses to renege. When sanctions do matter, the minimum, and therefore most cost effective, level of sanction needed to achieve demobilization is usually very small.

The given circumstances of conflict (the size of the country's labor force, the size of the rebel group, and diamond prices) are much more important than sanctions in determining the subgame perfect outcome (war or demobilization) that the rebel group chooses in the wake of a major civil war. When the conditions are "right" for demobilization, sanctions may influence the Rebel's decision to demobilize; however, these sanctions do not need to be very large or costly in order to achieve this outcome.

The two-stage Cournot game that I model in this paper is based on several assumptions. Firstly, I assume that both the Rebel and the Government have perfect information about the size of sanctions prior to making their labor allocation decisions. I also assume, for the sake of simplicity, that the country's total labor supply does not change over time periods. An interesting extension to this paper could examine the effects of a dynamic labor supply on the Rebel's incentive to demobilize. How would the subgame perfect outcome change when population changes caused the labor force to shrink or grow over time? The paper's findings could also be further extended by modeling a situation in which the Rebel and the Government must make their allocation decisions in the face of uncertainty. In this situation, the parameter δ becomes a random variable that determines the probability that the Rebel would choose to demobilize in the second time period.

My model also assumes that diamond production and warfare are mutually exclusive activities that could be substituted for each other according to their relative opportunity costs. In reality, however, rebel groups often capture the necessary inputs for diamond production – such as labor, land and diamond mines- through warfare. In such a situation, diamond production and warfare are actually complements in generating rebel revenue. The impact of sanctions on rebels' utility and demobilization decisions in the absence of a perfect substitution effect between diamond production and warfare is an important concept that my model does not address. By analyzing this scenario in further detail, future studies could greatly extend our comprehension of the role of diamond sanctions in civil conflict resolution.

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