Abstract: Procrastination is defined as the behavior of postponing. Although procrastination may result in psychological discomfort, this paper shows that it could also be the result of utility maximization. This paper finds out that an individual will procrastinate less if he/she is more outcome-oriented, if the importance of everyday inputs is stressed, if time-inconsistent preference is realized. In addition, this paper finds that an individual will carry out work plan steadily in the steady state with adjacent substitution or low degree of adjacent complementarity, but the steady state will become unstable with adjacent complementarity. Besides, this paper also looks into the coexistence of procrastination and promptness from a time-inconsistency perspective and a two-procrastination-capital perspective.

Procrastination, according to the Merriam Webster Dictionary, means the act of putting things off intentionally or habitually. Burka and Yuen (2008) referred to procrastination as “the behavior of postponing”. We may observe individuals who procrastinate almost all the time but seem really “efficient” and work overnight for a few days before the deadline. You may wonder why and how procrastination and promptness can coexist? However, the stories of Elaine and Sarah below may seem familiar to you:

It is the start of the spring semester. Elaine enrolls into one graduate economics course, which requires a final paper at the end. Since it is a new start, Elaine makes up her mind that she will start working on this paper right from the first week of class and will never repeat what she did in her game theory class. Elaine remembered that in her game theory class, she planned to start writing the course paper after midterm, but it turned out that she did not start writing the paper until the last two weeks before the deadline and she almost failed this class. Her painful memory about the game theory paper incentivizes Elaine to work on her paper as soon as possible. However, when she sits down, she feels it just seems so difficult to start writing and always reminds herself that it is okay to leave this paper for tomorrow since she still has enough time, so she procrastinates. Time goes by and it comes to the midterm. Elaine writes nothing yet and she is tortured all the time by the terrible memory of the game theory paper and the guilt of not working. Midterm makes the fear of repeating the story game theory paper and the guilt of not working dominate over her instantaneous satisfaction of avoiding writing the paper, so she starts writing. As time is not adequate, Elaine carries out the double daily workload of what she planned to do at the beginning of this semester to manage to meet the deadline. Elaine’s roommate Sarah also enrolls into the same class, and Sarah does start working from the
beginning of this semester. However, Sarah is faced with a different situation: she works less than her actual plan and tells herself that she will work more tomorrow. In the last month before the deadline, Sarah realizes she will never be able to finish her paper with her current progress, so she starts building up her workload quickly in order to finish her paper on time.

The behavior of both Elaine and Sarah is not uncommon among us. This paper is interested to understand what procrastination is, what drives an individual to procrastinate and how procrastination and promptness can coexist. This paper models Elaine’s and Sarah’s behaviors by introducing what I call “procrastination capital” and time-inconsistent preference, respectively.

There are generally two types of procrastination: one will cause us to suffer little because we postpone doing something that we tend to think is trivial, unimportant, and not worth putting time and effort on, such as replying to a market survey call or finishing a supermarket customer satisfaction form to win a 10-dollar coupon; the other will cause us to suffer disutility because we postpone doing something that we believe we should do, such as washing the dishes after dinner or filling out and mailing our tax returns. This paper shows that the behavior of both types of procrastination actually can be the result of maximizing utility of a forward-looking individual because procrastination itself helps us to ameliorate or avoid undesired suffering for the moment, even though we know that we may suffer in the next second.

When we procrastinate, we form experience and memory about our behavior at the same time, which are defined as “procrastination capital” in this paper. Memory can be either good or bad, and experience can be either successful or unsuccessful. Suppose Elaine procrastinates during most of the time of last semester but crammed before finals. Elaine had to work overtime or even stay up for several nights before finals to survive, which forms painful memory about procrastination. Such painful memory will drive Elaine to start study earlier this semester, which means that procrastination capital discourages procrastination. If Elaine survived last semester, which she takes as successful experience, then she might tell herself that it is okay to procrastinate during this semester and cram before finals. Successful experience like this could drive Elaine to procrastinate again in this semester, which means procrastination capital will encourage procrastination.

Akerlof (1991) brought up a cost-benefit analysis framework and analyzed procrastination as a result of cost minimization. He pointed out that procrastination is irrational because individuals tend to focus on today’s costs, which is more evident than the future’s costs, without anticipating their future behavior. However, Fischer (2001) assumed time as an exhaustible resource and set up a time-consistent framework to
discuss that procrastination is a rational behavior as the result of utility maximization. This paper follows Fischer (2001) to view time as an exhaustible resource but relaxes the assumption about finishing the task by incorporating utility from final outcome. Becker and Murphy (1988) set up a framework to analyze a time-consistent and forward-looking individual’s consumption path of addictive goods. They found that when the addictive goods have adjacent complementarity, addicts would either consume a large amount of addictive goods or reduce consumption rapidly. Chaloupka (1990) used empirical evidence to support Becker and Murphy’s (1988) findings. This paper applies the rational addiction theory (1988) to procrastination and extends the discussion to both adjacent complementarity and adjacent substitution. Besides, this paper also explains how an individual builds up workload in a multi-stage task and reaches the same conclusion as Fischer (2001) from the procrastination capital perspective. Gruber and Kőszegi (2001) also tested Becker and Murphy’s (1988) conclusion empirically, but they introduced a discussion about time-inconsistent behavior. O’Donoghue and Rabin (2001) developed a model about procrastination with the assumption that individuals only partially realize their self-control problems. Ariely (2008) conducted a behavioral experiment on setting deadlines for three tasks and comparing students’ overall performance afterwards, and what he found out supported O’Donoghue and Rabin’s (2001) conclusion that an individual has self-control problem and their awareness of self-control problems varies. Banerjee and Mullainathan (2010) focused on one extreme time-inconsistent case about temptation goods. Thus, this paper compares the time-inconsistent preference case with the time-consistent preference case and finds out that time-inconsistent individuals procrastinate more at the beginning of the work process and build up workload much more quickly as well than time-consistent individuals, which is another explanation of the coexistence of procrastination and promptness.

This paper discusses how an individual allocates time to doing his/her work and procrastination, based on several simple models. Unlike the one-time task in Akerlof’s (1991) model, this paper follows Fischer (2001) to assume that the time budget constraint can be either spent on work or procrastination, which is a normal good, and closes down the difference between procrastination and leisure for simplicity. In the first section, this paper addresses how the individual allocates time between working and not working in a two-period model, with time input requirement or considering the utility from final outcome based on their efforts – their time devoted to the work. In order to understand how procrastination and promptness can coexist, this paper gives answers from time-inconsistent preference and procrastination capital perspectives. In the second section, this paper differentiates procrastination and leisure to compare the individual’s allocation of time in a time-inconsistent framework to that in a time-consistent framework. In the third section, this paper frames the analysis as a
continuous-time dynamic optimization question and takes what I call “procrastination capital” into account. In the last section, the paper concludes and discusses how to distinguish different models empirically.
Table 1. Key notations of this paper

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Period $t$</td>
</tr>
<tr>
<td>$c_t$</td>
<td>The amount of procrastination in period $t$, a normal good</td>
</tr>
<tr>
<td>$u_t, U_t$</td>
<td>The continuously differentiable instantaneous utility function in period $t$</td>
</tr>
<tr>
<td>$\mathcal{U}$</td>
<td>The discounted utility of all periods</td>
</tr>
<tr>
<td>$H$</td>
<td>The total amount of time at each period $t$</td>
</tr>
<tr>
<td>$M$</td>
<td>The required amount of time to finish the task</td>
</tr>
<tr>
<td>$\rho$</td>
<td>The subjective discount rate to measure an individual’s time preference</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>The shadow value of time</td>
</tr>
<tr>
<td>$O$</td>
<td>The outcome function, which is an increasing function of time input</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
<td>The effect of one unit time in period $t$ on the final outcome, and $\vartheta$ is a weakly decreasing function of $t$</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>The parameters in the utility function</td>
</tr>
<tr>
<td>$y_t$</td>
<td>The leisure time, which is the time spent neither on procrastination nor on the task</td>
</tr>
<tr>
<td>$v_t$</td>
<td>The instantaneous utility from the temptation good in each period $t$</td>
</tr>
<tr>
<td>$D_t$</td>
<td>The time spent on working</td>
</tr>
<tr>
<td>$T$</td>
<td>The deadline date</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The degree of time-inconsistency, and $\delta \in [0,1]$</td>
</tr>
<tr>
<td>$S_t$</td>
<td>The procrastination capital in period $t$</td>
</tr>
<tr>
<td>$c^<em>, S^</em>$</td>
<td>The steady state levels of procrastination or procrastination capital</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The depreciation rate of procrastination capital</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>The shadow price of procrastination capital in period $t$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The characteristic root of the ordinary differential equation, or the speed of converging towards the steady state</td>
</tr>
</tbody>
</table>

I. A two-period framework

i. The model with a certain work input requirement

Following the analysis of Fischer (2001), the time of a day is divided into two parts — time spent on work and time not spent on work, where the latter one is a normal good and the measurement of procrastination in this paper. The time not spent on work, is denoted as $c_t$. $t$ denotes time period 1 or 2. $H$ is the total time of each period — 24 hours so $c_t \in [0, H]$ — and $M$ is the total amount of time required to finish an assigned job.

We first assume that time spent on the job generates no utility, and only time spent not on work generates utility. As the individual is a forward-looking rational individual, the individual’s utility maximization problem becomes

$$
\max_{\{c_1, c_2\}} \mathcal{U} = u(c_1) + \frac{u(c_2)}{1+\rho}
$$

subject to
\[(H - c_1) + (H - c_2) \geq M \tag{2}\]

where \(\rho\) denotes the subjective time preference, the larger \(\rho\) is, the more present-oriented the individual is. And \(u\) denotes the instantaneous utility function of each period, and \(u\) is continuously differentiable. Equation (2) can be interpreted as the work input requirement constraint.

Thus, the individual wants to solve the Lagrangian equation

\[\mathcal{L} = u(c_1) + \frac{u(c_2)}{1+\rho} + \lambda[(H - c_1) + (H - c_2) - M] \tag{3}\]

where \(\lambda\) stands for the shadow value of time spent on the work.

**Result 1:** Given a risk-neutral or risk-averse individual,

(a) Having a deadline is better than having no deadline;
(b) The individual always procrastinates more today than tomorrow.

Intuitively, Result 1 (a) tells that with a deadline \((M > 0)\), the time budget forces the individual to conduct the work. Result 1 (b) tells that due to the time preference, the individual will prefer to consume more procrastination today than tomorrow. Because working provides no utility, the individual will procrastinate as much as possible to maximize his/her welfare. This behavior may also be explained by hyperbolic discounting if the individual believes that “tomorrow may never come”, which will be discussed in the time-inconsistency model part.

If the individual is a risk-neutral agent, which means the individual does not care about smoothing of procrastination, but he/she cares today more than tomorrow \((\rho > 0)\), then we can get the results as the follows:

If \(H \geq M, c_1 = H, c_2 = H - M\); If \(H < M, c_1 = H - M, c_2 = 0\).

But in reality, individuals tend to value today more than tomorrow. Thus, we can also assume the individual is a risk-averse agent, which means that the individual cares more about today than tomorrow, and his/her instantaneous utility function is strictly concave.

By taking the first order condition, we can get

\[u_{c_1} = \frac{u_{c_2}}{1 + \rho} \tag{4}\]

The interpretation of equation (4) would be that the marginal benefit coming from one unit of time allocated to procrastination today equals the discounted marginal benefit of one unit of time allocated to procrastination tomorrow. Because the instantaneous utility function is strictly concave, in order to satisfy equation (4), we must have \(c_1 > c_2\).
Therefore, no matter whether the individual is risk-neutral or risk-averse, under the setting of the basic model, the individual will always tend to procrastinate more today than tomorrow.

ii. The model with utility from doing work

In the basic model following Fischer (2001), it seems very strange that doing the work does not generate utility, so what is the rationale for us to do the work? In terms of utility, assuming no utility from doing the work will make the individual only devote the minimum time – that is, M – to get the job done.

In this part, we make a small modification and assume that the effort of getting the work done also matters and the outcome captures the effort inputs. The outcome depends on the time devoted into doing the work, therefore we define an outcome function

\[ O = O'[\theta_1(H - c_1) + \theta_2(H - c_2)] \]  
where \( O' > 0, \theta_1 \geq \theta_2 > 0. \)

The outcome is only realized in the end of the second period and we assume that the individual has rational expectation, that is, his/her expectation of the realized outcome in period 1 matches what he/she actually gets at the end of period 2. Equation (5) implies that the more time inputs, the better the outcome will be. The parameters \( \theta_1 \) and \( \theta_2 \) capture the fact that the effect of one unit of time today may be different from the effect of one unit of time tomorrow on the final outcome. If \( \theta_1 > \theta_2 > 0 \), then the sooner the individual starts working, the better the outcome will be.

**Result 2:** Given a forward-looking individual who anticipates his/her final outcome, when there is no required amount of time input to get the work done, he/she may procrastinate less if either that he/she is heavily outcome-oriented or that he/she realizes that the sooner he/she starts working, the better the outcome.

Intuitively, Result 2 tells us that an outcome-oriented individual who values the final outcome a lot will tend to procrastinate less and get the work done as better as possible. And if an individual realizes that one unit of time input tomorrow is inferior to one unit of time input today in terms of improving the final result, he/she will start working soon.

The assumption we make here is that the utility function is continuously differentiable. Then the individual’s utility maximization problem becomes

\[ \max_{\{c_1, c_2\}} U = u(c_1, O) + \frac{u(c_2, O)}{1+\rho} \]  
Thus, the individual wants to maximize

\[ \mathcal{L} = u(c_1, O) + \frac{u(c_2, O)}{1+\rho} \]
By taking first order conditions with respect to \( c_1, c_2 \), we can get

\[
u_{c_1} = u_o o' \vartheta_1 + \frac{u_o o' \vartheta_1}{1 + \rho}
\]

\[
u_{c_2} = u_o o' \vartheta_2 + \frac{u_o o' \vartheta_2}{1 + \rho}
\]

(Equations (8) and (9) mean that the individual will choose to procrastinate up to the point where the marginal benefit from procrastination equal the forgone utility from final outcome.

Rearranging the terms, we can get

\[
u_{c_1} = \frac{\vartheta_1}{\vartheta_2} \frac{u_{c_2}}{1 + \rho}
\]

The smaller \( \vartheta_2 \) is, then given the amount of procrastination in the second period constant, the individual will procrastinate less in the first period.

However, there may be interactions between procrastination in both periods. Thus, whether the individual will actually procrastinate less depends on the shape of utility function. Here we give three types of utility function as examples.

**EXAMPLE 1:**

Assume the utility function is linear in \( c \) and \( O \), which means that \( c_1, O \) and \( c_2 \) are perfect substitutes. Thus,

\[
u_t(c_t, O) = \alpha c_t + \beta O
\]

\[
O = \vartheta_1 (H - c_1) + \vartheta_2 (H - c_2)
\]

**Table 2. The final outcome**

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of ( \vartheta_1 ) and ( \vartheta_2 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \vartheta_1 \geq \vartheta_2 &gt; \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \vartheta_1 &gt; \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} \geq \vartheta_2 &gt; \frac{\alpha}{(2 + \rho)\beta} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \vartheta_1 &gt; \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} \geq \frac{\alpha}{(2 + \rho)\beta} \geq \vartheta_2 )</td>
<td>0</td>
<td>H</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} \geq \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} \geq \vartheta_2 )</td>
<td>H</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{(1 + \rho)\alpha}{(2 + \rho)\beta} \geq \frac{\alpha}{(2 + \rho)\beta} \geq \vartheta_2 )</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{\alpha}{(2 + \rho)\beta} \geq \frac{\alpha}{(2 + \rho)\beta} \geq \vartheta_2 )</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>
As shown in the six cases in Table 6, the result shows that either the effect of work efforts on final outcome – that is, $\beta$ – or the time devotion on final outcome – that is, $\vartheta_1$ or $\vartheta_2$ – is larger enough, then the individual will procrastinate less in each period.

EXAMPLE 2:

Assuming the utility function is concave in $c$ but generates constant marginal utility from the outcome $O$, then,

$$u_t(c_t, O) = \alpha tnc_t + \beta O$$

$$O = \vartheta_1(H - c_1) + \vartheta_2(H - c_2)$$

At the optimum,

$$c_1 = \min\left\{ \frac{(1 + \rho)\alpha}{(2 + \rho)\vartheta_1\beta}, H \right\}$$

$$c_2 = \min\left\{ \frac{\alpha}{(2 + \rho)\vartheta_2\beta}, H \right\}$$

The result is mainly driven by the fact that the outcome function is linear in time inputs of each period.

If $\vartheta_1$ is bigger, the individual will procrastinate more in the first period but won’t change his/her procrastination in the second period.

If $\vartheta_2$ is bigger, the individual won’t change his/her procrastination in the first period but will procrastinate less in the second period.

EXAMPLE 3:

Assuming the utility is concave in both $c$ and $O$,

$$u_t(c_t, O) = \alpha tnc_t + \beta tlnO$$

$$O = K[\vartheta_1(H - c_1) + \vartheta_2(H - c_2)]$$

where $K$ is a constant.

At the optimum,

$$\frac{\alpha}{c_1} = \frac{2 + \rho}{1 + \rho \beta} \frac{\beta}{K \vartheta_1}$$

$$\frac{\alpha}{c_2} = (2 + \rho) \frac{\beta}{O K \vartheta_2}$$

$$\frac{1}{c_1} = \frac{1}{1 + \rho} \frac{1}{\vartheta_1}$$

$$\frac{1}{c_2} = \frac{1}{1 + \rho \vartheta_2} \frac{1}{\vartheta_2}$$
Thus, we can get

\[
c_1 = \min \left\{ \frac{(1 + \rho)(\vartheta_1 + \vartheta_2)H\alpha}{(2 + \rho)(\beta + \alpha)\vartheta_1}, H \right\}
\]

\[
c_2 = \min \left\{ \frac{(\vartheta_1 + \vartheta_2)H\alpha}{(2 + \rho)(\beta + \alpha)\vartheta_2}, H \right\}
\]

If the individual is more outcome-oriented – that is, \( \beta \) is larger, he/she will procrastinate less in both periods.

As shown above, given \( \vartheta_1 \), if \( \vartheta_2 \) is smaller – that is, the individual knows that cramming before finals is worse than studying ahead of finals, he/she will procrastinate less in period 1 but procrastinate more in period 2 relatively. Similarly, given \( \vartheta_2 \), if \( \vartheta_1 \) is bigger – that is, the final results put more weights on every day’s effort – then he/she will procrastinate less in period 1 but procrastinate more in period 2.

EXAMPLE 4:

More generally, we assume a general concave Cobb-Douglas utility on \( c_1, c_2 \) and \( O \),

\[
u(c, O) = \alpha_1 c_1 + \alpha_2 c_2 + \beta \ln O
\]

\[
O = K[\vartheta_1(H - c_1) + \vartheta_2(H - c_2)]
\]

Then we can get

\[
c_1 = \min \left\{ \frac{(\vartheta_1 + \vartheta_2)H\alpha_1}{(\alpha_1 + \alpha_2 + \beta)\vartheta_1}, H \right\}
\]

\[
c_2 = \min \left\{ \frac{(\vartheta_1 + \vartheta_2)H\alpha_2}{(\alpha_1 + \alpha_2 + \beta)\vartheta_2}, H \right\}
\]

As shown above, if the individual is more outcome-oriented – that is, \( \beta \) is larger, he/she will procrastinate less in both periods.

Given \( \vartheta_1 \), if \( \vartheta_2 \) is smaller, he/she will procrastinate less in period 1 but procrastinate more in period 2. Similarly, given \( \vartheta_2 \), if \( \vartheta_1 \) is bigger, then he/she will procrastinate less in period 1 but procrastinate more in period 2. The general case matches the conclusions in the example above.

As illustrated above, what is interesting is that only the relative weights – the weights of time inputs into outcome, or the weights of outcome and procrastination in utility – matter, not the absolute terms. When an individual is deciding whether to procrastinate or work, either a deadline or a utility from outcome or both are necessary to drive a utility maximization solution. Suppose there are no required efforts, nor does an individual care about the final outcome – a channel of transfer work into utility, there is no rationale for the individual to work at all. Thus, in the following part of this paper,
both the effort requirement constraint and utility from working are adopted to explain an individual’s utility maximization behavior.

II. The time-inconsistent framework

In part I, we discuss the case where the individual’s preference is consistent over time and his/her utility maximization problem over time. However, people who procrastinate today will tell themselves that they should not procrastinate tomorrow, but they will procrastinate again when tomorrow actually comes, as Sarah’s story at the beginning of this paper. Thus, procrastination is more like a self-control problem (Burka and Yuen, 2008; Fischer, 2001; O’Donoghue and Rabin, 2001; Ariely, 2008). Self-control problem can be analyzed as time-inconsistency framework. This part first discusses the time-inconsistency problem in the temptation good framework brought up by Banerjee and Mullainathan (2010) to compare the time-consistent and time-inconsistent individuals’ choices of time allocation on leisure, procrastination and doing the work. In addition, the paper discusses the more general time-inconsistency issue of the procrastination behavior.

i. Procrastination as a temptation good

Temptation goods, defined by Banerjee and Mullainathan (2010), is a type of good that consumption of it will increase the utility today but will not provide utility tomorrow. Here we use \( c_t \) to denote the temptation good – procrastination, \( y_t \) denotes the leisure time, \( D_t \) denotes the time spent on the work, and \( t \) denotes the period 1 or 2.

It is necessary to differentiate procrastination from leisure in this part. In other parts of this paper, this paper treats leisure and procrastination as a whole to describe time not spent on work for simplicity. But procrastination and leisure are fundamentally different from a psychological perspective. Leisure can be described as a good that is desired to be consumed at each period to bring positive utility, such as sleeping, eating in this paper’s setting. However, procrastination is a good which is desired to be consumed in the current period but not any more in future periods (Banerjee and Mullainathan, 2010). Procrastination can be treated as the sum of behaviors that prevent or postpone doing some work. One possible setting could be writing a paper. We can refer necessary time spent on sleeping, eating, exercising and relaxation as leisure time, but time spent on sorting out files or checking Facebook just to look busy and avoid writing paper is clarified as procrastination. In order to model how an individual generally chooses among procrastination, leisure and work, this part introduces utility coming from work, procrastination and leisure time.

In a time-consistent framework, the individual’s utility maximization problem is
\[
\max_{\{c_1,c_2,D_1,D_2,y_1,y_2\}} U = u(D_1, y_1) + v(c_1) + \frac{u(D_2,y_2)+v(c_2)}{1+\rho} \tag{8}
\]

where \( u \) and \( v \) are instantaneous utility functions, subject to

\[
D_1 + D_2 \geq M \tag{9}
\]
\[
H \geq D_1 + y_1 + c_1 \tag{10}
\]
\[
H \geq D_2 + y_2 + c_2 \tag{11}
\]

Assume there is interior solution, then we can get

\[
u_{y_1} = v_{c_1} \tag{12}
\]
\[
u_{y_2} = v_{c_2} \tag{13}
\]
\[
u_{y_1} - \nu_{d_1} = \frac{u_{y_2} - u_{d_2}}{1 + \rho} \tag{14}
\]

Equations (12) – (14) means that at the optimal point, the marginal utility of one more unit of procrastination must equal the marginal utility brought by one unit of leisure in that period, and the marginal utility of idleness or leisure net of the foregone marginal utility of working in period 1 should equal the discounted marginal utility of procrastination or leisure net of marginal utility of working in period 2.

In the temptation framework, as the individual expects no consumption of procrastination in the future, the individual’s utility maximization problem becomes

\[
\max_{\{c_1,D_1,D_2,y_1,y_2\}} U = u(D_1, y_1) + v(c_1) + \frac{u(D_2,y_2)}{1+\rho} \tag{15}
\]

where \( u \) and \( v \) are instantaneous utility functions, subject to Equations (10) and (11) and

\[
H \geq D_2 + y_2 \tag{16}
\]

Assume there is interior solution, we can get

\[
u_{y_1} = v_{c_1} \tag{17}
\]
\[
u_{y_1} - \nu_{d_1} = \frac{u_{y_2} - u_{d_2}}{1 + \rho} \tag{18}
\]

The interpretations are similar, Equation (17) means that at the optimal point, the marginal utility of one more unit of procrastination must equal the marginal utility brought by one unit of leisure in that period. And Equation (18) means the marginal utility of leisure net of the foregone marginal utility of working in period 1 should equal the discounted marginal utility of leisure net of marginal utility of working in period 2.

Although the equations in the time-consistent framework and the temptation good framework look similar, the results differ. To illustrate, here gives an example.

EXAMPLE 5: Suppose the instantaneous utility functions are

\[
u(D, y) = a\ln D + b\ln y \tag{19}
\]
\[
u(c) = \gamma\ln c \tag{20}
\]

In a time-consistent framework, we can solve
\[ y_1 = \frac{\beta}{\gamma} c_1 \]
\[ y_2 = \frac{\beta}{\gamma} c_2 \]
\[ \frac{\alpha - \beta}{y_2} = \frac{\alpha}{D_2} - \frac{\beta}{y_1} \]

Rearranging terms, combined with the constraints, we can get
\[ \frac{\alpha}{M - D_1} - \frac{\beta + \gamma}{1 + \rho} = \frac{\alpha}{D_1} - \frac{\beta + \gamma}{H - D_1} \]

Similarly, by doing the same thing in the temptation good framework, we can get
\[ y_1 = \frac{\beta}{\gamma} c_1 \]
\[ \frac{\alpha - \beta}{y_2} = \frac{\alpha}{D_2} - \frac{\beta}{y_1} \]
\[ \frac{\alpha}{M - D_1} - \frac{\beta}{1 + \rho} = \frac{\alpha}{D_1} - \frac{\beta + \gamma}{H - D_1} \]

We can obtain at equilibria \(D_{1,\text{consistent}} > D_{1,\text{temptation}}\). Thus, the time-inconsistency will induce the individual to do less work in period 1 than the individual would do in a time-consistent framework.

ii. Time-inconsistent problem

In the former discussion, temptation good is a relatively extreme case of the time-inconsistent problem. This part of the paper gives a more general discussion of time-inconsistency.

**Result 3:** A time-inconsistent risk-averse individual will procrastinate more and work less at the beginning of the work process and build up workload more quickly than the case where he/she is time-consistent.

Intuitively, because the individual is time-inconsistent, at any period \(t\), he/she will discount days after \(t\) much more heavily, thus he/she will procrastinate more. However, because he/she procrastinates more at the beginning of the work process, due to the time constraint of every day, he/she must build up workload quickly to be able to finish the work before the deadline.
In a time-inconsistency framework, following Fischer (2001) and Gruber and Kőszegi (2001), the time-consistent model is

\[ u_t = \sum_{i=0}^{T-i} \left( \frac{1}{1+\rho} \right)^i u(c_{t+i}) \tag{19} \]

Equation (19) means the total present value of utility in period \( t \) is the flow of discounted value from period \( t \) until the deadline.

The time-inconsistent model is

\[ u_t = u(c_t) + \delta \sum_{i=1}^{T-i} \left( \frac{1}{1+\rho} \right)^i u(c_{t+i}) \tag{20} \]

where \( T \) is the deadline and \( \delta \) measures the degree of the time-inconsistency: the smaller \( \delta \) is, the larger the time-inconsistent degree.

Equation (20) means that the total present value of utility in period \( t \) is the instantaneous utility in period \( t \) plus the discounted flow of instantaneous utilities from period \( t+1 \) until the deadline. Equation (20) shows that in period \( t \), the discount rate of between \( u(c_{t+s}) \) and \( u(c_{t+s+1}) \) is \( \frac{1}{1+\rho} \), however, the discount rate becomes \( \frac{\delta}{1+\rho} \) when the individual is in period \( t+s \). The time-inconsistency model captures the fact that the individual tends to overvalue the current period while undervaluing the future.

Both equations (19) and (20) are subject to the budget constraint

\[ \sum_{i=t}^{T} (H - c_i) \geq M_t \tag{21} \]

where

\[ M_t + \sum_{i=0}^{t-1} (H - c_i) = M \tag{22} \]

Solving equation (19) subject to equations (21) and (22), we can get

\[ u_{c_t} = \left( \frac{1}{1+\rho} \right)^s u_{c_{t+s}} \tag{23} \]

Solving equation (20) subject to equations (21) and (22), we can get

\[ u_{c_t} = \delta \left( \frac{1}{1+\rho} \right)^s u_{c_{t+s}} \tag{24} \]

Comparing equations (23) and (24), we can generally tell that \( c_{t,\text{inconsistent}} > c_{t,\text{consistent}} \) when \( t \) is small.
Figure 1. An illustration of working load everyday with $u(c) = Inc$ function

As shown in Figure 1, the time-inconsistent individual procrastinates longer and builds up workload more quickly than the time-consistent individual. Thus, given a multi-stage task, the individual will procrastinate at the beginning of each period, but build up their workload quickly enough to meet the deadline, which explains how procrastination and promptness could coexist.

III. Procrastination capital

Following Becker and Murphy (1988), we can also find some similarity between an addictive good and the behavior of procrastination. The current behavior of procrastination, denoted as $c_t$, will increase current utility. However, the past memory or experience of procrastination can induce the individual to suffer from disutility in the current period – for example, in the form of psychological regret – or, conversely, may induce the person to procrastinate habitually. As defined at the beginning of this paper, procrastination capital is the past memory or experience about procrastination behavior, which can be either encourage or discourage current procrastination behavior. This paper defines procrastination capital $S_t$ as the state variable of the dynamic optimization method. $D_t$ denotes the time spent on getting work done. $t$ denotes the time period $t$ and $T$ denotes the deadline date of the task.

i. Single procrastination capital

In the single capital case, we have
\[ u(t) = u[c_t, D_t, S_t] \]  

(25)

where the instantaneous utility \( u \) is continuously differentiable and depends on the time spent and not spent on the work \( - D_t \) and \( c_t \) accordingly – and procrastination capital \( - S_t \). Here we do not differentiate leisure and procrastination for simplicity. We also assume that the instantaneous utility function is separable in \( c_t, D_t, S_t \) at each period \( t \), but \( c_t \) and \( S_t \) are correlated across time. In addition, there is a required amount of time input \( - M \) – to get the work done.

Thus, the individual’s utility maximization process becomes

\[
\max_{\{c_t\}} \int_0^T e^{-\rho t} u[c_t, D_t, S_t] \, dt 
\]

(26)

where \( \rho \) stands for time preference or subjective discount rate, subject to the budget constraint and the work effort requirement

\[
H \geq c_t + D_t 
\]

(27)

\[
\int_0^T D_t \, dt \geq M 
\]

(28)

And the procrastination capital accumulation function is

\[
\dot{S}_t = c_t - \gamma S_t 
\]

(29)

Equation (29) tells us that procrastination capital depends on current procrastination behavior of each period \( t \) – that is, \( c_t \), net of the depreciation of procrastination capital. \( \gamma \) is the procrastination capital depreciation rate. \( \gamma \) can capture how fast the individual “forgets” his/her past procrastination behavior. The larger \( \gamma \) is, the less procrastination capital will be accumulated.

Then we can get the results by setting up the current value Hamiltonian

\[
CVH = u[c_t, D_t, S_t] + \lambda_1 (H - c_t - D_t) + \lambda_2 \left[ \int_0^T D_t \, dt - M \right] + \mu_t (c_t - \gamma S_t) 
\]

(30)

where \( \lambda_1 \) stands for the shadow value of one unit of time, \( \lambda_2 \) stands for the shadow value of time spent on working, and \( \mu_t \) stands for the shadow price of procrastination capital.

Then we can get

\[
\mu_t = \int_t^T e^{-(\gamma + \rho)(\tau - t)} u_{c_t} \, d\tau 
\]

(31)

Equation (31) shows that the shadow price of procrastination capital is the discounted flow of marginal utility brought by procrastination capital from period \( t \) to the deadline date \( T \).

\[
u_{c_t} = \lambda_1 - \mu_t 
\]

(32)
Equation (32) shows that at the optimal point, the marginal benefit of procrastination will equal the marginal cost, which consists of both the shadow value of time and the shadow price of procrastination capital. The $\mu_t$ term captures the fact that the procrastination behavior today will transform into procrastination capital and impact the individual’s utility from period $t$ and onwards.

$$u_{D_t} + \lambda_2 = \lambda_1$$

Equation (33) tells that the utility brought by one unit of time devoted to work and the shadow value of time devoted to work will equal the shadow value of time at the optimum.

In order to see how procrastination behavior evolves across time, following Becker and Murphy (1988), this paper drops time subscripts and makes following additional assumptions:

(A1) $T = \infty$, such that the deadline is far away enough

(A2) The instantaneous utility function $U_t$ is in quadratic forms of $c_t, D_t, S_t$

(A3) There is no interaction between $D_t$ and $S_t$, or between $D_t$ and $c_t$

$$U = \beta_c c + \beta_s s + \beta_D D + \frac{\beta_{cc}}{2} c^2 + \frac{\beta_{ss}}{2} s^2 + \frac{\beta_{DD}}{2} D^2 + \beta_{cs} c s + \beta_{cD} c D + \beta_{SD} S D$$

By optimizing out the time spent on the task $D$ (see Appendix), we can get the value function

$$F = \beta_c c + \beta_s s + \frac{\beta_{cc}}{2} c^2 + \frac{\beta_{ss}}{2} s^2 + \beta_{cs} c s - \lambda_1 c$$

Subject to

$$\lim_{t \to \infty} e^{-\rho t} S_t^2 = 0$$

The transversality condition means that at the deadline or the end of the planning horizon, procrastination capital should be zero.

The Euler equation is

$$\ddot{S} - \rho \dot{S} - BS = \frac{(\rho + \gamma) \beta_c c + \beta_s s - (\rho + \gamma) \lambda_1}{\beta_{cc}}$$

where

$$B = \frac{(\rho \gamma + \gamma^2) \beta_{cc} + \beta_{ss} s + (\rho + 2 \gamma) \beta_{cs}}{\beta_{cc}}$$

Following the same reasoning of Becker and Murphy (1988) and taking the transversality condition into consideration, we can tell there is only one characteristic root $\theta$ and one unique steady state $(c^*, S^*)$, that is (see Appendix)
The optimal path of procrastination capital is

$$\theta = \frac{p - \sqrt{p^2 + 4B}}{2}$$  \hspace{1cm} (39)

The optimal path of procrastination capital is

$$S = (S_0 - S^*)e^{\theta t} + S^*$$  \hspace{1cm} (40)

Where $S_0$ denotes the procrastination capital of last period.

Take time derivative of equation (40) and substituting back into equation (29) to get

$$c = (\theta + \gamma)S - \theta S^*$$  \hspace{1cm} (41)

Rearranging the terms, we can get

$$c = \theta(S - S^*) + \gamma S$$  \hspace{1cm} (42)

One interesting question to discuss here is why the case with procrastination capital differs from models without procrastination capital. The answer is that the feedback channel differs. In the models without procrastination capital, procrastination behavior gives feedback to a forward-looking individual by squeezing out the time spent on work. However, in the procrastination capital case, the procrastination behavior gives feedback not only by squeezing out time spent on work, but also by influencing the utility gained from procrastination itself. Based on this difference of channeling, we can differentiate these models empirically, which is discussed in the last section.

1. **Procrastination with positive feedback on $u_c$**

Can procrastination capital be interpreted as an addictive good? In this case, one additional assumption is that $\frac{\partial^2 u}{\partial c \partial s} = \beta_{cs} > 0$.

Intuitively, this interpretation is possible because past experience of successfully managing a task will increase the possibility of procrastination when the individual is dealing with the next task. That is,

$$S_0 = E_{-0}$$

where $E_{-0}$ stands for the past survival experience of procrastination before the new task starts ($t=0$), and $S_0$ denotes the stock of procrastination capital in period 0. When the individual successfully survived his/her past projects in spite of procrastination, then $E_{-0}$ is positive and will increase the initial stock of procrastination capital, resulting in a higher probability of procrastination during the ongoing task. Thus, in this sense, more procrastination capital will make the individual more likely to procrastinate by increasing the individual’s marginal utility of procrastination.

However, the positive feedback of procrastination capital only implies that more procrastination capital will increase the marginal utility of procrastination and then the possibility of procrastination, but the individual will actually procrastinate more only when the positive improvement of more procrastination capital on marginal utility of
procrastination is large enough. This makes sense because the individual has to compare the marginal benefit gained from more procrastination with the marginal cost of more procrastination through the form of procrastination capital and forgone time value.

**Result 4:** Let \( \beta_{cs} \in \left( \frac{\rho y + y^2}{\rho + 2y}, \frac{\rho y + \rho y^2 + \rho y S}{\rho + 2y} \right) \), which means more procrastination capital will increase the marginal utility of procrastination, but within a range. Then the procrastination behavior will finally reach at the steady state \((c^*, S^*)\). The optimal path of \((c, S)\) is stable.

Stability means that when \( S < S^* \), \( S \) will increase to \( S^* \); when \( S > S^* \), \( S \) will decrease to \( S^* \). Intuitively, Result 4 tells that when procrastination capital does not accumulate too rapidly, the individual will procrastinate more and finally reach \((c^*, S^*)\). If the past positive history of procrastination up to today is substantial, then the individual will reduce procrastination in the next period and finally reach \((c^*, S^*)\).

![Figure 2. The stable optimal path of \((c, S)\)](image)

As shown in the equation (42) and Figure 2, we can find that the optimal path of \((c, S)\) is stable: when \( S < S^* \), \( S \) will increase to \( S^* \) and \( c \) will increase to \( c^* \); when \( S > S^* \), \( S \) will decrease to \( S^* \) and \( c \) will decrease to \( c^* \).

In the setting of writing a paper, as we do not differentiate procrastination from leisure for simplicity, stability means that the individual will work more or less according to how much he/she works in the last period, and in the steady state, he/she will work for a certain amount of time everyday.
**Result 5:** Let the positive feedback that more procrastination capital will increase the marginal utility of procrastination be large, and specifically \( \beta_{CS} \in \left( \frac{(\rho \gamma + \gamma^2) \beta_{cc} + \beta_{SS}}{\rho + 2\gamma}, \infty \right) \). The procrastination behavior will then exhibit adjacent complementarity and the optimal path of the procrastination will become unstable.

Result 5 tells that when the positive feedback of procrastination capital on current marginal utility of procrastination is large enough to weigh more than its marginal cost, the individual will tend to procrastinate more as procrastination capital stock becomes large. When procrastination capital stock is small, there is not enough positive feedback to encourage the individual to procrastinate more – that is, procrastination habit is not formed yet – thus, the behavior will decrease over time.

![Figure 3. The unstable optimal path of \((c, S)\)](image)

If \( \beta_{CS} \left( \frac{(\rho \gamma + \gamma^2) \beta_{cc} + \beta_{SS}}{\rho + 2\gamma} \right) \), B < 0, then \( \theta > 0 \). By Equation (42), we can tell that the optimal path of \((c, S)\) is unstable. As illustrated in Figure 3, when \( S > S^* \), S will increase to infinity, although c will increase but must be bounded by the time budget constraint. Thus, S is also bounded; when \( S < S^* \), S will decrease to \( \frac{\theta S^*}{\theta + \gamma} \), and c will decrease to 0. This is the case described as adjacent complementarity as described in Becker and Murphy (1988).

Intuitively, adjacent complementarity means that an individual will procrastinate at a certain level, or when there is some perturbation, he/she will either not procrastinate and get the work done or will procrastinate as much as possible and finally give up the
work altogether. In this case, if we assume specific functional forms of utility functions, we could have multiple equilibria.

2. Procrastination with negative feedback on $u_c$

In this part, the additional assumption is that $\frac{\partial^2 U}{\partial c \partial s} = \beta_{cs} < 0$. We can interpret procrastination capital as the bad memory or unsuccessful experience about past procrastination behavior up to the current period. When past procrastination behavior makes the individual suffer, bad memory about past procrastination behavior — procrastination capital — will remind the individual of his/her past behavior, thus reducing the net marginal benefit of procrastination in the current period.

**Result 6:** If procrastination capital stock has a negative effect on the marginal utility of procrastination, which means $\beta_{cs} < 0$, the individual will procrastinate to reach a steady state $(c^*, S^*)$. The optimal path of $(c, S)$ is stable.

Intuitively, when bad memory of past procrastination behavior causes psychological discomfort, the individual will feel less comfortable when he/she procrastinates in the current period, as he/she is reminded of his/her past behavior. When the bad memory of past procrastination behavior is weak, the individual will procrastinate but will procrastinate less and less as the memory builds up. However, when the bad memory is already substantial, the individual will hesitate to procrastinate, which reduces memory because of natural forgetting process, which is referred as depreciation in this model. As memory decreases, the individual will feel better about procrastinating today, thus he/she will again procrastinate more.

As analyzed above, if $\beta_{cs}$ is negative, $\theta < 0$. We can get a stable path and the procrastination capital will exhibit adjacent substitution, that is,

$$\theta + \gamma < 0 \iff 0 < \frac{\beta_{ss} + (\rho + 2\gamma)\beta_{cs}}{\beta_{cc}} \quad (43)$$

From equation (43) we can see that $\beta_{cs} < 0$ guarantees that $\theta + \gamma < 0$, which means the more procrastination capital up to the period, the less procrastination will be in this period. This is the case of adjacent substitution, in contrast to the addictive goods with adjacent complementarity as analyzed in Becker and Murphy’s (1988) model.
As illustrated in Figure 4, starting from a positive amount of procrastination, consumption of procrastination will decrease over time as the marginal utility of procrastination decreases when procrastination capital is accumulated, until reaching the steady state \((c^*, S^*)\). When there is some perturbation leading \((c, S)\) above the steady state \((c^*, S^*)\), the procrastination capital decreases due to the fact that the consumption of procrastination is too low to offset the depreciation of procrastination capital, and then the consumption of procrastination will increase as procrastination capital decreases, thus \((c, S)\) will finally return to the steady state \((c^*, S^*)\). In sum, when \(S < S^*\), \(S\) will increase to \(S^*\) and \(c\) will decrease to \(c^*\); when \(S > S^*\), \(S\) will decrease to \(S^*\) and \(c\) will increase to \(c^*\). Therefore, the optimal path of \((c, S)\) is stable.

Another interesting question to discuss is that what will happen if \(\beta_{cs}\) changes signs as procrastination capital accumulates? In this case, we interpret \(S\) as the general past procrastination history. Suppose \(\beta_{cs}\) is large enough to generate adjacent complementarity and the individual has high stock of procrastination capital when the task is assigned. As the individual can gain high level of utility from procrastination, he/she will choose to procrastinate at the very beginning and accumulate procrastination capital, following the unstable path. However, when the procrastination capital is large enough to make him/her feel really bad about not working, he/she will gradually reduce procrastination and start working to a steady state. In addition, we can frame this “turning point” effect of procrastination capital as the result of two-procrastination-capital case with conflicting effects on procrastination.

ii. **Two procrastination capitals**
In reality, general past memory and experience about procrastination have two conflicting effects: the positive effect due to past successful survival experience induces the individual to procrastinate; the negative effect due to past bad memory of such behavior results in the individual’s psychological discomfort, thus reducing the incentive to procrastinate. Following Becker and Murphy (1988), this paper discusses the two-procrastination-capital case. The procrastination capital with positive feedback on marginal utility of procrastination, such as past successful survival experience, is denoted as $S_{1,t}$; the procrastination capital with negative feedback on marginal utility of procrastination, such as bad memory about past procrastination behavior, is denoted as $S_{2,t}$. Then our model can change into

$$u(t) = u[c_t, D_t, S_{1,t}, S_{2,t}]$$

(44)

where the instantaneous utility $u$ is continuously differentiable and depends on the time spent and not spent on the work and procrastination capitals $S_{1,t}, S_{2,t}$. We also assume that the instantaneous utility function is separable in $c_t, D_t, S_{1,t}, S_{2,t}$ at each period $t$, but $c_t$ and $S_{1,t}$, $c_t$ and $S_{2,t}$ are correlated across time.

Thus, the individual’s utility maximization process becomes

$$\max_{\{c_t\}} \int_0^T e^{-\rho t} u[c_t, D_t, S_{1,t}, S_{2,t}] dt$$

(45)

where $\rho$ stands for time preference or subjective discount rate, subject to the budget constraint and the work effort requirement

$$H \geq c_t + D_t$$

(46)

$$\int_0^T D_t dt \geq M$$

(47)

And procrastination capital accumulation functions are

$$S_{1,t} = c_t - \gamma_1 S_{1,t}$$

(48)

$$S_{2,t} = c_t - \gamma_2 S_{2,t}$$

(49)

Equations (48) and (49) tell us that procrastination capital depends on the procrastination behavior of each period $t$—that is, $c_t$, net of the depreciation of procrastination capital, $\gamma_1, \gamma_2$ are the procrastination capital depreciation rates for $S_{1,t}, S_{2,t}$, respectively. The larger $\gamma_1$ or $\gamma_2$ is, the less corresponding procrastination capital will be accumulated. Although we assume $S_{1,t}, S_{2,t}$ have similar accumulation process, the initial stocks of $S_{1,t}, S_{2,t}$ vary.

Then we can get the results by setting up the current value Hamiltonian

$$CVH = u[c_t, D_t, S_{1,t}, S_{2,t}] + \lambda_1 (H - c_t - D_t) + \lambda_2 \left[ \int_0^T D_t dt - M \right]$$

$$+ \mu_{1,t}(c_t - \gamma_1 S_{1,t}) + \mu_{2,t}(c_t - \gamma_2 S_{2,t})$$

(50)
where \( \lambda_1 \) stands for the shadow value of one unit of time, \( \lambda_2 \) stands for the shadow value of time spent on working, \( \mu_{1,t} \) stands for the shadow price of the procrastination capital \( S_{1,t} \), and \( \mu_{2,t} \) stands for the shadow price of the procrastination capital \( S_{2,t} \).

Then we can get

\[
\begin{align*}
\mu_{1,t} &= \int_t^T e^{-(\gamma_1+\rho)(t-\tau)}u_{s_{1,\tau}} d\tau \\
\mu_{2,t} &= \int_t^T e^{-(\gamma_2+\rho)(t-\tau)}u_{s_{2,\tau}} d\tau
\end{align*}
\]  

Equations (51) and (52) show that the shadow price of each type of procrastination capital is the discounted flow of marginal utility from procrastination capital from period \( t \) to the deadline date \( T \).

\[
u_{c_t} = \lambda_1 - \alpha_{1,t} - \alpha_{2,t}
\]  

Equation (53) shows that at the optimal point, the marginal benefit of procrastination will equal the marginal cost, which consists of the shadow value of time and the shadow prices of two procrastination capitals \(- S_{1,t}, S_{2,t} \). One unit of procrastination in period \( t \) will increase the stocks of two procrastination capitals, hence the marginal cost of procrastination consists of the shadow prices of both procrastination capitals.

\[
u_{D_t} + \lambda_2 = \lambda_1
\]  

Equation (54) is the same as Equation (33), which tells that the marginal utility of time devoted to work plus the shadow value of time devoted to work equals the shadow value of time at the optimal point.

Like the analysis of binges in Becker and Murphy (1988), two procrastination capitals could also form cycles of procrastination and promptness. When \( S_{1,t} \) exhibits adjacent complementarity and \( S_{2,t} \) exhibits adjacent substitution, whether the individual will procrastinate depending on the magnitudes of the coefficients – that is, \( \theta \) – of adjacent complementarity and adjacent substitution and the quantities of the two capital stock \( S_{1,t}, S_{2,t} \). Suppose when the result of the last task is really successful in spite of procrastination, the stock of \( S_{1,t} \) increases. Thus, the adjacent complementarity effect will dominate and increase the consumption of procrastination. In this case, the procrastination behavior will either go to zero or, conversely, explode to the maximum level of procrastination at every period bounded by the time budget. Suppose the individual is really likely to regret on his/her memory about past procrastination behavior. In this case, the adjacent substitution effect will dominate and induce the individual to procrastinate less when the procrastination capital \( S_{2,t} \) is high. In sum, with the two conflicting forces of these two procrastination capitals, the individual will procrastinate when procrastination capital with adjacent complementarity dominates, and will procrastinate less when procrastination capital with adjacent substitution.
dominates. This could be helpful to understand why students do not spend much time on study everyday but stay up overnight to cram before midterms and finals.

IV. Conclusion

This paper shows that the individual chooses to procrastinate as a result of utility maximization. Based on a simple two-period model, this paper finds that a forward-looking individual will procrastinate more in the current period than in the next period; the deadline may induce less procrastination; when the individual can anticipate the final outcome and the more outcome-oriented he/she is, the less he/she will procrastinate in the current period, holding the amount of procrastination in the next period constant. This paper also reaches the conclusion that a time-inconsistent individual will procrastinate more at the beginning of the work process and build up his/her workload more quickly than the situation where the individual’s preference is time-consistent. In addition, this paper discusses procrastination in the framework of addictive goods (Becker and Murphy, 1988): when procrastination exhibits adjacent complementarity, the individual will not procrastinate and get the work done, or the individual will conversely procrastinate as much as possible and finally give up the task altogether; when procrastination exhibits adjacent substitution, the individual will consume a certain level of procrastination at a stable steady state. In a two-procrastination-capital setting, the individual is likely to behave cycles of procrastination and promptness. Therefore, in Elaine and Sarah’s story, their behavior could be understood either from the time-inconsistency preference perspective, or from the case of two procrastination capitals.

Empirically, we can test these models by behavioral experiments. We can observe participants’ actual schedules of getting work done to see whether individuals will procrastinate more today than they would tomorrow. To see if individuals care about final outcome, we can observe how the participant’s work schedules actually change by changing final reward rules. As for time-inconsistency, we can give each experimental participant a multi-stage task and observe how they assign their workload. More specifically, we could ask them to submit their plans for getting the work done every day. If the schedules, which are actually carried out, turn out that participants procrastinate more and build up their workload more quickly than their initial plans in the previous periods, then we can tell that there is time inconsistency. As for the procrastination capital model, we can analyze their actual procrastination behavior and their psychological activities when they choose to work or procrastinate. As analyzed in Section III, the main difference between the procrastination capital model and models without procrastination capital is the feedback channel. Thus, by analyzing the incentives to procrastinate or avoid procrastinating, we can differentiate models with or
without procrastination capital empirically. In details, we can randomly pick students from the same course with final paper requirement at the beginning of class into different groups, and assign each group with different rules and track students’ writing progress. We can have one group as control, one group in which students will be reminded of their past writing grades for their other courses, one group in which students will be frequently reminded of their current progress on the paper, and one group in which students will be frequently reminded of the potential danger of procrastination. At the end of semester, we can compare students’ writing performance to check if past writing experience, writing progress reminders and knowledge of procrastination will impact students’ writing progress and writing performance.

In order to reduce procrastination, the paper tells us that (1) deadlines help fight against procrastination; (2) the more outcome-oriented an individual is, the less he or she will procrastinate; (3) if he/she learns that one unit of time tomorrow is not the same as one unit of time today to improve the results, the individual will start working sooner; (4) an individual can also cut down on the inference from past successful survival experiences or adopt the cold turkey strategy (Becker and Murphy, 1988) when the procrastination behavior exhibits adjacent complementarity.

In terms of realistic implication, this paper has both subjective and objective suggestions. In order to avoid procrastination, this paper implies that the importance of everyday effort should be stressed and time-inconsistency should be identified. In a course design setting, this paper implies that we can assign course workload into multiple reports, tests or homework across the whole semester in the syllabus, instead of just simply giving out midterms and finals.

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Appendix

Following Becker and Murphy (1988) and Chaloupka (1990), we drop the time subscripts and assume a quadratic form of the utility function

\[ U = \beta_c c + \beta_s S + \beta_D D + \frac{\beta_{cc}}{2} c^2 + \frac{\beta_{ss}}{2} S^2 + \frac{\beta_{DD}}{2} D^2 + \beta_{cs} cS + \beta_{cD} cD + \beta_{SD} SD \]
Because we assume the instantaneous utility function is separable in $S$, $D$ and $c$, then $\beta_{CD} = \beta_{SD} = 0$. Then we can get

$$U = \beta_c c + \beta_s S + \beta_D D + \frac{\beta_{cc}}{2} c^2 + \frac{\beta_{ss}}{2} S^2 + \frac{\beta_{DD}}{2} D^2 + \beta_{cs} cS$$

Then the utility maximization problem becomes

$$V^* = \max_{c,D} \int_0^T [e^{-\rho t} U + \lambda_1 (H - c - D) + \lambda_2 (\int_0^T D dt - M)] dt$$

subject to $\dot{S} = c - \gamma S$.

At the optimal point,

$$U_D = \beta_D + \beta_{DD} D = \lambda_1 - \lambda_2$$

$$\lambda_1 = \frac{\partial V^*}{\partial H}$$

$$\lambda_2 = - \frac{\partial V^*}{\partial M}$$

Then we can get

$$D = \frac{\lambda_1 - \lambda_2 - \beta_D}{\beta_{DD}}$$

By substituting $D$ back into the $V^*$ function, we can get

$$V^* = K + \max_c \int_0^T [e^{-\rho t} F(c, D)] dt$$

where $K$ is a constant and $F$ is the following form

$$F = \beta_c c + \beta_s S + \frac{\beta_{cc}}{2} c^2 + \frac{\beta_{ss}}{2} S^2 + \beta_{cs} cS - \lambda_1 c$$

Notice that at the optimal point,

$$\dot{S} = c - \gamma S$$

Thus substituting $c = \gamma S + \dot{S}$ into $e^{-\rho t} F(c, D)$, so we can get

$$l = e^{-\rho t} \left[ (\beta_c \gamma + \beta_s - \lambda_1 \gamma) S + (\beta_c - \lambda_1) \dot{S} + \frac{\beta_{cc}}{2} \dot{S}^2 + \left( \frac{\beta_{cc}}{2} \gamma^2 + \frac{\beta_{ss}}{2} + \gamma \beta_{cs} \right) S^2 \right.$$

$$+ \left( \beta_{cc} \gamma + \gamma \right) SS \right]$$

Following Madden and Turaeva (2015), Euler equation must satisfy
\[
\frac{\partial I}{\partial S} - \frac{d}{dt} \frac{\partial I}{\partial S} = 0
\]

Then we can get the Euler equation
\[
\dot{S} - \rho \dot{S} - BS = \frac{(\rho + \gamma)\beta_c + \beta_s - (\rho + \gamma)\lambda_1}{\beta_{cc}}
\]
where
\[
B = \frac{(\rho\gamma + \gamma^2)\beta_{cc} + \beta_{ss} + (\rho + 2\gamma)\beta_{cs}}{\beta_{cc}}
\]

In order to solve the differential equation, we have
\[
\theta = \frac{\rho \pm \sqrt{\rho^2 + 4B}}{2}
\]

Due to the transversality condition, there is only one solution for \(\theta\), that is,
\[
\theta = \frac{\rho - \sqrt{\rho^2 + 4B}}{2}
\]

And in steady state where \(\ddot{S} = \dot{S} = 0\), thus we can get the steady state procrastination capital and procrastination are \((c^*, S^*)\), where
\[
S^* = -\frac{(\rho + \gamma)\beta_c + \beta_s - (\rho + \gamma)\lambda_1}{B\beta_{cc}}
\]
\[
c^* = \gamma \frac{(\rho + \gamma)\beta_c + \beta_s - (\rho + \gamma)\lambda_1}{B\beta_{cc}}
\]

References


