
Extending the Possibilities of Kidney Exchange with Compatible Pairs

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Abstract

A patient needing a kidney cannot receive a transplant from an incompatible donor. But, if there are multiple patients with incompatible donors, then they may be able to exchange donors and each get a compatible kidney. This trade is called a *kidney exchange*. Unlike incompatible pairs, compatible pairs are pairs in which the patient has a compatible donor; these patients could get a transplant from their own donors, but could get a better-matching transplant through kidney exchange. The present study examines the matching outcome when incompatible and compatible pairs are matched in cycles of 2 or more pairs, and uses HLA match to measure the match quality of the transplant. When incompatible *and* compatible pairs trade with one another, compatible pairs see an average improvement in match quality of 2/3 of one HLA match. Further, 50-67% more incompatible pairs can be matched than would be in a pool of only incompatible pairs, which at the national level would amount to 1000-1330 more transplants per year. ¹

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1 Introduction

1.1 The National Problem of Kidney Failure

Kidneys perform an essential role in the body: they filter the blood of wastes, reabsorb basic nutrients like water and glucose, and produce hormones critical for normal body function. They are so effective at their job that an adult human only needs one functioning kidney to sustain himself, although most of us have two. Unfortunately, various conditions, especially diabetes and high blood pressure, can cause long-term failure of both kidneys. In the United States alone, 115,198 individuals contract end-stage kidney failure each year. Kidney failure brings a host of symptoms, including anemia, the accumulation of acidity, potassium, and wastes in the blood, and if untreated, these symptoms can result in death. A treatment called dialysis, which uses either a machine or the abdominal cavity lining to filter the blood of waste, can keep kidney failure patients alive, but requires treatments at least 3 times a week for a few hours at a time. Additionally, dialysis is costly, and can lead to increased susceptibility to cardiovascular disease and other complications.

Since most adults only need one healthy kidney, they can donate one kidney to a person in need. Compared to dialysis, receiving a kidney transplant yields better outcomes both medically and economically. As shown by Oniscu, Brown and Forsythe (2005), who compared patients who had received a kidney transplant to those who were undergoing dialysis, transplant recipients did have an elevated risk of death in the first 30 days after the procedure, but in the long term they had a life expectancy of 17.19 years, almost tripling the 5.84 years that dialysis patients could expect. There are also a host of quality of life benefits: you have a much lower risk of death from cardiovascular disease than you would under dialysis, and you can work, travel, eat a fairly unrestricted diet, become pregnant, engage in athletic training, and engage in other activities that you simply cannot do when on dialysis. Not everyone is healthy enough to receive a transplant, and with a transplant you do need to take a drug regimen to prevent your immune system from rejecting (destroying) the kidney. But, if you are healthy enough for a transplant, it is clearly the superior treatment to dialysis.

The high frequency of kidney failure and the benefit from receiving a kidney transplant create an enormous demand for kidney donors. Donations

come from both living and deceased² donors, but the demand for kidney donors far outstrips this supply. As of June 9, 2011, there are 89,201 patients actively waiting for a kidney on the national waitlist. In 2010, there were 16,898 kidney transplants conducted, with 10,622 from deceased donors and 6,276 from living donors; but, 34,422 individuals were added to the waitlist, and 4,634 individuals died while waiting for a kidney³. If kidneys were a typical private good, a market could arise, and kidney prices would rise until enough donors were found; however, to prevent unscrupulous individuals from abusing unwitting or unwilling donors, the purchase and sale of organs is illegal in most countries, and so voluntary and altruistic donation is the only source of kidneys.

Making the shortage even more acute, even if donations were to increase fivefold and thus match the size of the waitlist, it might not suffice. It takes more than the will to donate to successfully donate a kidney. Donors must be blood-type compatible with their intended recipients, and some recipients are very highly sensitized and so cannot match to most donors. It used to be that if a patient could not find a close relative or friend that is compatible, they would have to join the waitlist. But today, kidney exchange, also known as “kidney paired donation” or “kidney swap”, offers another alternative.

1.2 Using Kidney Exchange to help Alleviate the Shortage

Quite recently, the concept of kidney exchange has emerged to help alleviate the issues of compatibility and high sensitivity. The idea is, while your close relatives or friends may not be compatible with you, they may be compatible with another potential recipient; if one of their close relatives/friends can donate to you too, then you and the other recipient can “exchange” donors and get matched.

The particular exchange just described is called a “2-way exchange” or “2-cycle” since two donor-recipient pairs are involved; theoretically exchanges can involve an arbitrary number of pairs, but all the transplants in an exchange must be done simultaneously, to prevent donors from renegeing after their recipient has received an organ. Thus, it is difficult to conduct exchanges larger than 3 pairs because each transplant requires at least one

²with personal and family consent

³<http://optn.transplant.hrsa.gov/data/>, accessed June 9, 2011

transplant team - this difficulty is called the “simultaneity constraint”. To avoid this issue, some hospitals will also conduct altruistic-donor chains - first an altruistic donor gives a kidney to a recipient, then the recipient’s paired donor gives to another recipient, and so on. Chains rely on the goodness of the donors to not renege, but this way transplants can be conducted one at a time, and there is always the potential to conduct another one.

Over the past decade, hospitals have begun kidney exchange programs, and a few regional kidney exchange programs have emerged, linking transplant programs across various hospitals to increase the chances of conducting exchanges. Some of these hospitals include UCLA and Johns Hopkins, and regional entities include New England Program for Kidney Exchange (NEPKE), the Alliance for Paired Donation (APD), and the National Kidney Registry. It is estimated that up to 2000 additional transplants could be conducted annually by conducting exchanges of size 2 and larger (Sönmez and Ünver 2009), but the current level is well below this figure - for example, as of June 9, 2011, NEPKE has facilitated 83 transplants, the National Kidney Registry 289, and each has been around for at least 3 years. The “shortfall” has many potential explanations, but a significant issue is reach of the programs, as each is only working with a few hospitals. If there were a national program, the combined pool would yield many more matches than the separate pools of each program.

Considering the national reach of the organization, if there were to be one single national kidney exchange system, the United Network for Organ Sharing (UNOS) would be the entity best poised to manage it. According to its website, “UNOS is a private non-profit organization that manages the US organ transplant system under contract with the federal government.” In the case of kidneys, it manages the national waitlist, allocating organs from deceased donors to patients according to a complex priority system that accounts for compatibility, time on the list, region, medical urgency, and other factors. UNOS had not been involving itself with kidney exchanges, but in the fall of 2010, it debuted a kidney exchange pilot program. The first matching run matched one 2-way exchange, and the second run, conducted in December 2010 across 62 pairs, matched four 3-way exchanges⁴.

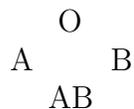
To date, mechanisms have been designed to incorporate more complicated ways of matching pairs: multi-way exchanges, matching over time, and al-

⁴<http://www.ama-assn.org/amednews/2011/01/10/prsa0110.htm>, accessed June 9, 2011

truistic donor chains are among the key extensions. And real-life exchange programs like NEPKE and APD have incorporated them into their matching process, with great success given their confined reach. But, there is one extension that thus far has proved too controversial to be incorporated in most programs, including the UNOS pilot: adding *compatible* pairs to the pool. The donor-recipient pairs discussed up to this point are *incompatible* pairs - each recipient is incompatible with her paired donor. In a compatible pair, a recipient is compatible with her paired donor, and so doesn't medically need to participate in the pool. But, by participating, they benefit the incompatible pairs in the pool by creating more potential matches, and the recipient in the compatible pair can get a better quality match.

1.3 Kidney Transplantation - Compatibility and Match Quality

Before we can fully appreciate the impact of adding compatible pairs, we have to understand what factors make a donor compatible with a recipient, and what makes a donor of higher or lower quality. Compatibility is driven by two factors: blood-type compatibility and crossmatch. There are four blood types: O, A, B, and AB. To understand blood-type compatibility, one just needs to remember the diagram below. Namely, types A and B are not compatible with each other, and types on higher levels can donate to types on lower levels, but cannot receive from them. Thus, O is the universal donor and AB the universal recipient:



Crossmatch relates to the tissue type, which is determined by the human leukocyte antigen (HLA) proteins. If the recipient has an antibody to one of the donor's HLA proteins, the donor's kidney will cause a reaction inside the recipient and be destroyed by the immune system. This reaction is called a *positive crossmatch*. Thus, if the crossmatch is *negative*, the recipient's immune system will not have a bad reaction to the donor's kidney's tissue type. For most recipients, the chance of a crossmatch with a random donor is 11%, but certain recipients (termed *highly-sensitized*) can have a crossmatch probability as high as 100%! To avoid crossmatch with a highly sensitized

recipient, most if not all of the donor's HLA proteins need to match those of the recipient.

In sum, a donor is *compatible* with a recipient if he is blood-type compatible and has a negative crossmatch with the recipient. If a donor is compatible with a recipient, it is then relevant to consider the match quality of that donor.

Given that you have a compatible donor, match quality is a composite of factors that can affect how long a donated organ will survive: there are around 24 such factors, but principal among them are HLA match and donor age (Gjertson and Cecka 2000). With donor age, the younger the donor, the harder the donated organ tends to be, so it can survive longer. With HLA match, survival rates are slightly higher if all of the HLA proteins match than if only some match or none match. There are many HLA proteins, but for match quality, 6 of them matter most: the two HLA-A, two HLA-B, and two HLA-DR proteins. So, going forward, an HLA match of 6 will mean that all 6 of these proteins match between recipient and donor, and an HLA match of 0 will mean that none of them match. Match quality can also have other benefits, including improved transplant function, lower doses of medication needed after the transplant, and so on.

1.4 The Controversy of Using Compatible Pairs in Kidney Exchange

Why incorporate compatible pairs? As mentioned earlier, one benefit is that more pairs in the pool allow for more potential matches. Adding compatible pairs particularly benefits incompatible pairs that otherwise might not be matched. Let X-Y represent a pair whose recipient is of blood type X and donor is of blood type Y. Then, for example, an incompatible pair might be of type O-AB. In a 2-way exchange, the only pair that could match to it is of type AB-O. Most AB-O pairs are compatible, but as with any donor-recipient pair, about 11% of AB-O pairs are incompatible because the recipient has a positive crossmatch with their donor. In a group of only incompatible pairs, then, it is rare for the O-AB pair to find a match. But by incorporating compatible pairs, the O-AB pair has much better odds of being matched.

This benefits the incompatible pairs, but the recipient in the compatible pair also stands to gain personally, because of match quality. Specifically, if the recipient in the compatible pair has a paired donor who is relatively old

or has an HLA match of 0, by entering the pool she might find a donor who is younger or offers higher HLA match, and the recipient in the incompatible pair will be happy to get a donor in the first place.

There are two principal objections to incorporating compatible pairs. Medically, if a recipient can find a compatible donor, as an individual they have little reason to try to find a “better organ” on the market, since all compatible live-donor kidneys have about the same probability of survival (Gjertson and Cecka 2000). And ethically, if one proposes to a compatible recipient-donor pair to enter an exchange for the sake of incompatible pairs, and the recipient has complications after the exchange, they would lament that complications “would not have happened” had they not participated in the exchange and just done the transplant directly (Ross 2006). To the first, one could counter that there are other medical benefits of match quality besides the probability of the kidney’s survival, including better kidney function and less need for medication post-transplant. As for the ethical concerns, the risk of complications from any transplant is the same, so complications should not be attributed to using a non-related donor. But, the combination of these types of arguments has been enough to keep compatible pairs out of most matching programs, including the UNOS pilot program.

Controversy aside, the incorporation of compatible pairs into kidney exchange has been studied from various perspectives. Roth, Sönmez, and Ünver (2005a) used the mechanism⁵ developed in Roth, Sönmez, and Ünver (2004) to evaluate the possible number of transplants that could be enabled by kidney exchange, and found that if all the compatible pairs in the population were to participate, 98.83% of patients with paired donors would receive a matching kidney. But, they cautioned that this is an upper bound to the possibilities, as many compatible pairs would not want to participate in exchange.

Gentry et al. (2007) studied the incorporation of compatible pairs as well, but unlike the Roth, Sönmez, and Ünver studies, they relax the assumption that all donors and recipients are unrelated. They find that at the national level, incorporating compatible pairs into programs at individual transplant centers could add 1316 transplants per year, and if these compatible pairs were to join a nationwide program, an additional 948 transplants would be facilitated, for a total of 2071 transplants. They examine the benefits to

⁵See the next section for discussion of the mechanism in Roth, Sönmez, and Ünver (2004).

compatible pairs in terms of a) finding a younger donor and b) avoiding child-to-mother and husband-to-wife transplants, which have higher risk of positive crossmatch. The chance that the recipient benefits in either of these ways is 34% at the center level and 48% at the national level if just one compatible pair participates, but declines to 11.7% and 14.7%, respectively, if all compatible pairs participate, because an increasing number of compatible pairs are competing for a fixed number of incompatible pairs. Finally, they provide an important estimate: nationally, every month there would be approximately 250 incompatible pairs and 539 compatible pairs in the pool. The pool would be matched monthly to allow for a sufficiently large pool over which to optimize; else it might be optimal to wait for more pairs to arrive before matching.

Kranenburg et al. (2006) interviewed Dutch compatible pair patients and donors to understand the reasons why compatible pairs would/would not participate in kidney exchange, given that the donor could directly donate to the recipient. The top two reasons for participating were to help another pair get a kidney, and the possible gain in quality of the kidney. The top reasons to not participate were emotional reasons - for example. donating directly to one's paired recipient has more meaning than donating indirectly. The study also found that 18% of pairs were definitely willing to participate in the exchange, 13% were probably willing to participate, 51% were probably or definitely unwilling to participate, and 18% were unsure. Though this was over a sample of 96 participants (48 pairs), and Dutch attitudes on the issue may be distinct from those in the US, it is perhaps the only study that has examined this issue. Thus, going forward, it will be assumed that all of the "definitely willing" and 75% of the "probably willing" compatible pairs would participate in the kidney exchange pool, leading to an overall participation rate of $(18\% + .75 * 13\%) = 27.7\%$.

The analysis of compatible pairs thus far is thorough, but has not yet investigated the benefits of incorporating compatible pairs in the pool when improved HLA match is the incentive. Gentry et al. (2007) explicitly avoided examining HLA match, asserting that the debate on the medical value of HLA match was split. Roth, Sönmez, and Ünver (2004) do incorporate compatible pairs, but permit cycles larger than size 4, allow for other types of kidney exchange⁶, and assume patients have strict preferences over all compatible

⁶Other types of kidney exchange include altruistic chains and list-paired donation; see Sönmez and Ünver (2009) for an overview

donors. Given that the UNOS pilot program will only allow cycles, of size at most 3, the present study adds to the discussion by focusing just on the HLA match aspect of match quality, and examining the benefits of incorporating compatible pairs in the pool when only cycles are allowed. Compatible pairs benefit because they may obtain a better HLA match, and incompatible pairs benefit because they are more likely to find a match. I find that when compatible pairs participate at a rate of 27.7%, 50-67% more incompatible pairs can be matched than would be in a pool of only incompatible pairs, and compatible pairs see an average improvement in match quality of 2/3 of one HLA match. If kidney exchange with incompatible pairs could create up to 2000 transplants (Sönmez and Ünver 2009), then incorporating compatible pairs would facilitate another 1000-1330 transplants. In sum, compatible pairs should be incorporated into the nationwide kidney exchange program, and HLA match should be accounted for in the prioritization of matches.

1.5 Static Kidney Exchange Mechanisms

This study employs a static kidney exchange mechanism, which is intended to optimally match a pool of donor-recipient pairs at one instant in time, with no accounting for the arrival of future pairs. As Sönmez and Ünver (2009) explained, the initial obstacle to conducting kidney exchanges was to make an efficient and incentive-compatible mechanism for the purpose. Roth, Sönmez, and Ünver (2004) proposed the first such mechanism. The innovation was to extend algorithms from other mechanism design literature, specifically “top-trading cycles” and “you-get-my-house-I-get-your-turn” from the housing allocation research, and adapt them into something usable for kidney exchange. The result, the top-trading cycles and chains algorithm, has the important properties of Pareto-efficiency and of strategy-proofness - the latter meaning it is optimal for recipients to reveal their true preferences over the set of compatible kidneys as well as the full set of donors available to them. Unfortunately, this algorithm does not account for the simultaneity constraint in its optimization, and in simulations it created many very large cycles. Additionally, the algorithm incorporates compatible pairs, which as was discussed earlier is a controversial move.

Roth, Sönmez, and Ünver (2004) also accounted for HLA match and donor age in their simulations by constructing two different sets of preferences for patients. The first type, termed “rational,” sets patient utility to be a function of donor age and number of HLA matches, ignoring personal

connections recipients may have with one donor compared to another. The second type, called “cautious,” dictates that a recipient prefers another donor to the donor they are paired with only if their paired donor is incompatible, or if the paired donor is compatible but the other donor is a better match by the equivalent of more than one HLA match. To clarify the second case, if the paired donor has an HLA match of 2 with the recipient, and the other donor has an HLA match of 4, or a combination of better HLA match and lower age that is equivalent to an HLA match of 4, the other donor would be preferred.

The next major advance in the literature was by Roth, Sönmez, and Ünver (2005b), which developed two 2-way exchange mechanisms, both of which were Pareto-efficient. The first was a priority mechanism: patients are prioritized by how sensitive they are to others’ tissue types (termed panel reactive antibody, or PRA), and then the mechanism matches recipients in descending order of priority. The second was an egalitarian mechanism: defining utility to be the probability of being matched, it maximizes utility for each recipient constrained by the fact that not every type can be matched. To do so, one must first graph the pool of incompatible pairs and the lines of compatibility between pairs, and classify each recipient as one of three types: over-demanded, perfectly matched, and under-demanded. Under-demanded types are recipients for which there exist a Pareto-efficient matching that leaves them unmatched; over-demanded types are recipients that are not under-demanded but are mutually compatible with under-demanded type(s); perfectly matched types are recipients that are neither over- nor under-demanded.

Then, one performs a mechanism design analysis called the *Gallai-Edmonds decomposition*, which breaks down the compatibility graph into components - subsets of the graph where any pair can be reached from any other by traversing compatibility edges. The results are that in a Pareto-efficient matching, each perfectly-matched pair is matched to another perfectly-matched pair in the same even component, each over-demanded pair is matched to an under-demanded pair, and each component with an odd number of pairs matches all but one of its pairs together. In this last case, the remaining pair is an under-demanded pair, and is either matched to an over-demanded pair or left unmatched. There are many possible Pareto-efficient matchings. Some vary “trivially” from each other by changing who is matched to who without affecting who is matched, while others vary “non-trivially” in that different under-demanded pairs are matched to the same over-demanded pair. The

egalitarian mechanism randomizes over the set of Pareto-efficient matchings that vary non-trivially, with two results. First, over-demanded and perfectly matched recipients are always matched. Second, under-demanded recipients are matched with the maximum of 2 probabilities: the probability they are matched within their component, and the probability they are matched to an over-demanded pair.

From here, the next step was to examine the added benefits of conducting exchanges larger than size 2, which was done by Roth, Sönmez, and Ünver (2007). This paper keeps the category of over-demanded and under-demanded as it was in Roth, Sönmez, and Ünver (2005b), but drops perfectly matched for the new categories of self-demanded and reciprocally-demanded: self-demanded pairs are pairs like A-A and B-B in which donor and recipient have the same type, and the reciprocally-demanded pairs are A-B and B-A. Multi-way exchanges allow odd numbers of self-demanded pairs to do an exchange, and enable over-demanded pairs to facilitate an exchange with multiple other pairs. With the key assumptions of ignoring tissue-type incompatibility, arbitrarily many under-demanded pairs, and ignoring self-demanded types, they determined the theoretical maximal number of matches for 2-way; 2- and 3-way; and 2-,3-, and 4-way exchanges. Under their assumptions, 3- and 4-way exchanges captured all of the gains of multi-way exchanges, and when they conducted simulations and restored the tissue-type incompatibility aspect, they still found that 3- and 4-way exchanges captured the vast majority of these gains.

Next, the model, kidney exchange mechanism, and simulation specifics used in this study are presented.

2 The Model

2.1 Exchange pool

The exchange pool is a set of donor-recipient pairs that are available to be matched to one another, and consists of both compatible and incompatible pairs. For simplicity, pairs are assumed to be unrelated. Start with the predicted compatible pairs participation rate (27.7%) based on Kranenburg et al (2006), and apply it to the pool composition of 250 incompatible pairs, 539 compatible pairs from Gentry et al. (2007), to get a ratio of 250 incompatible pairs to 150 compatible pairs ($539 * 27.7\% = 150$).

In this study, three pool compositions are tested. The main pool composition of study was Pool A, which consists of 15 compatible pairs, 25 incompatible pairs, in agreement with the ratio above. Pool B, which consists of 5 compatible pairs, 25 incompatible pairs, was used to test for sensitivity to decrease in the participation rate. Pool C, which consists of 10 compatible pairs, 50 incompatible pairs, was compared to Pool B to show the sensitivity of results to changes in the pool size when the participation rate was held fixed.

Kidneys are indivisible, so the only acceptable exchange that a pair can have with the pool is to have the donor donate her kidney into the exchange pool and the recipient receive a different kidney in return. Pairs are characterized by the following traits, defined as they were in the introduction:

1. The blood types of the recipient and the donor
2. The tissue types of the recipient and the donor
3. Whether or not the pair is compatible
4. The HLA match between the recipient and the paired donor. This is only relevant for a compatible pair.

In this model, we assume that the distribution of pair types is just determined by multiplying the distribution of blood types by itself. According to the American Red Cross, in the US population, the frequency of blood type O is 45%, of A is 40%, of B is 11%, and of AB is 4%⁷. For simplicity we assume that all patients and donors have this same blood type distribution. We also assume that for each pair, the probability of a positive crossmatch within the pair is 11%, and is 0% between the recipient of one pair and the donor of another pair. As discussed in Zenios, Woodle, and Ross (2001), the 11% figure is a standard used in the medical literature. In reality, highly sensitized recipients have a far higher chance of a positive crossmatch, but to obtain a simple upper bound on the possible utility of the exchange, we assume a uniform probability of crossmatch within pairs and between pairs. The reason why we assume 0% crossmatch between pairs, instead of 11% for any possible combination, is to simplify the problem and provide a useful upper bound.

⁷<http://www.givelife2.org/aboutblood/faq.asp>, accessed April 18, 2011

Given these probabilities, pools are generated as follows: For a given pool composition, generate pairs and add them to the pool until you either accumulate the desired number of incompatible pairs, or the desired number of compatible pairs. Then, continue to generate pairs, but only add them to the pool if they add to the group that did not have all the pairs it needed. For example, suppose you want to generate an instance of Pool A, and in the first 30 pairs you generate 15 compatible pairs and 15 incompatible pairs. At this point, the quota for compatible pairs is filled, so keep generating pairs but only add the incompatible ones. Once you add 10 additional incompatible pairs, you have the 25 incompatible pairs needed and the pool is complete.

The blood type distribution used is different than the distributions of kidney recipient and donor blood types for compatible and incompatible pairs given in Gentry et al (2007) and based on UNOS data. But, generating pairs according to the Red Cross probabilities, then categorizing by compatible and incompatible, donor and recipient, should yield similar distributions to the population numbers used in Gentry et al (2007), because kidney failure is independent of blood type, and so it should be distributed evenly across the blood types.

2.2 Preferences and Utility

Let the endowment refer interchangeably to the kidney of a recipient's paired donor, and to the utility that the recipient would get from that kidney. The key difference between incompatible and compatible pairs, then, is that only compatible pairs can match to their own endowment; further, the worst possible outcome for an incompatible pair is to be unmatched, while the "worst" a compatible pair can do is match to their own endowment or a kidney of equal quality.

Preferences are as follows. For recipients in an incompatible pair, the recipient prefers any compatible kidney with HLA match degree 6, then degree 5, etc. down to a kidney with 0 HLA match, then to be unmatched. For recipients in a compatible pair, a recipient prefers any compatible kidney that is better than his endowment by 6 HLA matches, then 5 HLA matches, etc. down to better by 1 HLA match, and finally any compatible kidney as good as his endowment, including the endowment itself.

To represent the benefits for compatible pairs, utility must now take match quality into account, instead of just whether or not a match occurred. Earlier I stated that HLA match and donor age were the principal drivers

of match quality, but in this study I confine the model to measuring match quality with just HLA match, to simplify the analysis. Assume utility is only obtained by a recipient when he is matched, never before and never after. The utility experienced by a recipient i from kidney j is

$$u_i(j) = 1 + \delta * k(i, j)$$

if a recipient is matched, and

$$u_i = 0$$

if the recipient is not matched. $k(i,j)$ is the HLA match between the recipient i and donor j , and δ is a parameter measuring the value of one degree of HLA match relative to the value of receiving a kidney at all. We expect that $\delta < 1/6$, since receiving a matching kidney in the first place extends life far more than even a perfect HLA match (degree 6). Thus, in the simulations we set $\delta = .05$. Observe that in the static setting, when δ satisfies $\delta < 1/6$, changing the value of δ will never change the exchanges selected, just the utility they generate. For example, for a compatible pair with an endowment of HLA match 2, when $\delta = .05$, the endowment is worth $1+2*(.05) = 1.1$. Another donor with HLA match 1 is worth $1 + .05 = 1.05$. Now suppose we set $\delta = .1$. Now the other donor is worth 1.1, but the endowment is now worth 1.2. Either way, the exchange would match the compatible pair to its own endowment, the only difference is in the total utility generated from the pair.

At any time t , the mechanism either matches recipient a_i to kidney o_j , creating utility of $u_i(j)$, or leaves a_i unmatched, in which case $u_i = 0$. Then aggregate utility enjoyed by society at time t , called $U(t)$, is just the sum of the individual utilities generated by the matching mechanism at time t .

2.3 Kidney Exchange Mechanism

2.3.1 General Setup

The mechanism that follows takes the pool of pairs and finds the set of exchanges that maximizes the aggregate utility, subject to:

1. Each pair can be matched only to itself or to one other pair; kidneys cannot be divided.
2. Make no pair worse off than their endowment. For incompatible pairs, this means only match to another pair if the match is compatible. For

compatible pairs, this means only match to another pair if the match is compatible and of quality at least as good as the endowment.

3. The limit on size of exchanges.
4. The *exchange regime* for the mechanism.

2.3.2 Cycle Size Limit

In this study, the cycle size limits tested are 2 and 3, with limit 4 and No Limit tested only for Pool B. As the cycle size limit is increased, more matches are made possible, so the total utility of the optimum under a larger limit is weakly better than the total utility under a smaller limit. The no limit case is not practical because of the simultaneity constraint, but gives the maximal possible utility of any matching for a given pool and exchange regime.

2.3.3 Exchange Regimes

An exchange regime is said to be *individually rational* if, after conducting the exchanges according to that regime, each recipient is matched to a kidney that does not violate his preferences. Matches that would violate preferences include matching any recipient to an incompatible donor, or match a compatible pair's recipient to a donor worth less than her endowment. All of the exchange regimes below are individually rational.

The first exchange regime is "Autarky" (no trade), in which no kidney exchanges take place. Incompatible pairs remain unmatched, and each compatible pair matches to itself - the recipients receive from their own donors. Note that this regime gives the same result regardless of the cycle size limit, because in this case the largest cycle is of size 1. Because incompatible pairs aren't matched, "Autarky" gives the total utility of the compatible pairs when they don't participate in the pool.

The other three exchange regimes exist for every size limit. For "No Mixing," start by self-matching the compatible pairs as in "Autarky", then optimally match incompatible pairs in the pool by finding the set of incompatible-pair-only exchanges that maximizes their aggregate utility. Essentially, "No Mixing" measures the best possible outcome when compatible pairs are not mixed in the pool, so it provides a baseline against which to measure the mixing outcome. Observe that the difference in utility between the "No Mixing" and "Autarky" outcomes is the maximum possible utility of kidney

exchanges between just the incompatible pairs. Thus, the gains in the “No Mixing” outcome that occur when the cycle size limit is increased are solely because the incompatible pairs can be matched in larger cycles.

When compatible pairs are matched in the pool, it must be done in a way that is consistent with their preferences: prioritizing pairs of better match quality than the endowment over pairs that are as good as the endowment. Also, any matching must allow a compatible pair to match to its endowment, or the pair would not participate. Consider a particular compatible pair. For simplicity, group together all the pairs that are compatible and are better than the endowment by at least 1 HLA match. Then, this group plus the endowment is the highest ranked set of pairs that is preference-compatible and allows for mixing. Call this set the “Cautious” set. The full set of preference-compatible matches consists of the “Cautious” set, plus pairs that offer utility equal to the endowment; call this the “Maximal” set. In practice, some compatible pairs will only accept matches from their Cautious set, while others will be content with a match from their Maximal set. Rather than estimate the share that will choose one option or the other, this study tests the two extremes: the extreme in which all compatible pairs are matched from their Cautious set (called the “Cautious” regime), and the extreme in which all compatible pairs are matched from their Maximal set (the “Maximal” regime).

The “Maximal” regime pools together all pairs, and matches compatible pairs to pairs in their Maximal sets. Since the Maximal regime optimizes over all of the preference-compatible mixing outcomes, it gives the upper bound on the utility of mixing together compatible and incompatible pairs (the *mixing utility*). On the other hand, the “Cautious” exchange regime mixes all pairs in the pool, but matches compatible pairs only to pairs in their Cautious sets. This gives the lower bound on mixing utility, because it optimizes over the smallest subset of the preference-compatible matches; since we grouped together all the pairs better than the endowment, there can be no smaller set.

“Maximal” is similar to “rational preferences” and “Cautious” to “cautious preferences” from Roth, Sönmez, and Ünver (2004), but with a few key differences. In that study, it is assumed that each recipient has a strict preference ordering for all the donors, while in this study a recipient can be indifferent between two or more donors. Also, “cautious preferences” required that the other pair’s donor be better than the endowment by *more than* 1 HLA match, but under “Cautious” in this study, the other pair’s

donor need be better by *at least* 1 HLA match for the compatible pair to be able to match to it.

3 Simulations and Results

In the linear programming simulations conducted, random pools of compositions A, B, and C were generated using a Monte Carlo simulation with 500 trials, and optimally matched under all the exchange regimes and cycle size limits 2 and 3. Ideally, cycle size limits 4 and No Limit would have been tested for all three pool compositions, for the sake of comparison. But, for compositions A and C, the algorithm developed could not conduct 500 Monte Carlo simulations in a reasonable timeframe unless the limit-4 and no-limit cases were omitted, because in those scenarios, the number of possibilities over which to optimize was too many. Thus, only pool B has results for all four cycle size limits. As will be seen, for pool B, for most of the outcomes tested, the limit-4 and no-limit cases do not have much impact on welfare; combined with the fact that these cases result in exchanges that are impractical to conduct, it is sufficient to be able to compare the limit-2 and limit-3 cases across all three pool compositions.

The results are presented in the tables below, with comments preceding them to aid in interpretation. As there are many tables, and discussion of results will draw across many of them at once, discussion is reserved until the next section. The three outcomes analyzed are: total utility, number of incompatible pairs matched, and average utility of compatible pairs. The tables are each assigned two numbers; the first number corresponds to the outcome variable measured, while the second corresponds to the pool compositions whose results are shown. Again, Pool A has 15 compatibles, 25 incompatibles; Pool B has 5 compatibles, 25 incompatibles; Pool C has 10 compatibles, 50 incompatibles.

In all cells, the average values are given followed by the standard deviation. In some cases there is also an italicized percentage beneath these two numbers. These percentages are the percentage gain compared to the No Mixing outcome under the same cycle size limit. For example, in Table 1.1, in the 2-way case, the mean of the Cautious outcome represents a 12.2% improvement over the mean of the No Mixing outcome.

Tables 1.1-3 examine total utility.

TABLE 1.1: TOTAL UTILITY — POOL A

	2-way	3-way
Autarky	15.209 ± 0.095	15.209 ± 0.095
No Mixing	23.519 ± 3.4	24.903 ± 3.662
Cautious	28.273 ± 3.527 <i>20.2%</i>	30.235 ± 3.696 <i>21.4%</i>
Maximal	30.385 ± 3.567 <i>29.1%</i>	32.052 ± 3.586 <i>28.7%</i>

TABLE 1.2: TOTAL UTILITY — POOL B

	2-way	3-way	4-way	No Limit
Autarky	5.073 ± 0.057	5.073 ± 0.057	5.073 ± 0.057	5.073 ± 0.057
No Mix	13.3377 ± 3.26	14.6966 ± 3.557	14.8344 ± 3.56	14.8693 ± 3.572
Cautious	14.978 ± 3.381 <i>12.2%</i>	16.543 ± 3.673 <i>12.5%</i>	16.714 ± 3.687 <i>12.6%</i>	16.783 ± 3.702 <i>12.8%</i>
Maximal	15.992 ± 3.37 <i>19.9%</i>	17.375 ± 3.516 <i>18.2%</i>	17.492 ± 3.539 <i>17.9%</i>	17.557 ± 3.554 <i>18%</i>

TABLE 1.3: TOTAL UTILITY — POOL C

	2-way	3-way
Autarky	10.142 ± 0.08	10.142 ± 0.08
No Mixing	28.92 ± 4.601	31.323 ± 4.917
Cautious	32.762 ± 4.827 <i>13.2%</i>	35.422 ± 5.022 <i>13%</i>
Maximal	34.081 ± 4.844 <i>17.8%</i>	36.527 ± 4.978 <i>16.6%</i>

Tables 2.1-3 examine the number of incompatible pairs matched. Autarky is omitted because no incompatible pairs are matched in Autarky.

TABLE 2.1: INCOMPATIBLE PAIRS MATCHED — POOL A

	2-way	3-way
No Mixing	8.072 ± 3.302	9.408 ± 3.534
Cautious	12.438 ± 3.42 <i>54%</i>	14.088 ± 3.529 <i>49.7%</i>
Maximal	14.404 ± 3.483 <i>78.4%</i>	15.72 ± 3.434 <i>67%</i>

TABLE 2.2: INCOMPATIBLE PAIRS MATCHED — POOL B

	2-way	3-way	4-way	No Limit
No Mixing	8.032 ± 3.169	9.35 ± 3.44	9.454 ± 3.42	9.462 ± 3.417
Cautious	9.552 ± 3.288 <i>18.9%</i>	10.994 ± 3.528 <i>17.5%</i>	11.096 ± 3.516 <i>17.3%</i>	11.102 ± 3.507 <i>17.3%</i>
Maximal	10.526 ± 3.281 <i>31%</i>	11.766 ± 3.38 <i>25.8%</i>	11.808 ± 3.378 <i>24.8%</i>	11.808 ± 3.378 <i>24.7%</i>

TABLE 2.3: INCOMPATIBLE PAIRS MATCHED — POOL C

	2-way	3-way
No Mixing	18.148 ± 4.443	20.39 ± 4.706
Cautious	21.676 ± 4.662 <i>19.4%</i>	23.946 ± 4.779 <i>17.4%</i>
Maximal	22.89 ± 4.683 <i>26.1%</i>	24.936 ± 4.734 <i>22.2%</i>

Tables 3.1-3 discuss the average utility that compatible pairs obtain in Autarky and by matching in the pool. Here the Autarky result is omitted, because in both Autarky and No Mixing, the compatible pairs self-match, so showing Autarky would be redundant.

TABLE 3.1: AVERAGE UTILITY OF COMPATIBLE PAIRS — POOL A

	2-way	3-way
No Mixing	1.013 ± 0.006	1.013 ± 0.006
Cautious	1.035 ± 0.008	1.047 ± 0.008
Maximal	1.035 ± 0.007	1.044 ± 0.008

TABLE 3.2: AVERAGE UTILITY OF COMPATIBLE PAIRS — POOL B

	2-way	3-way	4-way	No Limit
No Mixing	1.014 ± 0.011	1.014 ± 0.011	1.014 ± 0.011	1.014 ± 0.011
Cautious	1.034 ± 0.015	1.045 ± 0.014	1.048 ± 0.014	1.049 ± 0.014
Maximal	1.031 ± 0.014	1.038 ± 0.014	1.042 ± 0.015	1.045 ± 0.014

TABLE 3.3: AVERAGE UTILITY OF COMPATIBLE PAIRS — POOL C

	2-way	3-way
No Mixing	1.014 ± 0.008	1.014 ± 0.008
Cautious	1.039 ± 0.01	1.053 ± 0.01
Maximal	1.037 ± 0.009	1.046 ± 0.01

All compatible pairs earn weakly better utility in the Cautious and Maximal outcomes compared to the No Mixing outcome. However, as Table 4 shows, when comparing the Cautious and Maximal outcomes, some compatible pairs earn more in one regime than the other, while other pairs are indifferent. “Cautious Better” means that the pair earned higher utility in Cautious, “Maximal Better” means the pair earned higher utility in Maximal, and “Indifferent” means that the pair obtained the same utility in both scenarios. The numbers given quantify the number of compatible pairs in each category, and the percents restate these numbers as a share of all the compatible pairs.

In Table 4, the results from all three pool compositions are put together, and distinguished by their letter.

TABLE 4. OPTIMAL EXCHANGE REGIME FOR INCOMPATIBLE PAIRS

	Better in Cautious	Better in Maximal	Indifferent
A: 2-way	1.684 (11.2%)	1.622 (10.8%)	11.694 (77.9%)
A: 3-way	2.596 (17.3%)	1.63 (10.8%)	10.774 (71.8%)
B: 2-way	0.518 (10.3%)	0.2 (4%)	4.282 (85.6%)
B: 3-way	0.888 (17.7%)	0.262 (5.2%)	3.85 (77%)
B: 4-way	0.842 (16.8%)	0.264 (5.2%)	3.894 (77.8%)
B: No Limit	0.604 (12%)	0.204 (4%)	4.192 (83.8%)
C: 2-way	1.116 (11.1%)	0.788 (7.8%)	8.096 (80.9%)
C: 3-way	1.912 (19.1%)	0.666 (6.6%)	7.422 (74.2%)

3.1 Discussion of Results

3.1.1 Total Utility

Looking at Table 1.1, the average gains to total utility from mixing compatible and incompatible pairs are sizeable, ranging from 20-29% in the 2-way case and 21-29% in the 3-way case. These are the classic gains from trade: create more potential matches by mixing compatible and incompatible pairs together, and the welfare is much higher than when incompatible pairs can only trade with each other and compatible pairs self-match.

In Pool B, the ratio of compatible pairs to incompatible pairs is $5/25 = 1/5$, while in Pool A the ratio is $15/25 = 3/5$. Since Table 1.2 reports Pool B

results and Table 1.1 Pool A results, comparing results from these two tables provides a sensitivity analysis to this ratio. Comparing the percent gains of Cautious and Maximal outcomes against the No Mixing outcome, Pool A always has higher percent gains from mixing than Pool B, ranging from a low of $20.2\% - 12.2\% = 8$ percentage points better in Cautious limit-2, to 10.5 percentage points better in Maximal limit-3. This is to be expected: mixing in Pool A creates $(39 + 38 + \dots 25) = 480$ new potential 2-way matches⁸, whereas mixing in Pool B creates only $(29 + 28 + \dots 25) = 135$ new potential 2-way matches, not to mention the new potential 3-way matches, which stack further in favor of Pool A. Thus, the chance that mixing helps unmatched pairs find a match, and matched pairs find a better match, grows more in Pool A. It is also possible to estimate the impact of a 1% increase in the compatible-incompatible ratio on the percent gains from mixing. Going from Pool B to Pool A, the percentage point increase in this ratio is $(3/5 - 1/5) = 40\%$. Comparing this to the minimum (8) and maximum (10.5) percentage point increase in mixing gains between Pool B and Pool A, the estimate is that a 1% increase in the ratio would increase the mixing gains by 0.2-0.25 percentage points.

This is not a minor point. If adding more compatible pairs only increased mixing gains in absolute terms, but mixing gains in percentage terms stayed the same or declined, then total utility would increase, but the average utility experienced by the pairs would stay the same or decline. It would *appear* that society was better off, but individuals would not see any of this growth. But this is not the case - as the compatible to incompatible pairs ratio increases from $1/5$ to $3/5$, mixing gains increase in percentage terms, so society gains and on average, individuals earn higher utility as well. This growth in the relative gains from mixing may taper off if the ratio of compatible to incompatible pairs rises past the 3:5 ratio studied in Pool A, but given the results from Kranenburg et al (2006) and Gentry et al (2007), it is unlikely that enough compatible pairs would participate to drive the ratio above 3:5. Thus, we can expect that most pairs will benefit in utility as more compatible pairs are added.

We would expect total utility to increase as the cycle size limit increases. Looking at Table 1.2, as the cycle size limit is raised within No Mixing,

⁸25 incompatibles plus 14 compatibles = 39 potential matches for the first compatible pair, $25 + 13 = 38$ potential matches for the second incompatible pair, ..., $25 + 0 = 25$ potential matches for the last incompatible pair

Cautious, or Maximal, total utility rises, so this expectation holds. Further, raising the limit from 2 to 3 contributes most of the gain, and limit-4 encompasses almost all of the utility of the no limit case, which agrees with the findings in Roth, Sönmez, and Ünver (2007). This is encouraging, as it means the simultaneity constraint⁹ does not obstruct society from obtaining most of the gains of multi-way exchange.

Looking more carefully at Table 1.2, the gains as the cycle size limit is increased are almost identical whether one looks at No Mixing, Cautious, or Maximal. Another way to see this is that the absolute difference between Cautious and No Mixing stays near 1.9, except for the 2-way case (1.64), and the absolute difference between Maximal and No Mixing stays near 2.7. In No Mixing, increases in utility as the cycle size increases are due to larger cycles of incompatible pairs forming. New, larger exchanges form in Cautious and Maximal as well when the cycle size limit is increased, but they do not create gains larger than the No Mixing gains. This trend holds for Pool C as well: examining Table 1.3, Maximal relative to No Mixing holds near 5.2, and Cautious relative to No Mixing is 3.9 in the 2-way case and 4.1 in the 3-way case, a “increase in gains” of only 0.2. Only in Pool A do Cautious and Maximal gain relatively on No Mixing: Cautious beats No Mixing by 4.8 in the 2-way case and 5.3 in the 3-way case, an increase in gains of 0.5; Maximal beats No Mixing by 6.9 in the 2-way case and 7.15 in the 3-way case, an increase in gains of 0.25. Thus, for the Cautious and Maximal regimes, higher compatible-incompatible pair ratios also amplify the gains from increasing the cycle size limit.

Finally, even though the No Limit case is infeasible, the 3-way case may capture most of the gains from raising the cycle size limit. Recall that the italicized percentages in Tables 1.1-3 represent the utility gains from mixing according to that exchange regime; Cautious can be thought of as the lower bound of these gains, Maximal as the upper bound, and the difference between Cautious and Maximal as the range of these gains. Looking at Tables 1.1-3, the range of utility gains from mixing tightens as the cycle size limit is raised from 2 to 3: the percentages for Maximal fall, and the percentages for Cautious either rise or change negligibly. Table 1.2 suggests that the range in the No Limit case is the limit to which the other ranges converge; that is, that in the No Limit case the percentages for Maximal are as low as possible, and the percentages for Cautious are as high as possible. If this hypothesis

⁹The constraint which makes exchanges larger than size 3 impractical

is true, then again looking at Table 1.2, the range for the 3-way case is about as small as possible, and the Cautious outcome is about as high as it can be raised; the former is useful because it reduces uncertainty in expectations, and the latter helps to assure a strong minimum level of gains. If this relationship between the 3-way case and No Limit case holds more generally, it would be a useful result - then we would know that the 3-way case has a range whose minimum and size are close to optimal, without explicitly having to test the 4-way or No Limit cases.

3.2 Incompatible Pairs Matched

Tables 2.1-3 make clear that most of total utility comes from matching pairs rather than HLA matches. For example, to get the total number of pairs matched in Pool A, just add 15 to each entry of Table 2.1¹⁰. For the No Mixing, 2-way case, this adds up to $8.072+15=23.072$ pairs matched, and since the utility of a match is 1 these matches create a utility of 23.072. The *total* utility in this case, looking at Table 1.1, is 23.519, so only about $(23.519-23.072) = 0.45$ units of utility originate from match quality in this case¹¹. For the Maximal, 3-way case, the utility from matches alone is $15.72+15=30.72$, and total utility is 32.052, so the contribution from quality is 1.332; this is certainly larger, but still a small contribution compared to the utility from creating matches.

Table 2.1 shows one of the key results of this study: in the 3-way case, mixing compatible pairs into the pool can cause 50-67% more incompatible pairs to be matched. Based on the estimate in Sönmez and Ünver (2009) that kidney exchange with incompatible pairs alone could create up to 2000 transplants, incorporating compatible pairs at 27.7% participation could facilitate another 1000-1330 transplants.

Going from Table 2.2 to 2.1, the percent gains in the number of incompatible pairs matched, which are given by the italicized percentages, increase by a factor of about 2.85 in the Cautious cases and 2.5 in the Maximal cases. For example, comparing the Cautious 2-way case, for Pool B the percent gain is 18.9%, and for Pool A the percent gain is 54%, so the percent gain increases by a factor of $(54/18.9)=2.86$. When going from Pool B to Pool A, the number of compatible pairs increases by a factor of 3. Thus, assuming

¹⁰There are 15 compatible pairs in Pool A, which are always matched; adding this to the number of incompatible pairs matched gives the total number of pairs matched

¹¹Given that $\delta=.05$, this is about 9 HLA matches across the pool

the number of incompatible pairs is held fixed, when the number of compatible pairs is increased by a certain factor, the percent gains in number of incompatible pairs matched increase by almost the same factor. As with total utility, the caveat is that the percent gains may stop “keeping pace” with the growth in compatible pairs as the number of compatible pairs continues to grow, but at least over this range of number of compatible pairs (5-15) this growth is promising.

There are also a few trends that are comparable to those from Total Utility. As the cycle size limit increases, the Maximal and Cautious regime always shrink in terms of percent gains relative to No Mixing. The fact that the Maximal regime gets weaker percent gains is no surprise, as the same trend happened with total utility. But, here the Cautious regime’s percent gains shrink or stay steady, where in the case of total utility they grew or remained steady. This may be because Total Utility gets boosts from higher quality matches and the utilities of compatible pairs, which could mask the declining percentage increase in number of incompatible pairs matched. Just like it was with total utility, looking at Table 2.2 shows that upgrading from limit-2 to limit-3 captures almost all of the utility gains from exchanges larger than size 2, and limit-4 and No Limit are almost the same. Thus, once more the limit-3 case has a range whose minimum and size are nearly optimal, where this time the range is of gains in incompatible pairs matched. Finally, as also occurs with total utility, the range of gains tightens on both sides when pool size is increased but the compatible-incompatible pair ratio is held fixed: for example, in Table 2.2, the range of gains in the 2-way case is 18.9%-31%, while in Table 2.3, the range of gains in the 2-way case is 19.4%-26.1%.

3.3 Average Utility of Compatible Pairs

In Tables 3.1-3, it is clear that both Cautious and Maximal offer better average utility than No Mixing¹². This must always hold, because in Cautious and Maximal, compatible pairs can either be matched to a pair in their respective set, or back to their endowment, so at worst they do as well as the endowment, and at best they improve by a couple HLA matches. Specifically looking at Table 3.1, in the 3-way case compatible pairs improve by an

¹²Since No Mixing and Autarky yield the same utility for every compatible pair, one can simply compare Cautious and Maximal to No Mixing

average of 0.034 between No Mixing and Cautious. Since $\delta=.05$, an increase of utility of 0.034 is about 2/3 of one HLA match, so on average compatible pairs will improve by 2/3 of an HLA match by participating in the pool, and they do so without giving up the option to receive from their own donor unless they are offered a better match.

Additionally, the average utility of compatible pairs under the Cautious regime is always better than the average utility of compatible pairs under the Maximal regime. This is predictable: by opting for Cautious, a compatible pair has a lower chance of only doing as good as the endowment, because the only option at that level is the endowment itself. On the other hand, by opting for Maximal, there are more possible matches that are as good as the endowment, so it is more probable that the compatible pair will not get an improved match. The dominance of Cautious over Maximal in this case is also shown in Table 4: for all three pool compositions, more pairs are better off in the Cautious scenario than are better off in the Maximal scenario, especially as the cycle size grows. But curiously, there are compatible pairs that do better in the Maximal scenario. This is likely because in the Cautious scenario, their “ideal” pair is matched in another exchange that generates more societal utility, but the Maximal scenario causes more compatible pairs to match in the pool and break up existing matches, freeing up the “ideal” pair for our original pair to match to. Given that there is this rare case, though, there is a strategic aspect to choosing between Cautious and Maximal, and future studies would do well to analyze this aspect of the problem. Granted, the strategic aspect might disappear if strict preferences were reintroduced, or if match quality was measured by a composite of characteristics including HLA match, donor age, donor-recipient relationship, etc. that practically created strict preferences, but neither of these changes guarantee to eliminate the strategic aspect. For average utility of compatible pairs, then, neither Cautious nor Maximal serve as true bounds on the averages, just approximate ones. The true upper bound would be “Greedy”, which would take Cautious to the extreme and maximize the gains of the compatible pairs first before doing the overall optimization, and the true lower bound would be Autarky, in which each compatible pair earns the “worst” they can possibly earn, their endowment.

A few other trends are worth noting. Average utility grows with cycle size for all three pool compositions, but Table 3.2 demonstrates that the Cautious regime gets most of its gain from upgrading from limit 2 to limit 3, while the gains for the Maximal regime are more spread out across the

upgrades to limit 3, limit 4, and No Limit. Given that limit 4 and No Limit are impractical, it becomes even more advantageous for compatible pairs to opt for the Cautious set, to guarantee most of the utility gains in cycles of at most size 3. Comparing Tables 3.2 and 3.1, Pool A offers better average utilities than Pool B for both exchange regimes and both cycle size limits. But, the gains are not large. Arguably, the main benefit is that the range between the Cautious and Maximal values is smaller in Pool A: for 2-way and 3-way size limit, the ranges are of size 0 and 0.03, respectively, for Pool A, but 0.03 and 0.07, respectively, for Pool B. Comparing Tables 3.1 and 3.3, Pool C strictly dominates Pool A across both exchange regimes and cycle size limits. Referencing back to Pool B, this suggests that increasing pool size while maintaining the compatible-incompatible ratio gives better yields to average utility than increasing the ratio does, which is intuitive because increasing pool size lets the compatible pairs have more potential matches without a disproportionately high increase in pairs that are competing for those matches.

4 Conclusion

There is an acute shortage of kidney donors in the US and internationally. Kidney exchange is a proven way of increasing the amount of live donors, but so far its potential has not been fully tapped, because compatible pairs have not been incorporated. When the compatible pair participation rate is 27.7%, compatible pairs see an average improvement in match quality of 2/3 of one HLA match, and 50-67% more incompatible pairs can be matched than would be in a pool of only incompatible pairs. Given that kidney exchange with incompatible pairs could create up to 2000 transplants, the increase from incorporating compatible pairs would facilitate another 1000-1330 transplants per year.

The average gain to compatible pairs may seem small, but some compatible pairs can improve by up to 2-3 HLA matches, without the donor being forced to give up the option to directly donate to his intended recipient. Further, as Kranenburg et al. (2006) showed, the first reason that compatible pairs choose to participate is altruism, and to obstruct such acts defies the spirit of kidney donation, as the system already relies on the charity of living and deceased donors to enable transplants at all. The case for incorporating compatible pairs is now even better established. But, why try to account

for HLA match in the matching prioritization? At the moment, the only way that match quality is incorporated in the prioritization of the UNOS pilot program is that a perfect HLA match (referred to as “zero antigen mismatch”) is assigned 200 points¹³. Without accounting for HLA match, the prioritization would allow compatible pairs to be matched to any pair as long as the match is of quality as good as the pair’s own donor. If HLA match is accounted for, the algorithm would prioritize the higher quality matches, incentivizing compatible pairs to participate in search of a better match, and generating higher compatible pair participation than altruism would alone. Prioritizing by HLA match could make it more difficult for some incompatible pairs to be matched, but given that compatible pairs will enable many more incompatible pairs to be matched, this is not a critical issue. Compatible pairs need to be incorporated into the nationwide kidney exchange program, and HLA match should be accounted for in the prioritization of matches.

Every day, kidney failure patients receive the gift of hope, a kidney transplant, owing to the kindness of donors around the nation. People are inherently altruistic, even if they personally stand to gain little from such behaviors. Allowing compatible pairs to participate in kidney exchange programs will facilitate such altruism, give many patients a better match, and help fight the kidney shortage in our country that has persisted for so long. Who are we to stand in the way?

¹³UNOS Pilot Program Proposal, http://optn.transplant.hrsa.gov/SharedContentDocuments/KPD_Briefing_Paper_508V.pdf, accessed April 18, 2011

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