Efficiency and Herd Behavior in a Signalling Market

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ABSTRACT

This paper extends a model of herd behavior developed by Bikhchandani and Sharma (2000) to establish conditions for varying levels of efficiency in a simplified equity-only financial market. By examining the valuations of three types of investors who receive market signals at separate times, as well as the price-setting behavior of an automatic then a profit-maximizing market mechanism, I consider the potential for arbitrage under a discrete-time process of information distribution. From simulations of a dynamic transaction model, I use a game-theoretic method to obtain an equilibrium for when each investor chooses to buy and sell. It turns out that stronger levels of market efficiency require large assumptions about the market's knowledge which may not be consistent with the reality of our present-day financial markets.

I. **Introduction**

The common prerequisite for successful participation in the equities market is information. Investors must be able to synthesize public information from financial statements, trading trends, and the news wire, as well as private information from their own sources, in order to make an informed decision about whether or not to invest in a particular stock.

Within this population of investors, there exists a hierarchy based on the timing of receiving one's information. Insiders with the means to obtain private signals before anybody else can potentially obtain arbitrage before the market is able to adjust. Fundamental traders then use signals based on the subset of the public information obtained from their own analysis, as well as the observed behavior in those insiders, to make their decisions. This influence of others' past actions on one's current actions is the simplest example of herd behavior. Besides Bikhchandani and Sharma (2000), papers by Hirshleifer and Teoh (2001), Cipriani and Guarino (2012), and Park and Sabourian (2011) also attempt to model the effect of this behavior in financial markets.

In the first part of this paper, I develop a model based on these considerations, simplified to the most basic one stock and one period with no selling, and set up a structure for when information is revealed and decisions are made for each type of investor. I then go through the three levels of efficient markets, and based on the stock valuations of each type of investor and nature of public and private information, establish the properties of the pricing mechanism required for each.

In the next part, I present an adjustment of the pricing model more aligned with reality in the form of a monopolistic market maker, who sets the buy and sell price and makes money off the spread. The role of market makers in efficiency is discussed by Rotthoff (2011). This addition also allows for the selling of stocks and ex ante ownership. I establish the market maker's profit-maximization incentive and the absence of a demanded premium as conditions for market efficiency.

I then incorporate herd behavior into the market maker model and try to determine the optimal time for each group of investors to buy and sell. Through a series of simulations, which involve more complicated profit-maximization procedures, I obtain a dynamic set of rules governing the traders' behavior when they have the freedom to enter the market whenever they want. Analyzing these preferences leads me to a Nash Equilibrium of investment timing in this extended model.

Finally, I provide an extension to the baseline model which considers the varying quality of the insider information. Under this framework, I find the conditions under which traders will want to change the source they base their herd behavior on. I also consider the option of paying for multiple signals, and compute the optimal coverage given the costs and other market variables.

My departure from Bikhchandani and Sharma is in incorporating their theory in a model to determine the strength of market efficiency when groups of traders obtain different information at different times. The key question this paper is trying to answer is whether or not it is possible to earn consistent positive returns in a stock market where investors herd. By considering the optimization incentives of all the market participants, I try to identify a general structure of investment timing in this type of economy. And by extending this strictly theoretical model using multi-period simulations of investor profit, I can achieve conclusions about herd behavior and efficiency more relevant to present-day financial markets.

II. The Efficient Market Hypothesis

Before we go over the specifics of this model, the Efficient Markets Hypothesis needs to be discussed. Clarke, Jandik, and Mandelker (2001) provides a concise overview of the concept.

The basic idea of efficient markets is that the market price for stocks or any financial instruments reflects the information available in the market. Thus, in theory, for all stocks, any indicators of a more likely future gain – for example, balance sheet ratios – that investors might want to exploit are already accounted for in a higher listed stock price. The same goes for negative indicators and a lower stock price.

There are generally believed to be three levels of efficient markets. A stronger efficient market automatically contains the properties of a weaker one.

In a weak-form efficient market, the price of a stock reflects all of its past price movements. Thus, an investor who invests based on past price movements (Group C investor in this model) will never obtain above-average returns in the long run.

In a semi-strong-form efficient market, the price of a stock reflects all publicly available information. Thus, an investor who invests based on financial statements and other public market information (Group B investor in this model) will never obtain above-average returns in the long run.

In a strong-form efficient market, the price of a stock reflects all private information. Thus, an investor who invests based on insider information (Group A investor in this model) will never obtain above-average returns in the long run.

III. **The Baseline Model**

We will develop our baseline model based on the basic information-based herding model in Bikhchandani and Sharma (2000). For that model, we normalized the current price of a stock to be 0 and made the simplifying assumption that it will either grow to 1 or fall to -1 with equal probability in the next period.

For this model, let us consider three types of investors. An investor in Group A does insider trading, so he receives his signal based on his private information. An investor in Group B only does fundamental analysis, so he receives his signal based on his subset of the public information. An investor in Group C only does technical analysis, so he does not use any public information besides past price movements and thus does not receive any signal. The investors in Group A, B, and C can choose to either invest or not invest based on the signals they receive. Note that investors can represent not only individual traders, but institutions and any type of single business entity.

From both private and public information, an investor receives a signal for either a gain (G) or a loss (L). A growing stock will give out the signal G as a proportion p of private signals and the signal L as a proportion $1 - p$ of private signals, where $p > 0.5$. Likewise, a falling stock will give out L as a proportion p and G as a proportion $1 - p$. We assume that signals from private information and those from an individual's subset of the public information are of similar quality, so the same proportions hold for public signals. We also assume that whether the stock grows or falls is independent of investor decisions.

Investors in Group A and Group B receive their signals as soon as they finish processing the private information and public information, respectively. In addition, each investor in Group B designates one random investor in Group A as his source, from whose behavior he will base his own investment decisions on. This is the basis of the herd behavior described in this model. Consider this in terms of an investor (B) who trusts the methods of a specific trading firm (A) with insider information, and thus will be inclined to follow its lead when it comes to buying or not buying a certain stock.

In this baseline model, Group A and Group B investors make their investment decision as soon as all the relevant signals and source information are received. This restriction will be relaxed in later sections.

Public information is made available at the beginning of the day at time 0. Define $x_{1t} + 1$ to be the processing time in period t, where $x_{1t} \in Bin(n_1, \delta)$. This is the time, in seconds, it takes for an investor in period t to process new public information (i.e., access the information source, download the information, and analyze it). Thus, all Group B investors will receive their signals from public information at time $x_{1t} + 1$. Due to the advanced information technology in finance, n_1 can be thought of as relatively small number. $0 < \delta < 1$ is a variable representing how fast information transfers; a smaller δ means faster transfer.

Any private information is also available at the beginning of the day at time 0. However, since it is obtained directly from insiders, fewer intermediate steps are involved and so the time it takes to process this information is less. And so, it takes a maximum of x_{1t} seconds for each Group A investor to process their private information.

Thus, each of the Group A investors will receive his signal at a uniformly distributed discrete time $a_{it} \in [0, x_{1t}]$ and then make his investment decision instantaneously.

Although Group B investors receive their signals at time $x_{1t} + 1$, they have to wait to process the new public information of their source's investment decision. Thus, the earliest they can invest is time $x_{1t} + 1$ and the latest time $2x_{1t} + 1$. And so each Group B investor will make his investment decision – based on his signal and the action of his source – at discrete time $b_{it} \in [x_{1t} + 1, 2x_{1t} + 1]$.

Finally, Group C investors are not doing insider trading or fundamental analysis, so they will not receive any type of signal or follow any source (they have no knowledge about the other investors). They will thus choose whether to invest at any random time after time 0, with no preference for a particular time. Formally, each Group C investor will make their investment decision at unformly distributed discrete time $c_{it} \in [1, y]$, where y is the length of the period in seconds and $y > 2x_{1t}$.

Relevant to this, we make the additional assumption that the change in stock price between two periods is independent of the change in stock price between any two previous periods. Also, in this simplified model, there is no existing stock ownership and the investors cannot sell.

Valuation

Based on their signals received, each investor has an expected value for the stock next period. For simplicity, let us assume no discounting across time. The valuations for Group A and B follow from the results in Bikhchandani and Sharma (2000).

Group A investors who receive signal G know there is a p probability of the future stock price S_1 being 1, and thus a $1 - p$ probability of $S_1 = -1$. This can be shown through Bayes' rule:

$$\begin{aligned} P(S_1 = 1|G) &= \frac{P(G|S_1 = 1)P(S_1 = 1)}{P(G)} \\ &= \frac{p \times \frac{1}{2}}{\frac{1}{2}} = p \end{aligned}$$

We can summarize the expected values in a table:

Table 1: Group A Investor's Valuations

Signal	Probability of Growth	Expected Value
G	p	$2p - 1$
L	$1 - p$	$1 - 2p$

Thus, the expected future value for that investor, and thus his valuation of the stock, is:

$$\begin{aligned} E(S_1|G) &= 1(p) + (-1)(1 - p) \\ &= 2p - 1 \end{aligned}$$

Since $2p-1>0$, the investor receiving G will have a positive valuation of the stock and invest in it for its market price 0. By symmetry, a Group A investor who receives a L signal will value the stock at $1-2p$, and thus not invest in it.

The Group B investor then makes his valuation based on both the decision of his source and his own signal.

Table 2: Group B Investor's Valuations

Source	Signal	Probability of Growth	Expected Value
Invest	G	$\frac{p^2}{p^2+(1-p)^2}$	$\frac{2p-1}{p^2+(1-p)^2}$
Invest	L	$\frac{1}{2}$	0
Not Invest	G	$\frac{1}{2}$	0
Not Invest	L	$\frac{(1-p)^2}{p^2+(1-p)^2}$	$\frac{1-2p}{p^2+(1-p)^2}$

Since the Group A investor receiving G will always invest and the Group A investor receiving L will never invest, the Group B investor knows that a source that did invest received G, and a source that did not invest received L.

If both the investor and his source receive G signals, then either the signals were correct both times (and the stock is going up) or incorrect both times (and the stock is doing down). Conditioning on the sum of those two probabilities, knowing that a signal is correct with probability p and incorrect with probability $1-p$, the probability that the stock will go up is:

$$P(S_1=1|GG)=\frac{p^2}{p^2+(1-p)^2}$$

Thus, the expected value of the stock is:

$$\begin{aligned}E(S_1|GG) &= (1)\frac{p^2}{p^2+(1-p)^2}+(-1)\frac{(1-p)^2}{p^2+(1-p)^2} \\ &= \frac{2p-1}{p^2+(1-p)^2}\end{aligned}$$

Likewise, we have:

$$E(S_1|LL)=\frac{1-2p}{p^2+(1-p)^2}$$

Under the same reasoning, we find that:

$$E(S_1|GL)=E(S_1|LG)=0$$

Finally, we consider the Group C investor:

Table 3: Group C Investor's Valuations

Signal	Probability of Growth	Expected Value
None	$\frac{1}{2}$	0

Since the Group C investor does not receive a signal, his probability that the stock will go up is always the unconditional probability.

$$P(S_1 = 1) = \frac{1}{2}$$

$$E(S_1) = 0$$

IV. **Pricing Under Efficient Markets**

Weak-Form Efficiency

Since investors in Group C do not receive any signals, they can only make investment decisions based on past stock price movements.

Theorem 1. The market is weak-form efficient iff stock price movements between each period are independent.

Proof: If the stock price movements between each period are independent, the movement next period is independent of movements in all previous periods. Denote the stock price movement from period t to $t+1$ by X_t. The stock has been on the market for n periods so far. We thus have:

$$P(S_1 = 1|X_{-1}, X_{-2}, ..., X_{-n}) = \frac{1}{2}$$

Thus, regardless of what the past stock price movements were, the probability of the stock moving up next period is $\frac{1}{2}$. And so, for the Group C investor:

$$E(S_1|X_{-1}, X_{-2}, ..., X_{-n}) = 0$$

Since this value is conditional on $X_{-1}, X_{-2}, ..., X_{-n}$, it can be said that the current market price of 0 reflects all previous stock price movements.[1]

Since the Group C investor values the stock at 0 and the market price of the stock is 0, the Group C investor, who relies on technical analysis, never has an arbitrage opportunity. Therefore, weak-form market efficiency holds.[2]

Now, suppose the stock price movements in each period were not independent. Then, since $P(S_1 = 1) = \frac{1}{2}$, there would exist sets of past movements $\{X_{-1} = x_1, X_{-2} = x_2, ..., X_{-n} = x_n\}$ and $\{X_{-1} = y_1, X_{-2} = y_2, ..., X_{-n} = y_n\}$ such that:

$$p' = P(S_1 = 1|x_1, x_2, ..., x_n) > \frac{1}{2}, p'' = P(S_1 = 1|y_1, y_2, ..., y_n) < \frac{1}{2}$$

For the first set, it follows that:

$$E(S_1|x_1, x_2, ..., x_n) = 2p' - 1 > 0$$

[1] For stronger forms of efficiency, assuming independent price movements gets us $E(S_1|X_{-1}, X_{-2}, ..., X_{-n}) \geq 0$. But since $E(S_1|X_{-1}, X_{-2}, ..., X_{-n})$ is still the same for any $X_{-1}, X_{-2}, ..., X_{-n}$, the same conclusion holds.

[2] Again, the market price of the stock is ≥ 0 for stronger forms of efficiency, under which the same conclusion holds.

In that case, the Group C investor has an arbitrage opportunity: when the stock is valued above 0, it is underpriced at 0 and thus the investor would buy it. $\square$

Semi-Strong-Form Efficiency

Previously, we assumed that the stock price during this period is constant at 0. As shown above, this leads to Investors A and B purchasing stocks with positive expected value at a price of 0, and thus obtaining arbitrage.

Theorem 2. The market is not semi-strong-form efficient if the market price of the stock during a period is constant.

Proof: Consider the current period, during which we assume the stock price is constant at 0. The proof for other periods is analogous.

If the stock price is moving up, there will be a proportion p^2 of the Group B investors who have observed their source invest as well as a signal G. As shown in Table 2, they will value the stock at $\frac{2p-1}{p^2+(1-p)^2}$. Since the market price of the stock is 0, they will purchase the stock and end up earning $1 - 0 = 1$.

There will also be a proportion $(p-1)^2$ of the Group B investors who have observed their source not invest as well as a signal L. They will value the stock at $\frac{1-2p}{p^2+(1-p)^2} < 0$ and thus will not buy it, ending up earning 0.

The other $2p(p-1)$ investors who receive a mixed signal will value the stock at 0 and thus be indifferent, buying it $\frac{1}{2}$ of the time and thus earning $\frac{1}{2}$.

If the stock price is moving down, there will be a proportion $(1-p)^2$ of the Group B investors who have observed their source invest as well as a signal G. As shown in Table 2, they will value the stock at $\frac{2p-1}{p^2+(1-p)^2}$. They will purchase the stock for 0 and end up earning $-1 - 0 = -1$.

It follows analogously that p^2 will not buy the stock (earning 0), and the other $2p(1-p)$ will buy it $\frac{1}{2}$ of the time (earning $-\frac{1}{2}$).

So each period, the investor can be either in the p^2, $(p-1)^2$, or $2p(p-1)$ section. Since the unconditional probability of the stock moving up is $\frac{1}{2}$, the investor's expected earnings if he is in the $2p(1-p)$ section is 0.

Since $p > \frac{1}{2} > 1-p$, an investor is more likely to be in the p^2 section than the $(1-p)^2$ section. Thus, over several periods, he will have more earnings of 1 than -1, and thus make a profit in the long run. Therefore, an arbitrage opportunity exists for Group B (fundamental analysis) investors, violating semi-strong-form efficiency. $\square$

In order to attain semi-strong-form market efficiency, we need to make an addition to our model that allows the market to adjust the price based on demand. We will make the important assumption in this case that the market has unlimited supply.

The market's pricing mechanism can only rely on public information. And the signals for Group A investors (insider traders) are not public information. Thus, initially, the market does not know what signal each Group A investor will receive, nor the relative proportion of each signal. So the market does not know what p is, and the initial market price is 0.

So in the case of a growing stock, p Group A investors will observe G and will want to buy it, valuing the stock at $2p-1$. $1-p$ Group A investors will observe L and not want to buy it, valuing the stock at $1-2p$. So from those who want buy the stock, the price demanded is $2p-1$.

So as the G investors buy undervalued stocks, the market price will adjust to the price being demanded. Eventually, the market price becomes $2p-1$.

The same thing applies for a falling stock, as $1-p$ Group A investors will demand a price of $2p-1$ and the market price will adjust to that amount.

Theorem 3. Assuming Theorem 1 holds, the market is semi-strong-form efficient if the market price of the stock adjusts immediately to demand and the market knows the total number of insider traders N_A and the sales volume V_s.

Proof: If the market price of the stock adjusts immediately to demand, it will become $M = 2p-1$ after time 0. Note that at this new adjusted price, Group C investors entering after time 0 will never invest, so they can be ignored. From that new adjusted price M which the market now knows, the market can then back out the value of p.

However, if the number of insider traders N_A is not known by the market, the market does not know when all the Group A traders have received their signals. Thus, when the first set of Group B traders receive their signals at time $x_{1t}+1$, the market price of the stock will still be $2p-1$

It was shown in Table 2 that a certain proportion of the Group B traders (p^2 if the stock is going up, $(1-p)^2$ if it is going down) will both imply a G signal from their source and receive a G signal on their own. Their valuation of the stock is $\frac{2p-1}{p^2+(1-p)^2}$. And since:

$$p^2+(1-p)^2<1 \implies \frac{2p-1}{p^2+(1-p)^2} > 2p-1$$

they will purchase the stock at the market price.

And their expected profit is positive. Following the same reasoning as in Theorem 2, it can be calculated to be:

$$\pi(p) = -4p^3+6p^2-2p$$

which is positive for all $p>0.5$. Thus, an arbitrage opportunity exists for Group B investors.

The only way to avoid arbitrage is to have the market price set at their valuation $\frac{2p-1}{p^2+(1-p)^2}$ before they receive their signal. However, the market needs to know when all the Group A investors are done deciding (at x_{1t}), so it can anticipate the Group B investors entering next second (at $x_{1t}+1$) and thus adjust to that price.

After time 0, the market knows what p is. The market's strategy is to adjust the price after time s to $\frac{2p-1}{p^2+(1-p)^2}$ when $V_s = pN_A$. Consider the two cases:

If the stock is going down and the proportion of buyers is $1 - p$, then all the insider traders will be done exactly when $V_s = (1 - p)N_A$. But since $(1 - p)N_A < pN_A$, the market will not know to adjust the price. Thus, the first Group B traders who receive GG will purchase the stock at $2p - 1$ as described above.

However, since the stock is going down, those investors will earn $-2p$ next period, and thus lose money, and so obtain no arbitrage. And given this change in price demanded, the market will adjust the price to the new equilibrium level.

If the stock is going up and the proportion of buyers is p, then the insider traders will be done exactly when $V_s = pN_A$, which is the same time the market will adjust the price. Thus, the Group B traders coming in right after will not have an arbitrage opportunity.

Thus, under this market mechanism where the pricing mechanism accounts for the market's knowledge, Group B investors cannot obtain arbitrage. Therefore, the market is semi-strong-form efficient. □

Strong-Form Efficiency

In a strong-form efficient market, neither investors in Group A, B, nor C can obtain above-average returns in the long run.

Theorem 4. Assuming Theorem 1 holds, the market is strong-form efficient iff the the market knows the value of p, the total number of insider traders N_A, and the sales volume V_s.

Proof: If the market knows p, then the market can initially set the stock price at $2p - 1$ at the beginning of the day, before any of the Group A investors receive their signals and invest. Since the expected earnings for those Group A investors who received G is $2p - 1$, they will not be able to obtain long-term above-average returns by investing in the stock at that price.

Without p being public information, the market has no way of knowing p before the investing begins and thus has no basis of setting an initial price to anything other than 0, which leads to arbitrage.

It was already shown in Theorem 3 that N_A and V_s being public information ensures that Group B investors will not be able to obtain arbitrage. Therefore, under this market mechanism the market is strong-form efficient. □

V. Market Making and Profit Maximization

So far, we have made extreme simplifications to the market mechanism for pricing stocks by disregarding selling and assuming the market has an infinite supply. In reality, the stock prices on the market are set by a market maker, who quotes a buy price and a sell price and makes a profit on the difference, or spread, between the two values.

In this market maker model, the supply for investors who wish to buy the stock comes entirely from other investors who have sold shares of the stock that they previously owned.

The fact that the stock might not have available sellers (buyers) when an investor would make his decision of whether or not to buy (sell) the stock necessitates another change from the original model. Instead of making decisions immediately after receiving their signal and source information, investors with information (i.e. Group A and B) can try to buy or sell at any time between then and the end of the day.

Standard Market Maker Model

Let us first consider the simplest setup where there are no separate investor groups or herd behavior. For the purpose of the proof, we will say that every trader has insider information and thus functions as a Group A investor (we can change this to Group B or C to prove weaker forms of efficiency). Now consider an economy where a proportion γ of all traders[3] already own a share of the stock. Assume that the signal an investor receives is independent of his ownership of the stock.

As the number of investors who hold a stock is much fewer than the number who don't in reality, we will assume $\gamma < \frac{1}{2}$. And as this proportion with holdings is usually very small – much smaller than the probability of a signal being wrong – we will further assume that $\gamma < 1 - p$, and so $\gamma < 1 - p < p < 1 - \gamma$.

Owners of the stock want to sell it for at least $q_j + \epsilon$, where q_j is their valuation of the stock and $\epsilon \geq 0$ is their demanded minimum premium. Similarly, those who do not own it want to buy it for at most $q_k - \epsilon$, where q_k is their valuation of the stock and $\epsilon \geq 0$ is the same premium. Assume no transaction costs.

To develop our main theorem, let us consider a profit-maximizing market maker, acting only in self-interest, who controls the market – all transactions must go through him. He has access to all public information, which we assume includes the value of p, the total number of traders N, and the sales volume V_s.

Consider the case of the stock moving up. The moving down case is analogous. Of the N investors, we have:

Type 1: $p\gamma N$ who own the stock and value it at $2p - 1$ (received G signal)

Type 2: $p(1 - \gamma)N$ who don't own the stock and value it at $2p - 1$ (received G signal)

Type 3: $(1 - p)\gamma N$ who own the stock and value it at $1 - 2p$ (received L signal)

Type 4: $(1 - p)(1 - \gamma)N$ who don't own the stock and value it at $1 - 2p$ (received L signal)

The market knows the value of p from the start. He also knows that he will be able to buy from or sell to any of the investors.

[3] γ is the same for all t, since each investor can hold at most 1 share

When setting the sell price (the price stock owners will sell it for), the profit-maximizing market maker will only choose $1 - 2p + \epsilon$ or $2p - 1 + \epsilon$, as those prices exactly capture the Type 3 and Type 1 sellers, respectively. Analogously, he will only choose the buy prices $1 - 2p - \epsilon$ or $2p - 1 - \epsilon$. Now consider the four combinations:

Table 4: Market Maker Profitability from Different Sell and Buy Prices for Growing Stock

Sell Price	Buy Price	Spread	Sellers	Buyers	Profit
$2p-1+\epsilon$	$2p-1-\epsilon$	-2ϵ	$(\gamma)N$	$p(1-\gamma)N$	≤ 0
$2p-1+\epsilon$	$1-2p-\epsilon$	$2-4p-2\epsilon$	$(\gamma)N$	$(1-\gamma)N$	< 0
$1-2p+\epsilon$	$2p-1-\epsilon$	$4p-2-2\epsilon$	$(1-p)(\gamma)N$	$p(1-\gamma)N$	$(4p-2-2\epsilon)B_1$
$1-2p+\epsilon$	$1-2p-\epsilon$	-2ϵ	$(1-p)(\gamma)N$	$(1-\gamma)N$	≤ 0

Note: $B_1 = \min((1-p)\gamma N, p(1-\gamma)N) = (1-p)\gamma N$

Doing the same thing for the case where the stock is going down, we get:

Table 5: Market Maker Profitability from Different Sell and Buy Prices for Falling Stock

Sell Price	Buy Price	Spread	Sellers	Buyers	Profit
$2p-1+\epsilon$	$2p-1-\epsilon$	-2ϵ	$(\gamma)N$	$(1-p)(1-\gamma)N$	≤ 0
$2p-1+\epsilon$	$1-2p-\epsilon$	$2-4p-2\epsilon$	$(\gamma)N$	$(1-\gamma)N$	< 0
$1-2p+\epsilon$	$2p-1-\epsilon$	$4p-2-2\epsilon$	$p(\gamma)N$	$(1-p)(1-\gamma)N$	$(4p-2-2\epsilon)B_2$
$1-2p+\epsilon$	$1-2p-\epsilon$	-2ϵ	$p(\gamma)N$	$(1-\gamma)N$	≤ 0

Note: $B_2 = \min(p\gamma N, (1-p)(1-\gamma)N) = p\gamma N$

Lemma 1. The market maker will earn no profit iff $\epsilon \geq 2p - 1$.

Proof: Proof is trivial. Since:

$$\epsilon \geq 2p - 1 \implies 4p - 2 - 2\epsilon \leq 0$$

And since $B_1, B_2 \geq 0$, this means that $(4p - 2 - 2\epsilon)B_1, (4p - 2 - 2\epsilon)B_2 \leq 0$. And so, the greatest profit the market maker can earn in either case is 0. $\square$

Thus, we come to our main result:

Theorem 5. Assuming stock price movements between each period are independent, in an economy with homogeneous investors and a single market maker, the market is strong-form efficient for all $\epsilon > 0$ iff the market maker knows p and is profit-maximizing.

Proof: We have gotten to this point in the model by assuming that stock price movements between each period are independent, the market maker is a monopoly, and the market maker knows the value of p and is profit-maximizing.

Let's consider the case where $\epsilon < 2p - 1$. The market maker thus maximizes profit by setting the sell price as $1 - 2p + \epsilon$ and the buy price as $2p - 1 - \epsilon$. Consider the investor's profitability in the four cases, with a buyer (B) or seller (S), and the market going up (U) or down (D).

$$\begin{aligned}
\pi_{BU} &= 1 - (2p - 1 - \epsilon) = 2 - 2p + \epsilon \\
\pi_{BD} &= -1 - (2p - 1 - \epsilon) = -2p + \epsilon \\
\pi_{SU} &= -1 + (1 - 2p + \epsilon) = -2p + \epsilon \\
\pi_{SD} &= 1 + (1 - 2p + \epsilon) = 2 - 2p + \epsilon
\end{aligned}$$

Then let Q denote the unconditional probability a random Group A investor will end up being a buyer or seller (and doing a transaction) in the specified situation in any given period:

$$Q_{BU} = \frac{1}{2}(1-p)\gamma,\ Q_{BD} = \frac{1}{2}p\gamma,\ Q_{SU} = \frac{1}{2}(1-p)\gamma,\ Q_{SD} = \frac{1}{2}p\gamma$$

From these, we can find overall expected profit:

$$\begin{aligned}
\pi &= \pi_{BU}Q_{BU} + \pi_{BD}Q_{BD} + \pi_{SU}Q_{SU} + \pi_{SD}Q_{SD} \\
&= \gamma(1 - 2p + \epsilon)
\end{aligned}$$

Thus, for all $\epsilon < 2p-1$, investors will never obtain above-average returns. And so, no arbitrage opportunity exists.

For completeness, we must consider the case where $\epsilon \geq 2p - 1$. If $\epsilon > 2p - 1$, all four other options have negative profit for the money-maker, and so he is better off setting unrealistic prices and making no transactions. Also, if $\epsilon = 2p - 1$, he is indifferent between the above pricing and unrealistic pricing; either way, the investors' expected profit is 0.

Thus, for all $\epsilon > 0$, no arbitrage can be obtained. Therefore, these investors with insider information cannot obtain positive expected returns and the market is strong-form efficient. $\square$

It is interesting to note that in calculating expected profit above, the γ terms cancelled out, meaning that the relative number of investors who own the stock is irrelevant to an investor's expected profitability in the long-term as well as market efficiency.

When considering all this in a real-world context, a few other results can be found.

Corollary 1. The market maker will obtain a higher profit when the stock is falling.

Proof: Since $\gamma < \frac{1}{2}$, then $\gamma < 1 - \gamma$.

And since $1-p < p$, $(1-p)\gamma_t$ will always be the smallest of the four terms in B_1 and B_2. Thus, $B_1 < B_2$, and it follows that the market maker's profit with a falling stock will be higher than his profit with a growing stock. $\square$

Also, just to have an idea of the range of values for p, we know that in reality the buy-sell spread x is always positive. Thus, $x = 2 - 4p - 2\epsilon > 0$. Rearranging terms, we get that the probability of a correct signal p can be written:

$$p = \frac{1}{2} + \frac{1}{4}x + \frac{1}{2}\epsilon$$

We have that a greater buy-sell spread x and minimum premium ϵ for investors is indicative of a higher signal correctness rate p. To make any further conclusions about the value of p will require empirical data on spreads and premiums relative to stock movements.

Market Maker Model with Herd Behavior

We can now extend this market maker model to include the three separate groupings of investors (A, B, C) as in the baseline model. We will assume that the proportion of each Group that holds the stock $\gamma_A, \gamma_B, \gamma_C$ is the same, and the market maker believes that proportion holds for any subset of each group.

Also, now that Group A investors can buy and sell beyond x_{1t}, we must specify that the content of their private information and the subset of their public information is the same – i.e. their private signal and public signal are the same. However, as they can do anything a Group B investor can, a Group A investor can designate another Group A investor as his source. Thus, for any time after x_{1t} a Group A investor has the same information as a Group B investor.

To simplify calculations, we will also assume that, ceteris paribus, a market maker would prefer to sell to a Group A or B investor over a Group C investor. Thus, if the market maker knows when the investors will change next, Group A and Group B investors have priority over Group C investors.

Let us simplify the problem at first by having the length of the period be $2x_{1t} + 1$.

We have a total N_A Group A investors and N_B Group B investors. A subset of $n_A \geq 0$ of the Group A investors try to trade before time $x_{1t} + 1$ (the earliest time a Group B investor can trade). There is also a subset $n_C \geq 0$ of the Group C investors that trade before time $x_{1t} + 1$.

Lemma 2. In a market maker economy with heterogeneous investors where the number of Group A market participants is public information and $\frac{p(1-\gamma)}{(1-p)\gamma} \leq 2$*, Group A investors who buy or sell during* $[0, x_{1t}]$ *cannot earn positive expected profit.*

Proof: Consider the market maker for that discrete length of time $[0, x_{1t}]$. He not only knows the value of n_A, but also knows γ_A and believes that proportion applies to any subset of Group A investors. His profit-maximization problem becomes the same as in the homogeneous case except we have two additional investor types representing Group C (moving up case, down is analogous):

Type 5: $\gamma_C n_C$ who own the stock and value it at 0.

Type 6: $\gamma_C n_C$ who don't own the stock and value it at 0.

The market maker now has two additional possible pricing levels: a sell price of ϵ captures Type 5 exactly, and a buy price $-\epsilon$ captures Type 6 exactly. Incorporating these, Table 4 extends to 8 total options. Only 3 of these are possible profit-maximizers[4]:

[4]The other 5 options yield ≤ 0 profit. If $\epsilon = 0$, the profit for these 3 are definitely positive and thus preferred. If $\epsilon > 0$, the other 5 options will yield < 0 profit; so the market maker would rather set unrealistic prices and have no trades than go with one of them. And so, outside of the trivial case of no trading in the market, one of these 3 will be profit-maximizing.

Table 6: Market Maker Profitability for Growing Stock

Sell Price	Buy Price	Spread	Sellers	Buyers	Profit
$1-2p+\epsilon$	$2p-1-\epsilon$	$4p-2-2\epsilon$	$(1-p)\gamma_A n_A$	$p(1-\gamma_A)n_A$	$(4p-2-2\epsilon)B_1$
ϵ	$2p-1-\epsilon$	$2p-1-2\epsilon$	$(1-p)\gamma_A n_A+\gamma_C n_C$	$p(1-\gamma_A)n_A$	$(2p-1-2\epsilon)B_3$
$1-2p+\epsilon$	$-\epsilon$	$2p-1-2\epsilon$	$(1-p)\gamma_A n_A$	$p(1-\gamma_A)n_A+\gamma_C n_C$	$(2p-1-2\epsilon)B_1$

Note: $B_3 = min((1-p)\gamma_A n_A + \gamma_C n_C, p(1-\gamma_A)n_A)$

Table 7: Market Maker Profitability for Falling Stock

Sell Price	Buy Price	Spread	Sellers	Buyers	Profit
$2p-1+\epsilon$	$1-2p-\epsilon$	$4p-2-2\epsilon$	$p\gamma_A n_A$	$(1-p)(1-\gamma_A)n_A$	$(4p-2-2\epsilon)B_2$
ϵ	$1-2p-\epsilon$	$2p-1-2\epsilon$	$p\gamma_A n_A+\gamma_{Ct} n_C$	$(1-p)(1-\gamma_A)n_A$	$(2p-1-2\epsilon)B_4$
$2p-1+\epsilon$	$-\epsilon$	$2p-1-2\epsilon$	$p\gamma_A n_A$	$(1-p)(1-\gamma_A)n_A+\gamma_{Ct} n_C$	$(2p-1-2\epsilon)B_2$

Note: $B_4 = min(p\gamma_A n_A + \gamma_C n_C, (1-p)(1-\gamma_A)n_A)$

Since $4p-2-2\epsilon > 2p-1-2\epsilon$, it follows that the third option in each table will never be profit-maximizing and thus can be ignored. Also, since we have $\frac{p(1-\gamma)}{(1-p)\gamma} \leq 2$, we know $B_3 \leq 2B_1$ and $B_4 \leq 2B_2$. And so the second options will never be profit-maximizing either.

Thus, the market maker will consider only the first option (Sell Price: $1-2p+\epsilon$, Buy Price: $2p-1-\epsilon$) in either case. This does not capture any of the Group C investors, so all the buyers and sellers will be Group A investors.

From Table 6, the total number of transactions (number of buyers and number of sellers) for growing stock is $b_1^* = (1-p)\gamma_{At} n_A$.

From Table 7, the number of buyers and the number of sellers for falling stock is $b_2^* = p\gamma_{At} n_A$.

Besides having n_A instead of N_A, the Group A investors' profits in each case are the same as in the original homogeneous market maker model. Thus, using the same calculations, we find that their expected profit is still ≤ 0. It can be shown from that calculation that the expected profit is also ≤ 0 for buyers and sellers separately.[5] $\square$

Now, as before, each Group B investor observes the action of his source. The source can either (1) buy or sell, or (2) do nothing. Thus, after he receives his subset of the public information at time x_{1t}, the Group B investor waits until either (1) his source buys or sells, or (2) time $2x_{1t}+1$ (when he knows his source received his signal and did nothing) to start buying or selling. If the source bought or sold, the probability of the stock growing is:

$$\begin{aligned}
\pi_U &= \frac{b_1^*}{b_1^* + b_2^*} \\
&= \frac{(1-p)\gamma_{At} n_A}{(1-p)\gamma_{At} n_A + p\gamma_{At} n_A} \\
&= 1-p
\end{aligned}$$

[5] That is necessary for this extension, since buyers and sellers know who they are. If being a buyer or seller led to positive expected profit, then the investor would only buy or sell when they're in that position. The proof is simple: it involves just adding the two terms for buyers, and the two for sellers, instead of all four at once.

Since buyers correspond directly to sellers, the same probability holds if the source sold. If the source did nothing, there are three possibilities:

(a) The source did not choose to buy or sell (market prices did not satisfy its valuation)

(b) The source tried to invest or sell, but could not (i.e. more buyers than sellers)

(c) The source did not yet make its decision to buy or sell

The sum of these is just the complement of the probability of the source making a transaction. Keeping in mind every transaction involves two investors, over these cases the probability that the stock is growing is:

$$\begin{aligned}\pi_U &= \frac{N_A - 2b_1^*}{(N_A - 2b_1^*) + (N_A - 2b_2^*)} \\ &= \frac{N_A - 2(1-p)\gamma_{At} n_A}{2N_A - 2\gamma_{At} n_A} > \frac{1}{2}\end{aligned}$$

Note: This information will only be known at time $2x_{1t} + 1$, when all the investors from the original period would have entered and the total number of market entrants from that period is received public information.

Lemma 3. All Group B investors who buy or sell after receiving source information before $2x_{1t} + 1$ *cannot earn positive expected profit, but earn greater expected profit than Group A investors in* $[0, x_{1t}]$.

Proof: n_A is not relevant in this calculation. Those n_B who see their source buy or sell will, as above, know the stock has a probability $1 - p$ of growing. We proceed in the same way as before (for growing stock):

Type 1: $p\gamma_A n_B$ who own the stock and value it at 0 (received G signal, source bought/sold)

Type 2: $p(1 - \gamma_A)n_B$ who don't own the stock and value it at 0 (received G signal, source bought/sold)

Type 3: $(1 - p)\gamma_A n_B$ who own the stock and value it at $\frac{1-2p}{p^2+(1-p)^2}$ (received L signal, source bought/sold)

Type 4: $(1 - p)(1 - \gamma_A)n_B$ who don't own the stock and value it at $\frac{1-2p}{p^2+(1-p)^2}$ (received L signal, source bought/sold)

Type 5: $\gamma_C n_C$ who own the stock and value it at 0.

Type 6: $(1 - \gamma_C)n_C$ who don't own the stock and value it at 0.

The profit-maximizing market maker must set Sell Price to be $\frac{1-2p}{p^2+(1-p)^2} + \epsilon$ and Buy Price to be $-\epsilon$. Again let's consider profit and probability for each case:

$$\pi_{BU} = \epsilon + 1$$
$$\pi_{BD} = \epsilon - 1$$
$$\pi_{SU} = \frac{1-2p}{p^2 + (1-p)^2} + \epsilon - 1$$
$$\pi_{SD} = \frac{1-2p}{p^2 + (1-p)^2} + \epsilon + 1$$

$$Q_{BU} = \frac{1}{2}(1-p)\gamma_B,\ Q_{BD} = \frac{1}{2}p\gamma_B,\ Q_{SU} = \frac{1}{2}(1-p)\gamma_B,\ Q_{SD} = \frac{1}{2}p\gamma_B$$

$$\begin{aligned}\pi_{early} &= \pi_{BU}Q_{BU} + \pi_{BD}Q_{BD} + \pi_{SU}Q_{SU} + \pi_{SD}Q_{SD} \\ &= \gamma_B(\frac{1-2p}{2p^2 + 2(1-p)^2} + \epsilon)\end{aligned}$$

These Group B investors only earn expected profit if $\epsilon > \frac{2p-1}{2p^2+2(1-p)^2}$, in which the market maker again would have negative profitability. The same result holds for buyers and sellers separately. Therefore, Group B investors who buy or sell before $2x_{1t}+1$ will earn no expected profit.

However, it can be shown that $2p^2 + 2(1-p)^2 \geq 1$ for all $\frac{1}{2} \leq p \leq 1$. This implies that:

$$\pi_{early} = \gamma_B(\frac{1-2p}{2p^2 + 2(1-p)^2} + \epsilon) \geq \gamma(1 - 2p + \epsilon) = \pi_A$$

Therefore, Group B investors who see their source buy or sell before $2x_{1t}+1$ will earn greater expected profit than Group A investors who buy or sell in $[0, x_{1t}]$. □

Lemma 4. If there was at least 1 Group A buyer and Group A seller in $[0, x_{1t}]$*, all investors who buy or sell at* $2x_{1t}+1$ *will earn higher expected profit than a Group A investor would trading in* $[0, x_{1t}]$.

Proof. With at least 1 buyer and 1 seller in the $[0, x_{1t}]$ period, we have $n_A > 0$. It is thus possible for these investors (either Group A or B) to observe their source buy or sell, in which the probability of moving up is:

$$\pi_U = 1 - p$$

And since these investors now know for certain that their Group A source did not invest in $[0, x_{1t}]$ (the entire period of decisions is now public information), they can find the probability of moving up in that case:

$$\pi_U = \frac{N_A - 2(1-p)\gamma_{At}n_A}{2N_A - 2\gamma_{At}n_A} > \frac{1}{2}$$

Combining this observation with their own signal, that investor can have four possible valuations for the stock:

Table 8: Time $2x_{1t}+1$ Investor's Valuation with Market Maker

Source	Signal	Probability of Growth	Expected Value
Buy/Sell	G	$\frac{1}{2}$	0
Buy/Sell	L	$\frac{(1-p)^2}{p^2+(1-p)^2}$	$\frac{1-2p}{p^2+(1-p)^2}$
Nothing	G	$\frac{pq}{pq+(1-p)(1-q)}$	$\frac{pq-(1-p)(1-q)}{pq+(1-p)(1-q)}$
Nothing	L	$\frac{(1-p)q}{(1-p)q+p(1-q)}$	$\frac{q-p}{pq+(1-p)(1-q)}$

Note: $q = \frac{N_A - 2(1-p)\gamma_{At} n_A}{2N_A - 2\gamma_{At} n_A}$

Using Matlab, I find that for all p, γ_{At}, N_A, n_A such that $\frac{1}{2} < p < 1 - \gamma_{At} < 1$:

$$\frac{1-2p}{p^2+(1-p)^2} < \frac{q-p}{pq+(1-p)(1-q)} < 0 < \frac{pq-(1-p)(1-q)}{pq+(1-p)(1-q)}$$

Next, we consider the case of the stock moving up and categorize those investors:

Table 9: Types of Time $2x_{1t}+1$ Investors with Market Maker

Type	Volume	Ownership	Demanded Value
1	$(1-p)\gamma_{At}[2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	Yes	$\frac{1-2p}{p^2+(1-p)^2}+\epsilon$
2	$(1-p)(1-\gamma_{At})[2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	No	$\frac{1-2p}{p^2+(1-p)^2}-\epsilon$
3	$(1-p)\gamma_{Bt}[1-2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	Yes	$\frac{q-p}{pq+(1-p)(1-q)}+\epsilon$
4	$(1-p)(1-\gamma_{Bt})[1-2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	No	$\frac{q-p}{pq+(1-p)(1-q)}-\epsilon$
5	$p\gamma_{At}[2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	Yes	ϵ
6	$p(1-\gamma_{At})[2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	No	$-\epsilon$
7	$p\gamma_{Bt}[1-2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	Yes	$\frac{pq-(1-p)(1-q)}{pq+(1-p)(1-q)}+\epsilon$
8	$p(1-\gamma_{Bt})[1-2(1-p)\gamma_{At}\frac{n_A}{N_A}]N_B$	No	$\frac{pq-(1-p)(1-q)}{pq+(1-p)(1-q)}-\epsilon$

From this, the market maker has 6 possible profit-maximizing pricing plans: (1) capture Type 1 and 4, (2) capture Type 1 and 6, (3) capture Type 1 and 8, (4) capture Type 3 and 6, (5) capture Type 3 and 8, and (6) capture Type 5 and 8.

Determining the profit-maximizing prices here follows a similar method, but is significantly more complicated than in previous cases. Thus, we need to perform an optimization in Matlab, which is described in detail in the Simulations section). Note that some of these pricing plans also capture Group C buyers and sellers, so we have account for them in the algorithm.

What the algorithm showed was that for any values of p, γ, and ϵ, the expected profits for these investors exceeded the expected profits for those in the homogeneous case (analogous to those buying and selling in $[0, x_{1t}+1]$, by Lemma 2). Again, the methodology and results can be found in Simulations.

Therefore, waiting to the end of the period at $2x_{1t}+1$ to buy or sell is always preferable to $[0, x_{1t}]$ for Group A investors. $\square$

Theorem 6. If there was at least 1 Group A buyer and 1 Group A seller in $[0, x_{1t}]$, then all Group A investors will wait until either: (1) they see their source buying or selling, or (2) time $2x_{1t}$ to start investing.

Proof. The proof follows easily. It was stated earlier that Group A investors can choose to function as a Group B investor by waiting for their source's action.

One possibility from waiting is that the investor will see their source buy or sell before $2x_{1t}+1$, which by Lemma 3 leads to a higher expected profit than investing in $[0, x_{1t}]$.

The other possibility is that they will not see their source buy or sell before $2x_{1t}+1$, which by Lemma 4 also leads to a higher expected profit than investing in $[0, x_{1t}]$.

Therefore, Group A investors have full incentive to wait until they see their source's action or the last possible time to invest. □

This result can be extended to a typical y period length for the following main conclusion:

Theorem 7. If Group A and Group B investors would prefer to buy or sell at an earlier time and minimize the waiting time ceteris paribus, then there is a strict equilibirium where:

(a) 1 owner and 1 non-owner in Group A start trading at $y - 2x_{1t} - 1$.

(b) The Group B investors and all other Group A investors start trading at $y - x_{1t}$.

Proof: To prove that this is a strict Nash Equilibrium, we just need to show that no Group A or Group B investor can benefit or be as well off by unilaterally deviating from the equilibrium.

If one of the Group A investors at time $y - 2x_{1t} - 1$ starts trading earlier, he will have to wait for his counterpart (owner/non-owner) to enter the same time in the market anyways. Thus, he will be doing the same trade and the same time, but will be waiting longer and thus be worse off. Note that until the Group B investors come, the market maker's his pricing scheme is the one in Lemma 2.

And if one of those Group A investors starts trading at a time in $[y - 2x_{1t}, y - x_{1t} - 1]$, the same trade will occur between the two, but at a later time – which makes the investors worse off by our condition. The intuition behind this earlier-time preference is that when investors accomplish the trade they want earlier in the day, they have more time to perform transactions that might be dependent on that trade being completed.

Lastly, if one of those two investors starts trading at a time in $[y - x_{1t}, y]$, him no longer trading at $y - 2x_{1t}$. means that there are no transactions for the investors at $y - x_{1t}$ or later to observe. Thus, they can only observe one thing – no transactions – outside their signal, which provides no additional information (The mathematical explanation is trivial). Thus, for those investors at $y - 2x_{1t}$ and after, the market maker's pricing reverts to what it would be with just insiders (since with no source information, every investor is relying on their own signal only – like an insider in $[0, x_{1t}]$). So he would earning the same expected profit, but on a later trade – and thus is worse off.

Now consider all the Group A and Group B investors trading at $y - x_{1t}$. Analogous to all the investors at $2x_{1t} + 1$ in the original example, they are earning expected positive profit due to herd behavior based on the trading of the two Group A investors at $y - 2x_{1t} - 1$. Keep in mind that after x_{1t} Group A and Group B are equivalent. By trading at earlier time (before $y-x_{1t}$), one of these investors could never receive the information from the first two (at $y-2x_{1t}-1$). Thus, with just the one signal, they would be subject to the pricing for an insider and thus earn no expected profit – and are thus worse off.

Conversely, if an investor starts trading after $y-x_{1t}$, they have the same information and will be trading in the same market – and thus will earn the same profit. Except they will be worse off because their trade is later. Note that the actions of the investors $y-x_{1t}$ becomes public information at $y+1$, after the market closes for the day. Thus, there is no way to take advantage of that by moving your trading back. □

It is important to note that the Group C investors were not a consideration in this equilibrium. Earlier in this section, we had already taken them into account in showing the profitability of trading in each period. And as was established in the baseline model, Group C investors only have knowledge of past stock movement – no public information, signals, or anything about the other traders. Thus, they are ignorant to the strategies of the other investors and their effects, and thus remain completely indifferent between when to invest – so are still discretely uniform in $[1, y]$.

Corollary 2. Strong-form efficiency holds in this equilibrium.

Proof: This can be shown in Simulations, where the expected profit of the investors at time $y-2x_{1t}-1$ is negative, as is the expected profit of the investors at time $y-x_{1t}$. Also, the expected profit of Group C investors is 0, since they never end up in any transactions (Further Intuition explains why this doesn't mean that they're better off).

And so, it is impossible for any investor to earn positive expected returns. Therefore, the market in this equilibrium is strong-form efficient. □

VI. **Simulations**

In this section we will run simulations of (1) the original market maker model with homogeneous investors, and (2) the equilibrium of the market maker model with herd behavior. For (1), as we already know positive expected returns are impossible, we want to find the specific distribution of returns over time. The simulation for (2) serves two purposes: we want to prove in the general case that Group A investors earn higher profit investing in $2x_{1t}+1$ (for Lemma 4), and we want to show that strong-form efficiency holds in our Equilibrium defined in Theorem 7.

We will also simulate an extended model where (3) instead of a binary gain or loss, stock movements and signal probabilities follow a normal distribution. Results from such a model align more closely from actual financial markets, allowing for continuous values and the inclusion of a volatility parameter, but are impossible to obtain analytically.

Standard Market Maker Model

In the original market maker model, all traders function as Group A investors with no herd behavior. Since the information is the same at different times of the day, there is no need to specify time in this simulation.

In the Matlab code, a loop is run to update two $N \times 1$ vectors (where N is the total number of investors) representing: (1) each investor's ownership of the stock (1 for owner, 0 for non-owner), and (2) his total profit earned so far. The values of σ, ϵ, and p are defined at the start, and can adjusted accordingly.

At each iteration, the stock's movement (increase or decrease) is randomly generated alongside a random number between 0 and 1 for each investor. Based on the movement and the value of p, this translates to a signal, which, alongside his ownership status, determines his valuation of the stock. From Theorem 5, the market maker will set a sell price of $1 - 2p + \epsilon$ and a buy price of $2p - 1 - \epsilon$ if $\epsilon < 2p - 1$, and will set out-of-range prices otherwise.

A count is then made of the buyers (valuation is higher) and sellers (valuation is lower) captured by this price, and a minimum is taken to find the total number of transactions T. Then, a sample without replacement of T of the captured buyers' indices are taken to determine which ones end up buying. Same is done for the sellers. Then, depending on the price and the actual movement, those investors will earn a positive or negative amount that is reflected in the total profit vector. Similarly, the investors who buy or sell will have their ownership status changed.

This can be run through any number of iterations, representing periods of trading. Within that, we can analyze the mean and standard deviation of daily returns for different p and γ. We will set ϵ at 0.05 for each case.

Table 10: Investor Profits Through 100 Periods

p	γ	ϵ	Average Return	Standard Deviation
0.55	0.2	0.05	-0.010097	0.0227
0.6	0.2	0.05	-0.029691	0.0252
0.7	0.2	0.05	-0.068530	0.0257
0.55	0.1	0.05	-0.005023	0.0165
0.6	0.1	0.05	-0.014976	0.0168
0.7	0.1	0.05	-0.035308	0.0196

Table 11: Investor Profits Through 10,000 Periods

p	γ	ϵ	Average Return	Standard Deviation
0.55	0.2	0.05	-0.010	0.0023
0.6	0.2	0.05	-0.0301	0.0022
0.7	0.2	0.05	-0.0695	0.0025
0.55	0.1	0.05	-0.0050	0.0016
0.6	0.1	0.05	-0.0150	0.0017
0.7	0.1	0.05	-0.0350	0.0019

A quick check will show that the simulated average profits converge to the relation from Theorem 5: $\mu = \gamma(1 - 2p + \epsilon)$ as the number of periods increases. It follows that the standard deviation of average returns σ also decreases as more and more periods are included in the simulation.

The distribution of the returns can also be shown in a histogram:

Figure 1: Investor Returns over 100 Periods ($p = 0.6, \gamma = 0.2, \epsilon = 0.05$)

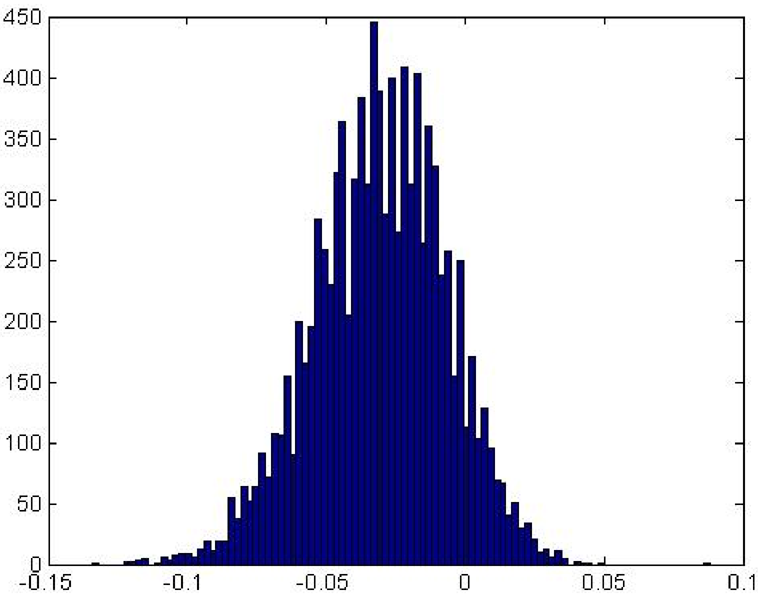
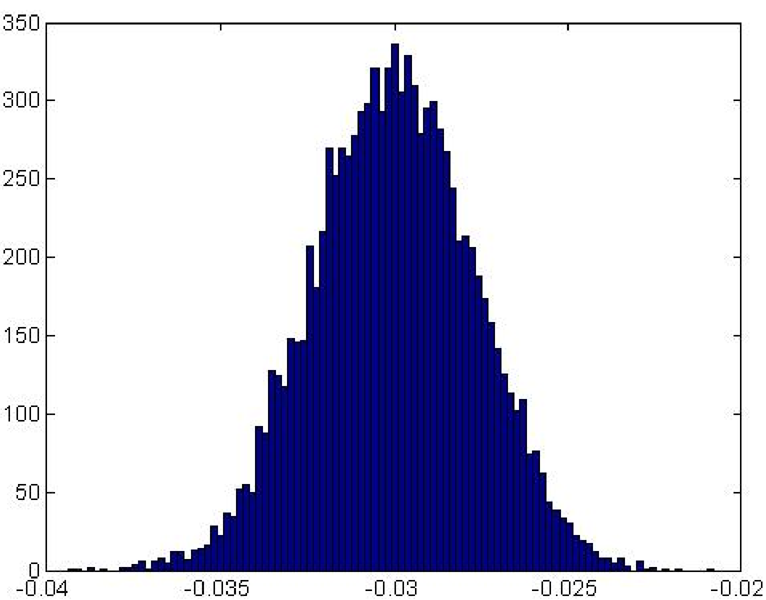


Figure 2: Investor Returns over 10,000 Periods ($p = 0.6, \gamma = 0.2, \epsilon = 0.05$)



This distribution of returns closely resembles a normal distribution. In can be seen that for a short time frame (100 periods), it is possible to obtain positive returns. However, in the long term (10,000), the variance of the daily returns becomes much smaller and positive returns become impossible. This further confirms our conclusion from Theorem 5 that an economy with homogeneous investors and a profit-maximizing market maker is strong-form efficient.

Equilibrium for Herd Behavior Model

In this simulation, we compute the returns for Group A and Group B investors under the equilibrium described in Theorem 7, which describes a market maker model with herd behavior where an investor can start trying to buy or sell at any time after he receives his information.

As with before, we update the profit and ownership vectors through a loop. However, here we have to update for two different sets of investors – the two investors buying and selling at $y - 2x_{1t} - 1$, the $N - 2$ buying and selling at $y - x_{1t}$ (where N is the total number of Group A and Group B investors), and the Group C investors who still invest at uniformly distributed times from 1 to y.

At each iteration, we first calculate the profits for the first two investors. As showed the Lemma 2, the market maker sets sell price $1 - 2p + \epsilon$ and buy price $2p - 1 - \epsilon$. The rest is the same as the original algorithm. It was also shown in that Lemma that Group C investors are never captured, and so their profits are no updated.

For the later investors, their valuation also must account for the action observed in their source. Thus, before the loop, we had to define a vector of random values representing who each investor is following. Based on this observation (which depends on the earlier investors' decisions) as well as their own signal, we can determine the valuation for each of these investors.

Next, we run a function, described in Lemma 4, to find based on the values of p, γ, ϵ, and n_A which pricing plan is optimal for the market maker by maximizing the product of spread and volume. Note that we also had to account for Group C investors in this optimization, though it turned out they were never captured by the optimal plan.

Based on this buy and sell price, we run the same procedure to find which investors end up trading, and then update each profit accordingly. We can observe the average profits across 10,000 iterations:

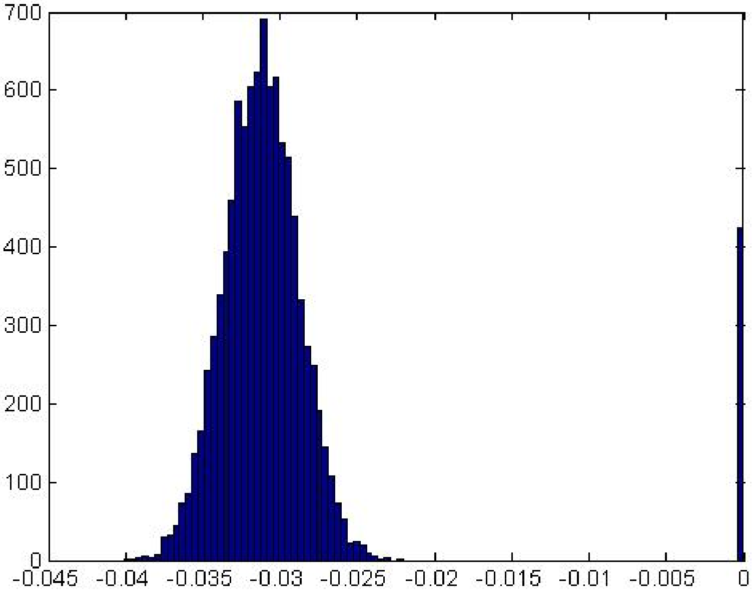
Table 12: Investor Profits Through 10,000 Periods

Time Invested	p	γ	ϵ	Average Return	Standard Deviation
$y - 2x_{1t} - 1$	0.6	0.2	0.05	-0.0298	0.0061
$y - x_{1t}$	0.6	0.2	0.05	-0.0354	0.0002824

It can be seen that the average profit of the later group (that invests at $y - x_{1t}$) is higher than the average profit of the earlier group, which supports our proof this being a Nash equilibrium. Also, it can seen that under the same p, γ, and ϵ, the average profit from the later group is slightly higher than it would be under homogeneous investors ($-0.0298 > -0.0301$). As the difference is even greater for larger general values of n_A, this leads directly to the conclusion of Lemma 4.

From the results, it can also be seen that Group C investors are never captured and thus never trade.

Figure 3: Investor Returns over 10,000 Periods ($p = 0.6, \gamma = 0.2, \epsilon = 0.05$)



Outside of those investors whose source is one of the first two investors (whose expected profit is 0), the average profits are normally distributed around a similar mean as in the homogeneous case. We are of course assuming that sources are chosen randomly and cannot be changed; the idea of changing sources will be explored further in the next section.

Extension: Normally Distributed Returns and Signal Accuracy

So far, our models have been developed under the extreme simplification that the stock's daily movements are binary: it can either go up or go down by a constant amount. In real-life markets, of course, a stock's daily changes follow a continuous distribution – whose variation by the volatility measure σ. As our goal is to bring our conclusions closer to real-life application, we will adjust the model for normally-distributed returns.

We have the current stock price of 0; and it has daily movements $S \sim N(0, 1)$. Each investor receives a (numerical) signal X with conditional distribution $X|S = s \sim N(s, 1)$; thus, the signals with highest probability will be those closest to the actual movement. Again, let us consider this in terms of original market maker model with homogeneous investors.

Lemma 5. An investor receiving signal x will value the stock at $\frac{1}{2}x$.

Proof. Knowing that the pdf of the normal distribution is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{\frac{-(x-\mu)^2}{2\sigma^2}\}$, we can find the distribution of $f(s|x)$ using conditional probability:

$$\begin{aligned} f(s|x) &\propto f(x|s)f(s) \\ &\propto \exp\{-\frac{(x-s)^2}{2}\}\exp\{-\frac{x^2}{2}\} \\ &\propto \exp\{\frac{-2x^2+2xy}{2}\} \\ f(s|x) &\propto \exp\{-\frac{2(x-\frac{1}{2}y)^2}{2}\} \end{aligned}$$

For this distribution, which is proportional to a normal distribution, we see that its mean is $\frac{1}{2}x$. And so $E[S|X=x] = \frac{1}{2}x$. □

Note that the above result holds in general for any standard deviation σ so long as it's the same for both S and $X|S$.

The simulation is set up the same way as before. At each iteration, the standard normal random variable is generated for stock movement; and taking that as the mean, each investor's signal is also computed. Based on the Lemma above (valuation is half the signal), and accounting for ϵ, their valuation of the stock can be determined.

Before that, however, we need to determine the prices that the market maker will set. Since s or the individual x each period is not public information, the market maker must rely on its knowledge of their distributions. Thus, what he can do is simulate 1000 different S values as well as every investor's valuation in each case. Then, using a constrained maximization, he can find the buy and sell price that would maximize his profit. Taking the average for all 1000 cases, he obtains a good approximation for the optimal buy and sell price.

With the optimizing price set, we proceed the same way as before to determine which of the investors end up buying and selling. Their total profits and ownership are also updated in the same way at each iteration.

Table 13: Investor Profits Through 100 Periods of Trading

σ	γ	ϵ	Sell Price	Buy Price	Average Return	Standard Deviation
0.5	0.2	0.05	-0.3280	0.4461	-0.0049	0.0059
1	0.2	0.05	-0.5825	0.7252	-0.0171	0.0151
2	0.2	0.05	-1.0436	1.3103	-0.0503	0.0360
0.5	0.1	0.05	-0.2583	0.4744	-0.0022	0.0041
1	0.1	0.05	-0.5433	0.7903	-0.0112	0.0138
2	0.1	0.05	-1.0231	1.5107	-0.0257	0.0306

Table 14: Investor Profits Through 10,000 Periods of Trading

σ	γ	ϵ	Sell Price	Buy Price	Average Return	Standard Deviation
0.5	0.2	0.05	-0.3280	0.4461	-0.0045	0.0005742
1	0.2	0.05	-0.5825	0.7252	-0.0185	0.0016
2	0.2	0.05	-1.0436	1.3103	-0.0410	0.0034
0.5	0.1	0.05	-0.2583	0.4744	-0.0040	0.00057999
1	0.1	0.05	-0.5433	0.7903	-0.0108	0.0013
2	0.1	0.05	-1.0231	1.5107	-0.0310	0.0030

As expected, higher variance in the signals leads to lower average profit. The relationship of the number of periods and the proportion of ownership γ seems to have a similar relationship to expected profits (and their standard deviation) as in the original model.

The spread of the market maker's prices also increases as σ increases, as greater variation means more buyers and sellers captured by extreme prices.

Figure 4: Investor Returns over 100 Periods ($\sigma = 1, \gamma = 0.2, \epsilon = 0.05$)

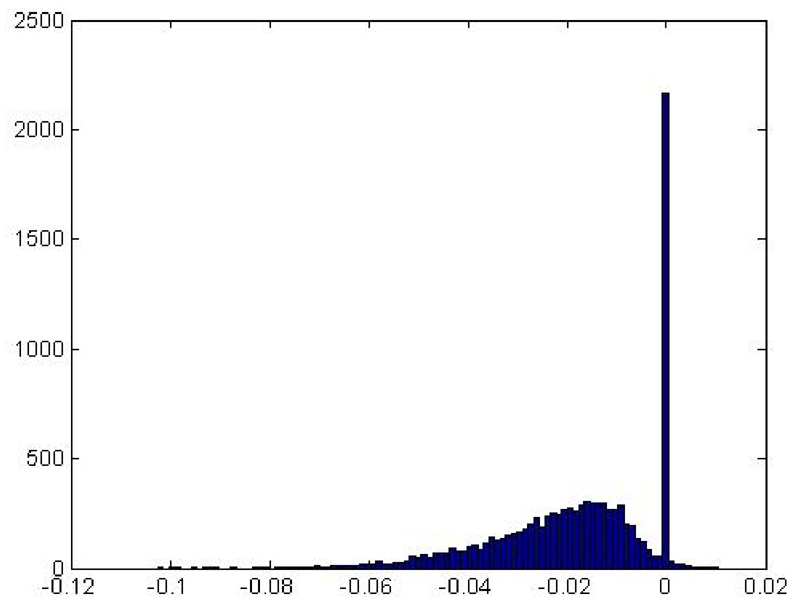
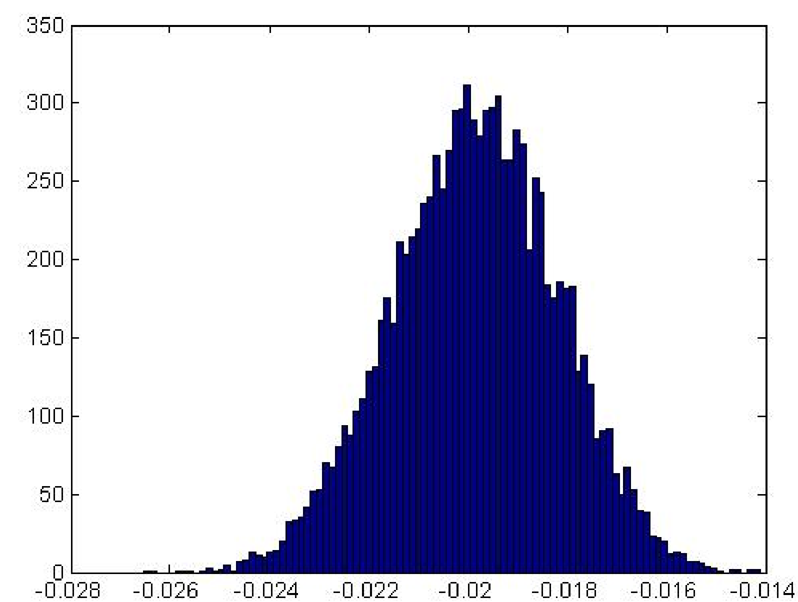


Figure 5: Investor Returns over 10,000 Periods ($\sigma = 1, \gamma = 0.2, \epsilon = 0.05$)



Over a large number of iterations, the distribution of returns closely approximates a normal distribution. However, when there are fewer iterations, there seems to be significant number of investors who never trade at all. Comparing this to Figure 1 suggests that much fewer transactions occur in this model than the original binary model.

The negative long-run returns again proves that strong-form efficiency holds with a profit-maximizing market maker in this normally-distributed case. The similarity of these results to those of the standard model suggests that there are still many potential avenues to make the results in this paper more applicable to real-world financial markets.

VII. **Sources**

Changing Sources

When investors engage in herd behavior in real life, several factors go into choosing who they want to follow – not the least of which is the insider's past performance. Insiders come with varying levels of quality, and if an investor feels like his is not very good he may decide to find a new one.

Now consider an extension to the baseline odel where out of the insiders in Group A, there are intelligent insiders I and unintelligent insiders U. Intelligent insiders receive signals that are correct $p + \alpha$ of the time, and unintelligent insiders receive signals that are correct $p - \alpha$ of the time, where $0 < \alpha < p - 0.5$. For simplicity, we will assume that p is the same each period. We will also assume that the market is only weak-form efficient.

Let's also assume that $\frac{1}{2}$ of the insiders are I and $\frac{1}{2}$ are U, so that the unconditional probability of being correct is $\frac{1}{2}(p+\alpha) + \frac{1}{2}(p-\alpha) = \frac{1}{2}$. Suppose the Group B and Group C investors receive their signals the same as before.

As you may recall, each Group B investor has a designated source in Group A. Suppose the probability of selecting each source is independent of whether the insider is I or U.

So, as before, the Group B investors will account for his source's decision in making his own decision. However, after observing the result in the first period and knowing what decision his source made, the investor then has the option of changing his source at a search cost s before committing to that source. Assume the pool of Group A investors N_A is large enough that a selected source from the rest of the investors has an equal probability of being I or U, so $\frac{N_U - 1}{N_A} \approx \frac{1}{2}$

Theorem 8. An investor will not change his source if his source was correct.

Proof: It is trivial. Use Bayes' rule to find the probability that the source is intelligent given that it was correct in the first period, where θ is the type of the insider.

$$\begin{aligned} P(\theta = I|C) &= \frac{P(C|\theta = I)P(\theta = I)}{P(C)} \\ &= \frac{(p+\alpha)\frac{1}{2}}{\frac{1}{2}(p+\alpha) + \frac{1}{2}(p-\alpha)} > \frac{1}{2} \end{aligned}$$

Thus, given this information, the source is more likely to be I than U, and so the investor will stay with his source. $\square$

Theorem 9. An investor with discount rate r will change his source iff his source was wrong and $s < \frac{\alpha^2}{r(4-4p)}$.

Proof: The Group B investor might only switch sources if his source was wrong. In that case, he would know the probability his source is U is:

$$\begin{aligned} P(\theta = U|W) &= \frac{P(W|\theta = U)P(\theta = U)}{P(W)} \\ &= \frac{(1-p+\alpha)\frac{1}{2}}{\frac{1}{2}(1-p-\alpha) + \frac{1}{2}(1-p+\alpha)} \\ &= \frac{1}{2} + \frac{\alpha}{2-2p} \end{aligned}$$

Since the probability of the source being U if he chose another one is $\frac{1}{2}$, the Group B investor would be reducing the probability of a U source by $\frac{\alpha}{2-2p}$.

Consider the investor's expected profit each period if his source was actually U.

$$\begin{aligned} \pi_U &= (p-\alpha)(p)(1)\frac{1}{2} + (1-p+\alpha)(1-p)(-1)\frac{1}{2} \\ &= p - \frac{\alpha}{2} - \frac{1}{2} \end{aligned}$$

Using the same calculation, we find the investor's expected profit with an I source is $p + \frac{\alpha}{2} - \frac{1}{2}$. So his expected profit each period, with no knowledge of the success of his source, is:

$$\pi = \frac{1}{2}(p - \frac{\alpha}{2} - \frac{1}{2}) + \frac{1}{2}(p + \frac{\alpha}{2} - \frac{1}{2}) = p - \frac{1}{2}$$

So his additional expected loss each period from a U source is $p - \frac{1}{2} - (p - \frac{\alpha}{2} - \frac{1}{2}) = \frac{\alpha}{2}$.

Thus, in switching to reduce the probability of a U source, he would be reducing his expected loss each future period by:

$$\frac{\alpha}{2-2p} \times \frac{\alpha}{2} = \frac{\alpha^2}{4-4p}$$

Now suppose the investor discounts the future at rate r. From the perpetuity formula, the reduction in expected loss would therefore be:

$$\hat{L} = \frac{\alpha^2}{r(4-4p)}$$

Therefore, the search cost s would have to be smaller than that amount $\hat{L}$ for the Investor B to change sources. $\square$

An interesting thing to note is that if we take the derivative of $\hat{L}$ with respect to p and with respect to r we get:

$$\hat{L}_p = \frac{\alpha^2}{r(2-2p)^2} > 0, \hat{L}_r = \frac{-\alpha^2}{r^2(4-4p)} < 0$$

So an investor is willing to pay a higher search cost s if the overall probability p that an insider is correct is higher, and if his discount rate r is lower.

Multiple Sources

Our existing model assumes that each Group B investor has exactly one insider source on which to base his behavior, when in reality he may refer to the investment decisions of multiple insiders. In this extension, we will allow Group B investors to designate up to an infinite number n of Group A investors as sources. However, each source chosen incurs a cost c, which is analogous to a search cost in that it represents resources foregone by processing the action that source. For simplicity, we will assume the original model with no different types of insiders or switching of sources.

As with before, the observed insider action corresponds directly to the signal that insider received; investing means a G signal, and not investing means a L signal. With n sources, plus his own signal, the Group B investor effectively receives $n+1$ signals. He will choose to invest if more than half of these signals are G, so when there are i G signals, where:

$$i > \frac{n+1}{2} = \frac{n}{2} + \frac{1}{2}$$

If n is odd, $\frac{n}{2} + \frac{1}{2}$ is an integer and the smallest i is $\frac{n}{2} + \frac{1}{2} + 1 = \lceil \frac{n}{2} \rceil + 1$.

If n is even, $\frac{n}{2} + \frac{1}{2}$ is a fraction, $\frac{n}{2}$ is an integer, and the smallest i is $\frac{n}{2} + \frac{1}{2} + \frac{1}{2} = \lceil \frac{n}{2} \rceil + 1$.

Thus, the Group B investor will invest for $\lceil \frac{n}{2} \rceil + 1 \leq i \leq n+1$. The corresponding number of L signals is $n+1-i$. Thus, from the binomial distribution, the probability of receiving i G signals when the stock is moving up, and down, is:

$$\pi_U = \binom{n+1}{i} p^i (1-p)^{n+1-i}$$

$$\pi_D = \binom{n+1}{i} (1-p)^i p^{n+1-i}$$

From everything above, we can write an equation for total expected profit:

$$\pi = -cn + \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n+1} \binom{n+1}{i} \frac{1}{2} (p^i (1-p)^{n+1-i} - (1-p)^i p^{n+1-i})$$

Defining this function in Matlab, we can find the optimal number of sources n^* for different combinations of p and c:

Table 15: Optimal Number of Sources for Different p and c

	p=0.525	p=0.55	p=0.575	p=0.6	p=0.65	p=0.7	p=0.8
c=0.00125	54	96	84	66	42	28	14
c=0.0025	14	40	46	42	30	20	10
c=0.00375	6	22	30	30	24	18	10
c=0.005	2	12	20	22	20	14	8
c=0.0075	0	6	10	14	14	12	6
c=0.01	0	2	6	10	10	10	6
c=0.02	0	0	0	2	4	4	4

From this table, a few simple conclusions can be drawn. As the optimal n^* is such that the marginal benefit (increased expected profit) from the next source becomes less than the marginal cost, it is expected that a smaller c means more insider sources.

Holding c constant, n^* peaks at a certain value of p depending on the value of c – higher cost means optimal number of sources is greatest for high p, while lower cost means it is greatest for low p. Also, note that all optimal n^* are even numbers.

Figure 6: Total Benefit of Finding n Sources ($p = 0.6, c = 0.005$)

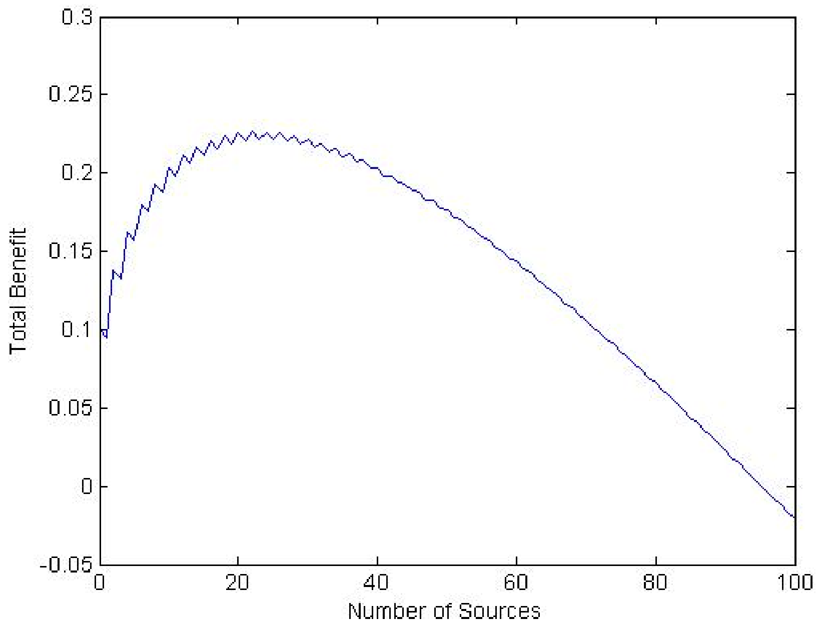
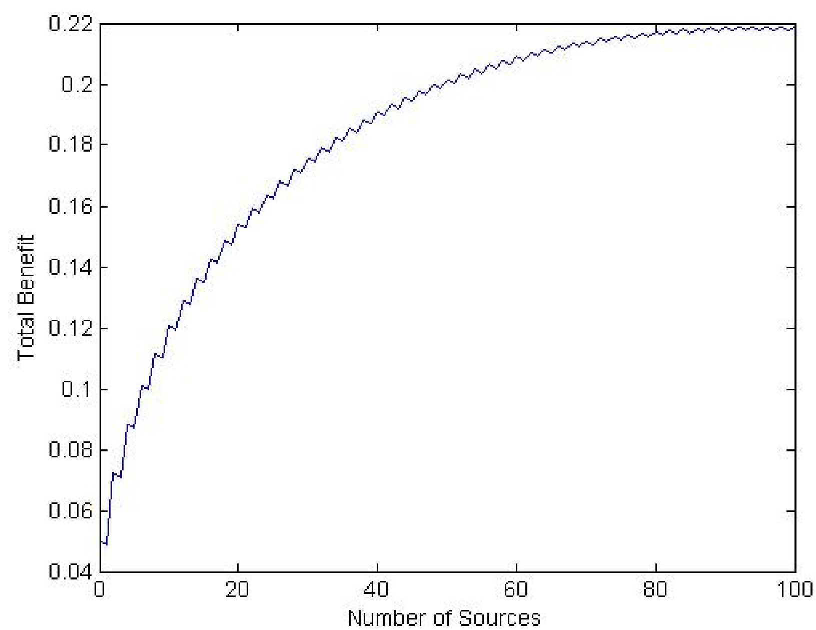


Figure 7: Total Benefit of Finding n Sources ($p = 0.55, c = 0.00125$)



These results can be applied in the real-world markets by insider traders who may wish to sell their predictions for a given cost c that will maximize their revenue. This notion can be extended to other types of services – premium company reports, advanced analytical software, and real-time news wires (i.e. Bloomberg) – that are based on insider research but can be sold to outside entities. On the traders' side, this may serve as a guideline for balancing accuracy with opportunity cost.

VIII. **Further Intuition**

Risk-Free Rates

In our market maker model and its several extensions, we showed that investors almost always end up with negative profits. An interesting question that arises from these results is: why would these investors trade at all if their returns were all negative? The simple answer is: in real terms, they are not.

Relating this to actual markets, all of the profits in these models are defined relative to a baseline risk-free rate, which the value of 0 represents (we omitted any numerical factor for simplicity). When the efficient market hypotheses says that it is impossible to earn above-average returns, it is saying that it is impossible to earn returns above the risk-free rate (that you earn by investing in T-bills, for example). This is why, by showing that average returns can never exceed our defined 0 value, we are effectively proving that market efficiency holds.

Thus, with 0 as the risk-free baseline, the nominal return of 0 that you get by not investing at all is defined in real terms as a negative number (i.e. loss from inflation). With that consideration, the simulation can be further extended by giving investors who do invest during a period a negative profit for that period. Under our market maker model, the return by definition must be less than the returns we have calculated so far in order for these investors to participate in the stock market in the first place.

By weighting non-trading as a significant negative number rather than 0, the truth about Group C investors becomes much more clear; they are losing more by not buying and selling than the other investors are by constantly buying and selling.

Risk Aversion and the Utility Function

An important consideration in behavioral finance is utility and risk-aversion – how much does an investor dislike uncertainty and how much is he willing to pay to get rid of or reduce it. Throughout our models, we assumed for simplicity that each investor's utility function is linear, while in truth they can vary from person to person.

Imposing, for example, a concave utility function could have a significant effect on profitability, as each investor's valuation is based entirely on he weights his potential gains relative to his potential losses. It would also play a role in the extended market maker model in terms of when is the optimal time to invest. For example, in Lemma 4, Group A investors are more willing to wait until $2x_{1t} + 1$ because of the small probability of a much higher return. But being risk-averse, they might rather take the marginally higher return on most days by staying in $[0, x_{1t}]$.

That said, incorporating this factor into our model seems like the next logical step to bring its results closer to applicability in actual financial markets.

The Nature of Signals

Throughout this paper I have been referring to "Gain" and "Loss" signals without really specifying what they involve.

As I mentioned several times, public signals are those obtained from "the investor's subset of the public information". What this means is that out of all the information out there, an investor can only process a limited amount – as much as he deems relevant. His signal is his single verdict (gain or loss) on the stock based on a holistic evaluation of all the public information he received. Of course, in our model we assumed that the investors are all equally skilled as their signals all have the same probability of being right.

Earlier, I had also mentioned that Group A investors have the same private signal and public signal. What this means is that technically they don't do fundamental analysis. Rather, they do a form on analysis that relies on insider sources to help them reach that verdict much sooner than their Group B counterparts. For them, everything relevant in valuing the stock (outside of the source's behavior) has been done when they received their private signal; so incorporating the public information when it comes out adds nothing.

IX. **Conclusion**

The purpose of this paper was to explore the nature of market efficiency in an environment of information timing heterogeneity, relating the variables for signal correctness in a herd behavior model to the conventional definitions of a weak, semi-strong, and strong-form efficient market. Most notably, I have shown that the presence of herd behavior forces several additional requirements on the market's knowledge in order to maintain semi-strong efficiency.

Also important is the result that a profit-maximizing market maker is necessary for market efficiency, which is supportive of the greater notion that common self-interest leads to a more effective economy. Finally, looking at the investor's problem of optimizing profit through investment timing in this type of model leads us to an efficiency-preserving equilibrium where traders with information cluster around the later times of the day.

Ultimately, the goal of any paper like this is the applicability to modern financial markets, and for that the use of empirical data and more complex models of pricing and investor behavior will be required.

X. **Appendix – Table of Variables**

Variable	Explanation
p	Proportion of investors in a group receiving the correct signal
x_{1t}, x_{2t}	Times at which Group A, B investors start or stop investing
y	Number of seconds in each period
n_1, n_2	Maximum time for full signal penetration in minutes for Group A, Group B
δ	Slowness of information diffusion
a_{it}, b_{it}, c_{it}	Time each Group A, Group B, Group C investor makes decision
S_t	Stock price in period t
t	Period (currently $t = 0$)
X_t	Stock price movement from period t to $t + 1$
M	Market price of the stock after adjustment in a semi-strong efficient market
N_A, N_B, N_C	Number of investors in Group A, Group B, Group C
m	Minute of the period
V_m	Sales volume in the period up to and including minute m
π	Expected profit
$\gamma_A, \gamma_B, \gamma_C$	Proportion of Group A, Group B, Group C investors owning the stock
q_j, q_k	Owner's, buyer's valuation of stock
ϵ	Buyer and seller's minimum desired premium
n_A, n_C	Number of Group A, Group C investors buying/selling before $x_{1t} + 1$
n_B	Number of Group B investors whose source buys/sells before $x_{1t} + 1$
α	How much above (below) average is intelligent (unintelligent) insider
S	Daily stock movement in normal distribution extension
X	Investor's signal in normal distribution extension
θ	Insider's type (intelligent or unintelligent)
s	Search cost for finding new insider source
r	Discount rate

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