The Formation of Small Cities: Adopting a Model to Include Preference Endogeneity and Population Mobility

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Abstract

This paper adopts some assumptions in Alesina and Spolaore's 1997 paper to investigate the formation of cities under small economic shocks. People's preferences are endogenous and they have limited mobility in this model. When deciding whether and where to move, people will evaluate the change in their utility, the possible profit from the shocks, and the costs of moving. Thus, the population and structure of cities can be different when a shock occurs in the capital, within the country, or on the border. Another variable will be the tax system. Different tax systems will lead to different welfare effects and also to different city size distributions. An economic border war may also be a consequence of two neighboring countries enacting different tax policies.

Keywords

City formation; Endogenous preferences; Tax systems; Economic border war

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Contents

1. Introduction

In this paper I try to model how new cities form as a consequence of small economic shocks. I begin with the model found in Alesina and Spolaore's 1997 paper, which explains how nations are formed, and apply it to city formation in response to economic shocks. In doing so, I change the Alesina and Spolaore model in two way to make it more realistic. One, preferences are endogenous in my model, because people will change their preferences through interactions with surrounding people. Two, people are not immobile. They can move, but suffer from high moving costs.

Other economists before me have proposed theories to explain the formation of cities. Two classic models in the literature are those by Mills (1967) and Henderson (1974). Mills (1967) views city formation as a balance between benefits and costs. Businesses in a larger city can enjoy scale economies in traded good production, but their employees will have to bear increasing commuting costs. Henderson (1974) lists two conditions for city formation under perfectly mobile labor and capital. The first condition is the existence of technological economies of scale in production and consumption; the second is that economic activities are not land intensive.

In the two theories mentioned above, economies of scale are considered the important factor for city formation. However, one of the flaws in Henderson's paper is that the location of the city can be random as long as there are enough labor and capital coming together. This does not jibe with the fact that some economic shocks may happen at specific locations, as in my model, and thus make these sites different, attracting people to form cities there. Moreover, both theories assume that production factors are perfectly mobile, which is also unrealistic. Moving costs can be high both economically and psychologically. In this paper, I will investigate a situation in which people are almost fixed at their location due to large moving costs. Thus, only a huge shock of profit can attract them to move and then the congregation of profit-seekers may lead to the formation of a city.

Thus, I am adopting Alesina and Spolaore's assumptions in investigating the formation of cities, and trying to correct some unrealistic assumptions in Mills (1967) and Henderson (1974). The main finding of my paper is that the population and structure of cities will vary with the location of the shock: at the capital, within the country, or on the border. The design of tax systems will also influence the city size distribution and welfare effects. Two neighboring countries enacting different tax policies may even lead to an economic border war.

The paper is organized as follows: Section 2 will present the assumptions, some of which I will modify. Section 3 will discuss how a small city can be formed when people are attracted by the profit of the shock and make moving decisions. Section 4 includes some discussion of the effect of income taxes, the economic border war and larger shocks. The last section presents conclusions.

2. Basic Assumptions

Following are the basic assumptions of the model.

1. Total population and world area are normalized. Alesina and Spolaore normalize the world population to 1 and make it uniformly distributed on the segment $[0,1]$.

2. There are at least one government and only one public good in the world. In their model, they deny the state of anarchy and believe that people must live with public services provided by a government. Thus, there must exist at least one government in the world. They also simplify all public services to only one public good, which is the "government". The government thus includes all the administrative services provided publicly, such as the legal system, education, and defense. And naturally, each government needs taxes to support its operation. Alesina and Spolaore assume here that each government will cost k, regardless of the size it controls and the number of services it provides.

3. The income distribution is homogeneous. Without further information about the composition of people in the world, people are assumed to earn the same exogenous income y and pay the same amount of tax. The model did not clarify what type of tax is assigned here. We will first treat the tax system as a poll tax and then consider what would happen if it was an income tax system. Therefore, everyone in a nation is economically homogenous.

4. Individual utility is decided by preference and economic gain. The utility of the individual is expressed as

$$U_i = g(1 - al_i) + y - t_i \qquad \text{------(1)}$$

in this model. l_i is the distance of individual from his government in preference. The parameter g represents the degree to which an individual's utility depends on public goods and a is the marginal utility loss in preference when an individual becomes farther from his government.

5. Population and preferences are distributed uniformly. To make the model tractable, Alesina and Spolaore assume coincident geographical and preference distribution. Thus, l_i will indicate the distance of an individual from his government both in preference and geographically. This assumption is reasonable in that it will simplify the model, and at the same time guarantee connected countries. It is also compatible with our intuition that the farther the individual is from the government, the less or poorer public services s/he may be able to enjoy.

Preference formation has been a much-discussed topic in political science, and researchers hold very different ideas about preferences than are found in Alesina and Spolaore's paper. Aaron Wildavsky (1987) points out that preferences are endogenous instead of exogenous, as often appeared in economic papers. Culture theory explains preferences as endogenous because preferences are formed through social interaction when people are defending or opposing different ways of life. To be more explicit, people's perspectives are influenced by the culture and they can change the culture as

well, since humans are social creatures. Druckman and Lupia (2000) believe that preferences are basically the ranking of attitudes and are derived from evaluations. People's evaluations are generally biased, because the underlying beliefs are the results of interactions between people and their surroundings. Political actions, such as persuasive and strategic communications, are attempts to change people's beliefs and thus preferences. As a result, preferences are formed through interactions between people and their environment.

Assumption (modified) 1: Preferences are **endogenous**. People in the model will change their preferences to where they are once they move, as a result of the interaction with people surrounding them and the public services enjoyed at the new location.

Endogenous preference can explain why preferences are distributed continuously on the segment [0,1]. On one side, people with similar preferences may choose to live together as a group. On the other side, people's preferences can be molded by their environment, which will assimilate people's preferences within a group. Moreover, people living together can share the same natural resources and public services, and can be influenced by the same shock; thus, they are more prone to have similar preferences. When there is a newcomer, the person's preference will be affected gradually by the surroundings and ultimately become the same as his or her neighbors. Thus, in the model, an individual's preference will be coincident with his or her geographical location and will change accordingly when the individual moves to another location.

Another assumption that I will modify is to make each individual's location mobile but with high moving costs. Immobility of people is becoming extremely difficult even under the strict household registration system used in some countries. Instead, we can set up a cost function of moving but making the cost very high to fix most people.

Assumption (modified) 2: People can move with high costs; the cost of moving for individual *i* is

$$C_i = F + cd_i \qquad \text{------(2)}$$

There is always a fixed cost F when moving, because people will have to sell their houses and adjust to their new environment psychologically. The parameter c represents the degree of cost related to distance; it is positive, since moving farther will inevitably involve more transportation costs. More importantly, people can also predict that moving farther means adjusting is more difficult, because the gap in preference with their new neighbors will be wider. In summary, transportation and adjustment costs together make moving costs increase with distance.

The full list of variables and their definitions can be found in table 1.

VARIABLE	DESCRIPTION
$\boldsymbol{k}$	the operation cost of a nation
$\boldsymbol{U_i}$	Individual i's utility

y	Individual's income
t_i	Individual i's tax
g	the degree of individual's utility depends on public goods
a	the marginal utility loss of larger preference distance
l_i	Individual i's preference distance
C_i	Individual i's moving cost
F	Fixed cost of moving
c	the degree of cost related to distance
d_i	Individual i's moving distance
s	the size of a nation
M	the magnitude of the shock
r_i	the distance of people from the shock
r	the radius of the shock
W	total welfare
h	the distance between the shock and the capital
t	the tax rate

Table 1: List of variables.

3. Basic Model of Small City Formation

3.1 Reformulation of the A&S Model

In Alesina and Spolaore's model, every nation will be of equal size s in equilibrium and every individual will have to pay the same tax. The stable number of nations in the world is $\widetilde{N}$, which is the largest integer smaller than $\sqrt{ga/2k}$.

People living in the capital will not choose to move since they are the median voters and gain the most utility in a nation. In contrast, people living on the border of two countries are the ones who want to move the most. They have the maximal distance from the capital, which is half the length of the nation, $\frac{s}{2}$. They can reduce their distance in preference by moving nearer to the capital. If they can gain more utility from moving, they will choose to do so, even though they have to pay for the cost of moving. The original utility of individual i on the border is

$$U_i = g\left(1 - a\frac{s}{2}\right) + y - t_i \qquad \text{------(3)}$$

If the person moves distance d towards the capital, and d is in the interval $[0, s/2]$,

s/he will have utility

$$U_i' = g\left[1 - a\left(\frac{s}{2} - d\right)\right] + y - t_i - F - cd \quad \text{------(4)}$$

People are immobile in the model; thus people will choose not to move, and the inequality $U_i' \geq U_i$ turns into

$$F \geq (ag - c)d \quad \text{------(5)}$$

The moving cost is the sum of fixed costs and distance-increasing cost. If the fixed cost is extremely high, no one will move regardless of the gain from reduced distance in preference. If the fixed cost F is zero, the marginal utility gain from reduced distance, ga, must be less than the marginal moving cost increasing with distance, c, to prevent people from moving. The intuition here is simple: as long as the cost is larger than the benefit people can gain from reduced distance in preference, people will stay on the border. Thus, the fixed position condition changes into $F \geq (ag - c)d$ in the A&S model.

3.2 The Formation of a Small City

In urban economic theory, cities can be formed either by a land developer who buys land in a national market to build a city and manage in the first stage, or people can congregate and form a self-organizing agglomeration (Henderson and Becker, 2000). There is no land developer in this model, but people can congregate to form a city. Though the model assumes that it is hard for people to move, people will possibly change location to gain more utility when the opportunity is attractive enough. In this section, I will investigate how a small city can be formed under a sudden shock.

There can always be a sudden shock that will lead to local economic growth at least in a short period. Such a shock can be a new resource discovery, for example the gold fields in San Francisco in 1849, or technological progress, such as the spinning Jenny in Lancashire in 1764. These shocks can be seen as a source of economic growth, and their effect on the economy weakens as the distance gets larger. This may be caused by some transportation costs or transmission loss.

As Henderson and Becker (2000) point out, there must exist centripetal forces and centrifugal forces working together to attract people within a range and repulse people to limit a city in size. The shock will attract people within a radius r coming to the point but will be indifferent for people outside the radius because they are too far away and the moving costs will not be fully compensated by the profit gained from moving.

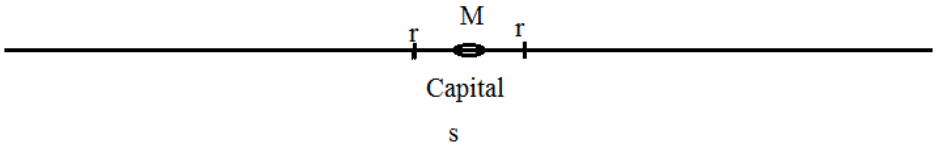
The definition of a small city here may be vague; we just use "small" to refer to a city that has a short radius. Without loss of generality, we assume $r < s/20$. We also assume the additional profit people can gain from living in the radius of a shock is $M/(r_i + 1)$, with r_i being the distance of people from the shock. People living at the center will gain profit M, and the profit is the same for everyone on the central point regardless of the number of the people there. People living outside the radius will gain

nothing from the shock. This assumption is basically consistent with the cases of the San Francisco gold fields and the new spinning Jenny machine.

There can be many small shocks at the same time in the world, and thus many small cities may be formed simultaneously. We focus on one, since they will not interfere with each other. Shocks happening at various locations will produce different effects on people and form cities of different structures. We now discuss three situations: they are when shocks are (1) at the capital, (2) between the capital and the border, and (3) on the border of two countries.

3.2.1 A Shock at the Capital

The simplest case is when the shock coincides with the capital geographically. The capital attracts people originally by short distance in preference. However, with the new discovery of resources or technological progress, people can gain more profit from moving toward the capital.



The individual originally living at the capital will not choose to move. Indeed, s/he is the median voter and can earn more profit from the shock than anyone else in the nation. For people living outside the capital but within the radius of the shock, individual *i* choosing not to move will gain utility

$$U_i = g(1 - al_i) + y - t_i + \frac{M}{l_i+1} \qquad \text{------(6)}$$

But if the person chooses to move toward the capital by distance d, $d \in [0, l_i]$, the individual will gain utility

$$U_i' = g[\,(1 - a(l_i - d)] + y - t_i + \frac{M}{l_i+1-d} - F - cd \qquad \text{------(7)}$$

The individual will move if and only if $U_i' \geq U_i$, that is

$$M \geq \frac{(l_i+1)(l_i+1-d)(F+cd-agd)}{d} \qquad \text{------(8)}$$

If the profit M is large enough to satisfy the condition above, individual *i* will choose to move distance d. $F + cd > agd$ is the condition in the reformulation of the A&S model to fix individuals when there is no shock. l_i is the original distance in preference. This means that the magnitude of the shock, M, and the gain in preference utility (agd) must compensate people for their moving costs ($F + cd$).

M has to be larger to attract people living farther, as they have larger l_i. The extreme case is that $l_i = d = r$, that is, all the people within the radius are moving to the capital. We assume that we can have a building tall enough to shelter all the people. Substitute $l_i = d = r$ in to equation (8); M will be

$$\frac{(r+1)(F+cr-agr)}{r} \qquad \text{------(9)}$$

As Mills and Muth (1987) propose, population density should decrease exponentially from the center of a city in standard location theory. Various factors may work together to form such a city. The most important one is the rent. When people can gain from moving to the city center, the owner can definitely charge more rent. This makes people living in the city center choose smaller houses, leading to a denser population there. The equilibrium will make the tenants indifferent to all the locations economically. Another factor is that the construction cost of a tall building will be much more than that of building one-story houses to hold the same number of people. People, therefore, choose to live around the city, instead of crowding into one tall building at the center. Additionally, the pollution and crowdedness in the city center may drive away to the suburbs some people who value living environment. What is also possible is that M there may be less than $\frac{(r+1)(F+cr-agr)}{r}$, which violates the condition we get above.

To limit the size of the city, M cannot be too large. Otherwise, people living outside the radius will seek to live within the radius to gain profit. The individual living nearest the city border originally has the utility

$$U_i = g(1 - al_i) + y - t_i \qquad (l_i \geq r) \qquad \text{------(10)}$$

If the person chooses to move distance d toward the border, $d \in [0, l_i]$, s/he will get

$$U_i' = g[\,(1 - a(l_i - d)] + y - t_i + \frac{M}{l_i + 1 - d} - F - cd \qquad \text{------(11)}$$

The condition $U_i \geq U_i'$ turns into

$$M \leq (F + cd - agd)(1 + l_i - d) \qquad \text{------(12)}$$

When d goes to zero and $l_i = r$, the condition is $M \leq F(1 + r)$

Therefore, if M lies in the interval $[\frac{(r+1)(F+cr-agr)}{r}, F(1 + r)]$, the shock will attract all people within the radius r to the capital but keep other people from moving. r can also be expressed as a function of M, $r=r(M)$.

With our endogenous preference assumption, when people come to live at the capital, their preferences will change to be the same with the median voter through interaction with their surroundings. The other parts of the nation and other parts of the world remain unchanged by the shock. Consequently, more people act as median voters in the nation. The location of the government is not changed; thus, the size of the nation will not be influenced. The configuration of nations in the world will remain the same.

When considering the welfare effect, the shock becomes a source of welfare improvement, but subsides with distance. When people are making a moving decision, they will consider the changed distance in preference, the cost of moving, and the profit gained from the shock. For the individual living in the center, he will

not pay the moving cost but enjoy the most profit. For other people in the radius, they will gain more utility in preference and profit but have to pay for the moving. The total welfare improvement is

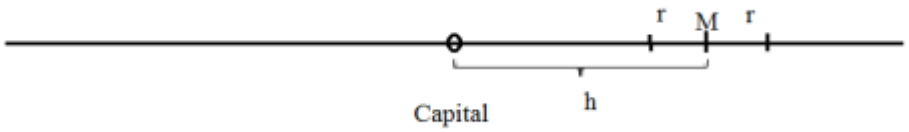
$$\Delta W = 2\int_0^r (agl_i + M - \frac{M}{l_i+1} - F - cl_i)\, dl_i = (ag - c)r^2 + 2(M - F)r - 2Mln(r + 1) \quad \text{------(13)}$$

if all people within the radius move to the capital.

We cannot easily determine to what extent small shocks such as those considered in our model played a role in a capital city's formation, since the capital cities that do exist came into existence for any number of reasons. Some capitals, such as Brasilia, Abuja, and Pretoria, are solely as administrative centers and thus lack powerful private industries. However, most capitals are crowded with people and private businesses and may have a greater chance of experiencing sudden technological progress. This may in turn attract more people to the capital.

3.2.2 A Shock between Capital and Border

When the small shock occurs in the capital, the influence is simple to estimate. However, the shock can happen within the country but have a distance of h from the capital. We made the assumption that the shock is small in that it will only produce a small source of profit M within a small radius r. It is theoretically possible that the capital will fall into the radius, but we have made the radius very small to exclude this situation.



When the shock is of distance h from the capital, the individual in the center will not choose to move since the moving cost will not be compensated by the reduced distance from the capital. Furthermore, the person will lose profit when moving away from the point of shock. This is the same when the shock is in the capital. However, the city will possibly be formed asymmetrically from left and right.

First, we consider the people on the right-hand side of the shock. Since the shock is assumed to happen on the right of the capital, people within the radius moving to the shock will gain utility from reduced distance from the capital and also from the profit of the shock, $M/(r_i + 1)$. The original utility of individual i living within the radius of the shock is

$$U_i = g(1 - al_i) + y - t_i + \frac{M}{l_i - h + 1} \quad \text{------(14)}$$

If s/he moves distance d towards the shock, $d \in [0, l_i - h]$, the person's utility will be

$$U_i' = g[\,(1 - a(l_i - d)] + y - t_i + \frac{M}{l_i + 1 - h - d} - F - cd \quad \text{------(15)}$$

If people living within the radius r are all moving to the location of the shock, the condition will be $U_i' \geq U_i$, that is

$$M \geq \frac{(l_i-h+1)(l_i-h+1-d)(F+cd-agd)}{d} \quad \text{------(16)}$$

This condition is similar to what we get in the last part. If we move the capital of distance h to the right, it is just what we get there. We have $l_i = h + r$ and $d = r$; then

$$M \geq \frac{(r+1)(F+cr-agr)}{r} \quad \text{------(17)}$$

And again we limit the size and get

$$M \leq F(1+r) \quad \text{------(18)}$$

This case is almost the same with setting capital at the location of the shock. Because people gain the same marginal utility in preference and marginal profit when moving, people will choose the same behavior as in the first case.

However, people in the radius who are on the left side of the shock have a different situation. When they move towards the shock, they gain profit from the shock, but they are paying moving costs and are increasing their distance in preference, thus losing utility in preference. For people on the left side in the radius, the original utility is

$$U_i = g(1-al_i) + y - t_i + \frac{M}{h-l_i+1} \quad \text{------(19)}$$

If s/he moves distance d towards the shock, $d \in [0, h-l_i]$, one's utility will be

$$U_i' = g[\,(1-a(l_i+d)] + y - t_i + \frac{M}{h-l_i-d+1} - F - cd \quad \text{------(20)}$$

To make people move, that is $U_i' \geq U_i$, the condition is

$$M \geq \frac{(h-l_i+1)(h-l_i-d+1)(F+cd+agd)}{d} \quad \text{------(21)}$$

The right-hand side of this inequality is larger than the right-hand side of (16), $\frac{(l_i-h+1)(l_i-h+1-d)(F+cd-agd)}{d}$. The differently signed part *agd*, the utility change in preference of moving distance d, is exactly what people on the left will lose and what people on the right will gain. If we have people within radius r on the right of the shock, and all move to the location of the shock, we can have M as large as $M = F(1+r)$.

However, this is not enough for people on the left-hand side at the radius r to move to the shock, because $F(1+r) < \frac{(r+1)(F+cr+agr)}{r}$ (r is assumed much smaller than 1).

The marginal people choosing to move either from the left or from the right will have the same overall utility change. Thus, the left boundary of the city must be closer to

the center than the right boundary. The city will not be symmetric because people on the left will suffer from the loss of utility in preferences, which is opposite to people on the right of the shock.

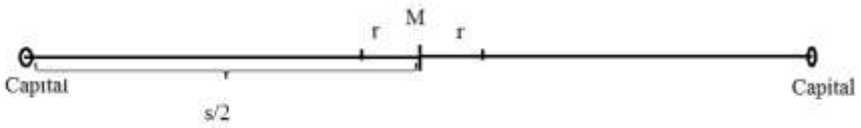
Given the endogenous preference assumption, people moving to the shock will ultimately form the same utility with people who originally live there. However, this will not influence the location of capital by majority rule. As long as the capital is not within the radius of the shock, the median voter will not move, making this nation and all the other nations unchanged. The configuration of the world should still be the same as it was before the shock.

The welfare change will be similar to that in the capital situation. An individual makes a moving decision based on the profit gain, the moving costs, and the change in preference utility. Welfare improvement declines with the distance from the location of the shock, so people living where the shock occurred gain the most welfare. The total welfare change should be less than in the last part, since people on the left are losing utility in preference when moving.

A city other than the capital formed after a shock is very common in the real world. There are countries that have a political capital and an economic capital, such as the Netherlands. More countries are having economic centers not the same as their capitals. For example, China has Beijing as its political capital, but Shanghai is more economically active because of transportation advantages and historical reasons. New York City, Sydney, Toronto and Mumbai are all economic centers in their own countries, but they are not capitals. Durham, North Carolina, can also be used as an example here. Since its recent economic development, more people are attracted to this town. People living to the west are willing to move to Durham, while people living between Raleigh and Durham are naturally less influenced. People living near Raleigh may hesitate to move because they also enjoy the accessibility of Raleigh.

3.2.3 A shock on the border

Another situation that will happen is that the shock can be on the border between two countries. This case can be seen as the opposite side of the first situation, which is when the shock occurs in the capital. A symmetric city will be formed, but people attracted to the shock for profit will have to bear the moving cost and an increasing distance in preference at the same time.



As in all the previous situations, the individual living at the location of the shock has no incentive to move. People living in the radius r of the shock gain some profit $M/(r_i + 1)$. When individual i is living in the radius but chooses not to move, the person has utility

$$U_i = g(1 - al_i) + y - t_i + \frac{M}{\frac{S}{2} - l_i + 1} \qquad \text{------(22)}$$

When the individual chooses to move distance d, $d \in [0, \frac{S}{2} - l_i]$, s/he will have utility

$$U_i' = g[\,(1 - a(l_i - d)] + y - t_i + \frac{M}{\frac{S}{2} - l_i - d + 1} - F - cd \qquad \text{------(23)}$$

If people living in the radius r will be attracted to the city, we have $U_i' \geq U_i$, that is

$$M \geq \frac{(\frac{S}{2} - l_i + 1)(\frac{S}{2} - l_i - d + 1)(F + cd + agd)}{d} \qquad \text{------(24)}$$

The condition states that the profit gained from the shock must at least fully compensate the moving costs and the loss of utility in preference to make people move. Thus, people living near the capital will be less willing to move. If we make all people within the radius move to the point of hock, we have

$$M \geq \frac{(r+1)(F + cr + agr)}{r} \qquad \text{------(25)}$$

This condition is the same with the condition happening on the left side of the shock in the asymmetric city formation case. We use the inequality $M \leq F(1 + r)$ to limit the size of the city.

This city is formed naturally from utility analysis, but it is politically different from the previous cities, in that it will enjoy public services from two countries. This means people are from two neighboring countries under their own government, but they live together in one city to share the economic profit. The two countries may compete by offering different services to attract the people in the city, which will be discussed in the extension. The radius is so small that it will not be able to attract more people and form a new country. People will not choose to establish their own government due to high taxes.

The formation of such a city will only influence the border; the median voter will not change in either nation. The capital of the two countries will be unchanged and the configuration of the world remains the same.

The welfare effect is similar to the previous cases: individuals living near the shock will gain more utility. But compared with the two previous cases, given the same magnitude of shock, *M*, this one does the worst in improving total utility. The reason lies in that when the shock improves people's utility by providing extra profit, it also pulls people away from the capital, causing utility loss. When the shock happens in the capital, all the people moving are gaining utility in preference and profit at the same time. People on the right side are gaining these two benefits simultaneously when the shock happens in the country. Therefore, the shock on the border is the least welfare improving case.

Few cities are exactly on the border between two different countries. And the city can be quite unstable. A small positive shock in the city on either side may change the

structure of the city. Thus, some cities lying between two countries do not have symmetric structures. There are many real-world examples of cities near the border of two countries. Some are formed in the process of trading such as those on the ancient Silk Road in Asia. And some of the cities were established originally because of the resources. For example, the city of Baishan on the border between China and North Korea was formed based on its mining products and forest. The city of Altay on the border between China and Kazakhstan first attracted people for the gold mines. There are some American cities lying on the border between two states, such as Kansas City, on the border of Missouri and Kansas, and Danville, on the border of Virginia and North Carolina. In these border cities, people from the neighboring two countries or states live together to share profit while enjoying their own country or state's government services, just like what we predict in this part.

As the world is assumed as a segment $[0,1]$, the shock can also happen on the "end" of the world, which is on the point 0 or 1. This will be the same with shocks on the border except that people will come from one country and the diameter of the city will be half the size we discussed above.

In this part, all the three cases we discussed will not affect the location of the capital. When the shock is in the capital, the shock will attract the most people in the three cases, because all people moving to the capital will gain utility in preference and economic profit at the same time. When the shock is strictly within the country, an asymmetric city may be formed because people nearer the capital will lose utility in preference when moving, but people farther from the capital will gain utility in preference when moving. When the shock is on the border, it will provide the least welfare, since all people are sacrificing utility in preference for forming a city. But it is the special case in which the city will be under the governance of two countries.

4. Extensions

4.1 Income Tax

In the previous parts, we treat the tax an individual should pay as a poll tax or head tax. However, taxes are in the form of income taxes in most countries, such as the US and Canada. It seems morally more acceptable to have the rich pay more taxes; and many countries have a progressive tax rate to levy more taxes on richer people. We now assume a constant tax rate for the income tax system.

Under this tax system, when a shock appears, people trying to move to exploit the profit will calculate the taxes that will be deducted from their income $M/(r_i + 1)$. Given that the operation of the government will only need k, the government can adjust the tax rate, based on the prediction of how many people will move to the location of the shock and how much profit they will make. That means this is a complete information case: each player knows all the information others know. Suppose t_i is the tax an individual pays before any shock happens. The original tax

rate is $t = \frac{t_i}{y}$, and the population in the country is $N = \frac{k}{t_i}$.

We first consider the situation when the shock happens in the capital and can then infer the effect in the other two situations. When there is a shock in the capital, the individual living in the capital will not choose to move, just as in the poll tax situation. The people living in the radius will choose between staying and moving but they will take the taxes into consideration this time. Individual *i*'s original utility is

$$U_i = g(1 - al_i) + \left(y + \frac{M}{l_i+1}\right) * (1 - t') \qquad \text{------(26)}$$

If the person chooses to move d towards the capital, $d \in [0, l_i]$, s/he will gain utility

$$U_i' = g[\,(1 - a(l_i - d)] + \left(y + \frac{M}{l_i - d + 1}\right) * (1 - t') - F - cd \qquad \text{------(27)}$$

To attract people to form a city, the condition is $U_i' \geq U_i$, that is

$$M \geq \frac{(l_i+1)(l_i+1-d)(F+cd-agd)}{(1-t')*d} \qquad \text{------(28)}$$

In the extreme case, all the people in the radius *r* are attracted to the capital, that is, $l_i = d = r$, if we have

$$M \geq \frac{(r+1)(F+cr-agr)}{(1-t')*r} \qquad \text{------(29)}$$

All the results we derive are very similar to what we derived in the previous part. However, we have a $(1 - t')$ in the denominator in all inequalities. Since the tax rate lies in the interval $(0,1)$, the magnitude of the shock, M, has to become larger to attract the same amount of people. Equivalently, we can consider the shock to be multiplied by $(1 - t')$. The effect of the change in the tax system implies heavier taxes on people in the city and people are thus less likely to move to the city. The condition to limit the size of a city also changes proportionally to

$M \leq \frac{(F+cd-agd)(1+l_i-d)}{(1-t')}$ and $M \leq \frac{F(1+r)}{(1-t')}$.

We take the least M required in this case, that is, $M = \frac{(r+1)(F+cr-agr)}{(1-t')*r}$, to derive the relationship between M and the corresponding r.

As the total population in a nation is $N = \frac{k}{t_i}$, the population in the radius is $n = \frac{2kr}{t_i s}$, under the assumption that people are uniformly distributed.

Since all the people living in the radius now congregate at the capital, they have the same income: $y + M$.

The tax rate can be calculated as the operation cost over the new total income of people, that is

$$t' = \frac{k}{y*N+M*\frac{2kr}{t_i s}} = \frac{t_i s}{ys+2Mr} \qquad \text{------(30)}$$

The new tax rate is obviously smaller than the original one, $t = \frac{t_i}{y}$, since people forming the city are earning more and paying more. That means all people are benefiting from the shock, even people living outside the radius. The equation shows that a larger shock will induce a lower tax rate, since M and t' are negatively correlated.

What the tax system can also influence is the distribution of cities. Consider two countries with a poll tax system and an income tax system, respectively. The same shock will generate two cities of different sizes in the two countries, because the magnitude of a shock can be seen as multiplied by $(1 - t')$ in the country with an income tax system. That is, a country with an income tax system will generally have smaller cities, though all people can benefit from the prosperity. In contrast, cities will be larger in a country with a poll tax system, while only a small portion of people can enjoy the profit.

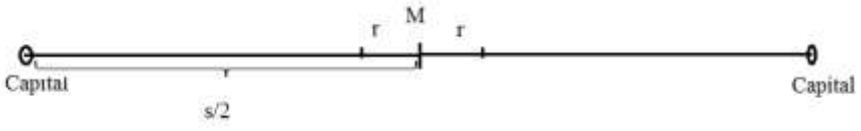
From the welfare side, the income tax redistributes the income from the rich to the poor, generating a wider effect in the nation; it will not only benefit people in the radius by endowing them with profit, but also decrease the taxes people outside the radius have to pay. However, the income tax makes the shock less attractive for people in the radius. This can be explained by the fact that, given the same magnitude of shocks, the radius r will be shorter, meaning less people will come to the city. The total income is decreased by the income tax system.

What is more important in this case is that if people on the border are paying less taxes, more people may be attracted to this country from the borders and the configuration of the world may change under such a shock. This will lead us to the discussion in the next part.

When the shock happens within the border or on the border, the change will be similar as we have discussed.

4.2 The Economic Border War

As we analyzed in Part 3.2.3, a city on the border of two neighboring countries can be formed when there is a shock exactly on the border. People living within the radius will choose to move to the border to share the economic gain, but still enjoy the public services from their own countries. This is the situation when the two countries take no reaction to the economic boom on the border.



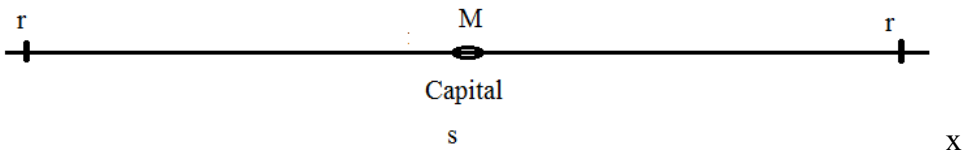
However, if either country decides to attract all the people on the border into one

country, an economic border war may start. First of all, one of the two countries, country A, has the incentive to do so. Assimilating more people into country A can reduce the tax per person, because we assumed the total operation cost of a government to be k. Moreover, the people living on the border are willing to take the offer. They are now of equal distance to the two capitals, leading them to have equal utility in preference from the two countries. Their all moving to country A will not change their utility in preference or economic profit, but they will have to pay less tax. However, the other country, country B, will be harmed by the migration. Since half the population of the city moves out of country B, the rest of the population will have to pay more tax per person for public services. Similarly, country B can offer lower tax rates to people on the border to attract them. Then the two countries are competing in tax rates to enlarge their own borders. They can even levy more taxes in other parts to subsidize the people on the border. They can equivalently provide better public services to people on the border.

This is just what is happening in Kansas City. Kansas enacted tax cuts concerning small business taxes, which influenced hundreds of lawyers and accountants in the city in 2012 and in 2013. Thus, Missouri's legislators were encouraged to pass tax cuts and a subsidies plan in 2013 to prevent small businesses from moving to Kansas. What makes the situation more complicated is that less fiscal income may have affected public services such as education in Kansas. The government can operate with a budget less than k, but the poor public services may drive some businesses away. Actually, the number of registered firms in Kansas has decreased, even though new firms have been established. This implies that many firms maybe move to other states because of the deteriorating public services. Since Kansas is facing a budget deficit, the long-term result of the tax cuts may not be favorable to taxpayers.

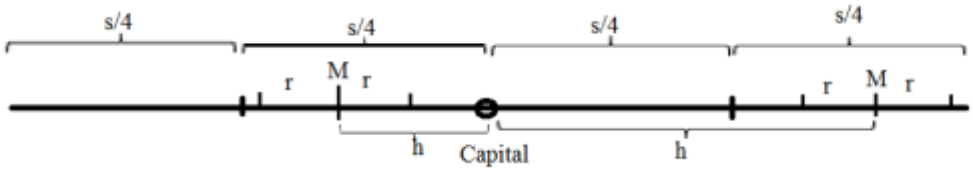
4.3 Larger Shocks

We have been focusing on small shocks that are both small in radius and in magnitude; thus they will not change the location of the capital. In this part, we will consider the effect of a large shock.



When the shock coincides with the capital in location, as long as $r \leq s/2$, the shock will attract more and more people in the nation to the capital but will not influence the location of the capital or the world's configuration. City-states like Singapore and Monaco were formed for many intertwined reasons, but this may serve to explain their initial formation to some extent.

However, when the shock is large enough to attract people from other countries, that is, $M \geq \frac{\left(\frac{s}{2}+1\right)(F+c\frac{s}{2}-ag\frac{s}{2})}{\frac{s}{2}}$, the size of this country will change and thus so will that of neighboring countries. Since the change of population in one country will affect directly the tax levied on individuals, all the countries in the world will probably be changed in a chain effect. The configuration of the world can be totally different after this shock.



When the shock happens strictly within one nation but not in the capital, as long as $r > s/4$, there can be a chance of changing the whole world. If the distance between the shock and the capital, h, is larger than $s/4$, the upper limit of the radius is $r \leq s/2 - h$ for the shock to remain a domestic one. Otherwise, the shock may attract people from the neighboring country and affect all the countries one by one. This may partly explain why some countries like to create cities near the border. One main reason can be to prevent the people from being attracted to neighboring countries when a shock occurs. People living near the border enjoy little utility in preference. When living in a prosperous city, people may wish to stay in the country because of economic gains. This can serve as a way to improve political stability. However, the existence of such cities may also cause some conflicts between two neighboring countries. One example is Strasbourg, which was a part of Germany and now belongs to France.

Then if $h \leq s/4$, and the radius is larger than h, the median voter will be attracted to the shock and a different city will become the capital. This will make people on the left and right border asymmetric from the capital and the change in this country will probably reshuffle the configuration of the world.

If the shock is on the border, enough people attracted from the two countries may vote to establish their own country. The effect of a new country will be transmitted to all the countries and then reshuffle the structure of the world.

This part highlights the effect that new discoveries and technological progress can have on the configuration of the world. This may not be obvious in real world but some modern inventions, such as the television, the computer, and the Internet, have changed people's daily life so profoundly that the whole world has been changed. The improved transportation network definitely had significant and long-lasting effects on the configuration of the world and international relationships.

5. Conclusion

This paper adopts and modifies some basic assumptions in Alesina and Spolaore's paper to investigate how a new city is formed under a small shock of a new discovery or technological progress. Preferences are now endogenous in the model, a characteristic that is supported by theory in political science. And individuals can now move, although their mobility is limited because of high moving costs both economically and psychologically.

The magnitude and the location of the shock work together to decide the population and the structure of the city. This is achieved by the analysis of individuals near the shock. When people are making moving decisions, they will consider changes in preference, possible profit, and moving costs. People will not congregate on one point because of the rent, construction costs and crowdedness. The tax system can have different effects. With an income tax, all people in the nation can benefit from the shock, in contrast to that in the poll tax system, where only people living in the radius gain. Cities are thus formed differently in the two tax systems. Two countries may even use different tax strategies to enlarge their border by assimilating border cities.

This paper ignores the possibility of the formation of local government. It assumes that only national governments exist and therefore that people enjoy public services only from the latter, which is unrealistic. In addition to the central government, local governments also provide public services. Moreover, the assumption that all governments have the same operating costs is questionable. Nations of different sizes should have different operation costs. And even when nations are of the same size, the efficiency can be very different among countries. We may use a function $K = f + ks$ to represent the cost of a government. f serves as the fixed cost of setting up a government and k is the costs increasing with size. $k > 0$ and $k' < 0$ are reasonable assumptions here. We may expect larger countries to have greater efficiency in administration due to economies of scale. However, in reality, that assumption is not completely convincing, since large countries, like Russia, China, and Italy, do not necessarily have a more efficient government than smaller countries, such as Singapore and Switzerland, do.

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