

**Enhanced versus Traditional Indexation for International Mutual Funds:
Evaluating DFA, WisdomTree and RAFI PowerShares**

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Abstract

This paper uses style analysis to compare the performance of traditional international index funds and enhanced international index funds. It attempts to measure the value added beyond classic indexation by the consideration of fundamentals. By employing Sharpe's style analysis, I formulate a synthetic portfolio composed of DFA traditional funds to imitate each enhanced index fund portfolio's performance. Then I compare the return and volatility of each portfolio. The result shows that two enhanced fund portfolios tested in this paper outperform their traditional synthetic portfolio, while four underperform.

JEL classification: G11, G15

Keywords: Enhanced index fund, Fundamental indexation, Style analysis

I. Introduction

The basic element of most economic debates stems from the premise of market efficiency. Whether to believe in market rationality or not is the key component that divides the entire economic academia. The debate over active and passive investment is along the same line. The point of contention comes from whether smart enough investors can defeat the market by buying selective baskets of stocks.

With an alleged tendency to outperform the market and more sophisticated technologies, the appearance of the fundamental index fund marks another interesting identity, the hybrid of active management and passive management. Fundamental indexation is passive in that it comprises rules in conjunction with market indices. It incorporates company fundamentals instead of or in addition to market capitalization. Fundamentals, measures of firm size, used for indexation include dividends, earnings, cash flow, sales, book-equity values, and so on.

Jeremy Siegel said in the interview with the New York Times in 2006, traditional index funds overweight overvalued stocks while they underweight undervalued stocks, causing investors to buy fashionable assets at high price (Anderson, 2006). Also, during the periods of boom and bust, the big swings of those securities can harm investors. The Enhanced index fund, in this sense, was invented not only to outperform its traditional counterpart but to protect investors from bigger volatilities.

The opponents of index funds, however, claim that the excess return generated from fundamental indexation is minor and insignificant that the additional costs, including transactional costs, turnover costs, and tax inefficiencies, would cancel out the advantage of this alternative index fund, if there is any.

Tower & Yang (2013) looks at DFA, RAFI, and WisdomTree US enhanced index funds in comparison to Vanguard portfolios. This paper extends their work. It seeks to compare the performance of traditional index funds and fundamental index funds of

international stocks. Instead of using Vanguard classic index funds to benchmark, the paper employs DFA traditional index funds to do its job. Using a modified “style analysis” of Sharpe (1992), I attempt to measure the value added beyond classic indexation by the consideration of fundamentals. Thus, this paper aims to see if a particular enhanced indexation type can be said to defeat classic indexation for international stocks, and if so, what can be concluded in terms of returns and risks.

Section II contains the literature review on traditional and enhanced indexation and the debates about their relationship. Section III discusses the methodology employed in this paper, section IV describes the data source and data set that are used throughout the study. Section V presents the results, which is followed by conclusion in section VI. Appendix includes further analyses on individual enhanced funds.

II. Literature Review

The comparison between active management and passive management has been prolific since the former's introduction, but not many have produced conclusive opinion in one type's favor. Reinker and Tower (2004) compare U.S. and international index and managed funds. Their paper creates Vanguard synthetic portfolios based on the assets of Vanguard's mutual funds to explore which investment strategy works best for the average Vanguard investor. Running the model on different time spans, they demonstrate conflicted results. While they could not conclude the absolute predominance of one fund type, managed funds looked as though they provided some protection against stock bubbles, especially around the end of the 1990s.

Despite the short history of enhanced indexation, there is a good amount of analyses that attempts to evaluate its performance over its traditional counterpart. Bernstein (2006) briefly introduces the motivation behind the invention of fundamental indexation scheme and analyzes the effect of factor exposure in the fundamental indexation's excess return, which was shown through running the data from 1962 to 2004. Although the annualized return of the composite RAFI portfolio, suggested by Arnott, Hsu, and Moore (2006), outperformed S&P 500 by 197 basis points from 1962 to 2004, the style-adjusted performance of the RAFI index only underperformed by 0.14%. Thus, he concludes that small difference in returns after adjusting Fama-French factors indicates that the excess return of fundamental indexing is not much explained by the technique of fundamental indexation itself. Rather, it depends on expenses, fees, transactional costs as well as application of fundamentalist indexing scheme. Thus, as he states, "the prospective shareholder needs to consider not only the selection paradigm used, but just who is executing it."

In Bogle (1998) and Bogle (2002), John Bogle compares actively managed mutual funds and index funds, respectively high-cost and low-cost funds, by adjusting them into nine "style boxes." The funds in the low-cost quartile outperform the funds in the high-cost quartile, and this relationship was kept when he took account of risk-

adjusted returns by employing the Sharpe ratio. The takeaway from both studies is that costs matter and, more importantly, that the index funds which are low-cost funds prevail over active managers which induce high costs.

In addition, Bogle and Malkiel ridicule “beating the market” as a loser’s game (Bogle & Malkiel, 2006). According to them, beating the market should be a zero-sum game, for all the stocks in the market are to be held by someone. The zero-sum game, however, fails to earn the market-level return after the additional investment costs of enhanced index fund are deducted. Similar to other relevant studies, Bogle and Malkiel conclude that there could be circumstances when fundamental indexing tends to do better than its counterpart – one of which is when small-cap stocks and value stocks tend to outpace. But this does not tell anything about absolute superiority of enhanced indexation.

Tower (2009) thoroughly discusses classic and enhanced index funds, describes the investment strategies of three fund families, Vanguard, DFA, and WisdomTree, and compares their performances using the “style analysis” developed by Sharpe (1992). Essentially, Tower (2009) lays out basic background and strong foundation for this paper’s development. He utilizes continuously compounded daily returns of each fund manager and applies Sharpe’s style analysis to compare first DFA and Vanguard funds only to find that there is not much confidence in ranking them in any order. For the comparison between WisdomTree and Vanguard, as well, there is no statistically significant conclusion about the relative performance between two fund families. The most clear-cut conclusion from his study is best reflected from his sentence “all one can say is over a particular period one firm beat the other for a particular simulation and for a particular asset class.” Considering higher costs of enhanced index funds, this conclusion can be useful to make investors rethink their strategies and to encourage fund families keep their costs low.

In continuation, Tower and Yang (2013) compare Vanguard, a traditional index fund manager, with leading enhanced index fund families, DFA, RAFI, and WisdomTree. The analysis is composed of two Fama-French models as well as Sharpe’s style analysis

(Sharpe, 1992). With Fama-French factor models, Tower and Yang constrain factor loads of the Vanguard portfolio to equal those of the enhanced portfolio. By applying style analysis (1992), they create Vanguard mutual fund portfolios that mimic the style of each enhanced index fund. The analysis focuses on US funds only. Instead of supporting one type or the other, their study produces the conflicted result: “two cheers for enhanced indexation and one for traditional.”

Sharpe (1992) introduces a very useful analysis methodology in comparing fund performance. By providing a simple regression structure for an asset's return, the asset class factor model starts from the premise that the return can be separated into two main components: style and selection. To determine the investor's exposures to each key asset class, he finds the set, or the style, that minimizes the variance of the difference between the return on the fund and the synthetic portfolio. The synthetic portfolio is a combination of asset classes and reflects the manager's style and effective asset mix.

III. Methodology

In order to compare the two different types of funds, traditional index funds and enhanced index funds, this paper constructs a synthetic portfolio and runs the “style analysis” introduced by Sharpe (1992). More precisely, I use a modified version of style analysis as Tower (2009) more comprehensively explains, replacing indices with index funds. With 11 DFA international traditional index funds, this paper creates the synthetic portfolio that can best simulate the each enhanced funds’ movement.

How do I create synthetic portfolio?

This paper uses Microsoft Excel Solver to find the set of weights that allow the portfolio to best mimic each enhanced fund or enhanced fund portfolio. Solver is instructed to find the weighted sum of each traditional fund’s return to produce the portfolio’s return. By allowing the weights, which must sum up to 1, to move around, I can find the particular set of weights that minimizes the standard deviation of return differentials between the synthetic portfolio and the compared object. In short, it finds the set of coefficients that minimizes the mean square error of prediction (aside from a constant term), which means to minimize the standard deviation of the return differentials between the portfolio and enhanced fund.

For example, if the coefficients for Mutual Fund A are 30% of Stock I, 20% of Stock II, and 50% of Stock III, Mutual Fund A moved as if it had a 30% allocation in Stock I fund, 20% in Stock II fund, and 50% in Stock III fund. The difference between the actual movement of Mutual Fund A and the portfolio of 3 stocks with given allocations can be explained by the manager’s active selection of securities, market timing, and statistical error (Lucas and Riepe, 1996).

Why do I use a synthetic portfolio and style analysis?

The goal of this paper is to explore whether a mutual fund outperforms a basket of index funds with the same style as an international investment scheme. By maintaining the same style, in other words the same exposure to asset factors, I can describe the return of any enhanced index portfolio as the return of a bundle of traditional index funds plus a constant term and a random term. As the random term is an exogenous factor that is not correlated among portfolios, it does not carry any meaning in comparison. The constant term, however, indicates the relative advantage, or disadvantage, of enhanced indexation over classical indexation.

In short, the paper compares enhanced index fund portfolios with the best tracking DFA traditional funds. Then, by looking at alpha, which is the annualized geometric average excess return, I can determine how much of value has been added by more active management through enhanced indexation.

Why do I use continuous compounding and geometric average return?

In order to measure the average rate of return over multiple time periods, this paper employs continuously compounded geometric average return. While the average of arithmetic average returns does not measure the average return over a span of life, the average of geometric average returns is the average multi-period return.

Why do I use equally-weighted portfolio and risk-averse portfolio for comparison?

To compare the performance of traditional indexation and enhanced indexation, this paper generates two types of portfolio composed of enhanced index funds from each fund family. This approach is more realistic and applicable than looking at individual funds, for typical investors keep a variety of funds, which are composed of multiple equities. Thus, instead of looking at each fund separately, I create an equally-weighted

portfolio and a risk-averse portfolio to compare the returns with a traditional synthetic portfolio.

While the equally-weighted portfolio is not necessarily optimal, it is by far the simplest way to look at the overall performance of the assets in a fund family. The choice of how to construct a fund family enhanced index portfolio is not a problem, for the DFA traditional synthetic portfolio can always be created corresponding to any specific fund family enhanced index portfolio to mimic its style.

The Risk-averse portfolio, however, is more useful when I take account of the investment atmosphere after the financial crisis. As the whole finance world has undergone a huge loss, there have been growing tendencies toward safer decisions. Therefore, in order to reflect the social preference on less risk, I create a risk-averse portfolio of each fund family. The weights of each fundamental index fund were given in a way that minimizes the standard deviation of the return of a portfolio, using Microsoft Excel Solver again.

What does it mean to allow short-selling?

The Sharpe style analysis constrains the weights to be non-negative. In other words, it does not allow holding short positions on any of fund to create a clone portfolio. However, in some cases, the investors need to hold some traditional funds short to imitate enhanced funds. For example, if the enhanced fund is more focused on small or value companies than any of the DFA classic funds, the investor can short large cap companies in order to better mimic the enhanced index fund (Tower & Yang, 2012). Holding a short position on some funds essentially means that an investor sells some of its holdings to diversify the portfolio and purchase the enhanced fund. Therefore, a negative coefficient signals the case where the enhanced fund is leveraged. It is common to expect the smaller absolute value of alpha when the short portfolio is granted, for the model has more freedom to approximate the style. While traditional index funds cannot be sold short, ETFs can be sold short, and some ETF's mimic traditional index funds.

IV. Data

As an extended study from Tower and Yang's paper (2012), which evaluates the performance of enhanced indexation compared to classic indexation for US equities, this paper looks at international funds under each indexation. The study is done through looking at DFA, RAFI, and WisdomTree funds. They are leading enhanced indexers with relatively broad portfolio and long history. The ideal pick of fund choices is to make them mutually exclusive while exhaustive (Sharpe, 1992). The funds with redundant components were eliminated to conform these criteria. The data on returns was collected from the Center for Research in Security Prices (CRSP) and some missing data was filled with Yahoo Finance and Morningstar. In this paper, DFA has 50 monthly returns, from November 2008 to December 2012, WisdomTree has 58 observations, from March 2008 to December 2012, and RAFI has 63 monthly returns, ranging from October 2007 to December 2012.

i. DFA

This paper uses both traditional index funds and enhanced index funds of the Dimensional Fund Advisor (DFA). DFA uses a combination of traditional and fundamental indexation. Throughout the study, the DFA traditional index funds constitute the synthetic portfolio to compare with enhanced funds.

The overall investment strategy of DFA on international investment can be found at DFA homepage Non-US Equity section¹. More details of each fund, either traditional index fund or enhanced index fund, are described in DFA's most recent prospectus².

¹ <http://www.dfaus.com/strategies/non-us-equities.html>

² http://www.dfaus.com/pdf/prospectuses/idg_equity_i_pros.pdf

DFA traditional index funds

DFA traditional index funds are selected from DFA funds. 11 traditional funds were selected after excluding the similar funds. They are listed alphabetically in tickers as follows:

Emerging Market Small Cap Portfolio I (DEMSX)

Large Cap International Portfolio I (DFALX)

Emerging Markets Portfolio I (DFEMX)

Emerging Markets Value Portfolio I (DFEVX)

International Small Company Portfolio I (DFISX)

International Value Portfolio I (DFIVX)

Japanese Small Company Portfolio I (DFJSX)

Asia Pacific Small Company Portfolio I (DFRSX)

United Kingdom Small Co Portfolio I (DFUKX)

International Small Cap Value Portfolio I (DISVX)

DFA fundamental index funds

DFA enhanced index funds are called DFA core and DFA vector funds. They are determined by fundamentals and market capitalization. Among fundamentals, growth and value are usually assessed by additional factors, such as price-to-cash flow or price-to-earnings ratios (DFA, 2013).

6 DFA enhanced index funds were selected:

CSTG&E International Social Core Equity Portfolio (DFCCX)

Emerging Markets Core Equity Portfolio I (DFCEX)

Emerging Markets Social Core Equity Portfolio (DFESX)

International Core Equity Portfolio I (DFIEX)

International Sustainability Core Portfolio I (DFSPX)

International Vector Equity Portfolio I (DFVQX)

Although August 14th, 2008 marks the latest inception of DFA enhanced index funds, the data was obtainable only from November 2008. As the paper utilizes monthly returns data, which is calculated by the end of each month, we have 50 observations: from November 2008 to December 2012.

ii. **WisdomTree**

As described on its website, WisdomTree considers fundamentals like dividends and earnings to better measure and reflect a company's value and profitability (WisdomTree, 2013). So the funds have focus either on dividends or earnings, which the company believes to generate higher returns and less risk.

14 enhanced index funds were selected from WisdomTree. They are followed by CRSP Identifier in each parenthesis:

Asia Pacific ex-Japan (032506)
Australia Dividend (032505)
DEFA (032494)
Emerging Markets Equity Income (035773)
Emerging Markets SmallCap Dividend (036484)
Europe SmallCap Dividend (032510)
Global ex-US Growth (032508)
India Earnings (036970)
International Dividend ex-Financials (032502)
International LargeCap Dvidend (032503)
International MidCap Dividend (032501)
International SmallCap Dividend (032500)
Japan Hedged Equity (032509)
Japan Small Cap Dividend (032507)

The evaluation of WisdomTree funds starts from February 8th, 2008, from which all the WisdomTree enhanced funds were active. As the paper utilizes monthly returns data, which is calculated by the end of each month, we have 58 observations: from March 2008 to December 2012.

iii. RAFI

RAFI funds are the PowerShares FTSE RAFI portfolios. These portfolios incorporate four fundamental factors – dividends, cash flow, sales, and book equity value – to determine each fund’s weights.

There were four RAFI enhanced index funds that were tested in this paper:

FTSE RAFI Developed Market ex-US Small-Mid Portfolio (036221)

FTSE RAFI Emerging Market Portfolio (036223)

FTSE RAFI Asia Pacific ex-Japan Portfolio (043623)

FTSE RAFI Developed Markets ex-US Portfolio (043624)

The evaluation of WisdomTree funds starts from October 31st, 2007, from which all the RAFI enhanced funds were active. As the paper utilizes monthly returns data, which is calculated by the end of each month, we have 63 observations: from October 2007 to December 2012.

V. Results

i. Style analysis on DFA performance

Table 1. Sharpe style analysis on DFA enhanced index portfolios from September 1, 2008 to December 31, 2012

| Fund name | | DFA Equally-weighted portfolio | DFA Risk-averse portfolio |
|--|---------------------------------|--------------------------------------|---------------------------------|
| Initial date | | 8/14/08 | 8/14/08 |
| α (%/year) | | -0.162 | -0.039 |
| Significance of α , 1 tail t-test (%) | | 27.7 | 45.5 |
| Observations (months) | | 50 | 50 |
| Correlation | | 0.999 | 0.999 |
| Excess standard deviation of fund (%/month) | | 0.002 | -0.005 |
| Standard deviation of prediction error | | 0.002 | 0.002 |
| DFA traditional index fund | | Coefficients | |
| DEMSX | Emerging Markets Small Cap I | 0.063 | 0.005 |
| DFALX | Large Cap International I | 0.152 | 0.548 |
| DFCSX | Continental Small Company I | 0.002 | 0.044 |
| DFEMX | Emerging Markets I | 0.160 | 0.000 |
| DFEVX | Emerging Markets Value I | 0.112 | 0.000 |
| DFISX | International Small Company I | 0.114 | 0.052 |
| DFIVX | International Value I | 0.277 | 0.207 |
| DFJSX | Japanese Small Company I | 0.012 | 0.027 |
| DFRSX | Asia Pacific Small Company I | 0.019 | 0.003 |
| DFUKX | United Kingdom Small Co I | 0.023 | 0.020 |
| DISVX | International Small Cap Value I | 0.066 | 0.095 |

Interpreting the values

As this paper extends the idea from Tower and Yang (2013), the result tables contain the similar components and interpretations. Table 1 presents the results of my simulation on DFA equally-weighted and risk-averse portfolio.

- 1) α is the annual average continuously compounded return differential between the synthetic portfolio and enhanced portfolio over the entire period.

- 2) **Significance of α** denotes the probability that α , the return differential, delivers the opposite sign relationship between the two portfolios. Thus, in this case, significance of 28% for α at -0.16 means that there is 28% probability in which the return of the DFA equally-weighted enhanced index portfolio outperforms the DFA traditional index fund bundle in the future repeated trials. In short, the number tells you the chance that the universe from those returns were drawn has DFA under-returning the DFA enhanced portfolio.

Thus, one-tailed test tells us if the average return from one portfolio is greater than that of the other. The statistical significance test is done through Paired 2-Sample T-Test from Microsoft Excel Data Analysis Package, using continuously compounded monthly returns.

- 3) **Observations** denotes the number of months since the introduction of the newest enhanced index fund in each fund family used in the simulation.
- 4) **Correlation** indicates the correlation of two continuously compounded monthly returns.
- 5) **Excess standard deviation of fund** is the excess volatility of the enhanced portfolio compared to that of the traditional portfolio.
- 6) **Standard deviation of prediction error** measures the standard deviation of the return differentials. During modeling, I constrained this value to be minimized through Solver.
- 7) **Coefficients** denote the weights given to each DFA traditional index fund to make the portfolio that mimics the compared enhanced fund, or portfolio. In every column, they add up to 1.

Observations

- 1) Both DFA portfolios, equally-weighted and risk-averse, underperformed the DFA traditional index funds basket, respectively by 0.162% and 0.039%.
- 2) The excess standard deviation of both funds compared to DFA traditional index bundle was very close to zero, so I could only see negligible difference in volatilities, or in risks.
- 3) The correlations of both DFA portfolios were close to 1, showing that the DFA traditional index funds successfully mimicked the styles of two compared portfolios, giving me better confidence in the results.
- 4) There are respectively 27.7% and 45.5% of probabilities that the returns from two portfolios can show different inequality relationship. This is the chance that DFA enhanced index portfolios can outperform traditional index portfolios if the study is repeated in the future. These high numbers can again be attributed to limited sample size. Also, it can simply mean that we cannot draw any significant conclusion about whether fundamental indexation would add value onto classic indexation.

No significant conclusion could be drawn from the comparison between DFA traditional index funds and enhanced index funds. With the average significance of alpha on a one tailed t-test at 36.6%, the conclusion that DFA enhanced fund portfolios underperform the classic index funds is not compelling enough. In addition, there was almost no reduction or increase in the risk, for the excess standard deviation of both simulations was trivial.

ii. **Style analysis on WisdomTree performance**

Table 2 presents the results for WisdomTree equally-weighted and risk-averse portfolio returns.

Table 2. Sharpe style analysis on WisdomTree enhanced index portfolios from March 1, 2008 to December 31, 2012

| Fund name | | WT Equally-weighted portfolio | WT Risk-averse portfolio |
|--|---------------------------------|--|---|
| Initial date | | 2/22/08 | 2/22/08 |
| α (%/year) | | -1.351 | -0.103 |
| Significance of α , 1 tail t-test (%) | | 18.3 | 48.1 |
| Observations (months) | | 58 | 58 |
| Correlation | | 0.991 | 0.964 |
| Excess standard deviation of fund (%/month) | | -0.176 | -0.471 |
| Standard deviation of prediction error | | 0.009 | 0.013 |
| DFA traditional index fund | | Coefficients | |
| DEMSX | Emerging Markets Small Cap I | 0.087 | 0.000 |
| DFALX | Large Cap International I | 0.282 | 0.147 |
| DFCSX | Continental Small Company I | 0.082 | 0.000 |
| DFEMX | Emerging Markets I | 0.223 | 0.000 |
| DFEVX | Emerging Markets Value I | 0.000 | 0.000 |
| DFISX | International Small Company I | 0.000 | 0.000 |
| DFIVX | International Value I | 0.000 | 0.000 |
| DFJSX | Japanese Small Company I | 0.174 | 0.767 |
| DFRSX | Asia Pacific Small Company I | 0.000 | 0.000 |
| DFUKX | United Kingdom Small Co I | 0.152 | 0.086 |
| DISVX | International Small Cap Value I | 0.000 | 0.000 |

Observations

- 1) Both WT portfolios, equally-weighted and risk-averse, underperformed the DFA traditional index funds basket, respectively by 1.351% and 0.103%.
- 2) Both WT enhanced index portfolios are less volatile than DFA synthetic portfolio, for they show negative excess standard deviation among each continuously

compounded return. This means smaller risk by incorporating fundamental consideration.

- 3) With large enough correlation and small enough prediction error standard deviation, the DFA portfolio successfully imitated WT portfolios.
- 4) There are respectively 18.3% and 48.1% of probabilities that the universe that the returns were drawn from actually has better returns from WisdomTree enhanced indexation. These high numbers can be partially explained by small sample numbers due to relatively short history of enhanced indexation. The more data we can collect in the future, the more conclusive will be the result.
- 5) Risk-averse portfolio seems to beat equally-weighted portfolio in that it underperforms classic index portfolio by smaller gap than the other does - compare -1.351% vs. -0.103. Besides, it lowers the risk approximately as 2.5 times much as equally-weighted one does. The result is not surprising though because standard-deviation minimization is the way the risk-averse portfolio was constructed in the beginning.

In conclusion, the Sharpe style analysis tells us that DFA classic indexation might be better than WisdomTree fundamental indexation, in either portfolio. However, with negligible α at -0.103% and low statistical significance, the WT risk-averse portfolio does not show much difference from DFA traditional index basket in terms of profitability. Equally-weighted portfolio's excess return is also not significantly different from zero at the 10% level.

iii. Style analysis on RAFI performance

Table 3 shows the style analysis simulation of RAFI portfolios.

Table 3. Sharpe style analysis on RAFI enhanced index portfolios from November 1, 2007 to December 31, 2012

| Fund name | | RAFI Equally-weighted portfolio | RAFI Risk-averse portfolio |
|--|---------------------------------|--|-----------------------------------|
| Initial date | | Oct/07 | Oct/07 |
| α (%/year) | | 4.387 | 0.869 |
| Significance of α , 1 tail t-test (%) | | 1.4 | 28.4 |
| Observations (months) | | 63 | 63 |
| Correlation | | 0.985 | 0.990 |
| Excess standard deviation of fund (%/month) | | 0.016 | -0.017 |
| Standard deviation of prediction error | | 0.013 | 0.010 |
| DFA traditional index fund | | Coefficients | |
| DEMSX | Emerging Markets Small Cap I | 0.000 | 0.178 |
| DFALX | Large Cap International I | 0.513 | 0.000 |
| DFCSX | Continental Small Company I | 0.237 | 0.052 |
| DFEMX | Emerging Markets I | 0.000 | 0.000 |
| DFEVX | Emerging Markets Value I | 0.023 | 0.000 |
| DFISX | International Small Company I | 0.016 | 0.003 |
| DFIVX | International Value I | 0.011 | 0.047 |
| DFJSX | Japanese Small Company I | 0.000 | 0.159 |
| DFRSX | Asia Pacific Small Company I | 0.013 | 0.000 |
| DFUKX | United Kingdom Small Co I | 0.154 | 0.131 |
| DISVX | International Small Cap Value I | 0.034 | 0.430 |

Observations

- 1) Both RAFI portfolios outperformed the DFA traditional index portfolio, respectively by 4.387% and 0.869%. While the outperformance of the risk-averse portfolio looks minimal, the extent by which equally-weighted RAFI funds outperformed DFA traditional index funds is relatively higher than other cases. The annualized return differential at 4.387% means that if you invest \$1,000 a year, you will make \$43.87 less by investing in traditional index funds.

- 2) The equally-weighted portfolio was more volatile with 0.016 percentage points in excess standard deviation of continuously compounded monthly returns. The risk-averse portfolio, however, showed smaller volatility: higher return by higher risks.

- 3) There are respectively 1.4% and 28.4% of probabilities that the returns that I can have underperforming RAFI portfolios from future repeated trials. 0.014 of significance means that the positive alpha of RAFI average portfolio is statistically significant at standard levels, so I can confidently conclude that the portfolio with equal weights on RAFI enhanced index funds outperform DFA traditional funds.

Throughout all these simulations, DFA and WisdomTree enhanced index funds had lower or similar returns compared to portfolios of DFA traditional index funds while RAFI outperformed DFA traditional index funds. Table 4 summarizes all three simulations.

Table 4. Sharpe style analysis on three family funds with no funds held short. DFA and WT enhanced funds underperform DFA traditional funds while RAFI portfolios beat them.

| Fund Family | DFA Equally- weighted portfolio | DFA Risk- averse portfolio | WT Equally- weighted portfolio | WT Risk- averse portfolio | RAFI Equally- weighted portfolio | RAFI Risk- averse portfolio |
|---|--|-------------------------------------|---|------------------------------------|---|--------------------------------------|
| Initial date | 8/14/08 | 8/14/08 | 2/22/08 | 2/22/08 | Oct/07 | Oct/07 |
| α (%/year) | -0.162 | -0.039 | -1.351 | -0.103 | 4.387 | 0.869 |
| Significance of α , 1 tail t-test (%) | 27.7 | 45.5 | 18.3 | 48.1 | 1.4 | 28.4 |
| Observations (months) | 50 | 50 | 58 | 58 | 63 | 63 |
| Correlation | 0.999 | 0.999 | 0.991 | 0.964 | 0.985 | 0.990 |
| Excess standard deviation of fund (%/month) | 0.002 | -0.005 | -0.176 | -0.471 | 0.016 | -0.017 |
| Standard deviation of prediction error | 0.002 | 0.002 | 0.009 | 0.013 | 0.013 | 0.010 |
| DFA traditional index funds & Coefficients | | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.063 | 0.005 | 0.087 | 0.000 | 0.000 | 0.178 |

| | | | | | | |
|---|-------|-------|-------|-------|-------|-------|
| Large Cap International I (DFALX) | 0.152 | 0.548 | 0.282 | 0.147 | 0.513 | 0.000 |
| Continental Small Company I (DFCSX) | 0.002 | 0.044 | 0.082 | 0.000 | 0.237 | 0.052 |
| Emerging Markets I (DFEMX) | 0.160 | 0.000 | 0.223 | 0.000 | 0.000 | 0.000 |
| Emerging Markets Value I (DFEVX) | 0.112 | 0.000 | 0.000 | 0.000 | 0.023 | 0.000 |
| International Small Company I (DFISX) | 0.114 | 0.052 | 0.000 | 0.000 | 0.016 | 0.003 |
| International Value I (DFIVX) | 0.277 | 0.207 | 0.000 | 0.000 | 0.011 | 0.047 |
| Japanese Small Company I (DFJSX) | 0.012 | 0.027 | 0.174 | 0.767 | 0.000 | 0.159 |
| Asia Pacific Small Company I (DFRSX) | 0.019 | 0.003 | 0.000 | 0.000 | 0.013 | 0.000 |
| United Kingdom Small Co I (DFUKX) | 0.023 | 0.020 | 0.152 | 0.086 | 0.154 | 0.131 |
| International Small Cap Value I (DISVX) | 0.066 | 0.095 | 0.000 | 0.000 | 0.034 | 0.430 |

iv. DFA, WisdomTree, RAFI (shorts allowed)

The same simulation was run after short-selling is allowed. Table 5 represents the results for three fund families with short positions allowed.

Table 5. Sharpe style analysis on three family funds with short positions allowed. DFA and WT equally weighted portfolio underperform DFA traditional funds while WT risk-averse portfolio and RAFI portfolios outperform DFA traditional synthetic portfolios.

| Fund Family | DFA Equally-weighted portfolio | DFA Risk-averse portfolio | WT Equally-weighted portfolio | WT Risk-averse portfolio | RAFI Equally-weighted portfolio | RAFI Risk-averse portfolio |
|--|--------------------------------|---------------------------|-------------------------------|--------------------------|---------------------------------|----------------------------|
| Initial date | 8/14/08 | 8/14/08 | 2/22/08 | 2/22/08 | Oct/07 | Oct/07 |
| α (%/year) | -0.162 | -0.184 | -2.152 | 0.418 | 4.281 | 0.770 |
| Significance of α , 1 tail t-test (%) | 27.7 | 29.0 | 4.6 | 39.3 | 1.2 | 30.4 |
| Observations (months) | 50 | 50 | 58 | 58 | 63 | 63 |
| Correlation | 0.999 | 0.999 | 0.993 | 0.977 | 0.987 | 0.990 |
| Excess standard deviation of fund (%/month) | 0.002 | -0.007 | 0.029 | 0.145 | -0.040 | -0.043 |
| Standard deviation of prediction error | 0.002 | 0.002 | 0.008 | 0.010 | 0.012 | 0.010 |

| DFA traditional index funds & Coefficients | | | | | | |
|---|--------------|---------------|---------------|---------------|---------------|---------------|
| Emerging Markets Small Cap I (DEMSX) | 0.063 | 0.041 | 0.372 | 0.272 | -0.099 | 0.305 |
| Large Cap International I (DFALX) | 0.152 | 0.541 | 0.459 | 1.057 | 0.607 | -0.077 |
| Continental Small Company I (DFCSX) | 0.002 | 0.036 | 0.155 | -0.200 | 0.231 | 0.057 |
| Emerging Markets I (DFEMX) | 0.160 | 0.001 | 0.441 | 0.131 | 0.063 | -0.008 |
| Emerging Markets Value I (DFEVX) | 0.112 | -0.046 | -0.385 | -0.198 | 0.036 | -0.137 |
| International Small Company I (DFISX) | 0.114 | 0.061 | -0.381 | -0.089 | 0.021 | 0.003 |
| International Value I (DFIVX) | 0.277 | 0.220 | -0.187 | -0.670 | 0.010 | 0.129 |
| Japanese Small Company I (DFJSX) | 0.012 | 0.020 | 0.144 | 0.508 | -0.076 | 0.161 |
| Asia Pacific Small Company I (DFRSX) | 0.019 | 0.006 | -0.051 | -0.308 | 0.014 | 0.005 |
| United Kingdom Small Co I (DFUKX) | 0.023 | 0.012 | 0.111 | -0.042 | 0.159 | 0.118 |
| International Small Cap Value I (DISVX) | 0.066 | 0.108 | 0.321 | 0.539 | 0.034 | 0.445 |
| Sum of shorts | 0.000 | -0.046 | -1.004 | -1.507 | -0.175 | -0.222 |

By allowing short positions, the results had smaller prediction error, as presented by Table 6. Besides, α decreased, which means worse performance from enhanced index portfolio's side, for four portfolios – DFA risk-averse portfolio, WT equally-weighted portfolio, and two RAFI portfolios. And three out of four aforementioned cases, the confidence level increased.

In addition, note that DFA equally-weighted portfolio does not contain any shorts position even if it is allowed to do so – sum of shorts is zero, here. One possible explanation for this result is that under the same manager family, DFA traditional index funds may cover as diverse assets as DFA average enhanced fund looks at, so there is no need to sell the holdings and purchase the enhanced fund. This is consistent with the fact that the sum of shorts for DFA risk-averse portfolio is very low, compared to that of others.

Table 6. Style analysis of DFA, WisdomTree, and RAFI with no shorts and shorts allowed.

| Fund Family | DFA Equally- weighted portfolio | DFA Risk- averse portfolio | WT Equally- weighted portfolio | WT Risk- averse portfolio | RAFI Equally- weighted portfolio | RAFI Risk- averse portfolio |
|---|--|-------------------------------------|---|------------------------------------|---|--------------------------------------|
| Initial date | 8/14/08 | 8/14/08 | 2/22/08 | 2/22/08 | Oct/07 | Oct/07 |
| α (%/year) | -0.162 | -0.039 | -1.351 | -0.103 | 4.387 | 0.869 |
| α (%/year), shorts | -0.162 | -0.184 | -2.152 | 0.418 | 4.281 | 0.770 |
| Significance of α , 1 tail t-test (%) | 27.7 | 45.5 | 18.3 | 48.1 | 1.4 | 28.4 |
| Significance of α , 1 tail t-test, shorts (%) | 27.7 | 29.0 | 4.6 | 39.3 | 1.2 | 30.4 |
| Observations (months) | 50 | 50 | 58 | 58 | 63 | 63 |
| Observations (months), shorts | 50 | 50 | 58 | 58 | 63 | 63 |
| Correlation | 0.999 | 0.999 | 0.991 | 0.964 | 0.985 | 0.990 |
| Correlation, shorts | 0.999 | 0.999 | 0.993 | 0.977 | 0.987 | 0.990 |
| Excess standard deviation of fund (%/month) | 0.002 | -0.005 | -0.176 | -0.471 | 0.016 | -0.017 |
| Excess standard deviation of fund (%/month), shorts | 0.002 | -0.007 | 0.029 | 0.145 | -0.040 | -0.043 |
| Standard deviation of prediction error | 0.002 | 0.002 | 0.009 | 0.013 | 0.013 | 0.010 |
| Standard deviation of prediction error, shorts | 0.002 | 0.002 | 0.008 | 0.010 | 0.012 | 0.010 |
| Sum of shorts | 0.000 | -0.046 | -1.004 | -1.507 | -0.175 | -0.222 |

Appendix contains the results from style analysis simulation on each individual enhanced fund.

VI. CONCLUSION

Table 7 succinctly summarizes the test results from this paper.

Table 7. Comparison of α of three fund families – DFA, WisdomTree, and RAFI. Sharpe method was used and short positions were prohibited.

| Fund Family | DFA Equally- weighted portfolio | DFA Risk- averse portfolio | WT Equally- weighted portfolio | WT Risk- averse portfolio | RAFI Equally- weighted portfolio | RAFI Risk- averse portfolio |
|---|--|-------------------------------------|---|------------------------------------|---|--------------------------------------|
| Initial date | 8/14/08 | 8/14/08 | 2/22/08 | 2/22/08 | Oct/07 | Oct/07 |
| α (%/year) | -0.162 | -0.039 | -1.351 | -0.103 | 4.387 | 0.869 |
| Significance of α , 1 tail t-test (%) | 27.7 | 45.5 | 18.3 | 48.1 | 1.4 | 28.4 |
| Excess standard deviation of fund (%/month) | 0.002 | -0.005 | -0.176 | -0.471 | 0.016 | -0.017 |

Sharpe style analysis shows the DFA average enhanced fund, DFA risk-averse enhanced fund, and WisdomTree average enhanced fund underperforming the DFA traditional fund bundle. The other three portfolios, WisdomTree risk-averse, RAFI equally-weighted, and RAFI risk-averse one, out-returned the DFA traditional index portfolio. The results were, however, statistically significant only for WisdomTree and RAFI equally-weighted portfolio, both at 95% confidence level.

In terms of risk, RAFI outplayed two other enhancers and traditional indexation. Thus, with larger excess returns and lower volatilities, RAFI is a definite winner from this study. This rash conclusion, however, should be tested further with longer time span and rigorous regression analysis in the future. To revisit fundamental factors that three fund families consider: DFA puts in book-to-market ratio as a measure of growth and value; WisdomTree focuses on either dividends or earnings; RAFI incorporates dividends, cash flow, sales, and book equity value. Whether these are the driving factors of different returns and different risks, this paper does not tell anything substantial. The paper, however with mixed results, can be summarized by the same quotation that Tower and Yang concluded their study: “The prospective shareholder needs to consider not only the selection paradigm used, but just who is executing it.”

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Appendix

Table 8. Sharpe style analysis on 6 DFA enhanced index funds, no shorts

| CRSP identifier | DFCCX | DFCEX | DFESX | DFIEX | DFSPX | DFVQX | Average |
|---|---|--------------------------------|-------------------------------------|-----------------------------|-------------------------------------|------------------------------------|---------|
| Initial date | 8/3/07 | 4/5/05 | 8/31/06 | 9/15/05 | 3/12/08 | 8/14/08 | |
| Fund name | CSTG&E International Social Core Equity | Emerging Markets Core Equity I | Emerging Markets Social Core Equity | International Core Equity I | International Sustainability Core I | International Vector Equity Inst I | |
| α (%/year) | 0.20 | -0.27 | -0.07 | -0.32 | -0.19 | -0.35 | -0.17 |
| Significance of α , 1 tail t-test (%) | 33.41% | 24.58% | 43.09% | 9.97% | 31.15% | 23.70% | 27.65% |
| Observations (months) | 62 | 92 | 75 | 87 | 53 | 50 | 70 |
| Correlation | 0.9992 | 0.9992 | 0.9995 | 0.9996 | 0.9995 | 0.9992 | 0.9993 |
| Excess standard deviation of fund (%/month) | 0.09 | -0.03 | 0.04 | 0.01 | 0.01 | 0.04 | 0.03 |
| Standard deviation of prediction error | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DFA traditional index funds & Coefficients | | | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.000 | 0.212 | 0.190 | 0.000 | 0.001 | 0.000 | 0.067 |
| Large Cap International I (DFALX) | 0.322 | 0.000 | 0.000 | 0.382 | 0.526 | 0.000 | 0.205 |
| Continental Small Company I (DFCSX) | 0.112 | 0.000 | 0.000 | 0.004 | 0.039 | 0.000 | 0.026 |
| Emerging Markets I (DFEMX) | 0.000 | 0.472 | 0.415 | 0.004 | 0.000 | 0.000 | 0.148 |
| Emerging Markets Value I (DFEVX) | 0.000 | 0.303 | 0.381 | 0.000 | 0.000 | 0.017 | 0.117 |
| International Small Company I (DFISX) | 0.000 | 0.000 | 0.000 | 0.054 | 0.116 | 0.177 | 0.058 |
| International Value I (DFIVX) | 0.331 | 0.011 | 0.011 | 0.289 | 0.208 | 0.459 | 0.218 |
| Japanese Small Company I (DFJSX) | 0.071 | 0.000 | 0.000 | 0.014 | 0.014 | 0.016 | 0.019 |
| Asia Pacific Small Company I (DFRSX) | 0.000 | 0.001 | 0.001 | 0.029 | 0.000 | 0.047 | 0.013 |
| International Small Cap Value I (DISVX) | 0.120 | 0.000 | 0.000 | 0.188 | 0.078 | 0.245 | 0.105 |
| United Kingdom Small Co I (DFUKX) | 0.044 | 0.002 | 0.002 | 0.036 | 0.017 | 0.040 | 0.024 |

Table 9. Sharpe style analysis on 6 DFA enhanced index funds, shorts allowed

| CRSP identifier | DFCCX | DFCEX | DFESX | DFIEX | DFSPX | DFVQX | Average |
|---|---|--------------------------------|-------------------------------------|-----------------------------|-------------------------------------|------------------------------------|----------------|
| Initial date | 8/3/07 | 4/5/05 | 8/31/06 | 9/15/05 | 3/12/08 | 8/14/08 | |
| Fund name | CSTG&E International Social Core Equity | Emerging Markets Core Equity I | Emerging Markets Social Core Equity | International Core Equity I | International Sustainability Core I | International Vector Equity Inst I | |
| α (%/year) | 0.113 | -0.318 | -0.200 | -0.313 | -0.396 | -0.289 | -0.234 |
| Significance of α , 1 tail t-test (%) | 43.48% | 19.97% | 29.78% | 10.58% | 14.15% | 26.10% | 24.01% |
| Observations (months) | 62 | 92 | 75 | 87 | 53 | 50 | 70 |
| Correlation | 0.998 | 0.999 | 0.999 | 1.000 | 1.000 | 0.999 | 0.999 |
| Excess standard deviation of fund (%/month) | 0.067 | -0.036 | 0.006 | 0.014 | 0.011 | 0.021 | 0.014 |
| Standard deviation of prediction error | 0.004 | 0.003 | 0.003 | 0.002 | 0.002 | 0.003 | 0.003 |
| DFA traditional index funds & Coefficients | | | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.116 | 0.218 | 0.207 | -0.005 | 0.065 | -0.071 | 0.088 |
| Large Cap International I (DFALX) | 0.632 | -0.024 | -0.079 | 0.378 | 0.511 | -0.086 | 0.222 |
| Continental Small Company I (DFCSX) | 0.196 | -0.047 | -0.023 | 0.004 | 0.013 | -0.043 | 0.017 |
| Emerging Markets I (DFEMX) | -0.278 | 0.472 | 0.483 | 0.008 | 0.002 | -0.021 | 0.111 |
| Emerging Markets Value I (DFEVX) | 0.131 | 0.303 | 0.307 | 0.000 | -0.075 | 0.091 | 0.126 |
| International Small Company I (DFISX) | -0.429 | 0.026 | 0.011 | 0.058 | 0.172 | 0.393 | 0.039 |
| International Value I (DFIVX) | 0.150 | 0.067 | 0.068 | 0.291 | 0.237 | 0.518 | 0.222 |
| Japanese Small Company I (DFJSX) | 0.116 | -0.001 | -0.012 | 0.014 | -0.006 | -0.003 | 0.018 |
| Asia Pacific Small Company I (DFRSX) | -0.008 | 0.001 | 0.008 | 0.030 | -0.008 | 0.035 | 0.010 |
| International Small Cap Value I (DISVX) | 0.250 | -0.024 | 0.030 | 0.187 | 0.094 | 0.155 | 0.115 |
| United Kingdom Small Co I (DFUKX) | 0.124 | 0.008 | 0.000 | 0.036 | -0.004 | 0.032 | 0.033 |

Table 10. Sharpe style analysis on 11 WisdomTree enhanced index funds, no shorts

| CRSP identifier | 032506 | 032505 | 032494 | 035773 | 036484 | 032510 | 032508 | 036970 | 032502 | 032503 | 032501 | 032500 | 032509 | 032507 | Average |
|---|-----------------------|--------------------|---------|--------------------------------|------------------------------------|--------------------------|---------------------|----------------|--------------------------------------|---------------------------------|-------------------------------|---------------------------------|---------------------|-------------------------|---------|
| Initial date | 6/16/06 | 6/16/06 | 6/16/06 | 7/16/07 | 10/30/07 | 6/16/06 | 6/16/06 | 2/22/08 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | |
| Fund name | Asia Pacific ex-Japan | Australia Dividend | DEFA | Emerging Markets Equity Income | Emerging Markets SmallCap Dividend | Europe SmallCap Dividend | Global ex-US Growth | India Earnings | International Dividend ex-Financials | International LargeCap Dividend | International MidCap Dividend | International SmallCap Dividend | Japan Hedged Equity | Japan SmallCap Dividend | |
| α (%/year) | 1.661 | 1.534 | -0.354 | 4.082 | 1.584 | -2.177 | 1.268 | -8.130 | -0.571 | -0.484 | 0.287 | -0.155 | -13.817 | -0.461 | -1.124 |
| Significance of α , 1 tail t-test (%) | 24.67% | 32.52% | 39.91% | 7.53% | 28.29% | 8.66% | 38.06% | 15.30% | 38.37% | 37.93% | 41.29% | 45.46% | 7.44% | 38.42% | 28.85% |
| Observations (months) | 78 | 78 | 78 | 65 | 62 | 78 | 78 | 58 | 78 | 78 | 78 | 78 | 78 | 78 | 75 |
| Correlation | 0.968 | 0.949 | 0.987 | 0.967 | 0.977 | 0.989 | 0.837 | 0.893 | 0.978 | 0.983 | 0.989 | 0.986 | 0.667 | 0.966 | 0.938 |
| Excess standard deviation of fund (%/month) | -0.030 | 0.212 | -0.141 | -0.406 | -0.346 | 0.289 | -0.198 | 1.527 | 0.016 | -0.101 | -0.026 | -0.010 | -4.386 | -0.181 | -0.270 |
| Standard deviation of prediction error | 0.018 | 0.025 | 0.010 | 0.019 | 0.018 | 0.012 | 0.030 | 0.050 | 0.014 | 0.011 | 0.010 | 0.011 | 0.069 | 0.011 | 0.022 |
| DFA traditional index funds & Coefficients | | | | | | | | | | | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.000 | 0.000 | 0.000 | 0.000 | 0.542 | 0.000 | 0.000 | 0.591 | 0.006 | 0.000 | 0.049 | 0.064 | 0.000 | 0.000 | 0.089 |
| Large Cap International I (DFALX) | 0.221 | 0.270 | 0.888 | 0.000 | 0.000 | 0.000 | 0.684 | 0.000 | 0.555 | 0.973 | 0.344 | 0.015 | 0.000 | 0.000 | 0.282 |
| Continental Small Company I (DFCSX) | 0.000 | 0.050 | 0.045 | 0.001 | 0.000 | 0.541 | 0.000 | 0.000 | 0.050 | 0.000 | 0.259 | 0.182 | 0.000 | 0.000 | 0.081 |
| Emerging Markets I (DFEMX) | 0.532 | 0.148 | 0.000 | 0.738 | 0.259 | 0.000 | 0.000 | 0.000 | 0.012 | 0.000 | 0.000 | 0.005 | 0.000 | 0.000 | 0.121 |
| Emerging Markets Value I (DFEVX) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.409 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.029 |
| International Small Company I (DFISX) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.426 | 0.000 | 0.000 | 0.030 |
| International Value I (DFIVX) | 0.000 | 0.101 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.295 | 0.000 | 0.096 | 0.000 | 0.000 | 0.000 | 0.035 |
| Japanese Small Company I (DFJSX) | 0.081 | 0.000 | 0.000 | 0.098 | 0.027 | 0.000 | 0.262 | 0.000 | 0.000 | 0.000 | 0.109 | 0.135 | 0.000 | 0.955 | 0.119 |
| Asia Pacific Small Company I (DFRSX) | 0.166 | 0.396 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.112 |
| International Small Cap Value I (DISVX) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.021 | 0.000 | 0.000 | 0.001 |
| United Kingdom Small Co I (DFUKX) | 0.000 | 0.034 | 0.067 | 0.164 | 0.172 | 0.459 | 0.053 | 0.000 | 0.082 | 0.027 | 0.143 | 0.152 | 0.000 | 0.045 | 0.100 |

Table 11. Sharpe style analysis on 11 WisdomTree enhanced index funds, shorts allowed

| CRSP identifier | 032506 | 032505 | 032494 | 035773 | 036484 | 032510 | 032508 | 036970 | 032502 | 032503 | 032501 | 032500 | 032509 | 032507 | Average |
|---|-----------------------|--------------------|---------|--------------------------------|------------------------------------|--------------------------|---------------------|----------------|--------------------------------------|---------------------------------|-------------------------------|---------------------------------|---------------------|-------------------------|---------|
| Initial date | 6/16/06 | 6/16/06 | 6/16/06 | 7/16/07 | 10/30/07 | 6/16/06 | 6/16/06 | 2/22/08 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | 6/16/06 | |
| Fund name | Asia Pacific ex-Japan | Australia Dividend | DEFA | Emerging Markets Equity Income | Emerging Markets SmallCap Dividend | Europe SmallCap Dividend | Global ex-US Growth | India Earnings | International Dividend ex-Financials | International LargeCap Dividend | International MidCap Dividend | International SmallCap Dividend | Japan Hedged Equity | Japan SmallCap Dividend | |
| α (%/year) | 0.962 | 0.205 | -0.703 | 3.291 | -0.292 | -3.097 | -0.158 | -10.905 | -1.079 | -0.695 | 0.499 | -0.824 | -5.051 | -0.426 | -1.305 |
| Significance of α , 1 tail t-test (%) | 33.37% | 47.21% | 28.12% | 9.19% | 44.61% | 1.18% | 47.75% | 6.04% | 25.18% | 29.85% | 34.17% | 28.52% | 3.95% | 38.29% | 26.96% |
| Observations (months) | 78 | 78 | 78 | 65 | 62 | 78 | 78 | 58 | 78 | 78 | 78 | 78 | 78 | 78 | 75 |
| Correlation | 0.972 | 0.962 | 0.990 | 0.972 | 0.985 | 0.992 | 0.920 | 0.918 | 0.984 | 0.988 | 0.990 | 0.986 | 0.899 | 0.969 | 0.966 |
| Excess standard deviation of fund (%/month) | 0.170 | 0.180 | 0.044 | -0.058 | -0.038 | 0.006 | 0.378 | 0.999 | 0.063 | 0.042 | 0.060 | 0.086 | 0.487 | 0.069 | 0.178 |
| Standard deviation of prediction error | 0.016 | 0.021 | 0.009 | 0.016 | 0.014 | 0.010 | 0.020 | 0.044 | 0.012 | 0.010 | 0.009 | 0.011 | 0.021 | 0.010 | 0.016 |
| DFA traditional index funds & Coefficients | | | | | | | | | | | | | | | |
| Emerging Markets Small Cap I (DEMSX) | -0.039 | -0.355 | 0.223 | 0.264 | 0.955 | 0.234 | 0.616 | 0.736 | 0.285 | 0.213 | 0.272 | 0.222 | 0.557 | 0.204 | 0.313 |
| Large Cap International I (DFALX) | 0.181 | -0.274 | 1.057 | 0.247 | -0.021 | 0.067 | 2.544 | 0.299 | 0.496 | 1.236 | 0.349 | 0.001 | 1.067 | 0.117 | 0.526 |
| Continental Small Company I (DFCSX) | -0.185 | 0.482 | 0.325 | 0.001 | 0.049 | 0.639 | -0.168 | 0.871 | 0.482 | 0.389 | 0.213 | 0.198 | -0.433 | -0.225 | 0.188 |
| Emerging Markets I (DFEMX) | 0.998 | 1.276 | 0.133 | 1.202 | 0.840 | -0.015 | 0.213 | -0.031 | 0.309 | 0.145 | 0.073 | 0.215 | -0.030 | 0.090 | 0.387 |
| Emerging Markets Value I (DFEVX) | -0.462 | -0.823 | -0.245 | -0.498 | -0.605 | -0.213 | -0.596 | 1.374 | -0.436 | -0.253 | -0.214 | -0.341 | -0.392 | -0.224 | -0.280 |
| International Small Company I (DFISX) | 0.535 | -1.138 | -0.453 | 0.003 | -0.116 | -0.552 | -1.091 | -2.699 | -0.341 | -0.648 | 0.205 | 0.299 | -0.624 | 0.622 | -0.429 |
| International Value I (DFIVX) | 0.001 | 0.303 | -0.108 | -0.383 | -0.317 | -0.236 | -1.806 | -1.029 | 0.489 | -0.106 | 0.163 | -0.021 | -0.496 | -0.077 | -0.259 |
| Japanese Small Company I (DFJSX) | -0.035 | 0.203 | 0.072 | -0.056 | -0.055 | -0.026 | -0.128 | 0.650 | 0.118 | 0.097 | 0.032 | 0.115 | 0.450 | 0.776 | 0.158 |
| Asia Pacific Small Company I (DFRSX) | 0.259 | 0.877 | -0.055 | -0.197 | -0.241 | 0.032 | -0.304 | -0.171 | -0.023 | -0.042 | -0.150 | 0.033 | -0.223 | -0.182 | -0.028 |
| International Small Cap Value I (DISVX) | -0.185 | 0.327 | -0.062 | 0.378 | 0.417 | 0.555 | 1.772 | 0.520 | -0.530 | -0.161 | 0.000 | 0.168 | 1.164 | -0.028 | 0.310 |
| United Kingdom Small Co I (DFUKX) | -0.069 | 0.121 | 0.114 | 0.038 | 0.095 | 0.515 | -0.053 | 0.480 | 0.152 | 0.130 | 0.059 | 0.111 | -0.040 | -0.073 | 0.113 |

Table 12. Sharpe style analysis on 4 RAFI enhanced index funds, no shorts

| CRSP identifier | 036221 | 036223 | 043623 | 043624 | Average |
|---|--|---------------------------|---------------------------------|-----------------------------------|----------------|
| Initial date | 2007-Oct | 2007-Oct | 2007-July | 2007-July | |
| Fund name | FTSE RAFI Developed Market ex-US Small-Mid | FTSE RAFI Emerging Market | FTSE RAFI Asia Pacific ex-Japan | FTSE RAFI Developed Markets ex-US | |
| α (%/year) | 0.867 | -1.292 | 3.702 | -0.272 | 0.752 |
| Significance of α , 1 tail t-test (%) | 28.40% | 25.46% | 8.13% | 43.71% | 26.43% |
| Observations (months) | 63 | 63 | 66 | 66 | 65 |
| Correlation | 0.990 | 0.988 | 0.976 | 0.988 | 0.986 |
| Excess standard deviation of fund (%/month) | -0.017 | -0.018 | 0.108 | -0.031 | 0.011 |
| Standard deviation of prediction error | 0.010 | 0.013 | 0.018 | 0.011 | 0.013 |
| DFA traditional index funds & Coefficients | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.178 | 0.000 | 0.000 | 0.000 | 0.044 |
| Large Cap International I (DFALX) | 0.000 | 0.075 | 0.104 | 0.185 | 0.091 |
| Continental Small Company I (DFCSX) | 0.053 | 0.000 | 0.000 | 0.016 | 0.017 |
| Emerging Markets I (DFEMX) | 0.000 | 0.704 | 0.674 | 0.000 | 0.344 |
| Emerging Markets Value I (DFEVX) | 0.000 | 0.222 | 0.000 | 0.000 | 0.055 |
| International Small Company I (DFISX) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| International Value I (DFIVX) | 0.047 | 0.000 | 0.065 | 0.623 | 0.184 |
| Japanese Small Company I (DFJSX) | 0.160 | 0.000 | 0.000 | 0.000 | 0.040 |
| Asia Pacific Small Company I (DFRSX) | 0.000 | 0.000 | 0.157 | 0.000 | 0.039 |
| International Small Cap Value I (DISVX) | 0.431 | 0.000 | 0.000 | 0.065 | 0.124 |
| United Kingdom Small Co I (DFUKX) | 0.131 | 0.000 | 0.000 | 0.110 | 0.060 |

Table 13. Sharpe style analysis on 4 RAFI enhanced index funds, shorts allowed

| CRSP identifier | 036221 | 036223 | 043623 | 043624 | Average |
|---|--|---------------------------|---------------------------------|-----------------------------------|----------------|
| Initial date | 2007-Oct | 2007-Oct | 2007-July | 2007-July | |
| Fund name | FTSE RAFI Developed Market ex-US Small-Mid | FTSE RAFI Emerging Market | FTSE RAFI Asia Pacific ex-Japan | FTSE RAFI Developed Markets ex-US | |
| α (%/year) | 1.076 | 0.088 | 3.017 | -0.097 | 1.021 |
| Significance of α , 1 tail t-test (%) | 23.63% | 48.01% | 8.84% | 47.30% | 31.94% |
| Observations (months) | 63 | 63 | 66 | 66 | 65 |
| Correlation | 0.990 | 0.990 | 0.983 | 0.992 | 0.989 |
| Excess standard deviation of fund (%/month) | -0.055 | -0.017 | 0.046 | -0.047 | -0.018 |
| Standard deviation of prediction error | 0.010 | 0.012 | 0.015 | 0.010 | 0.011 |
| DFA traditional index funds & Coefficients | | | | | |
| Emerging Markets Small Cap I (DEMSX) | 0.301 | -0.120 | -0.166 | 0.135 | 0.037 |
| Large Cap International I (DFALX) | -0.147 | 0.385 | -0.191 | 0.741 | 0.197 |
| Continental Small Company I (DFCSX) | -0.047 | -0.305 | 0.035 | 0.020 | -0.074 |
| Emerging Markets I (DFEMX) | 0.007 | 0.587 | 1.553 | -0.193 | 0.488 |
| Emerging Markets Value I (DFEVX) | -0.157 | 0.544 | -0.723 | 0.078 | -0.065 |
| International Small Company I (DFISX) | 0.364 | 0.407 | -0.383 | -0.702 | -0.079 |
| International Value I (DFIVX) | 0.203 | -0.041 | 0.352 | 0.263 | 0.194 |
| Japanese Small Company I (DFJSX) | 0.093 | -0.164 | -0.021 | -0.051 | -0.036 |
| Asia Pacific Small Company I (DFRSX) | -0.049 | -0.232 | 0.414 | -0.073 | 0.015 |
| International Small Cap Value I (DISVX) | 0.377 | 0.031 | 0.168 | 0.610 | 0.296 |
| United Kingdom Small Co I (DFUKX) | 0.056 | -0.090 | -0.037 | 0.173 | 0.025 |