The VIX as a Fix: Equity Volatility as a Lifelong Investment Enhancer

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Abstract

The VIX, a measure of the implied volatility of S&P 500 index options, is the premier gauge of investor sentiment and market volatility. This analysis examines the effectiveness of adding the VIX to passively managed equity-bond portfolios. Furthermore, this study extends the existing literature by examining the efficacy of the VIX in a life-cycle investing context. Due to the large negative correlation between the VIX and the major equity indices, we find that a relatively small allocation to the VIX would have significantly improved the risk-return profile of standard equity-bond portfolios from 1986 through 2007. Additionally, we find that younger investors (i.e. investors with higher risk tolerances and thus more exposure to equities rather than fixed income) will benefit from having greater exposure to the VIX.

1. Introduction

The goal of any good asset manager is to find a combination of assets that reduces risk without significantly affecting portfolio returns. Although riskier assets generally offer greater returns over the long-run, combining assets that are negatively correlated with one another can provide tremendous diversification benefits as the volatility of the portfolio returns is ultimately reduced. There are several asset classes that have historically exhibited negative correlations with the S&P 500 index portfolio, such as gold and oil, but these asset classes do not always display a consistent negative correlation and may come at a relatively high cost if they require active management. In the search for passive investments that reduce the risk of the overall portfolio without significantly affecting returns, it appears that equity volatility may offer a solution. In this paper, we examine the risk-return benefits of adding volatility to a portfolio comprised of equities and fixed income investments in the context of life-cycle investing.

Equity volatility as a *tradable* asset class is a relatively recent development. Although it has long been possible to gain exposure to volatility by trading options, these types of trades are not "pure play" trades on volatility. Option positions have exposure to market direction, in addition to volatility. Therefore, option traders must spend considerable time and financial resources "delta hedging" their position if they wish to rid themselves of directional risk. Furthermore, vanilla options are subject to theta decay. In other words, their value decreases simply with the passage of time, and therefore options cannot be included as an asset in a passively managed portfolio.

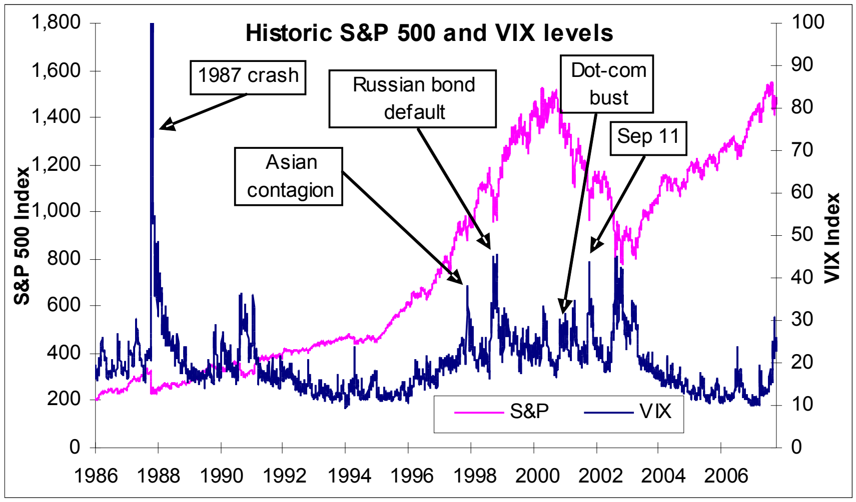
With the introduction of VIX futures (ticker symbol VX) in March 2004, however, the Chicago Board Options Exchange (CBOE) presented market participants with the ability to take a position on implied volatility independent of the level and direction of stock prices. VIX futures can realistically be included as an asset in a passively managed portfolio as the futures can be "rolled" relatively cheaply from one contract to the next as each contract expires. The following analysis will therefore examine the effectiveness of adding equity volatility in the form of the VIX as an additional asset to *passively* managed equity-bond portfolios.

2. Literature Review

2.1 VIX

The CBOE's trademark Volatility Index, or VIX, was introduced as an index in 1993 in a paper by Professor Robert E. Whaley (Whaley, 1993). The VIX is an implied volatility index; it measures the market's expectation of 30-day volatility as implied by the prices of S&P 500 index options. The method for calculating the VIX was updated in 2003. Although both the new and the old methodologies focus on 30-day implied volatility, the new VIX uses option prices of the S&P 500 (instead of the S&P 100) and is more robust, as all options traded in the first two contract months are included in the calculations (Black, 2006). Implied volatility often indicates financial instability, and as a result, the VIX has been referred to as the "investor fear gauge" (Whaley, 2000). When market participants are apprehensive, VIX levels tend to be elevated as the price of options, especially put options, are bid upwards to reflect increasing demand for protection against large negative market moves. Similarly, when the market is relatively stable, VIX levels tend to fall. In fact, the Wall Street Journal has gone as far as to say that the VIX represents "the fear and loathing in the markets at any given moment" (Gaffen, 2007). It is important to note that implied volatility is not the same as realized volatility and is typically higher than realized volatility because of the risk premium built into equity options. The average realized volatility of the S&P 500 from 1985 through 1999, for example, was 14.7% while the average implied volatility on the VIX over the same period was 19.8% (Traub, Ferreira, McArdle, and Antognelli, 2000). Implied volatility is a biased estimate of future realized volatility and not necessarily an accurate predictor of future realized volatility.

One of the most important features of the VIX in the context of portfolio diversification is its high negative correlation with the S&P 500 and other major equity indices. Although VIX futures were only introduced as a tradable asset in 2004 and the VIX itself was introduced as an index in 1993, we can analyze the historic performance of the VIX over the last 22 years, from Whaley (1993), which retroactively calculated the index to 1986. As noted earlier, the VIX often functions as an investor "fear gauge;" the index tends to rise sharply in response to major negative market events. *Graph 1* depicts the historic S&P 500 and VIX levels from 1986 to 2007. It clearly illustrates elevated VIX levels during five major negative market events of the last 20 years: the 1987 crash, the Asian currency crisis of 1997 and 1998, the Russian bond market default in 1998, the dot-com bust in March 2000, and September 11, 2001. From 1990 through 2006, the correlations of the monthly and daily returns of the S&P 500 and the spot VIX index are -61% and -65%, respectively (Moran and Dash, 2007).



Graph 1

(included with the permission of Rasiel, Temple, and Jacobs, 2008)

2.2 Research on the Value of Volatility as an Asset Class

Most investors have long equity positions and as a result are implicitly short volatility because of the negative correlation between equities and equity volatility (Hill and Rattray, 2004). The VIX, therefore, serves as a very powerful diversification tool and risk management aid. Numerous studies have supported this claim, concluding that adding volatility as a separate asset class to an S&P 500 portfolio reduces risk without significantly affecting return. Moran and Dash (2007) compared a portfolio made up of 100% equities to one composed of 95% equities and 5% VIX from 1990 through March 2007. They discovered that the 5% VIX allocation lowered the overall volatility of the portfolio by 92 basis points and increased the Sharpe ratio. Total return was only reduced by six basis points. In a 2003 Merrill Lynch report, Bowler, Ebens, Davi, and Kolanovic (as cited in Moran, 2004) found that a portfolio consisting of 90% S&P 500 and 10% VIX reduced risk by 25% and actually enhanced return by 5% when rebalanced on a weekly basis since 1986. Daigler and Rossi (2006) also examined the risk-return profile of an S&P 500-volatility portfolio compared to that of an S&P 500 only portfolio. They looked at data from 1992 through 2002 and concluded that adding VIX to the equity-only portfolio significantly reduced risk without a consequential affect on return. Furthermore, they determined that using the previous year's optimal weight in VIX to determine the next year's weight in VIX generated an S&P-VIX portfolio that was almost identical to the minimum risk portfolio. During this 11 year period, the daily correlation between the S&P 500 and VIX ranged from -45.4% to -82.4%. Rather than adding volatility to long-only equity portfolios, Moran and Dash (2005) examined the effect of adding volatility to hedge fund portfolios. They found that the VIX also has a relatively high negative correlation with hedge fund returns and therefore can be a very powerful diversification tool and risk management aid for hedge funds.

Although the VIX generally provides diversification benefits in all market conditions, it is most useful when the market performs extremely poorly and thus market participants require the most protection. In other words, it has an "asymmetric correlation profile" (Moran and Dash, 2005), with the correlation being more negative in contracting equity market periods. In 2003, Bowler et al. found that the VIX generally rises faster in sharply declining markets, while tending to fall more gradually in expanding markets. For example, during the Long Term Capital Management crisis in 1998 when Russia defaulted on its debt, investors who lost 15% in their stock portfolio during July and August would have been sufficiently hedged with a relatively small allocation to the VIX as it rose from 19.70 to 44.28 (Black, 2006). A CSFB report (Toikka, Tom, Chadwick, and Bolt-Christmas, 2004) found a correlation between one-week S&P and VIX returns of -0.49% when there was a market return between -1% and +1%, but the correlation had an even higher magnitude when the S&P 500 return was below -3%. Moran and Dash (2005) observed asymmetrical correlations between the VIX and long/short hedge funds. The correlation was -5% during months when the CSFB/Tremont Hedge Fund Index (HFI) delivered positive returns, but the correlation increased in magnitude to -44% in months when hedge funds lost money.

Another advantage of trading VIX futures is that they do not require active management. As Lawrence McMillan (2007) discusses, the VIX provides dynamic protection in which the hedge does not need to be consistently rebalanced when the market moves. Unlike vanilla options, which require frequent delta hedging to maintain directional neutrality, VIX futures permit a passive hedging strategy as they have no directional exposure. The futures can be "rolled" relatively cheaply from one contract to the next as each contract expires. Furthermore, because of the extreme volatility of the VIX itself, McMillan explains that a relatively low allocation to the VIX (~10%) is sufficient to hedge an equity portfolio, and thus insurance costs are kept at a minimum. It is important to note that the behavior of VIX futures (VX) does not precisely mirror that of the VIX index. In fact, VIX futures tend to be less volatile than the VIX index. However, the correlation between the two is very high (92% during the three-year period from December 2004 through December 2007). Rasiel, Temple, and Jacobs (2008) extrapolated VIX futures back to 1986, based on the relationship between VIX futures and VIX from 2004 to 2007, and showed that the futures offer very similar hedging properties in an all-equity portfolio.

2.3 Research on Life-cycle Investing

Although previous studies have illustrated the advantages of adding the VIX to equity portfolios, this is the first study to examine the benefits of adding the VIX in a life-cycle investing context. The theories on modern portfolio investing started with the theory of diversification, which concluded that it was possible to extend the efficient frontier (i.e. reduce risk without decreasing expected return) by adding assets that are not perfectly correlated to the portfolio (Markowitz, 1952). Based on this theory, many economists have developed models to determine how an investor should allocate funds throughout the course of an investor's life-cycle. Four specific approaches have been popularized as efficient ways to allocate investors' money over their lifetimes: the 100-minus age rule, the Malkiel approach (1990), the Shiller plan (2005), and the new "L-fund plan" offered to federal employees through a Thrift Savings Plan that is similar to a 401(k) (Kintzel, 2007). Each of these approaches is based on the principle that one should shift money from stocks to bonds as retirement approaches. This principle, in turn, is based on several demographic and economic factors affecting investors' abilities to make and spend money over the course of their lives. Human capital (ability, education, experience, etc.) is the largest and most important one of these factors. A good proxy for measuring the value of human capital is the present value of wages over an individual's remaining working life, and because this is generally much less variable than equity returns, one should shift asset allocation from stocks to bonds as one ages in order to maintain a consistent risk exposure (Kintzel, 2007).

While there has been substantial research on the value of adding the VIX to an S&P portfolio, our research analyzes the benefit of including the VIX in a combination equity and bond portfolio, and one in which the allocation between these assets varies over time.

3. Methodology

Daily price levels of the S&P 500, 10-year US Treasury notes, and the VIX from 1986 through the end of 2007 were collected and converted into daily log returns r_i, where:

$$(1)\quad \mathrm{r_i} = \ln(\frac{\mathrm{p_i}}{\mathrm{p_{i-1}}}) \quad where\ i = each\ day \text{ and } p = asset\ prices$$

The daily log returns for each three month period (non-overlapping) were then used to calculate quarterly returns and volatility over the sample period (88 quarters).

$$(2)\quad r_q = \{1 + (\frac{\sum_i^{66} r_i}{66})^{260}\} - 1 \quad where\ q = quarter\ (1,2,..88)$$

$$(3)\quad \sigma_q = \{\sqrt{\frac{66\sum r_i^2 - (\sum r_i)^2}{66 * 65}}\} * \sqrt{88}$$

After quarterly returns and volatility were calculated, assets were combined to create portfolios with various weights *w* of the three assets. The quarterly portfolio returns, volatilities, and Sharpe ratios for two and three asset portfolios were calculated according to the following formulas:

$$\begin{aligned} \mathrm{r_{qs}} &= \text{quarterly return S \& P 500} \\ \mathrm{r_{qb}} &= \text{quarterly return 10 — year U.S. Treasury note} \\ \mathrm{r_{qv}} &= \text{quarterly return VIX} \end{aligned}$$

$$(4)\quad r_p = \sum_j w_j r_{qj} \quad where\ j = (s, b, v)$$

$$(5)\quad \sigma_p = \sum_j \sum_k w_j w_k \sigma_{qj} \sigma_{qk} \quad where\ j, k = (s, b, v)$$

Note: Sum(w_j) = 1, and $w_j > 0$ for all j

$$(6)\ Sharpe\ Ratio = \frac{r_p}{\sigma_p}$$

The average Sharpe ratio of each portfolio over the 88 quarters was then calculated. Going forward, we refer to the S&P 500 as "equities" and the 10-year US Treasury notes as "bonds."

We created a *base case* portfolio consisting of 80% equities and 20% bonds: the BC_{80} portfolio (where the subscript number indicates the fixed portfolio weight in equities). While maintaining 80% in equities, we then varied the amount allocated to bonds by permitting some proportion of the remaining 20% to be in the VIX, using the three methodologies outlined below.

For methodology A, we maximized portfolio return in each quarter by varying bond and VIX allocations, with the constraint that the volatility of the overall portfolio could be no greater than the volatility of the BC_{80} portfolio for that particular quarter. Thus, the amount allocated to the VIX for each quarter, w_v, varied between 0% and 20%, with $w_b = 20\% - w_v$. The additional constraint in this optimization was that neither bonds nor the VIX could be sold short ($w_V, w_b > 0$). In some quarters, therefore, the allocation to the VIX is zero rather than a negative position. We call methodology A the Quarterly Maximum Return portfolio, or QMR_w (QMR_{80}, for example, is the portfolio with 80% in equities, with the remaining 20% allocated optimally between bonds and the VIX).

For methodology B, the percent allocated to the VIX was held constant for every quarter and was determined by taking the average optimal VIX allocation for each of the 88 quarters from methodology A. For QMR_{80}, the average VIX allocation from methodology A was 7.35%, so 12.65% of the portfolio was allocated to bonds (with 80% still in equities). We label this methodology the Average Optimal VIX portfolio, or

AOV_w. Since we do not permit any short selling, methodology B is not necessarily the optimal VIX allocation across time.

For methodology C, the percent allocated to the VIX was held constant for every quarter and was determined by an iteration procedure that established the asset allocation combination that would maximize the average Sharpe ratio over the 88 quarters. We label this methodology the Maximum Average Sharpe ratio portfolio (MAS_w). We chose to include this variation since portfolio managers are often evaluated based on their Sharpe ratios. For MAS_{80}, the VIX allocation that maximized the average Sharpe ratio over the 88 quarters was 6.5%, leaving 13.5% allocated to bonds, and again 80% in equities.

The quarterly portfolio return, volatility, and Sharpe ratios for each quarter were calculated for all three methodologies across all 88 quarters. The average return, volatility, and Sharpe ratio for the quarters was then calculated for all portfolios.
In addition to the portfolios with 80% equity allocations, we also created portfolios with allocations to equities of 60%, 40%, and 20%. As with the 80% equity portfolios, we used the three methodologies to vary bond and VIX allocations. In each case, the allocation to equities remained constant (at either 60%, 40%, or 20%), while the allocation between bonds and the VIX was permitted to vary based on the optimization methodologies discussed above.

4. Data

	Avg. Quarterly Return	**Avg. Quarterly Volatility**	**Avg. Quarterly Sharpe Ratio**
S&P 500	2.53%	7.63%	0.514
10Y US T-Bonds	0.61%	3.66%	0.209
VIX	2.99%	44.66%	0.003

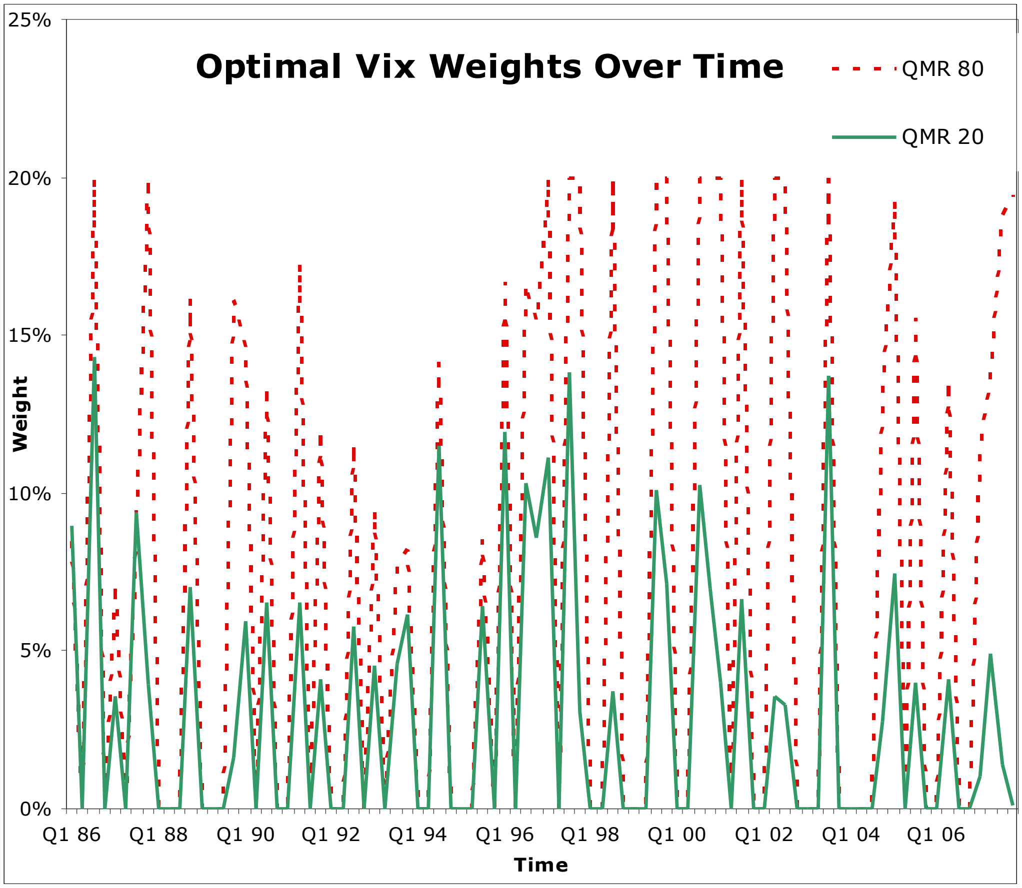
Table 1- Macro Data

The data presented in *Table 1* represent the index macro data that was used to create the portfolios in the study. One of the most interesting features of the data is that the average quarterly return of the VIX is greater than that of the S&P 500 from the beginning of 1986 through the end of 2007 (2.99% vs. 2.53%). As Dash and Moran (2007) noted, the VIX is a volatility series and therefore has no intrinsic value (unlike say a share of stock or an ounce of gold). Therefore, over time, not only should the VIX return be less than the return of the S&P 500, but the VIX should be mean reverting and have a long-term return of zero. However, there are several explanations for why we do not see this in our data. First, the 22-year sample is not a particularly long one. It is essentially a random sample that became the entire sample of this study because the VIX has only been calculated back to 1986. There are myriad sub-periods within the data in which the average quarterly return is positive, negative, or close to zero. Second, the average quarterly standard deviation of the VIX is very large (44.66%). With volatility this large, a mean of 3% is not significantly different from zero. Finally, there may actually be a "secular" upward trend in volatility. This possibility remains to be seen as more data is required to determine whether this is a long-term or periodic trend.

5. Results

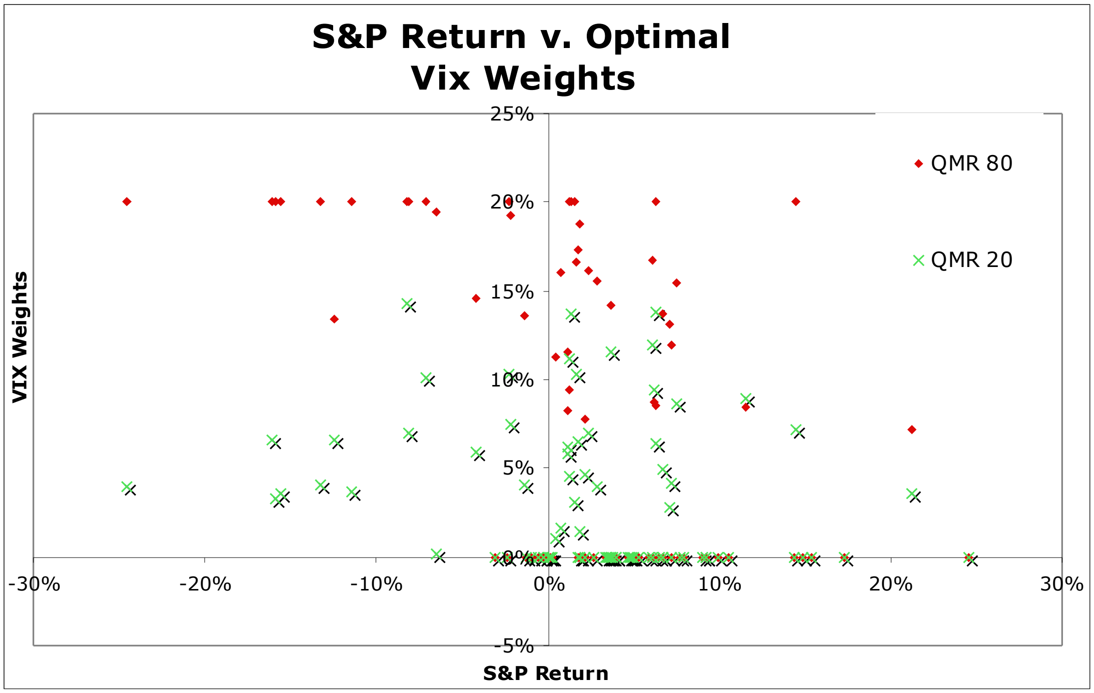
The addition of the VIX to standard equity-bond portfolios improved the risk-return profile of the respective portfolios. Since 1986, the average returns and Sharpe ratios of the four equity-bond portfolios would have all increased if some small percentage of the portfolio was allocated to the VIX (instead of bonds). *Graph 2* shows the optimal VIX weights in each quarter for both the QMR_{80} and the QMR_{20} portfolios. It is clear from the graph that the portfolio with a greater allocation to equities (QMR_{80})

always required a greater allocation to the VIX and that the optimal VIX allocations were fairly volatile themselves.



Graph 2

Graph 3 below illustrates the inverse relationship between S&P returns and optimal VIX allocations for both the 80% equity and 20% equity scenarios. Clearly, a smaller allocation to the VIX was generally more favorable during expanding markets, while a greater allocation to the VIX was preferable during contracting markets. Further, a greater portfolio weight in equities (80% versus 20%) required a commensurate increase in allocation to the VIX; however, the absolute VIX allocations were still relatively small.



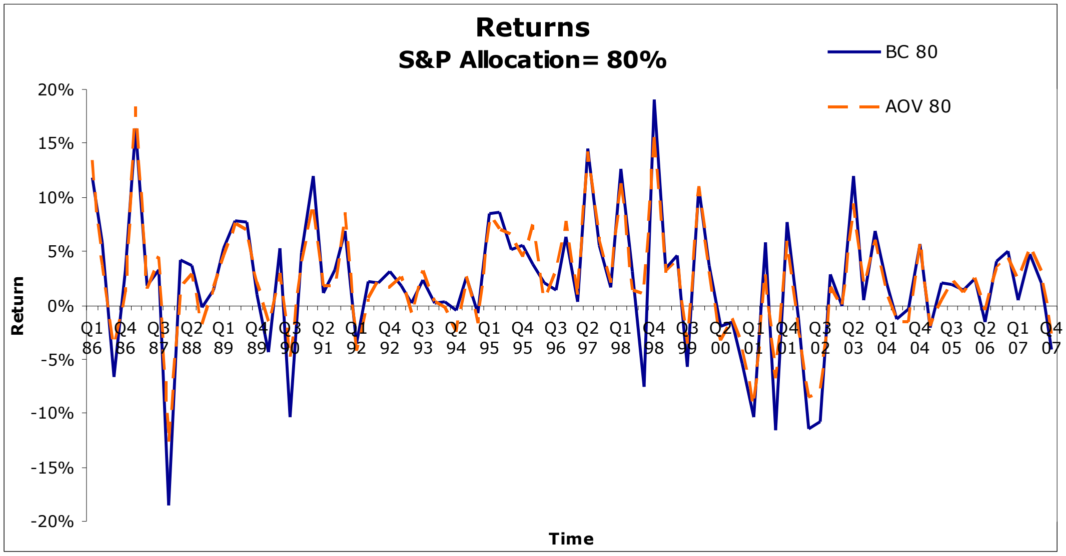
Graph 3

The average of these optimal VIX allocations was then used to determine the weighting for the VIX in the AOV portfolios (See *Table 2*).

	80% S&P Portfolio	60% S&P Portfolio	40% S&P Portfolio	20% S&P Portfolio
Average Optimal VIX Allocation	7.35%	6.86%	4.94%	3.01%

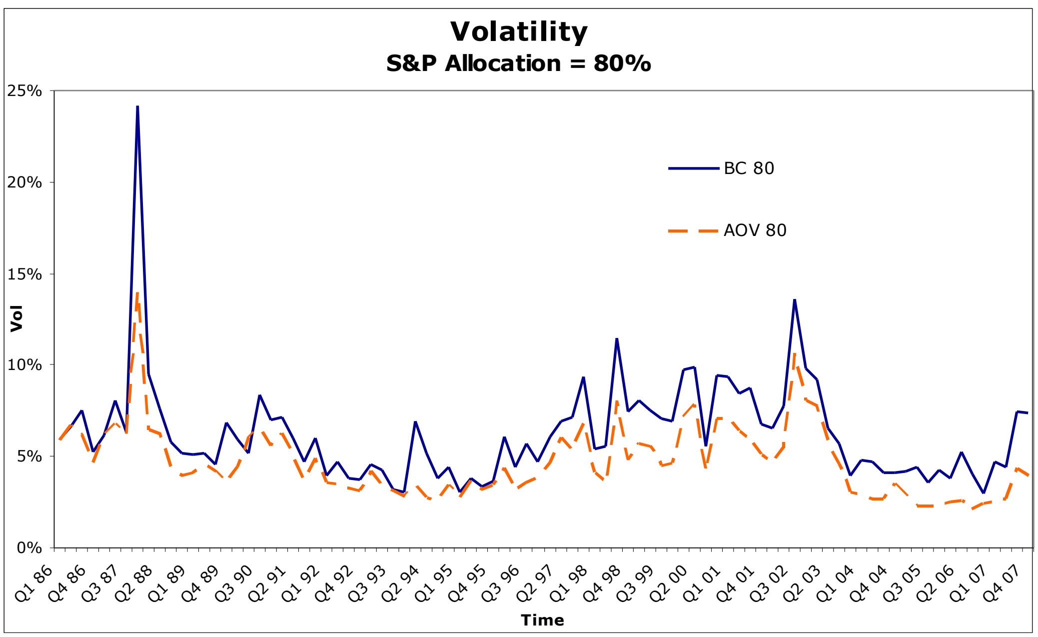
Table 2- VIX Allocations for AOV Portfolios

Graph 4 illustrates that the AOV_{80} portfolio marginally outperformed the BC_{80} portfolio. The average quarterly return of the BC_{80} portfolio is 2.14%, while that of the AOV_{80} portfolio is 2.32%. Importantly, the AOV_{80} outperformed the BC_{80} portfolio by the greatest amount in the quarters in which there were large negative returns in the equity-bond portfolio. For example, in the fourth quarter of 1987 when the BC_{80} portfolio returned -18.52%, the re-weighting of 7.35% from bonds to the VIX yielded a return of -2.98%, which represents a substantial reduction in the negative return.



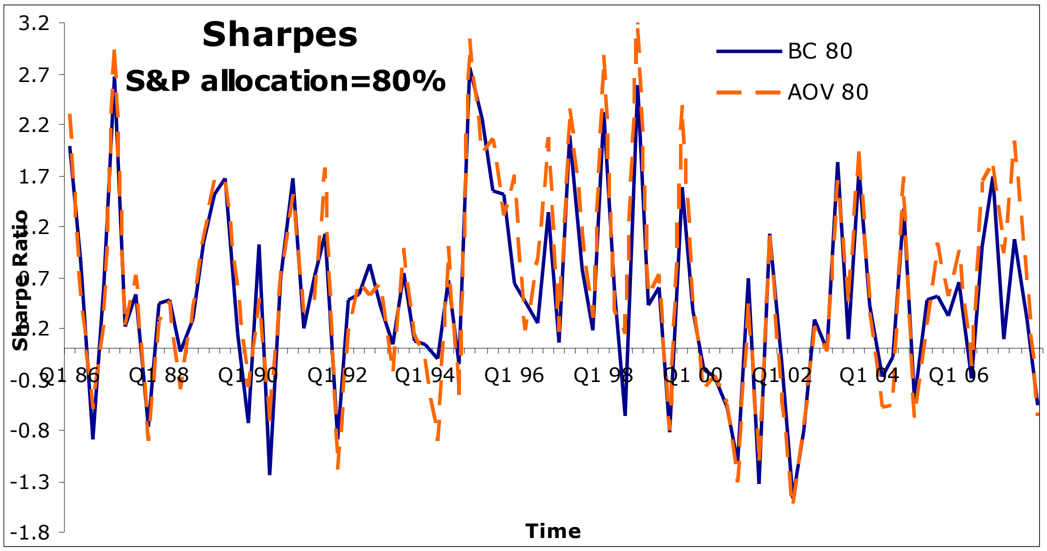
Graph 4

The true benefit of allocating a small proportion of the portfolio to the VIX is made clear when the portfolio volatilities are compared (See *Graph 5*). The quarterly volatility of the AOV_{80} portfolio is always lower than the volatility of the BC_{80} portfolio. The average quarterly volatility of the BC_{80} is 6.25%, while the average quarterly volatility of the AOV_{80} is 4.68%, a considerable reduction in quarterly volatility.



Graph 5

Finally, the quarterly Sharpe ratios were compared over time (see *Graph 6*). The Sharpe ratio for the AOV_{80} portfolio was greater than the Sharpe ratio of the BC_{80} portfolio in 60% of the quarters, and the average Sharpe ratio of the three-asset portfolio was approximately 20% higher (0.60 vs. 0.50).



Graph 6

While adding the VIX as an asset to an equity-bond portfolio had the greatest positive effect on portfolio returns and volatilities when 80% of the portfolio was weighted towards equities, the addition of the VIX to portfolios with S&P allocations of 60%, 40%, and 20% also increased the average quarterly returns, decreased the quarterly volatilities, and increased the average quarterly Shape Ratios of these portfolios. An estimation of the QMR asset weights for the portfolios with 60%, 40%, and 20% S&P allocations determined that the VIX weights for the AOV portfolios should be 6.86%, 4.94%, and 3.01%, respectively. *Table 3* below shows the average returns, volatilities and Sharpe ratios of these portfolios relative to their base cases.

	Average Return	Average Volatility	Average Sharpe Ratio	% Improvement in Sharpe
BC 80	2.14%	6.25%	0.50	
AOV 80	2.32%	4.68%	0.60	20%
BC 60	1.76%	5.00%	0.47	
AOV 60	1.92%	3.73%	0.55	17%
BC 40	1.38%	3.96%	0.41	
AOV 40	1.49%	3.15%	0.49	20%
BC 20	0.99%	3.39%	0.35	
AOV 20	1.07%	3.10%	0.39	11%

Table 3- Comparison of BC and AOV Portfolios

Thus we have shown that the three-asset AOV portfolios performed better than the BC portfolios. However, we noted earlier that average is not necessarily the best measure of determining the optimal allocation in the VIX (due in part to the no short sale constraint). Therefore, we estimated the MAS portfolios, in which the VIX allocation was determined by maximizing the average quarterly Sharpe ratios of the portfolios. Although the allocations to the VIX for both the AOV (Average Optimal VIX) and MAS (Maximum Average Sharpe ratio) portfolios improved the average quarterly Sharpe ratios substantially, it is worth noting that the VIX allocations for these portfolios were not the same. In fact, the VIX allocations for the MAS portfolios were between 0.5% and 1.7% lower than VIX allocations in the AOV portfolios (see *Table 4*). This result is most likely due to the no short sales constraint that was put in place to determine VIX allocations for the QMR (Quarterly Maximum Return) portfolios. We determined that it would have been optimal to have no allocation to the VIX for 47 of the 88 quarters (53% of the quarters) for all four of the BC (Base Case) scenarios. If we had permitted short sales, VIX allocations for these quarters would have been negative rather than zero. Therefore, since AOV portfolio allocations were calculated by averaging the QMR VIX allocations, the AOV portfolio VIX allocations were skewed slightly higher.

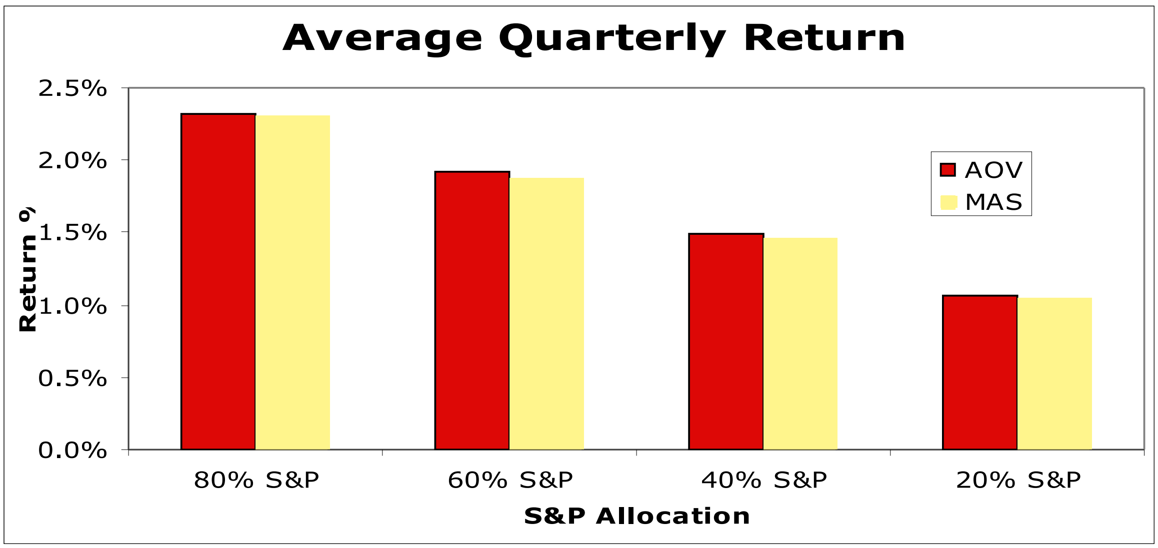
Table 4 also compares the average Sharpe ratios of the MAS portfolios with those of the AOV portfolios. The AOV portfolios had slightly higher average quarterly returns and slightly lower average quarterly volatilities, despite having lower average Sharpe ratios. This result is due to the overwhelmingly high volatility of the VIX as an asset class by itself.

		% Allocated VIX	**Avg. Return**	**Avg. Volatility**	**Avg. Sharpe Ratios**	**% Improvement from Base**
80% S&P Portfolio						
	BC 80	0%	2.14%	6.25%	0.499	N/A
	AOV 80	7.35%	2.32%	4.68%	0.600	20.2%
	MAS 80	6.50%	2.30%	4.76%	0.603	20.8%
60% S&P portfolio						
	BC 60	0%	1.76%	5.00%	0.468	N/A
	AOV 60	6.86%	1.92%	3.73%	0.554	18.4%
	MAS 60	5.25%	1.88%	3.82%	0.565	20.7%
40% S&P portfolio						
	BC 40	0%	1.38%	3.96%	0.412	N/A
	AOV 40	4.94%	1.49%	3.15%	0.491	18.3%
	MAS 40	4.00%	1.47%	3.19%	0.495	19.3%
20% S&P portfolio						
	BC 20	0%	0.99%	3.39%	0.354	N/A
	AOV 20	3.01%	1.07%	3.10%	0.396	11.9%
	MAS 20	2.50%	1.05%	3.10%	0.398	12.4%

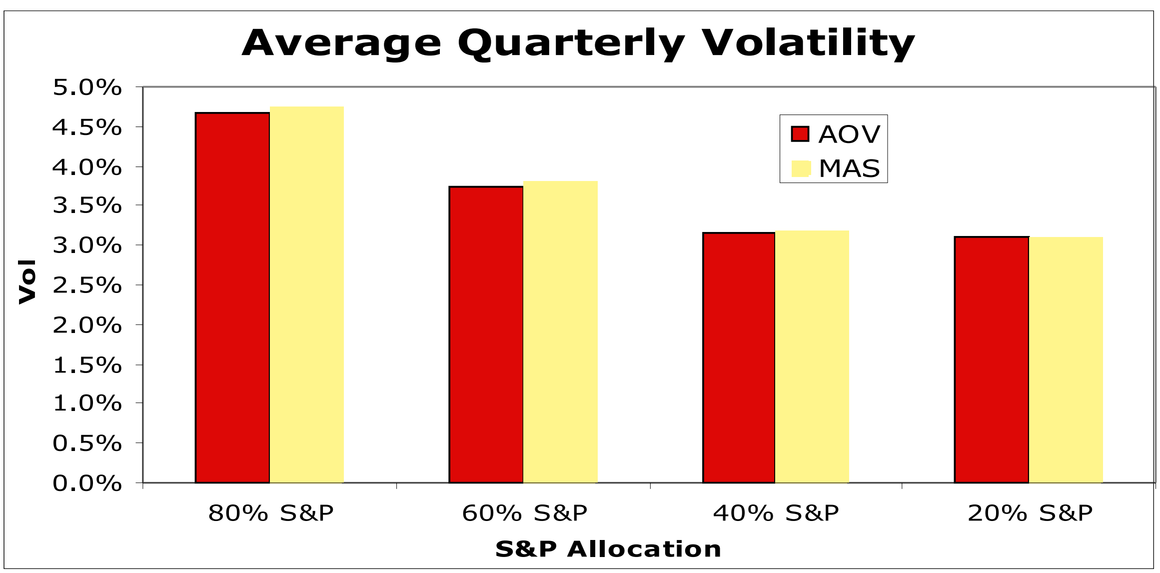
Table 4- Comparison of BC, AOV, and MAS Portfolios

While the return levels and volatilities for the AOV and MAS portfolios were fairly consistent across time, a comparison of the average Sharpe ratios shows a slightly better risk-return relationship for the MAS portfolios as compared to the AOV portfolios.

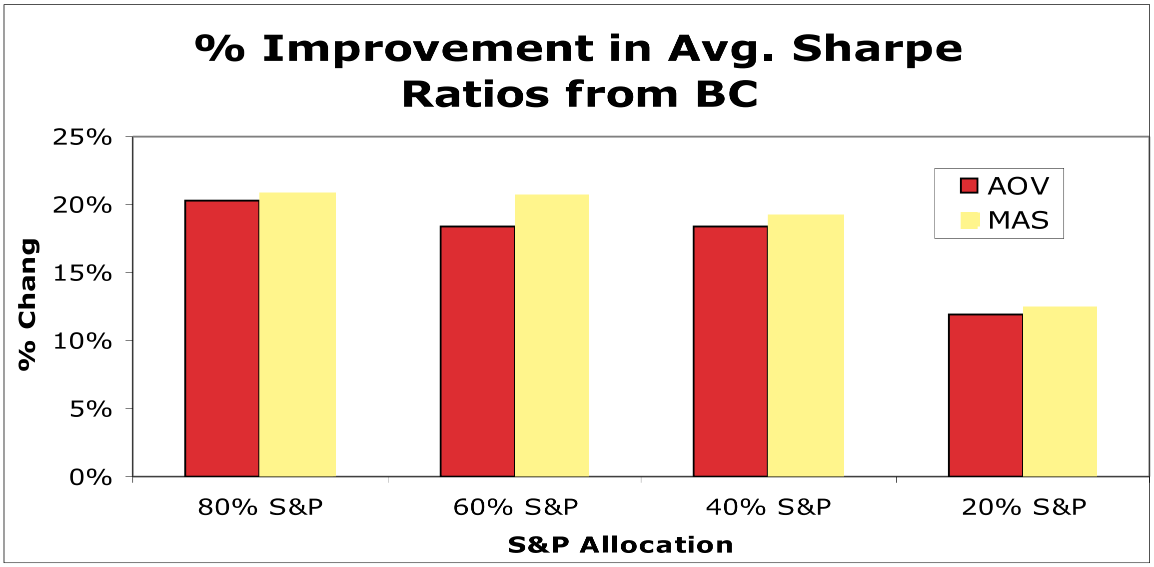
As noted earlier, the addition of the VIX had the largest impact on the portfolios' average Sharpe ratios when a larger portion of the portfolio was allocated to equities (See *Graphs 7, 8, and 9* below).



Graph 7



Graph 8



Graph 9

Of the two realistic passive strategies, the MAS portfolio improved the average Sharpe ratios by the greatest percentage.

6. Discussion

In past studies, the VIX has proven to be a very valuable asset in enhancing the risk-return profile of equity-only portfolios (Moran and Dash, 2007; Bowler et al., 2003; Daigler and Rossi, 2006). Due to liquidity and risk-aversion considerations, however, asset managers and other market participants, even those with longer term investment horizons, generally do not invest in equity-only portfolios. The current study, therefore, sought to take a more practical approach by examining the benefits of including the VIX in a portfolio with allocations to both equities and fixed income securities. The main conclusion of the study supported the conclusions of previous research: relatively small allocations to the VIX substantially lower the risk of the overall portfolio without significantly affecting the returns. This result is driven by the strong negative correlation between VIX and equity returns. Jeremy Siegel (2008) points out that this correlation may seem perplexing because one might expect market participants to demand greater protection when the market is high rather than low. However, when equity values are diminishing, investors are generally more eager to buy out-of-the-money puts, which drives up the implied volatility, and thus the value of these options. This increase in implied volatility is reflected in an increase in the level of the VIX. Additionally, as the prices of puts are driven up by investors who are looking for downside protection, arbitrageurs who sell puts must sell stocks in order to hedge their position and remain delta neutral (Siegel, 2008). This technical phenomenon may send stocks even lower, potentially increasing the magnitude of the negative correlation between VIX and equity returns. A third explanation for the strong negative correlation is that, historically, volatility has been greater in bear markets than in bull markets. Expectations based on historical volatilities will therefore cause implied volatilities to exhibit similar behavior.

We found a negative correlation between quarterly S&P 500 returns and optimal VIX allocations. In other words, a smaller allocation to the VIX was preferable during bull markets, while a greater allocation to the VIX was preferable during bear markets. This result is certainly to be expected due to the large negative correlation between S&P and VIX returns. However, it is important to note that our study mainly focused on how to improve the risk-return profile for passive (buy-and-hold) investors. If an investor was willing to take an active role in portfolio management or was willing to pay for active management, the VIX allocation could be adjusted based on forecasts of S&P performance. If the investor believed the S&P would increase in value, he/she might choose to reduce the allocation to the VIX, and vice-versa. Furthermore, other studies have shown that VIX levels can be used to time the market (for further discussion please refer to Traub, et. al, 2000; Chadwick, 2006; Connors, 2002).

The first step of the current study was creating the QMR portfolio described in the methodology section of the paper. While the QMR portfolio does appear to be the portfolio that increases the average Sharpe ratio by the greatest amount on a percentage basis, this is not a realistic investment portfolio. Since this portfolio is constructed retroactively after observing the quarterly returns and volatilities of equities, bonds, and the VIX, it is impractical to consider this to be a reliable investment strategy. While one could create a portfolio of these three assets based on market forecasts, the goal of this paper was not to come up with market timing investment strategies, but to create optimal *passively* managed portfolios.

The current study went beyond the scope of previous research by examining the advantages of including the VIX in a life-cycle investing context. Although the four principal life-cycle investing approaches (the 100-minus age rule, Malkiel approach, Shiller approach, and the Thrift Savings plan) vary with regards to what percentage should be allocated to each investment class at certain ages, all four are based on the principle that one should gradually shift money from stocks to bonds as retirement approaches in order to maintain a constant risk exposure. The current study examined optimal VIX allocations for investors with high risk tolerances/longer investment time horizons (portfolios with 80% S&P 500), for investors with intermediate risk tolerances/intermediate investment time horizons (portfolios with 60% and 40% S&P 500), and investors with low risk tolerances/shorter investment time horizons (portfolios with 20% S&P 500). Due to the consistently high negative correlation between equity indices and equity volatility, we found that that portfolios with greater allocations to the S&P 500 required greater allocations to the VIX in order to maximize average quarterly Sharpe ratios.

Another important result was that the VIX allocations in the MAS (Maximum Average Sharpe ratio) portfolios improved the average quarterly Sharpe ratio by the greatest percentage for the portfolio with the largest (80%) allocation to the S&P (and thus the greatest allocation to the VIX). The least improvement was seen in the portfolio with the lowest (20%) allocation to the S&P (and thus the lowest allocation to the VIX). This result underscores the importance of the VIX as an asset class to younger investors, who can substantially decrease portfolio volatility without significantly affecting returns. Further supporting the idea that the VIX is a more beneficial asset for investors with longer time horizons is that the volatility of the VIX is much greater than that of the S&P 500. Using 2006 as an example, the volatility was 94% for the VIX index, while it was just 10% for the S&P 500 Index (Moran and Dash, 2007). Investors with longer time horizons can afford to ride large short term market fluctuations, while older investors could get hurt by volatile markets to a much greater degree.

Despite the proven advantages of investing in the VIX, there are clearly limitations and risks associated with such an investing strategy. First of all, because the VIX is a volatility series, it has no intrinsic value and has an expected long-term return of zero (Moran and Dash, 2007). Therefore, although it may provide diversification benefits in many market conditions, it will not itself be a long-run source of return. Second, although the VIX has historically exhibited a high negative correlation with the S&P 500, there have been a number of instances where declines in the equity market have not been offset by significant positive returns from the VIX. When combined with a long equity position, a long volatility position may prove advantageous during market crashes, but when equity markets are suffering persistent losses during an extended bear market, both positions may suffer. For example, in the last mild bear market year of 2001, the S&P 500 fell 13% while the VIX itself declined 11.4% (Dizard, 2007). Furthermore, as all asset managers are inclined to tell their clients, past performance is not a guarantee of future results, and it may be that some technical or fundamental reason causes the VIX's high negative correlation with the S&P 500 to become less pronounced. Dizard (2007) points out the CBOE's disclosure regarding VIX futures: "VIX futures may also provide an effective way to hedge equity returns, to diversify portfolios, and to spread implied against realized volatility." "That statement," he says, "clearly suggests that VIX futures also may not do any of those things."

Another possible negative consequence of including the VIX in a passively managed, life-cycle portfolio are the negative tax consequences of investing in futures as compared to equities, bonds, and other vanilla products. Futures are 60/40 products, meaning that 60% of the gain/loss is long-term for tax purposes and the remaining 40% is short-term (Commodities Futures Modernization Act, 2000). However, the capital gains/losses from purchasing equities and bonds, if held for a minimum of one year, are all considered to be long-term. Given that there are many more years in which equity markets rise as compared to fall, in the up-market years, 60% of the losses on VIX futures would be long-term losses. Long-term losses are generally seen as less efficient for tax paying investors. Additionally, if profits were made from investing in VIX futures, 40% of the gains would be taxed as short-term, which is also less efficient for individual investors (Spiegelman, 2006).

6.1 Limitations to Study

We made several assumptions in this study. First, we assumed a risk-free rate of zero for all time periods. Given that 10-year US Treasury notes were included in the life-cycle portfolios, we decided not to use any varying benchmark treasury note as the risk-free rate. Additionally, nominal returns for all assets were used as opposed to real returns and thus inflation was not considered. Another limitation was that the analysis did not account for S&P 500 dividends. Finally, the study did not account for transaction costs or the liquidity of the different assets. Although equity indices and treasury bonds, especially macro benchmarks like the ones that were used, tend to be very liquid, VIX futures have proven thus far to be less liquid as the VIX index is a relatively recent development.

6.2 Areas for Further Study

There are myriad areas of further study that could contribute to our understanding of the diversification benefits of including the VIX and other volatility investments in traditional passively managed portfolios. First, one could determine optimal VIX weights in portfolios that include assets in addition to the S&P 500 and 10-year US Treasury notes. The risk-return profiles of portfolios that include allocations to such assets as commodities, international equities, corporate bonds, currencies, and alternative investments may be enhanced by allocations to the VIX. Studies such as these could be very beneficial to asset managers who have customers that would like portfolios to contain assets other than vanilla equities and bonds.

Another interesting study would be to examine the diversification benefits of other volatility indices (such as the VXN, RVX, etc.). One could also compare the contribution of other volatility investments (volatility swaps, variance swaps, exotic options, ATM straddles, etc) with volatility index futures.

Additionally, determining how often it is optimal to rebalance a portfolio with the VIX would add another dimension to this study (weekly, monthly, quarterly, etc). Although the VIX is dynamic and does not have to be rebalanced very often (McMillan, 2007), it may be optimal to rebalance after a certain time period. A few other approaches to investing in volatility could also be compared. For instance, one might compare pure bets on implied volatility (VIX) with bets on implied vs. realized volatility (variance swaps).

Finally, in this study we used VIX data instead of VIX futures data. While the VIX is not actually a tradable asset, the correlation between the VIX and VIX futures is very high. It has been found that VIX futures contribute to equity-based portfolios significantly, but at a slightly lower level than the VIX would (Rasiel, Temple, and Jacobs, 2008). Therefore, a study that used VIX futures instead of the VIX index may prove to be slightly more practical when it comes to trading implementation.

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