The Effect of the Learning Curve on the Optimal Dynamic Contract

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April 9th, 2008

Honors Thesis

Economics

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An increase in the marginal productivity of laborers due to the existence of a learning curve illustrates a potential dynamic that should affect the structure of the employer-employee contracts. This paper investigates the role of moral hazard in a dynamic setting given the presence of a learning curve and how this influences the shape of the optimal contract offered by the principal. By extending the standard principal agent problem to both a multi-action space for the agent and a dynamic setting, the analysis of the learning curve becomes tractable. The typical issues raised in the standard principal-agent problem including; the role of unverifiable information, the role of risk averseness, sufficient conditions for the use of first order conditions, and the role of the cost functions of each action form the basis of the analysis.

1. Introduction

The (negative) influence of unobservable actions and asymmetric information on the productivity of a particular contract represent issues that principals and agents alike have to deal with in a variety of situations from tenure contracts to executive bonuses. As the effort level of employees, athletes, teachers, etc. is often unobservable it would not be unwarranted to assume that some level of asymmetric information exists in any employee-employer relationship. The inability of the principal to contract an effort level, however, would not pose enough of a problem to warrant a critical analysis, because of the ability to observe the output/outcome, and thus the effort indirectly. Even if direct correlation between effort level and output did not exist the principal could still write an optimal contract. The effect risk averseness has on the feasibility of an optimal contract illustrates the final defining characteristic of the employee-employer relationship and the one feature that frames the "Principal-Agent problem". As soon as the agent possesses any aversion to risk the achievability of an optimal contract becomes questionable. This risk aversion creates a conflict between sheltering the agent from this risk and providing the agent with the proper incentives to exert the pareto optimal level of effort.

The standard problem outlined above consists of the agent taking one action and the principal writing a contract for one particular moment in time. Typically in the real world, contracts span over a significant period of time. In addition, the tasks assigned to the agent spread over a multi-dimensional action space. Examples of such multi-dimensional action spaces range from questions regarding the trade-off between "quantity and quality" to "productivity and maintenance". The multi-dimensional space framing the choice between exerting "effort towards output now" or "investment in human capital for output in the future" seems particularly interesting. The principal clearly wants the agent to engage in both of these particular actions, however, it is not clear how the principal will write an optimal contract if *both* of these actions are unobservable while at the same time influential for the outcome. By developing a series of optimization problems that face both the principal and agent, the learning curve will be shown to produce an optimal dynamic contract that becomes relatively more powerful in later periods, while remaining unresponsive to the outcomes in earlier periods.

2. Literature Review

The principal-agent problem and the models that have been developed have come in a variety of forms and have drawn several different conclusions. The topics and issues that will need more thorough attention include the influence of a dynamic perspective on the optimal contract, and the effect a multi-dimensional action space has on the shape of the optimal contract. Some ideas such as firm specific capital and reputation will also need surveying and the analysis of dynamic models will address both. Though some of the theoretical models and the conclusions regarding the implications of a multi-dimensional action space have their basis in a static perspective, many of the principles will still apply to the study of dynamic contracts in the principal-agent problem. A brief overview of the material that defines the principal-agent problem with emphasis on the implications of dynamic perspectives and multi-dimensional action space follows.

2.1 Standard Principal Agent Model

The standard one-period principal-agent problem models the role of the informational gap caused by the delegation of tasks from the principal to the agent. In the standard model, a principal offers a one-period contract to an agent to complete a particular task. While the particular action taken by the agent is not observable there does exist an observable outcome over which the principal can contract over. When the actions taken by the agent are "[not] observable, either by the principal who offers the contract or by the court of law that enforces it"[1] the situation possesses *moral hazard*. The "hidden action" taken by the agent, while not deterministic in the observable outcome, does result in a probability distribution for each of the observable outcomes. The given action of the agent endogenously determines the uncertainty and subsequent probabilities of the observable outcomes. While these observable outcomes maintain a certain level of uncertainty they do offer a "noisy signal" of the particular action taken by

[1] Laffont and Martimort, 2002.

the agent. If this uncertainty did not exist, no contractual problem would exist because of the ability for either the principal or courts to infer the agent's chosen action from the observed outcome.

Given that the principal and the agent will have differing objective functions, there exists a conflict in what action the agent should take. "The nonobservability of the agent's action may then prevent an efficient resolution of this conflict of interest, because no enforceable contract can ever stipulate which action should be taken by the agent."[2] The principal's goal, given the uncertainty of the observable outcome, then becomes to find the contract that will provide the largest gap between the expected wage and the expected gains of the effort level induced by that contract. What will constrain this optimization problem include a set of *Incentive Constraints* and a *Participation Constraint.* The *Incentive Constraints* ensure that the agent does not prefer taking any other action aside from the one the principal desires. The *Participation Constraint* ensures that the agent prefers engaging in the contract to some outside alternative.

> The first best situation is defined by the assumption that the Principal can observe the Agent's action. In that case he can order the Agent to choose the efficient action, and then choose the wages that achieve the optimal risk sharing…[however]…in the second-best situation we are concerned with, the Principal can only observe a variable correlated with the Agent's action: the outcome….Solving the moral hazard problem thus implies that the Principal offers the Agent a contract that trades off 1)risk sharing, which suggest that the Agent's wage should not depend too strongly on the outcome and 2) incentives, which induce the Principal to condition the Agent's wage on the outcome.[3]

[2] Laffont and Martimort, 2002.
[3] Salenie, B., 1997.

Grossman and Hart (1983) outline the feasibility of finding an optimal contract that yields a second best alternative and the methods in which to find and define this contract. The new approach the paper brings to identifying optimal contracts by "breaking up the principal's problem into a computation of the costs and benefits accruing to the principal when the agent takes a particular action"[4] illustrates one of the more illuminating aspects of the paper. While this approach generalizes the method of determining optimal contracts, Grossman and Hart were also the first to outline the conditions needed to guarantee the legitimacy of using first order conditions. Grossman and Hart (1983) define two important conditions restricting the type of stochastic behavior; monotone likelihood ratio condition (MLRC) and convexity of distribution function (CDFC). Rogerson (1985) determined that the satisfaction of these two conditions guarantee the use of first order conditions as a valid method to determine the optimal contract. These two conditions, though defined initially in a single action space, will prove influential in allowing first order conditions to govern the requirements of the optimal contract even when the agent makes decisions over a multi-action space. In order to guarantee the legitimacy of using first order conditions in the multi-dimension action space, convexity of the distribution function needs generalizing. Specifically MLRC and CDFC will be extended to restrict the stochastic behavior of a distribution function in $\mathcal{R}^2$.

As far as the particular shape of these optimal contracts Grossman and Hart (1983) illustrate no intuitive shape of the optimal contract will exist in general. Perhaps the most alarming result proves to be that the wage need not be increasing in outcome. Holmstrom (1979) also illustrated, using the *sufficient statistic theorem,* that the principal

[4] Grossman and Hart, 1983.

should condition the wage on all the signals received. The optimal contract in the presence of the learning curve, however, proves to be independent of some of the observed signals; most notably signals from earlier periods. This characteristic of the optimal contract represents a significant departure from the literature.

In addition, if the analysis allows for an infinite (usually continuous) set of outcomes (desired) then the contract will become a function, and the decision of the principal in selecting this contract lends itself to functional analysis[5]. In general, only a compact functional space guarantees the existence of an optimal contract. Compactness does not represent a typical feature of function spaces and thus will require restricting the functional space in order to ensure the existence of a solution (Page 1987).

One particular restriction that maintains a certain level of robustness outlined by Holmstrom and Milgrom (1987) suggests that optimal contracts may be simple (linear) contracts as opposed to highly complex ones. These results will lay the groundwork for the beginning of the analysis of a multi-action, dynamic principal-agent problem which will have the same functional analysis dilemmas that the standard problem has to negotiate.

The influence of renegotiation in even a static principal-agent problem demonstrates how dynamic implications can enter into the analysis of a one-period model. Fudenberg-Tirole (1990) illustrate that if the two parties may renegotiate after the exertion of the chosen effort an efficiency loss will result. Though renegotiation does not appear relevant in the immediate analysis of the learning curve, the dynamic implications of even one-period decisions remains influential. Renegotiation, in the context of the

[5] Salanie, Bernard, 1997.

learning curve, will turn out to represent a point of vulnerability to the robustness of the optimal contract.

2.2 Dynamic Extensions of the Standard Model

One of the more intuitive aspects of the dynamic models of the principal-agent problem includes the role reputation has on the optimal contract written by the principal and the action of the agent. The first discussion of the importance of reputation concerns in the principal-agent problem came from Eugene Fama.[6] The major breakthrough developed in this piece includes "the wage revision process imposed by the managerial labor market,"[7] given the "weight of the wage revision process is at least equivalent to full ex post settling up."[8] Fama attributes the survival of the modern corporation to these very concerns by the agents or managers in firms; however, Fama's paper taken at face value leaves little room for the possibility of dynamic principal-agent problem.

A more rigorous look at the role of reputation in the dynamic principal-agent problem comes from Holmstrom (1999). One of the defining features of Holmstrom's model versus most of the others mentioned in this survey comes in the form of his approach to the "ability" of the agent. In Holmstrom's model, the ability of the agent takes the form of a random variable, of which both the principal and the agent have expectations over a dynamic period of time. Even though the agent has concerns about the signal present actions will have on future wages (ala Fama) because of the uncertainty of the agent's ability Holmstrom concludes that "one can certainly not make any

[6] Fama, Eugene, 1980.
[7] Fama, 1980.
[8] Fama, 1980.

sweeping arguments about moral hazard problems in the long-run"[9]. This finding lends considerable weight to the notion that "contracts [in a dynamic model] clearly play an important role"[10], however, it does little in the way of suggesting a shape of that optimal contract, and what effect investment in human capital might have on the analysis.

Jovanic (1979) explores the role of human capital or learning on the job from experience. One of the more innovative characteristics of Jovanovic's model includes his approach of "firm specific capital" or the knowledge gained over time beneficial only to the current match. While Jovanovic model's use of imperfect information and a dynamic perspective (specifically when it comes to "firm-specific capital"), will prove to be incredibly applicable, because the agent bears all of the costs of searching for new employment some of the important aspects of the principal-agent problem are lost. So, though most of the conclusions supported by Jonanovic's model will indeed prove the basis of the initial framework, especially when it comes to the analysis of human capital, still other interpretations are needed. These other interpretations will include those that take a dynamic approach on the principal-agent problem exclusively. In this paper the dynamic contract's duration will be both certain and exogenous, and commitment by both the principal and agent will limit the uncertainty in the model to only the realized output.

2.3 Multi-dimensional Action Space Extension

Because in this analysis the agent has a decision over investing in human capital or exerting effort to present output, both literature dealing with the investment of capital

[9] Holmstrom, 1999.
[10] Holmstrom, 1999.

and multi-task jobs have applicability. The gains in marginal productivity from investment in human capital by an agent parallel the concept of regulating the investment in "cost saving" methods by a firm.

A significant number of theoretical models analyzing the effects of regulation on firms mimic the principal-agent problem and the various features of the relationship. Huseyin Yildirim (2002) develops a model where a regulator must create the incentives for a monopolist to minimize current costs of service while encouraging development of cost-saving innovations (ala investment in human capital). Yildirim finds that light-handed regulation encourages innovation and this innovation will occur in the absence of long-term agreements when private information recurs over time. Though the fundamental relationship being analyzed is different, the equilibrium solution in Yildirim's model will have parallels to the solution in this particular analysis because of the multi-dimensional action space of the agent. In Yildirim (2002) the state variable evolves stochastically not deterministically, however, which will cause significant differences in some of the fundamental principles of the analysis when examining the learning curve.

Another theoretical justification for less powered contracts in real life include Holmstrom-Milgrom's (1991) use of a multi-task model. If many tasks compete for the agent's effort the principal may have "to reduce the power of the incentives he provides to the agent"[11]. The cost curves of both actions including the costs relative to each other, illustrates an important aspect of the multi-task model that will need careful consideration when developing the learning curve model. Holmstrom-Milgrom's model also illustrates the importance of the observability of the each of the particular actions. Because

[11] Salanie, 1997.

observability of one action does not necessarily imply observability of the other action the possible contract space indeed becomes richer. Initially this paper will develop a model where both actions are unobservable. Later, this paper will examine how being able to observe investment in human capital will affect the efficiency of a particular dynamic contract.

In general the methods and conclusions found in the past will serve as the fundamental building blocks in order to build a theory that models the optimal contract and its relation to the learning curve. This paper suggests that such a model will be obtained by combining two extensions of the principal-agent problem; dynamic choice, and multi-action space for the agent.

3. The Principal Agent Problem with a Learning Curve

3.1 Two-Period, Discrete Multi-Action, Two Output Level, Principal Agent Problem

The first model will find the optimal contract given a two period horizon where the agent has a discrete multi-action space (effort and investment in human capital). If the agent chooses to invest in human capital in the first period, there will be human capital stock in the final period (initially there is no human capital stock). Both the effort level and the human capital stock in each period will affect the shape of the distribution function that will give the likelihood of the two discrete outcome levels for that particular period. The principal will thus have a maximization problem of finding the optimal wage given both the incentive constraints and the participation constraint.

The verifiable output/outcome that can be contracted will be defined by: $\{\underline{x}, \overline{x}\} \sim \{0,1\}$. The gains realized by the principal will be a function of x, given by: $S(x) = \{\underline{S}, \overline{S}\}$. The agent will have a discrete multi-action space where $e = \{0,1\}$, is effort; and $s = \{0,1\}$, is investment in human capital. In each period the agent will face the option of either exerting effort, investing in human capital (but not both), or shirking (doing neither). The utility function of the agent will be the constant absolute risk aversion (CARA) concave separable function used in Holmstrom-Milgrom (1987) illustrated by: $U(w,e,s) = -e^{-r(w)} - C(e,s)$. There also exists some function $h(u) = U^{-1}$.

The dynamic contract offered by the principal by the sufficient statistic theorem will take the form: $W(x_1,x_2) = \{w(x_1), w(x_1,x_2)\}$. In the context of a two possible outcome model, the resulting contract space will offer two values for period one, and four different values for period two (corresponding to the four different combinations of outcomes that can result). The total contract space can be depicted by the following:

$$W(x_1,x_2) = \{w(\overline{x}_1) = a_1,\ w(\underline{x}_1) = a_2; w(\overline{x}_1,\overline{x}_2) = b_1 + c_1, w(\underline{x}_1,\overline{x}_2) = b_2 + c_1, w(\overline{x}_1,\underline{x}_2) = b_1 + c_2, w(\underline{x}_1,\underline{x}_2) = b_2 + c_2\}.$$

Each action e and s has a respective cost function where each will be defined by: $C(e) = \{C(0) = 0, C(1) = C_e\}$ and $C(s) = \{C(0) = 0, C(1) = C_s\}$. The level of human capital stock, l, will exhibit the following equations: $l_i = \{l_1 = 0, l_2 = s_1\}$. Again, in the first period there will not be any human capital stock present. In the second period there will only be human capital stock present if the agent chose to invest in human capital in the first period. There exists a distribution function for the observed outcome x given by $\pi(x{:}\, e, l)$. This distribution will give probabilities to the likelihood of $\bar{x}$, given the effort level and the level of human capital stock, where the respective probabilities will be:

$$\pi(\overline{x}; 0,0) = \pi_0 \text{ and } \pi\left(\underline{x}; 0,0\right) = 1 - \pi_0$$

$$\pi(\overline{x}; 0,1) = \pi_0 \text{ and } \pi\left(\underline{x}; 0,1\right) = 1 - \pi_0$$

$$\pi(\overline{x}; 1,0) = \pi_1 \text{ and } \pi\left(\underline{x}; 1,0\right) = 1 - \pi_1$$

$$\pi(\overline{x}; 1,1) = \pi_2 \text{ and } \pi\left(\underline{x}; 1,1\right) = 1 - \pi_2$$

$$\text{With } 0 \leq \pi_0 \leq \pi_1 \leq \pi_2 \leq 1$$

It is important to note that these distributions exhibit the monotone likelihood ratio condition (MLRC) or that "the expected outcome increases in [effort and investment in human capital]"[12] as well as CDFC[13].

There will exist some outside alternative for the agent that will guarantee a reservation utility of, $\overline{U}$ for the two periods combined. A contract that gives a utility greater than or equal to this reservation utility will satisfy the *Participation Constraint*; $U(w, e, s) \geq \overline{U}$. This assumption, although common in the literature and essential for the subsequent analysis, implies some subtle generalizations that need noting. The participation constraint in this form makes a requirement on the *sum* of the two utilities achieved in each period. Accordingly, there will not be a requirement made on the minimum level of utility achieved by the agent in any one of the two periods. The principal may provide a wage contract that offers the agent a wage, $w_1 = h(\overline{U})$ in the first period and nothing in the second period. While this assumption increases the

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[12] Salenie,B., 1997.
[13] See Appendix 3.

flexibility of the contract space the principal optimizes over, it prevents concerns about the agent's minimal level of utility at each period from reaching into the analysis.

Another important consequence of this particular participation constraint includes its unresponsiveness to any of the endogenously determined values in the model. Because the outside alternative is exogenous, control and state variables such as the human capital stock, the realized output in the first period, or even the wage obtained in the first period cannot influence the value of this outside alternative. While keeping the outside alternative exogenous may not mimic reality, this assumption does have a strong connection with the notion of "firm-specific capital" initially cited in the literature by Jovanic (1979). If the human capital stock present does not have an influence on the respective outside alternative, then this particular human capital stock must only improve the marginal productivity of the agent in this particular job-match. Similarly, this assumption implies that the realized output in the first period does not send a signal to the other principals and increase/decrease the demand for the agent.

This model will assume full commitment from both the principal and agent. This assumption prevents the effects of renegotiation from impacting the analysis and the shape of the optimal contract. Though common in the literature, the possible effects of renegotiation after the first period will be noted before completing the analysis.

An illuminating Figure that represents the time-table of the renegotiation-proof dynamic contract follows:

t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t =7
Principal offers Wage = { $W(x_1), W(x_2)$}	Agent accepts or rejects the contract	Agent exerts an effort or invests in human capital or neither	Output is realized	Contracted wage is executed for period one	Agent exerts an effort or invests in human capital or neither	Output is realized	Contracted wage is executed for period two

Figure 1.

Now that the model has been fully specified it is time to develop the notion of the learning curve and begin to analyze its effect on the optimal dynamic contract.

Proposition 1: The ***learning curve*** *will be a distribution function that allows investing in human capital in period one by the agent to benefit the principal.*

The action space is defined as coordinates in $(e_1, s_1, e_2, s_2) \in \mathcal{R}^4$ such that $e_1, s_1, e_2, s_2 \in [0 \text{ } or \text{ } 1] \text{ } and \text{ } e_i + s_i \leq 1$. Proposition 1 rephrased, claims that in the presence of a *learning curve,* (0,1,1,0) must represent an optimal action choice for the principal to contract the agent to take.

In order to define the *learning curve* explicitly one must move the analysis to a scenario of verifiable information and determine when the principal contracting the action space(0,1,1,0) proves optimal. Given verifiable information one can assume that the wage offered for each period will be exactly $W = \{w_1 = h\left(\frac{\bar{U}+C_e+C_s}{2}\right), w_2 =$

$h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\}$ allowing for the satisfaction of the *Participation Constraint*. In this step of the analysis one can disregard the need to satisfy any incentive constraints because in presence of perfect and verifiable information, any deviation from the contracted action could be penalized by way of a fine either by the principal or a court of law.

So, if a *learning curve* exists then the principal must gain from having the agent engage in (0,1,1,0) as opposed to (1,0,1,0) or (0,0,1,0). Essentially the following inequalities must hold:

1) $$2h\left(\frac{\overline{U}+C_e+C_s}{2}\right) - 2h\left(\overline{U}/2 + C_e\right) \leq [(\pi_0 + \pi_2) - 2\pi_1](\overline{S} - \underline{S})$$

2) $$2h\left(\frac{\overline{U}+C_e+C_s}{2}\right) - 2h\left(\frac{\overline{U}+C_e}{2}\right) \leq (\pi_2 - \pi_1)(\overline{S} - \underline{S})$$[14]

Claim 1: There is a *learning curve* if and only if (1) and (2) hold.

These inequalities should make intuitive sense in that *marginal cost of investing in human capital* ~ $2h\left(\frac{\overline{U}+C_e+C_s}{2}\right) - 2h\left(\overline{U}/2 + C_e\right)$ or $2h\left(\frac{\overline{U}+C_e+C_s}{2}\right) - 2h\left(\frac{\overline{U}+C_e}{2}\right)$ must be less than the *marginal benefit of investing in human capital* ~ $[(\pi_0 + \pi_2) - 2\pi_1](\overline{S} - \underline{S})$ or $(\pi_2 - \pi_1)(\overline{S} - \underline{S})$, in order for investing in human capital to be optimal. The learning curve has two distinct parts: 1) the gain in *likelihood* of success and 2) the gain in *output* of success. Each of these distinct parts will have significantly different influences on both the feasibility as well as the shape of the optimal dynamic contract.

Now one must consider the Principal's optimization problem, given the presence of this *learning curve*. The optimization problem can be characterized as:

[14] See Appendix 1.

3) $$\text{Max } \pi_0(\overline{S}-a_1)+(1-\pi_0)(\underline{S}-a_2)+(\pi_0\pi_2)\left(\overline{S}-(b_1+c_1)\right)+\left(\pi_0(1-\pi_2)\right)(\underline{S}-(b_1+c_2)+\left((1-\pi_0)\pi_2\right)\left(\overline{S}-(b_2+c_1)\right)+(1-\pi_0)(1-\pi_2)(\underline{S}-(b_2+c_2))$$

With respect to $a_1, a_2, b_1, b_2, c_1, c_2$

Such that the incentive constraints are satisfied:

4) $$(\pi_1-\pi_0)(e^{-ra_1}-e^{-ra_2})+[\pi_2(e^{-rc_1})+(1-\pi_2)(e^{-rc_2})][\pi_0(-e^{-rb_1})+(1-\pi_0)(-e^{-rb_2})]-[\pi_1(e^{-rc_1})+(1-\pi_1)(e^{-rc_2})][\pi_1(-e^{-rb_1})+(1-\pi_1)(-e^{-rb_2})]\geq C_s-C_e$$

5) $$(\pi_1-\pi_0)(e^{-ra_1}-e^{-ra_2})+[\pi_2(e^{-rc_1})+(1-\pi_2)(e^{-rc_2})][\pi_0(-e^{-rb_1})+(1-\pi_0)(-e^{-rb_2})]-[\pi_0(e^{-rc_1})+(1-\pi_0)(e^{-rc_2})][\pi_1(-e^{-rb_1})+(1-\pi_1)(-e^{-rb_2})]\geq C_s$$

6) $$(\pi_2-\pi_1)[e^{-rc_1}-e^{-rc_2}][\pi_0(-e^{-rb_1})+(1-\pi_0)(-e^{-rb_2})]\geq C_s$$

7) $$(\pi_2-\pi_0)[e^{-rc_1}-e^{-rc_2}][\pi_0(-e^{-rb_1})+(1-\pi_0)(-e^{-rb_2})]\geq C_s+C_e$$

8) $$(\pi_2-\pi_0)[e^{-rc_1}-e^{-rc_2}][\pi_0(-e^{-rb_1})+(1-\pi_0)(-e^{-rb_2})]\geq C_e$$

By examining these incentive constraints one can conclude that: $a_2 \geq a_1$; $b_2 \geq b_1$; $c_1 \geq c_2$. These inequalities are supported by each of the incentive constraints. This intuitively should make sense because the principal desires to give positive incentives for good outcomes in the second period, while penalizing "good" outcomes in the first period, when the agent should be investing in human capital. Given that the above inequalities hold, the satisfaction of 6) and 7) will ensure that 4) and 5) hold respectively in addition to 8). Incentive constraints 6) and 7) thus become the focus of the analysis.

Both of these constraints are completely independent of the wage executed in the first period, and can only be satisfied if $c_1 \gg c_2$ (rewarding a "good" outcome in period two). Note also that each constraint depends on b_1, b_2 in a negative relationship unambiguously. This conclusion is consistent with the notion Holmstrom-Milrom (1991) describe when creating incentives for a multi-action space. The incentives of one action can in fact *dampen* the marginal effects of the incentives for the other because of the concavity of the utility functions. When these conclusions are combined with the alternative objective of the principal to minimize the wage bill, some very powerful simplifications can be made. First, $b_1 = b_2 = 0$, due to the binding of the non-negative constraints on the wage bill. Finally, $a_1 = a_2$, due to incentive constraints 6) and 7) being independent of period one's wage and the cost associated with subjecting the agent to any risk in period one. *The wage offered in period one will thus be a constant wage, and the wage offered in period two will be independent of the output realized in period one*. Unlike the dynamic contracts in the literature, the optimal dynamic contract in the presence of the learning curve will not exhibit a *memory*.

For the purpose of easing the analysis on the *strength of power* of the wage contract, the contract offered, from now on in the analysis, will be a linear combination of a constant term (wage) in addition to an incentive term (bonus): $w(x) = a + b(x)$. In addition the principal will choose a dynamic contract such that an affine wage is written for both periods; $W(x_1, x_2) = \{a_1 + b_1(x_1),\ a_2 + b_2(x_2)\}$. The last two significant steps in reducing the constrained optimization problem include noting that 7) guarantees 8), and 6) linearly depends on 7) and thus both cannot bind unless:

$$\frac{(\pi_2 - \pi_0)}{C_s + C_e} = \frac{(\pi_2 - \pi_1)}{C_s}$$

When all of these findings are combined a much simpler set of incentive constraints will result.

Optimization Problem #1:

Note it will be assumed that:

$$\frac{(\pi_2 - \pi_0)}{C_s + C_e} \geq \frac{(\pi_2 - \pi_1)}{C_s}$$

Max $\quad \pi_0\left(\overline{S} - (a_1)\right) + (1 - \pi_0)\big(\underline{S} - (a_1)\big) + \pi_2\left(\overline{S} - (a_2 + b_2)\right) + (1 - \pi_2)\left(\underline{S} - (a_2 + b_2)\right)$

With respect to $a_1, a_2, b_2 \geq 0$, such that:

$$(\pi_2 - \pi_1)(-e^{-ra_2})(e^{-rb_2} - 1) \geq C_s$$

$$(-e^{-ra_1}) - C_s + (-e^{-ra_2})[\left(\pi_2(e^{-rb_2})\right) + (1 - \pi_2)] - C_e \geq \overline{U}$$

The optimal wage contract then becomes $W = \{0, \frac{1}{r}\ln\frac{\pi_2 - \pi_1}{(1+\pi_2)C_s} + \frac{1}{r}\ln\frac{1+\pi_2}{\pi_2}x_2\}$[15]

Optimization Problem #2:

Note it will be assumed that:

[15] This optimal wage is based on the assumption that the Participation Constraint is not binding. This assumption allows for a much more concise closed form solution, however, the conclusions that are drawn from this solution are not specific to this assumption.

$$\frac{(\pi_2 - \pi_0)}{C_s + C_e} \leq \frac{(\pi_2 - \pi_1)}{C_s}$$

$$\text{Max } \pi_0 \left(\overline{S} - (a_1)\right) + (1 - \pi_0)\left(\underline{S} - (a_1\right) + \pi_2 \left(\overline{S} - (a_2 + b_2)\right) + (1 - \pi_2)\left(\underline{S} - (a_2 + b_2)\right)$$

With respect to $a_1, a_2, b_2 \geq 0$, such that:

$$(\pi_2 - \pi_0)(-e^{-ra_2})(e^{-rb_2} - 1) \geq C_s + C_e$$

$$(-e^{-ra_1}) - C_s + (-e^{-ra_2})[\left(\pi_2(e^{-rb_2})\right) + (1 - \pi_2)] - C_e \geq \overline{U}$$

The optimal wage contract then becomes $W = \{0, \frac{1}{r}\ln\frac{\pi_2 - \pi_0}{(1+\pi_2)(C_s + C_e)} + \frac{1}{r}\ln\frac{1+\pi_2}{\pi_2}x_2\}$[16]

Value of $b_2$vs. π_2

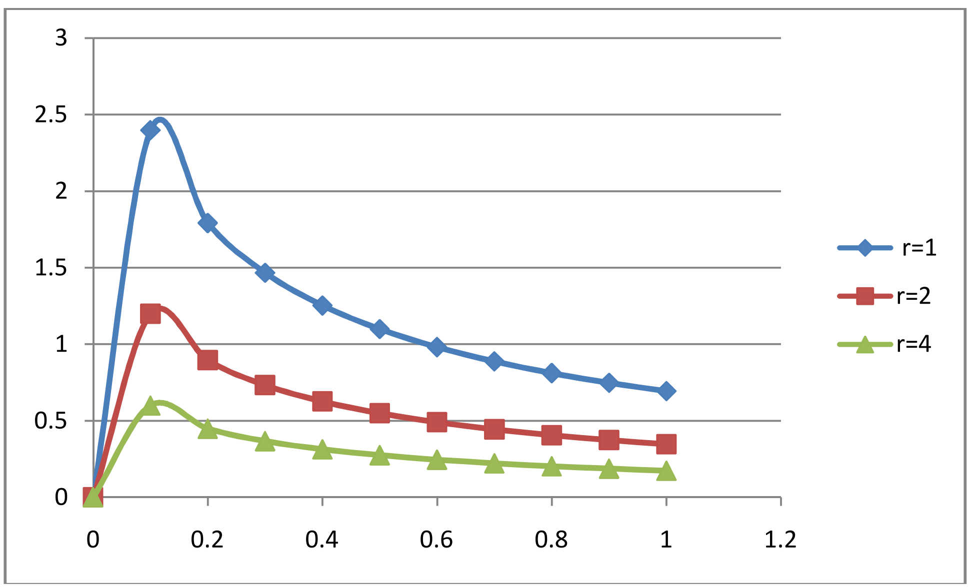

Figure 2

In the Figure 2 three different functions (each corresponding to three different levels of risk aversion) mapping the value of b_2 versus π_2 suggest that given a particular risk aversion, as the probability of success after investment in human capital gets larger,

[16]. This optimal wage is based on the assumption that the Participation Constraint is not binding. This assumption allows for a much more concise closed form solution, however, the conclusions that are drawn from this solution are not specific to this assumption.

the bonus required to satisfy the *Incentive Constraints* decreases. In addition, for a fixed likelihood of success, increases in risk aversion will also decrease the bonus term that satisfies the *Incentive Constraints.* The costs of investment in human capital, and effort as well as the outside alternative remain constant throughout this part of the analysis.

3.2 Two Period, Continuum Multi-Action Space, Infinite Outcome Principal-Agent Problem

In the next theoretical model the verifiable outcome as well as the action space for the agent will each be infinite continuum sets. Notation of these extensions will follow as:

$$x \in (\underline{x}, \overline{x})$$

$$e \in (0, +\infty)$$

$$s \in (0, +\infty)$$

In addition to the continuum extensions, the probability density function for the verifiable outcome (again a function of both effort and human capital stock) will become absolutely continuous. This density will be represented by:$p(x; e, l)$ where $x \in (\underline{x}, \overline{x})$. One important restriction on this density includes that the $Var(x) = \sigma^2$, constant for all values of e and s. In other words, the level of uncertainty remains the same regardless of the particular values of effort or human capital stock, or as the agent exerts effort or becomes better endowed with human capital the level of "noise" in the realized output holds constant. Graphically an example of the family of densities that might result from the multi-action space with constant variance is illustrated by Figure 3. Note that each density has differing expected values, dependent on e, l, but that the spreads of each remain fixed.

Probability Densities of X

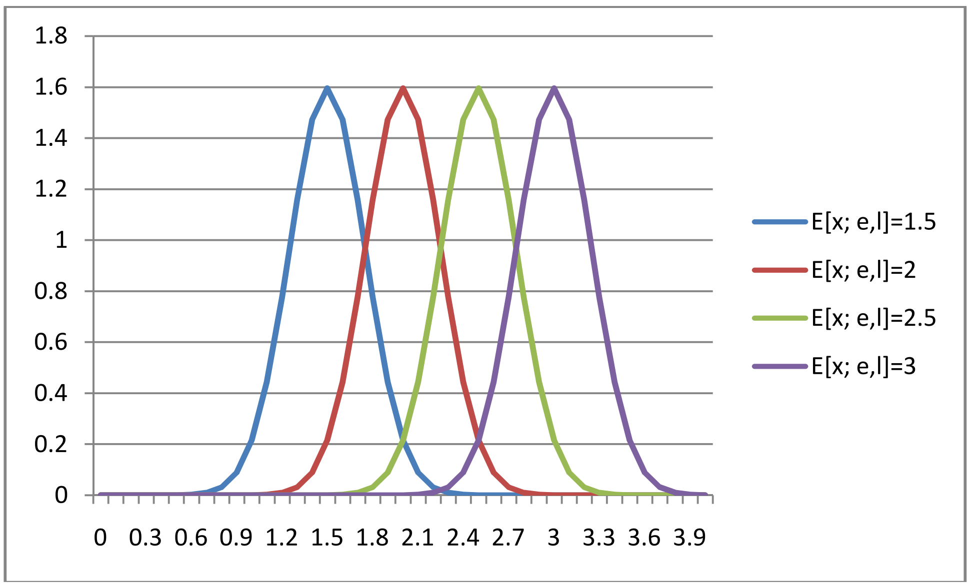

Figure 3.

The utility function of the agent will again be a constant absolute risk aversion concave separable function illustrated by: $U(w, e, s) = -e^{-r(w)} - C(e, s)$. There also exists some function $h(u) = U^{-1}$ that is also differentiable. The wage contract takes the form of a linear combination of a constant term in addition to a incentive term: $w(x) = a + b(x)$. The principal will choose a dynamic contract such that an affine wage exists for both periods; $W(x_1, x_2) = \{a_1 + b_1(x_1),\ a_2 + b_2(x_2)\}$. The respective costs of e and s combine to form a continuous convex cost function where the following requirements must be met: $C(e, s), C_e'(e, s), C_{ee}''(e, s), C_s'(e, s), C_{ss}''(e, s) \geq 0$ as well as $C''_{ee}(e,s)C''_{ss}(e,s) \geq C''_{es}(e,s)^{2}$. The level of human capital stock, l, will follow the equations: $l_i = \{l_1 = 0, l_2 = s_1\}$. There will again exist some outside alternative for the agent that will guarantee a reservation utility of, $\overline{U}$. A contract that gives a utility greater then or equal to this reservation utility satisfies the *Participation Constraint*; $U(w,e,s) \geq \overline{U}$.

The timetable of the dynamic contract will have the same structure as the first theoretical model, shown in figure 1.

In order for tractable analysis, and ultimately for the valid use of First Order Conditions, extensions of both MLRC and CDFC need to be defined and satisfied[17] moving their definitions from a single action space to a multi-dimensional action space. MLRC and CDFC, given absolute continuity of the distribution functions, require the distribution functions of the outcome ($F(x) = \int_{-\infty}^{x} p(u;e,l)du$) be convex in action. Given that the distribution functions in this model will be functions of both effort and human capital stock, extending MLRC and CDFC to multi-action spaces will require that the Hessian of the distribution function be positive semidefinite (CDFC), as well as that all partial derivatives of the distribution function are negative (MLRC).

Hessian:

$$D^2H = \{\begin{matrix} F''_{ee}(x;\ e,l) & F''_{le}(x;\ e,l) \\ F''_{el}(x;\ e,l) & F''_{ll}(x;\ e,l) \end{matrix}\}$$

The requirement that the Hessian matrix be positive semidefinite implies that:

9) $F''_{ee}(x;\ e,l) \geq 0\ \forall e,l\ \in (0,\infty)$

10) $F''_{ll}(x;\ e,l) \geq 0\ \forall e,l\ \in (0,\infty)$

11) $F''_{ee}(x;\ e,l)F''_{ll}(x;\ e,l) \geq\ F''_{le}(x;\ e,l)^2\ \forall e,l\ \in (0,\infty)$

[17] Rogerson,W, 1985.

These conditions supplement the requirements of all partial derivatives being negative:

12) $F_e'(x; e, l) \leq 0$

13) $F_l'(x; e, l) \leq 0$

Figure 4 illustrates an example of a possible surface (in 3-dimensions) that would exhibit both MLRC and CDFC in this extended form.

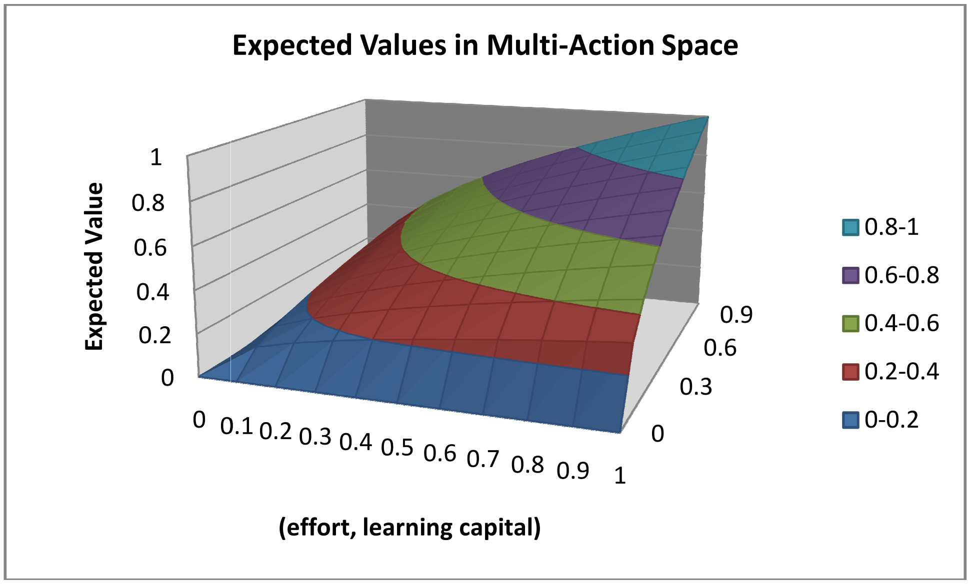

Figure 4.

Existence of an Optimal Wage Contract:

Before defining the optimal dynamic contract explicitly, the existence of the optimal contract needs to be ensured. Because the principal maximizes over a functional space sufficient conditions for the existence of an optimal solution would include: 1) compactness of the functional space and 2) continuity of the objective function.

Claim 2: There exists an optimal wage contract for the principal in Optimization Problem #3[18].

Proof: The objective function is a composition of continuous functions; $S(x), p(x; e, l), w(x) = a_i + b_i x$ are all continuous and thus there sum will also be continuous. The functional space the principal optimizes over can be explicitly described as (a_1, b_1, a_2, b_2): $a_i \in [0, S(\overline{x})], b_i \in \left[0, \frac{S(\overline{x})}{\overline{x}}\right]$. This functional space is indeed compact because it is closed and bounded, where the bounds are obtained by: 1) not allowing the wage to be negative and 2) not allowing the principal to expect negative returns. By the Weierstrass theorem there will exist an optimal wage contract given the above requirements of its obtainable values. ■

The next step in the analysis will be proving that given the existence of an optimal wage contract, the agent's maximization problem will be governed by the First Order Conditions with respect to the agent's action space. Proving the concavity of the agent's objective function in action justifies the use of First Order Conditions.

Claim 3: Given (1) MLRC and (2) CDFC are satisfied and that the principal offers a (3) linear wage contract then agent's optimal action will satisfy the First Order Conditions.

[18] Optimization problem #3 is fully described on page 30.

Proof: The necessary conditions for a function $f(x)$ to have a local maximum at $x^*$is that the Hessian (of $f(x)$) at $x^*$is negative semidefinite. If one replaces the semidefinite with *definite* in the requirement of the Hessian, then these prove to be sufficient conditions as well. The Hessian of the agent's objective function is:

$$D^2H = \begin{matrix} \int p''_{e_1e_1}(x;e,l)\left(-e^{-r(a_1+b_1x)}\right) - C''_{e_1e_1}(e_1,s_1) & 0 & 0 \\ 0 & \int p''_{e_2e_2}(x;e,l)\left(-e^{-r(a_2+b_2x)}\right) - C''_{e_2e_2}(e_2,0) & \int p''_{e_2s_1}(x;e,l)\left(-e^{-r(a_2+b_2x)}\right) \\ 0 & \int p''_{e_2s_1}(x;e,l)\left(-e^{-r(a_2+b_2x)}\right) & \int p''_{s_1s_1}(x;e,l)\left(-e^{-r(a_2+b_2x)}\right) - C''_{s_1s_1}(e_1,s_1) \end{matrix}$$

This matrix will be negative semi-definite as long as MLRC and CDFC are satisfied as well as the positive constraints on the wage contract; $a_1, b_1, a_2, b_2 \geq 0$. Thus, if the optimal action resides in the interior of the action space then satisfaction of the First Order Conditions guarantees the agent maximizes utility. ■

Now that the existence of an optimal contract has been proven as well as the governing conditions for the agent's maximization problem the principal-agent problem can now be defined.

Given the assumption of satisfaction of MLRC and CDFC the *Incentive Constraints* can be expressed as satisfying the First Order Conditions. The agent maximizes utility with respect to e_1, s_1, e_2.

$$14)\ \{\int -e^{-r(a_1+b_1x)}p(x;e_1,0)dx - C(e_1,s_1) + \int -e^{-r(a_2+b_2x)}p(x;e_2,s_1)dx - C(e_2,s_2)\}$$

This optimization can be shown as finding the $(e_1, s_1, e_2)^*$ that satisfies these three equations:

$$15)\ \int - e^{-r(a_1+b_1x)} p_e{}'(x; e_1, 0)dx = C_e'(e_1, s_1)$$

$$16)\ \int - e^{-r(a_2+b_2x)} p_s{}'(x; e_2, s_1)dx = C_s'(e_1, s_1)$$

$$17)\ \int - e^{-r(a_2+b_2x)} p_e{}'(x; e_2, s_1)dx = C_e'(e_2, 0)$$

The principal's complete optimization problem can be seen as:

$$\text{Max } \{\int p(x_1; e_1, 0)[S(x_1) - w(x_1)]dx + \int p(x_2; e_2, s_1)[S(x_2) - w(x_2)]dx\}$$

With respect to: $w(x)$

Subject to: (15)-(17) and

$$18)\ \int -e^{-r(a_1+b_1x)} p(x; e_1, 0)dx - C(e_1, s_1) + \int -e^{-r(a_2+b_2x)} p(x; e_2, s_1)dx - C(e_2, s_2) \geq \bar{U}$$

*Proposition 2: The **learning curve** will be defined as a distribution function that allows the principal to benefit from contracting the agent to invest in human capital in the first period.*

Again, in order to express the ***learning curve*** explicitly one needs to move the analysis to a situation of perfect and verifiable information, where the principal can contract the agent to take a particular action and set the wage contract that satisfies the participation constraint strictly. Given perfect and verifiable information the principal's constrained optimization problem becomes:

Max $\{\int p(x_1; e_1, 0)[S(x_1) - w(x_1)]dx + \int p(x_2; e_2, s_1)[S(x_2) - w(x_2)]dx\}$

With respect to e_1, s_1, e_2 such that

$w(x_1) = w(x_2) = h(\frac{(\bar{U}+C(e_1^*,s_1^*)+C(e_2^*,0))}{2})$ where e_1^*, s_1^*, e_2^* solve the FOCs:

19) $\int p_e'(x_1; e_1, 0)S(x_1) = h_e'(\frac{(\bar{U}+C(e_1^*,s_1^*)+C(e_2^*,0))}{2})$

20) $\int p_s'(x_2; e_2, s_1)S(x_2) = h_s'(\frac{(\bar{U}+C(e_1^*,s_1^*)+C(e_2^*,0))}{2})$

21) $\int p_e'(x_2; e_2, s_1)S(x_2) = h_e'(\frac{(\bar{U}+C(e_1^*,s_1^*)+C(e_2^*,0))}{2})$

Given MLRC and CDFC $s_1^* \gg 0\ iff$

22) $\int p_s'(x_2; e_2, 0)S(x_2) \gg h_s'(\frac{(\bar{U}+C(e_1^*,0)+C(e_2^*,0))}{2})$.

Claim 4: There is a *learning curve* iff (22) holds.

Again, the intuition behind this inequality is straight forward. The learning curve exists when *marginal benefit of contracting for investment in human capital* ~$\int p_s'(x_2; e_2, 0)S(x_2)$ exceeds the *marginal cost of contracting for investment in human capital*~$h_s'(\frac{(\bar{U}+C(e_1^*,0)+C(e_2^*,0))}{2})$ at $(e_1^*, 0, e_2^*)$. Because in this model both the action space and the outcome are an infinite set there does not exist the same appealing split of the differing effects of the learning curve, however, it should be noted that the shape of $S(x)$ (up to this point no restrictions have been made) will have a significant impact on both the likelihood of the learning curve having a significant effect on the optimal contract as well as the shape of this contract.

Now one must consider the Principal's optimization problem, given the presence of this *learning curve*. The optimization problem can be characterized as:

Optimization Problem #3:

$$\text{Max } \{\int p(x_1; e_1, 0)[S(x_1) - w(x_1)]dx + \int p(x_2; e_2, s_1)[S(x_2) - w(x_2)]dx\}$$

With respect to a_1, b_1, a_2, b_2 such that:

(15)-(17) and (22) are satisfied.

Here it will aid in the analysis to introduce the Expectation operator, denoted $E[*] = \int * \, p(*)dx$. Implementing this notation, the optimization problem becomes:

23)$$\text{Max}\{E[S(x_1);\ e_1, 0] - a_1 - b_1 E[x_1; e_1, 0] + E[S(x_2);\ e_2, s_1] - a_2 - b_2 E[x_2;\ e_2, s_1]\}$$

With respect to a_1, b_1, a_2, b_2 such that:

24) $E'_e\left[-e^{-r(a_1+b_1x)}; e_1, 0\right] = C'_e(e_1, s_1)$

25) $E'_s\left[-e^{-r(a_2+b_2x)}; e_2, s_1\right] = C'_s(e_1, s_1)$

26) $E'_e\left[-e^{-r(a_2+b_2x)};\ e_2, s_1\right] = C'_e(e_2, 0)$

27) $E\left[-e^{-r(a_1+b_1x)}; e_1, 0\right] + E\left[-e^{-r(a_2+b_2x)}; e_2, s_1\right] \geq \overline{U} + C(e_1, s_1) + C(e_2, 0)$

An explicit description of the optimal dynamic contract can be found using the following assumptions:

-$E[X; e, l] = \sqrt{el}$

-$Var[X] = \sigma^2$; constant

-$S(x) = x$

-$C(e, s) = (e + s)$

-$l_1 = 0$

The optimal contract takes the form: $W = \{0, b_2^* x\}$, where $b_2^* \gg 0 = b_1^*$ if there is a feasible contract.

3.3 Two Period Observable Learning Principal Agent Problem

In the next model the investment in human capital will be an observable action that subsequently the principal can include in the optimal contract. This model supplements model **3.1**.

All of the assumptions regarding the form of the utility function, the distribution functions as well as cost functions will be the same as model **3.1.** The one significant departure in this model will be the form of the optimal contract. As a result of the principal's ability to observe and verify that the agent has invested in human capital, the principal can contract for investment in human capital. Explicitly the change in the shape of the optimal contract will be represented by introducing an additional variable in period one's contract[19], c_1, which will represent a transfer of payments if investment in human capital takes place ($w_1(x, s) = a_1 + b_1(x) + c_1(s)$).

To ease the length of the analysis this model will be developed and compared exclusively to **Optimization Problem #1,** specifically that:

[19] It should be noted that an additional variable is not needed in the second period because of neither the principal nor the agent benefits from the agent investing in human capital in the final period.

$$\frac{(\pi_2 - \pi_0)}{C_s + C_e} \geq \frac{(\pi_2 - \pi_1)}{C_s}$$

.

All of the same conclusions regarding $b_1$and the set of incentive constraints generalize to this extended optimization problem. This extended optimization problem can be represented by the following program:

Optimization Problem #4:

28) $\text{Max}\{-(a_1 + c_1 + a_2 + \pi_2 b_2)\}$

s.th.

29) $(\pi_2 - \pi_1)(-e^{-ra_2})(e^{-rb_2} - 1) + (-e^{-ra_1})(e^{-rc_1} - 1) \geq C_s$

30) $(-e^{-r(a_1+c_1)}) - C_s + (-e^{-ra_2})\left[\left(\pi_2(e^{-rb_2})\right) + (1 - \pi_2)\right] - C_e \geq \overline{U}$

The optimal wage contract then becomes: W[20] $= \{c^*(s), a^* + b^*(x))$[21].

3.4 Infinite Horizon Utility Maximization with Learning Curve

In all of the previous models the duration of the contract occurred over a finite period. An important effect of the learning curve that has yet to be identified, however, is how the behavior of the agent would change if put in an infinite horizon. By extending model **3.2** into an infinite horizon one can identify the optimal action space for the agent.

---

[20] $c^* = \frac{1}{-\mathrm{r}}\ln\frac{(1-C_s)}{2}$, $a^* = \frac{1}{-r}\ln\frac{\frac{\pi_2(1-C_s)}{2}+C_s-\frac{(-1-C_s)}{2}}{(\pi_2-\pi_1)}$, $b^* = \frac{1}{-r}\ln\frac{\frac{\pi_2(1-C_s)}{2}}{\frac{\pi_2(1-C_s)}{2}+C_s-\frac{(-1-C_s)}{2}}$

[21] This optimal wage is based on the assumption that the Participation Constraint is not binding. This assumption allows for a much more concise closed form solution, however, the conclusions that are drawn from this solution are not specific to this assumption.

Some additional assumptions will need to supplement the structure of model **3.2** in order for the analysis to be tractable in an infinite horizon. First, the distribution function given effort and the level of human capital will be normally distributed with parameters: $\eta\left(\sqrt{el}, \sigma^2\right)$. A distribution function of this form exhibits both MLRC and CDFC. Two minor revisions will be made to the utility function as well; 1) the utility function will exhibit discounting by a term β^t (where $0 < \beta < 1$)and 2) the disutility of effort and investment in human capital will be expressed within the exponential operator. In addition, the cost functions will again be simple linear functions, $C(e, s) = (e + s)$. Although this cost function is convex, it also implicitly implies that the two actions are not substitutes[22].

The shape of the contract offered to the agent characterizes the last defining feature of this model. For this particular model, the agent faces the same wage $(a + b(x))$ in each period through out the infinite horizon. While this represents a significant departure from the contracts described in the other three models (where a's and b's are free to vary in differing periods), it is not immediately clear if and how the optimal contract would change given an infinite horizon. Though some important features of the principal-agent problem are subsequently lost, still some important generalizations regarding the agent's behavior in an infinite horizon can still be inferred. The resulting extended optimization problem thus becomes:

[22] As seen in Holmstrom-Milgrom (1987) when $C''_{es}(e, s) \geq 0$, the two actions are considered to be substitutes. .

Optimization Problem #5:

$$31)\ \text{Max}\ \{\sum_{t=0}^{\infty}\beta^t(-e^{-r\left(a+b\sqrt{e_t l_t}-e_t-s_t-\frac{rb^2\sigma^2}{2}\right)})\}^{23}$$

s.th.

$$32)\ l_{t+1}=\delta l_t+s_t,\ l_0=0$$

Where the optimal effort and investment in human capital policy functions are:

$$e^*=\frac{b^2 l}{4}$$

$$s^*=\ln 2\beta e^{l(1-\delta)}$$

4. Discussion

Even though all of the theoretical models developed have significant differences in their structure and the necessary steps for their analysis there do exist a few generalizations that apply to all of them.

First, in each of the finite horizon models once the principal moves from a situation of verifiable and perfect information to one of imperfect and unverifiable information there is an increase in the cost of contracting for each particular level of effort and/or investment in human capital. The cost results from the fact that in all of the models the principal writes a relatively *powerful* contract in the second period (i.e. $b_2 \gg 0$) in order to satisfy the incentive constraints and accordingly subjects the agent to some level of risk in the realization of the verifiable output. The Jensen inequality (given a concave utility function) proves this cost directly:

[23] The expected value of an exponential, E(e^{aX})= $e^{(aEX-\frac{a^2\sigma^2}{2})}$ where X is distributed η(μ,σ^2).

$$Eu(w(X)) \leq u(w(EX))$$

If investment in human capital becomes a verifiable and contractible act then the resulting optimal contract will pareto dominate the contract in the situation of unverifiable (in investment in human capital) information. As illustrated by the two optimal contracts found in **Optimization Problem #1** and **Optimization Problem #4** the agent also receives a *smoother* consumption path in a situation of verifiable information. Also, in the optimal contract found in **Optimization Problem #4** the agent is subjected to less risk. All of these findings suggest that both the principal and agent would benefit if able to move the contractual problem from a scenario of unverifiable information to verifiable information even if only in regards to the investment in human capital. This conclusion provides support for the introduction of "professional development programs" as well as providing pay increases for advanced degrees in a relevant industry in typical business employee contracts. Insurance premium discounts following the completion of an instructive course, or tutorial would also serve as examples of principals and agents attempting to move the level of human capital stock from a possibly unverifiable value to one that can be contracted (even as indirectly as some of these might seem).

Next, in all of the principal-agent problem models if a dynamic contract is feasible then the principal will provide a contract that both satisfies the incentive constraints but also binds at the participation constraint (only exception comes iff $w(x_i) = 0$ satisfies the participation constraint). Though the participation constraint does turn out to be binding the agent receives a "risk premium" given a "good/better than expected outcome". This "risk premium" is offset accordingly by the penalty dispensed following a "bad/worse than expected outcome".

The level of risk averseness will play a significant role in determining whether an optimal contract will be feasible given the set of incentive constraints and the participation constraint. There exists levels of risk averseness for given cost functions of effort and investment in human capital that will prevent a dynamic contract from being able to provide the proper incentives to engage the agent with the learning curve.

In all of the models there exists a strong likelihood that the *expected consumption* path of the agent will not be smooth and thus vulnerable to a pareto improving transfer of payments from periods. The solutions in **Optimization Problem #1 and #2,** where the entire wage offered is given in the second period, illustrate this conclusion most evidently. Clearly, the principal could make a transfer of payments from period two to period one in each of these examples making the agent strictly better off, leaving the total utility of the principal unchanged. This finding suggests that most optimal dynamic contracts will be pareto inferior, due to the high powered structure of later periods relative to earlier periods. The unsmooth consumption path of the agent makes the optimal contract vulnerable to renegotiation or lack of commitment.

In models **3.1, 3.2,** and **3.3** the principal writes a relatively more powerful contract in the second period compared to the first. This shape of the optimal dynamic contract results from two contributing factors. First, as the power of the contract in the second period increases the gains to investing in human capital for the agent will also increase. Next, as the power of the first period's contract increases the opportunity cost of investing in human capital for the agent also increases (assuming that the two actions are at least not complements). This result parallels the conclusions found in Holmstrom and Milgrom (1991). Combining the effects of both of these conclusions leads to the *optimal* *dynamic contract becoming relatively more powerful over the time period of the contract in the presence of a learning curve*.

Another important characteristic of the analysis of the learning curve is the difference in the effects of: the (1) gains in *likelihood* of successful outcomes and the (2) gains in *output* of successful outcomes. The first of these gains affect both the principal's and agent's optimization problem, while the second of these gains affects only the principal's optimization problem. The gains in the likelihood of successful outcomes denote a built-in incentive for the agent to invest in human capital (as long as $b_2 \gg 0$). Subsequently, as the gains in likelihood of successful outcomes increase, the need for the agent to reward successful outcomes (and create greater incentives to invest human capital) decreases. Figure 2 best illustrates this effect.

The gains in output of successful outcomes, on the other hand, is an effect of the learning curve that only affects the principal's optimization problem. In the analysis of the optimal dynamic contract the production function most directly influenced the particular functional space. The greater the gains in the successful outcomes the richer and larger the functional space became. This effect allows the principal greater capability to write a contract that will both satisfy the *Incentive Constraints* and *Participation Constraints* given some particular level of risk averseness. Consequently as the gains in successful outcomes increase (($\overline{S} - \underline{S}$) in **Optimization Problem #1, #2, and #4**) the more robust the optimal dynamic contract will be to increases in risk aversion from the agent as well as the particular cost functions of the agent and the outside alternative.

5. Conclusion

It has been the purpose of this paper to attempt to demonstrate that the presence of a learning curve can govern the shape of the optimal dynamic contract. In the presence of a learning curve one can expect that the optimal contract will get relatively more powerful in later periods. Holding other variables constant the greater the gains for the principal to investment in human capital the greater the likelihood an optimal contract will be feasible (even with high levels of risk aversion from the agent). In addition, the relative *power* of the optimal contract has an inverse relationship with the *marginal increase in expectation with respect to investment in human capital*.

Generally speaking one would expect to see jobs that have significant gains in output from learning curves to have dynamic contracts that get relatively more powerful. The most apparent illustration of this notion in the private sector comes in the form of "partner-tracked" contracts in law firms or equity firms. A significant amount of athletic contracts exhibit this affinity to becoming more incentive based further into the duration of the contract as well[24].

In order to make general statements regarding the effect the learning curve had on the shape of the optimal dynamic contract several assumptions were used to make the analysis significantly easier and more tractable. These assumptions serve as the most likely sources of further research.

The most restrictive assumption and most evident departure from the real world proves to be the structure of the participation constraint in the three principal-agent

[24] An additional argument exists that athletic contracts become more powerful later in the contract to shelter the principal from some of the risk of future injuries to the athlete.

problem models. As mentioned earlier, the exogenous characterization of the outside alternative suggests that the agent does not become more/less attractive to other employers after the realization of past verifiable outputs. Though there was a suggestion that this coincides well with Jovanic's "firm specific capital", it would be intriguing to determine what influence allowing the outside alternative to be function of past period realized outputs might have on the shape of the optimal contract.

Another exogenous factor in all of the models included the variance of the realized output. More specifically, this exogenously given variance remained constant throughout all levels of effort and human capital stock. This coincides with the intuition that even as effort or human capital stock increases the noise of the verifiable output still remains constant. It does not seem unreasonable, however, to suggest that perhaps the noise could decrease as the level of human capital stock increases; not only are there "marginal productivity gains" to human capital but there are also "decreased volatility gains". A possible distribution function that would exhibit this characteristic would be if $p(x;e,l) \sim \eta(e + \frac{l}{2}, \frac{1}{(4+l)^2})$for $(e,l) - (1,0), (1,1),$ and $(1,2)$:

Probability Distributions Exhibiting "Decreased Volatility Gains"

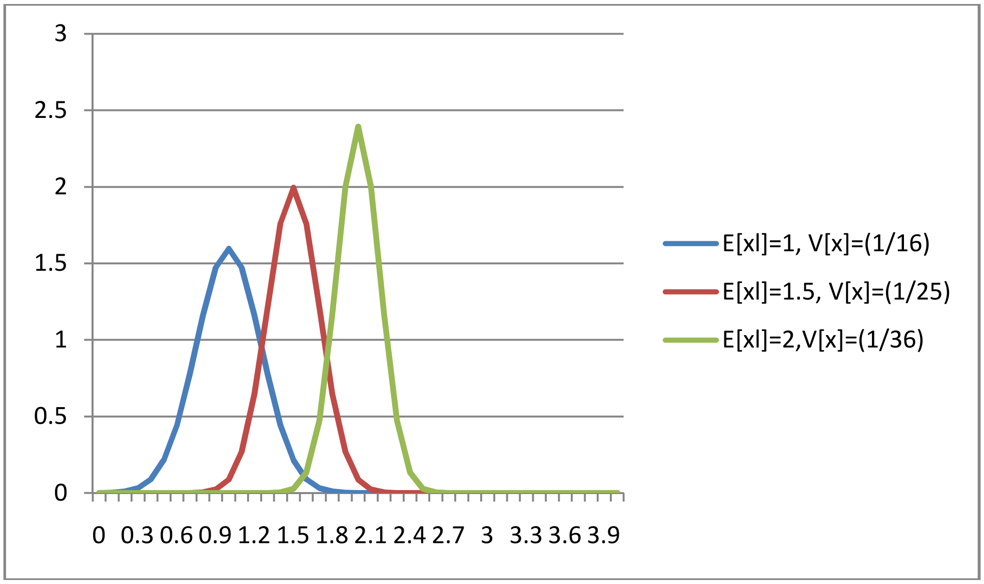

Figure 5

Initially it seems that if increased levels of human capital also decreased the variance of the realized output, and subsequently the cost of making a powerful contract, that one would expect the learning curve to play an even larger impact in such a model. Further analysis of the role gains in decreased volatility might have, would supplement the principal-agent literature both in a dynamic and static perspective.

Another significantly restrictive assumption used in both models was the constant absolute risk aversion of the utility functions. Again for tractability and ease of the analysis, wealth effects were not allowed to influence the shape of the optimal contract. Allowing for a more general class of utility functions would, however, increase the applicability of the result.

A common intuition and empirical regularity suggests that individuals become less risk averse as their wealth increases. In all of the models developed in this paper the risk averseness is both exogenous to the model as well as fixed over income levels.

Through out the analysis of the learning curve the risk averseness had a negative influence on the most powerful feasible contract. Allowing the risk averseness of the agent to decrease with wealth illustrates an interesting extension to the analysis conducted in this paper.

The shape of the utility function did not allow for savings or access to credit markets. In all of the models the agent was at the mercy of receiving utility from the wage bill received in that period and that period alone. In the literature the assumption of restricting the agent from access to credit markets has been both a debated topic as well as a significant source of complication in developing the analysis in the principal-agent problem. Weakening this assumption does not provide a clear intuition as to how the principal will write an optimal dynamic contract taking into account the ability of the agent to borrow or save at some exogenous real interest rate.

Finally, the most significant area for further research comes in the treatment of how time affects the shape of the optimal dynamic contract. In all three principal-agent models the contract was written for a fixed finite period of time. A significant finding would be to identify how moving the analysis to an infinite time horizon affects the shape of the dynamic contract. This paper makes a concerted effort to make some generalizations of the effect of an infinite horizon on an agent's behavior in the presence of the learning curve facing an unchanging wage contract. Once the wage contract is free to vary across time periods in an infinite horizon, however, the ability to use even the simplest dynamic programming tools such as the Bellman Value functions and the stationary principle no longer exists. A significant advancement for the principal-agent literature would be to solve one such problem where the objective function of the agent is not autonomous.

Allowing the terminal period of the contract to take the form of a random variable illustrates another possible extension of the standard approach to the principal-agent problem. Both principal and agent would need to possess some beliefs as to the likelihood of the contract terminating at each given period, but the particular form and structure of this belief could prove to vary widely.

The greatest source of further research in the principal-agent literature will most likely always have its foundations in various ways to treat time in the model. It should not be alarming that this is the case, however, due to the wide variety of ways time affects contracts in the real world. Finite horizon, infinite horizon, uncertainty in termination, as well as both the principal and/or agent controlling the termination of the contract all represent examples of types of contracts that can be found everyday in the real world. It was the hope of this paper to create the foundations of how the presence of a learning curve could possibly affect each of these types of contracts.

6. Appendix

1. $$2\left[\pi_1\left(\overline{S}-h\left(\overline{U}/2+C_e\right)\right)+(1-\pi_1)\left(\underline{S}-h\left(\overline{U}/2+C_e\right)\right)\right] \le \pi_0\left(\overline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+(1-\pi_0)\left(\underline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+\pi_2\left(\overline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+(1-\pi_2)\left(\underline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)$$

2. $$\pi_1\left(\overline{S}-h\left(\frac{\overline{U}+C_e}{2}\right)\right)+(1-\pi_1)\left(\underline{S}-h\left(\frac{\overline{U}+C_e}{2}\right)\right)+\pi_0\left(\overline{S}-h\left(\frac{\overline{U}+C_e}{2}\right)\right)+(1-\pi_0)\left(\underline{S}-h\left(\frac{\overline{U}+C_e}{2}\right)\right) \le \pi_0\left(\overline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+(1-\pi_0)\left(\underline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+\pi_2\left(\overline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)+(1-\pi_2)\left(\underline{S}-h\left(\frac{\overline{U}+C_e+C_s}{2}\right)\right)$$

3.

Expected Values versus Action Space for Model 3.1

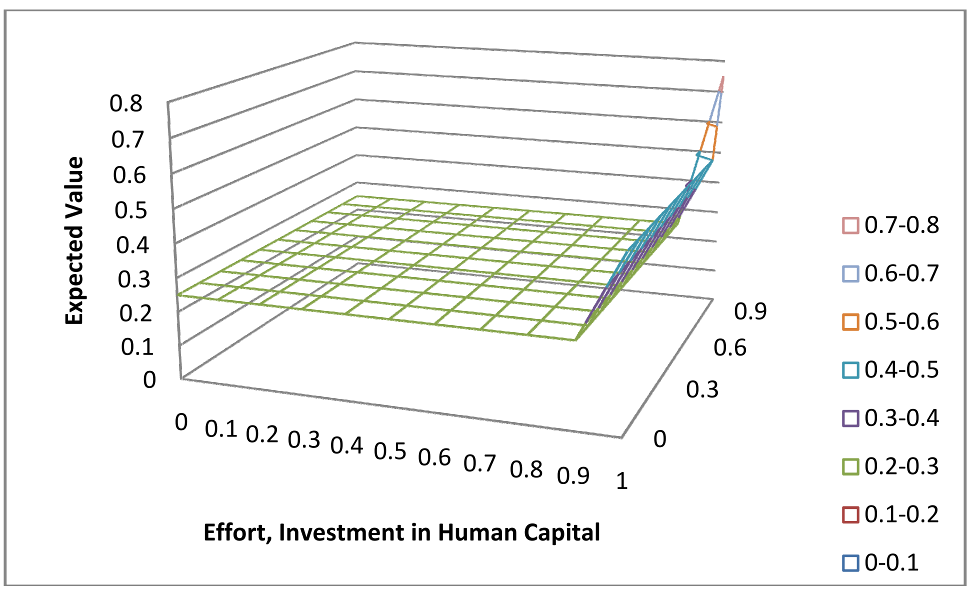

Figure 5

7. Works Cited

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