A THEORY OF OPTIMAL SICK PAY

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Abstract

Illness significantly reduces worker productivity, yet how employers respond to the possibility of illness and its effects on work performance is not well understood. The 2003 American Productivity Audit pegged the cost to employers of lost productive time due to illness at 225.8 billion US dollars/year. More importantly, 71% of that loss was explained by reduced performance *while at work*. Studies of worker illness have been up to this point empirical, focused primarily on characteristics which co-vary with worker illness and absenteeism. This paper seeks to understand how employers mitigate the impact of illness on profits through a microeconomic model, elucidating how employers influence workers through salary-based incentives to mitigate its associated costs, providing firms and policy makers with a comprehensive theoretical method for formulating optimal sick pay policies.

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Table of Contents

1 Introduction ... 11

2 Literature Review ... 13

3 A Model for Illness ... 18

Functions and Modeling ... 21

4 Modeling ... 23

The Basic Model ... 23

Introducing "Hidden Information" into the Basic Model ... 24

Conclusion 1. In the case of hidden information and exogenous probability of illness, employees are overcompensated. ... 26

Conclusion 2. Firms prefer severe illness over moderate illness in cases involving hidden information. Of course, they prefer light illnesses as well. ... 29

The Effort Model ... 29

Employer Chooses Effort ... 29

Conclusion 3. Firms prefer the ability to exert effort to reduce the probability of illness, *ceterus paribus*. ... 33

Employee Chooses Effort ... 34

Conclusion 4. The optimal effort when the employee selects the effort will always be less than in the case when the employer chooses effort. ... 37

Conclusion 5. When employees select their own non-zero effort, they are overcompensated, but their compensation diminishes as the inherent probability of illness increases. ... 38

Conclusion 6. Firms will only pay employees U_h (to select nonzero effort) if the intrinsic probability of illness α_0 is greater than a threshold. Namely, $\alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}$ ... 39

Conclusion 7. When employees select their own effort, firms receive the same profit in the face of different inherent probabilities of illness α_0 for all nonzero efforts regardless of the specific values of other exogenous variables. Also, profit in the case where employees select effort is always less than profit in the First Best with Effort....40

Conclusion 8. When employees select their own effort, firms will not give incentives for illnesses which have a low severity of illness, even if there is a high ratio of effectiveness of effort to cost....43

Employer Chooses Effort, but Employees can Lie....44

Conclusion 9. Firm profit is strictly higher for hidden information in the case where effort can be exerted and the employer selects it. At low inherent probabilities of illness α_0, where the employer would normally incentivize small amounts of effort, profit is reduced to the case of lying without effort. Alternately, when the firm would already pay a large premium for effort, at high values of α_0, the firm will see no loss at all due to hidden information....47

Conclusion 10. In cases in which firms can determine effort but cannot determine whether employee's report their state of health honestly, the employer will force excess effort....49

Employee Chooses Effort and Employee Can Lie....50

Conclusion 11. Unlike the previous case involving effort and hidden information in the case where employees select effort and can falsely report, the employer cannot achieve the same profit it achieves in the case where it can observe the employee's true state of health....51

Conclusion 12. Once employers are committed to incentivizing employees to exert effort to reduce their probability of illness, the fact that they can misreport makes only a small difference to the employer....53

Summary of Key Findings....54

5 Discussion....56

6 Conclusion....60

7 References 61

8 Appendix A (Cases) 64

First Case. Exogenous Probability of Illness, No Hidden Information 64

Second Case. Hidden Information , Exogenous Probability of Illness 65

Third Case: Introducing Effort....66

Fourth Case: Employee Chooses Effort 67

9 Appendix B (Additional Considerations) 71

Why Minimizing Expected Utility Maximizes Firm Profit....71

The Threshold where the Employer selects Nonzero Effort in the First Best 72

Why Employee Selected Effort is Less than or Equal to Employer Selected Effort....72

More On the Effort when Employee Selects Effort: Why the Employer will not always compensate for ε* 73

More On the Effort when Employee Selects Effort: Whether the constraint that $U_0 \geq 0$ will ever be binding....76

10 Appendix C (Methods of Numerical Optimization and Figure Generation) 78

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Table of Figures

Figure 1. The Timeline of an Employment Contract 19
Figure 2. Behavior of the production function 21
Figure 3. Behavior of the probability of illness function 22
Figure 4. Behavior of the cost of effort function 23
Figure 5. Profit in the case of Hidden Information is strictly less than in the First Best ($\pi \leq \pi_{FB}$) 28
Figure 6. Optimal Efforts over a range of effort effectiveness and production advantage 31
Figure 7. Firm profit in the new First Best. Notice that when the effectiveness of effort is low, $\pi_{FB} \approx \pi_{FB(\varepsilon)}$ 33
Figure 8. The firm does progressively better compared to the First Best with no effort as the inherent probability of illness rises. Notice $\pi_{FB} \leq \pi_{FB(\varepsilon)}$. 34
Figure 9. Contrasting the choices of effort when the employer selects effort vs. when the employee selects effort 36
Figure 10. Though firm profit continues to increase relative to the First Best without effort, as the inherent probability of illness increases, the utility the employee receives diminishes. 38
Figure 11. The dotted red line shows firm profit if the firm compensates the employee for effort whenever the employee is *willing* to exert effort, with the discontinuity representing the point at which the employee will work to reduce the probability of illness when offered a bonus. This is *not* the optimal solution however, as the first best with no effort offers the firm greater profit. .39
Figure 12. The dotted red line shows firm profit in the case when the employee selects effort. The firm will receive profit equivalent to the first best for most inherent probabilities of illness. However, the point at which the two diverge (circled) is the threshold point outlined in conclusion 7. At this point, it is more profitable for the firm to offer a bonus to the employee to select effort than to pay nothing. 41
Figure 13. Behavior of the Threshold Point as $\beta \to \infty$ and as $\eta \to \infty$ both make sense, as they take the threshold point $\to 0$. 43
Figure 14. The behavior of the threshold probability when the severity of illness is small, i.e. $c_s \approx c_h$. Notice that as the severity of illness falls, the threshold point rises to nearly 1 44

Figure 15. The firm makes profit very close to the profit in the hidden information with no effort case at low inherent probabilities of illness (graph upper left), since very little effort would be demanded. At high levels of inherent probability of illness (graph lower right), once the threshold point (denoted by a circle on the plot above) is crossed, the employer sees no reduction in profit from the first best with effort due to the possibility of hidden information..47
Figure 16. A plot of the effort in the case of hidden information. The effort demanded from employees in the case of hidden information is is greater than or equal to that demanded in the First Best. 49
Figure 17. The employer will not be able to incentivize effort, and thus can achieve only the profit above the case of hidden information with no effort, until the probability of illness reaches a threshold (circled) at which the utility bonus the firm wishes to offer to healthy workers is strictly larger than the bonus needed to prevent lying. Then the case behaves similarly to the case in which the employee selects his or her own effort (though at a lower profit)..........53
Figure 18. Expected profit for the firm across the range of employer-employee relationships considered in this model. 55

1 Introduction

People get sick and their illness affects their work performance. For employers, illnesses can lead to diminished profits through lost worker productivity, so in response to these potential losses, firms design sick-pay policies: incentives packages which offer employees reduced compensation during times of illness and encourage healthy workers to take steps to minimize their probability of contracting illnesses. So many different approaches to sick pay exist both within and across industries that a compelling argument can be made that the optimal incentives structure for sick pay is not well understood.

The significance of well designed sick pay incentives for the firm's bottom line cannot be understated. The flu proves a good example. According to the Centers for Disease Control and Prevention, 5% to 20% of the U.S. population is infected with the flu annually. Flu symptoms include extreme tiredness, muscle aches and fever, all of which make it difficult to work as easily as one would were he or she healthy. Flu is also highly contagious. Most healthy adults may be able to infect others beginning one day before symptoms develop and up to five days after becoming sick. This means that for the bulk of the time that they are sick, employees can take measures to minimize their flu transmission. ("Key Facts About Seasonal Influenza"). But flu is not the only culprit here. Employers must consider a host of illnesses with their own relatively high rates of transmission and infection, which together nearly guarantee that for any sufficiently large institution, someone is always going to be sick. In addition to flu, viral gastroenteritis or the "stomach flu," viral meningitis, the common cold (Bhatia 2008), and many other infectious diseases are transmitted in workplaces, schools and other public institutions through simple casual contact. The collective burden of just these common infectious illnesses is enormous and for employers the potential loss from several productive individuals becoming sick could be substantial, especially if the firm had no means of adjusting compensation in response to illness. Healthy individuals infected with illness face a significantly higher personal cost to complete work tasks. Employers thus possess a great incentive to develop policies which mitigate these impacts.

In addition to the burden of illness, state mandates also play a role in spurring employers to develop sick-leave policies. Internationally, national health insurance plans are coupled with mandatory paid sick leave. Sweden, for instance, has compulsory ‘sickness insurance’ as part of the national social insurance system. Insured individuals are entitled to benefits if their perception of their state of health is such that they consider that 'it does not permit them to do their regular work'. The regulations allow an insured person to be absent from work for up to eight days without a certificate from a physician. (Johansson and Palme 1996) More than ten states have already mandated that workers receive paid sick leave, with California passing a measure for universal paid sick leave within just the last year.

Then as was outlined, the task of the employer (and in the case of nation’s like Sweden, of the national health insurer) becomes to design incentives that make it attractive for workers to remain healthy and that minimize the cost to the firm of the worker’s diminished productivity. For institutions ranging from Wal-Mart to the Department of Defense these incentives packages take the form of sick leave policies and required employer health programs. Indeed, a survey of many of America’s largest employers reveals that the number of approaches to sick leave policies is nearly as varied as the number of illnesses which might cause workers to exercise them. Incentives packages range from paying large bonuses for perfect attendance to packaging all non-working days, like vacations and sick days, into a common category termed ‘flex’ or ‘personal’ time to requiring that a certain number of consecutive absences accrue before paying benefits.

Can all of these sick leave packages be efficient? Even accounting for differences in the types of labor demanded across industries, the sheer quantity of different sick day incentives packages strongly motivates an argument that these incentives are not well understood. Further bolstering this claim, in nations with national health insurance plans, employees widely abuse the sick day system. (Kangas 2004; Doherty 1979) As the literature makes clear, the ways in which researchers have approached sick-pay packages does not strike at the heart of the question: whether sick pay packages are actually optimal. Consequently, a robust theoretical model capable of differentiating between various sick policies could provide the key to unlocking whether certain sick-leave packages provide the desired outcomes for firms.

This paper first models a single-employee contracting to work for an employer in either a sick or healthy state. This case already raises economic implications for designing the employment contract. When given knowledge of its employee's health status the firm can pay a more efficient wage than when the health of the employee cannot be observed. This means when Wal-Mart requires that employees bring a doctor's note when they claim to be sick, Wal-Mart is free to pay an efficient wage to sick workers and can contract sick workers to produce an efficient quantity (at least in the absence of other workers to infect). Extending the model to the case in which effort can be exerted by the employee to reduce the probability of illness, additional implications arise.

This paper begins with a literature review (section 2), followed by the development and subsequent exploration of a theoretical model for sick-pay involving the development of an employment contract between an employee and employer (sections 3 and 4). The results of this analysis are then subsequently discussed, and limitations and implications of the model in this paper are noted (section 5). We conclude with suggestions for future work (section 6).

2 Literature Review

Unlike the present study, research on worker absenteeism and sick-leave has not sought to address the question of which incentives packages theoretically maximize firm profit or social utility. Instead, both in focus and methodology, authors have sought to answer narrower questions about employees and illness. In one rich area, researchers have analyzed what factors go into the decision to take an absence from work. In the other major branch, researchers have treated the cases when illness directly leads to unavoidable absence and analyzed the "work productivity" lost due to these illnesses. Across the literature in both branches however, the method has been to use empirical data to measure the impacts of incentives on real employee decisions. To the extent that these papers have constructed theoretical models of employee decision making, they have done so only to lend context to their data, inferring variables over which to regress based on theoretical approaches (i.e. they ask "What measurable characteristics might influence the choice to take a sick day beyond the state of illness?"). Perhaps constrained by methodology, little treatment has been given to the efficiency of incentives for the firm or for society. Even the few papers which have analyzed incentives and decisions together have viewed employee decisions as choices made in the face of static sick leave policies, ignoring firm strategies altogether. In essence, rather than seeking to evaluate underlying principles, previous work has sought to evaluate individual decisions induced by preexisting incentives.

As was noted, on one end of the spectrum of approaches to worker absence, researchers have sought to sift through data on absence and illness, together with many additional economic variables, in the hopes of bounding the 'moral hazard' which arises when illness benefits are given through an employer or government. Research in this vein is not so much concerned with when sick leave is properly used, but when it is misused. Doherty (1979) examined the British National Insurance system and whether variations in sickness absence could be explained by economic variables with a basic economic behavioral model. The subsequent regression found that the 'relative generosity' of a worker's benefits (i.e. the income of an individual compared to his or her potential sick leave payments) had a direct impact on the likelihood of his or her absence. Allen (1981, 1983) developed a model over another range of economic variables which potentially impact worker absence, focusing on the wage rate, the mix of compensation between wages and fringe benefits and employment hazards. Viewing absenteeism as one element in the bundle of commodities consumed in the course of employment, Allen established through his empirical work that wage rate strongly influences worker absenteeism. Drago and Wooden (1992) extended Allen's work, analyzing the causes of absence using data from a 1988 survey administered to workers in Australia, Canada, New Zealand and the United States. Their results indicated that many factors have a statistically significant impact on work absence. They found that male gender, short tenure, part-time status, higher wages and lower unemployment rates all affected absenteeism. Most importantly they found that greater sick leave entitlements led to higher rates of absence. Johansson and Palme (1996) studied whether 'economic incentives' affect work absence, modeling absence as an individual day-to-day decision where workers balance potential leisure and sick pay against compensation and cost of work. Their research arises directly from regressing days absent against many of the same variables set out by Allen, though in their case for a sample of Swedish blue-collar workers. The results confirmed those found by Allen and Drago: wage rate, unemployment rate and 'relative generosity' all affect absence.

The papers outlined above seek to answer the question of whether workers will lie about their true state of health and take sick days in the absence of illness. While a valuable question, the outcome is rather obvious. Workers will seek to conceal their state of illness if doing so increases their compensation. These studies thus lend little insight into which sorts of sick-leave packages are efficient for employers. Rather, they offer the key insight that employers must take steps to prevent cheating when offering sick days. Unfortunately, nearly all of these papers lack the specificity necessary to tease out the underlying motivation for absence or how this motivation was affected by the structure of incentives. This leaves it impossible to distinguish, within the studies themselves, which absences were due to illness and which were taken under false pretenses.

The above models also assume that employers do not care about and do not have the facility to verify what causes an absence, a matter which only complicates the question of whether sick leave is just a form of compensation, or whether it plays a legitimate role in minimizing the cost of illness to the firm. If sick days truly are for the sick, then employers granting them efficiently will care deeply about whether their employees are actually ill and infectious or not.

In perhaps the most relevant but least developed area of the literature, some authors have touched on the specific interplay between sick-leave incentives and sick-days taken. Denerley (1952) showed that absenteeism increases with the number of sick-leave days and with sick leave pay. More recently, Gilleskie (1998) looked specifically at absences taken by workers while acutely ill and found a 45% increase in absences during an episode of illness.

Yet of papers which look at a range of incentives and sick leave, little exists. A critical article comes from Winkler (1980) entitled "The Effects of Sick-Leave Policy on Teacher Absenteeism." Winkler notes the conspicuous dearth of other papers in the field in his paper writing, "The fact that absenteeism increases with the number of sick-leave days and with sick leave pay is one of the few empirical findings with respect to sick leave policy reported in the literature." (pp. 233) Winkler studied short-term absenteeism among public school teachers in California and Wisconsin. He found that policies requiring teachers to report every illness directly to the principal or to show proof of illness led to significantly lower rates of absenteeism. Winkler's data agrees well with this paper's theoretical findings.

There exists another approach to studying sick days and worker absence in the literature. Papers in this vein inherently assume a link between communicable illness and worker absenteeism and then try to assess the cost of absence due to illness. In contrast to papers seeking to tie together factors contributing to absence in addition to illness, articles here seek to pinpoint how much infectious or influenza like illnesses affect the economy through work lost because of absence. This perspective appears in public health literature when epidemiologists seek to determine whether state-wide vaccination regimens are more cost effective than treating individuals post-infection.

Keech (1998) made an important contribution in measuring the impact of influenza illness on work absence and productivity. He found that workers were incapacitated or confined to bed for 2.4 days, missing 2.8 days from work per episode of illness. On return to work, they reported reduced effectiveness and inability to resume normal activity until an average of 3.5 days after the onset of symptoms. Of note, Keech found that managers took significantly fewer sick days while acutely ill than secretarial or administrative staff, something confirmed by other authors (Briner 1996). Unfortunately, though all of the participants in the Keech study faced the same sick-leave policy at a large pharmaceutical company in the UK, Keech does not describe it in any detail, even though Keech himself notes in his conclusion that costs associated with illness depend in large part on the sick-leave policy of the firm.

Yet articles similar to Keech's are common in the literature. Akazawa (2003) for instance, attempts to quantify the association between lost workdays and influenza, controlling for other factors, with a secondary aim of assessing the net benefit of expanded vaccination in a workplace setting. Unlike Keech, Akazawa uses the 1996 Medical Expenditure Panel Survey Household Component to get a representative sample of U.S. households. Akazawa finds a substantially smaller number of sick days taken due to influenza infection in the general U.S. population than in Keech's pharmaceutical company study, though Akazawa notes the obvious difference: 35% of his study sample had no sick benefits at all. Indeed, Akazawa only confirms the trend which runs through this literature: workers will take an absence due to their illness far more frequently when they receive a paid sick-day.

The American Productivity Audit performed by Stewart (2003) picks up on this theme and raises important implications for the analysis of proper employment incentives. The study found that the vast majority of workers report their state of illness honestly when they take an absence, finding that 10% of workers were absent from work for a personal health reason and only another 2% were absent for a family health reason during a 'recall period' of 2 weeks. Though the study design—a telephone survey of a random sample of 28,902 U.S. workers—may have led to underreporting, the potential implications of the study are staggering. The audit found that 38.3% of workers reported unproductive time as a result of personal health on at least 1 workday during the recall period. As a share of lost productive time, reduced performance at work as a result of personal health accounted for 66% (1.32 hours per week) of the lost time, followed in order by work absence for personal health (0.54 hours per week) and work absence for family health (0.12 hours per week). In total, the audit found that on average, 71% of all health-related lost productive time was the result of reduced performance at work, not absences.

Stewart's results could imply that on the whole, American sick-leave packages are woefully inefficient. Gilleskie and others show that when workers take days off from work their time needed to recover is substantially reduced. Yet, Stewart shows that many workers choose to work at reduced capacity instead of taking absences, leading to far greater losses in firm productivity from their reduced usefulness than would have occurred had they simply taken time off to recover. Efficient sick leave packages should minimize this lost productivity by sending these workers home to get well. Just as important as the reduced productivity of individuals, if sick workers are at work and working less efficiently, they are also potentially infecting healthy coworkers, dragging down the productivity of the firm even further.

In all, there seems to be a fundamental conflict in the literature. On one end of the spectrum, empirical papers show that more sick leave benefits lead to more absences for non-illness. On the other end, analysis shows that more sick leave benefits lead to more absences due to illness. The problem is there is no metric in place for how to gauge productivity, cost and especially efficiency. This becomes more readily apparent when author's such as Heymann (1999) note that without paid sick days many parents will not take days off work to care for their children. In the calculus of developing sick leave incentives employers must balance the legitimate desire to prevent sick workers from costing the company additional wages with the need to offer modes of compensation which are not purely monetary, like flexible work time. Thus, they are faced with the difficult question of how to stop parents from cheating the sick-day system and caring for their children when they themselves are not sick.

This conflict in perspective introduces a new dimension into any model of optimal pay for absences: should employers be required to give some paid absences because it might raise the total utility of society (since caring for one's children likely raises employee utility substantially)? This is the question put to voters in California in November and another question onto which this paper seeks to shed light.

3 A Model for Illness

To make it clear exactly how employers respond to the possibility of employee sickness, a theoretical model for how illness affects employee utility and employer profit can be developed. First one observes that an employer contracts with an employee who can be either sick (s) or healthy (h) when he or she works. One can think of sickness as raising an employee's marginal cost of production from c_h—the marginal cost when he or she is healthy—to c_s, the marginal cost when he or she is sick. Mathematically, $c_s > c_h > 0$. The employee reports his or her state of illness to the employer and the employer has an (output, payment) pair designated for each state. Let (p_s, q_s) and (p_h, q_h) be the payment and output assignments for an employee who calls in sick or just reports to work as healthy, respectively. Additionally, let $U_s = p_s - c_s \cdot q_s$ and $U_h = p_h - c_h \cdot q_h$ be the utility for an employee in either state. Each represents the difference in payment from the employer and cost to the employee to produce a given quantity of output. Moreover, let $v(q)$ be payoff to the employer and let it have diminishing marginal returns. Mathematically, $v(0) = 0$, $v' > 0$, and $v'' < 0$.

Now the employer and employee agree on terms of payment and quantity in the manner described in Figure 1, a manner which reasonably reflects how a standard employment contract might be negotiated.

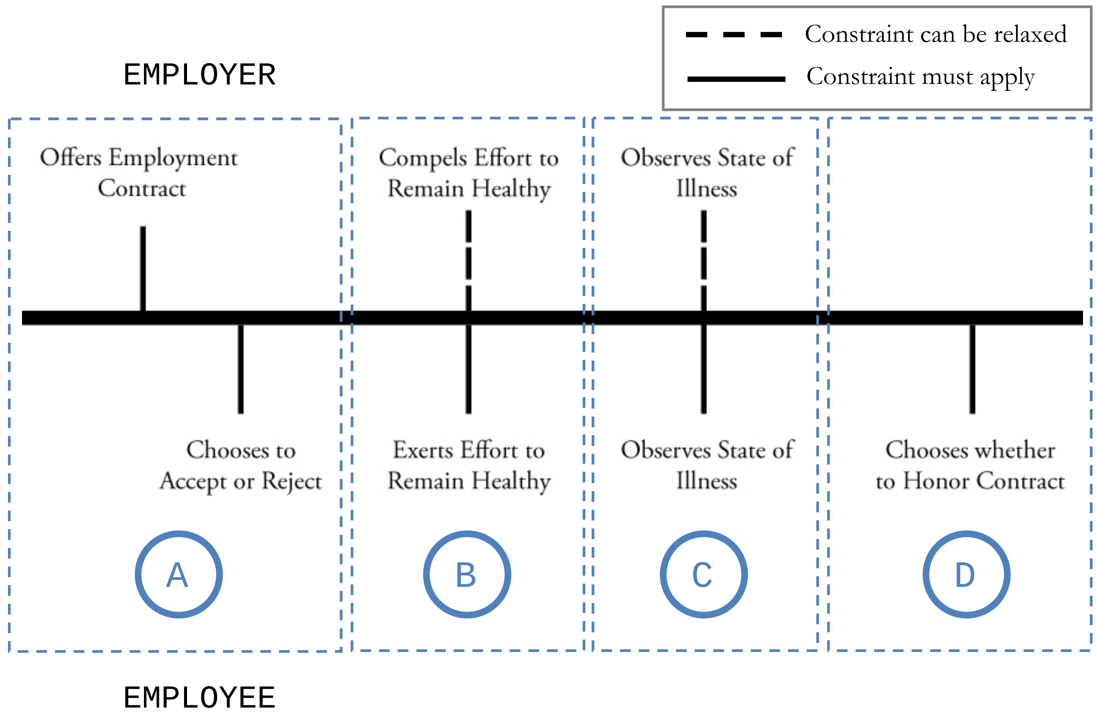


Figure 1. The Timeline of an Employment Contract

In the first step of the negotiation, (A), the employer and employee agree to an employment contract. At this point, neither the employer nor employee knows whether the employee will be sick or healthy when he or she comes to work. Rather, it is commonly known that the probability that the employee may get sick is between 0 and 1 ($\alpha(\varepsilon) \in [0,1]$) where ε is the effort an employee can exert to reduce α, the probability that he or she becomes sick.

The firm seeks to maximize the expected profit from the output of the employee,

$$\max_{(p_h, q_h, p_s, q_s)} \quad \pi = \alpha \cdot [v(q_s) - p_s] + (1-\alpha) \cdot [v(q_h) - p_h] \tag{4.1}$$

Where π is expected firm profit. The employee, in turn, also has an expected utility value tied to illness given by,

$$U_0 = \alpha(\varepsilon) \cdot U_s + (1 - \alpha(\varepsilon)) \cdot U_h - d(\varepsilon) \quad (4.2)$$

Where $d(\varepsilon)$ is the cost of effort to stay healthy and U_0 is the expected utility from signing a contract with the employer. This is the expected utility for the employee whether he or she becomes sick or not. Thus, for the employer and the employee to agree to a contract in (A) it must be the case that $U_0 \geq 0$.

In the next stage, (B), the employer could compel the employee to exert effort ε which maximizes the firm profit function. However, in some cases the employer may not have direct control, and so it may fall to the employee to exert effort which maximizes his or her own utility.

In part (C), the employee observes his or her own state of health. If the firm can directly observe the employee's state of health as well, then there are no additional constraints. However, this assumption is not always reasonable. If the employer cannot directly observe the employee's state of health, then the employer must make it unattractive to falsely report one's true state of illness. This requirement takes the form of the constraints, $U_s \geq p_h - c_s \cdot q_h$ and $U_h \geq p_s - c_h \cdot q_s$ which assert that the utility of reporting honestly must be greater than the utility of the (output, payment) pair one would be assigned were he or she to lie about his or her state of health.

The final stage, (D), is included as a formalism, meant only to remind us that the employee cannot be compelled to come to work if his or her state of illness makes working unattractive (yields negative utility). The employee would rather stay home at that point. This imposes two additional constraints on the problem, namely that for any incentives package it must be that $U_s \geq 0$ and $U_h \geq 0$.

Functions and Modeling

Abstract functions of the form $f(x)$ can be replaced by specific functions with the same desired properties. This is done only when necessary to elucidate an aspect of the model. In this section these model functions are defined and explained.

Definition 1. The amount of value produced for the firm by an employee, $v(q)$, is a function of the quantity of labor given by the employee and is defined to be $v(q)=2\sqrt{q}$. Moreover, we expect $v(0)=0$, $v'>0$ and $v''<0$. Note that $v(q)=2\sqrt{q}$ satisfies these requirements.

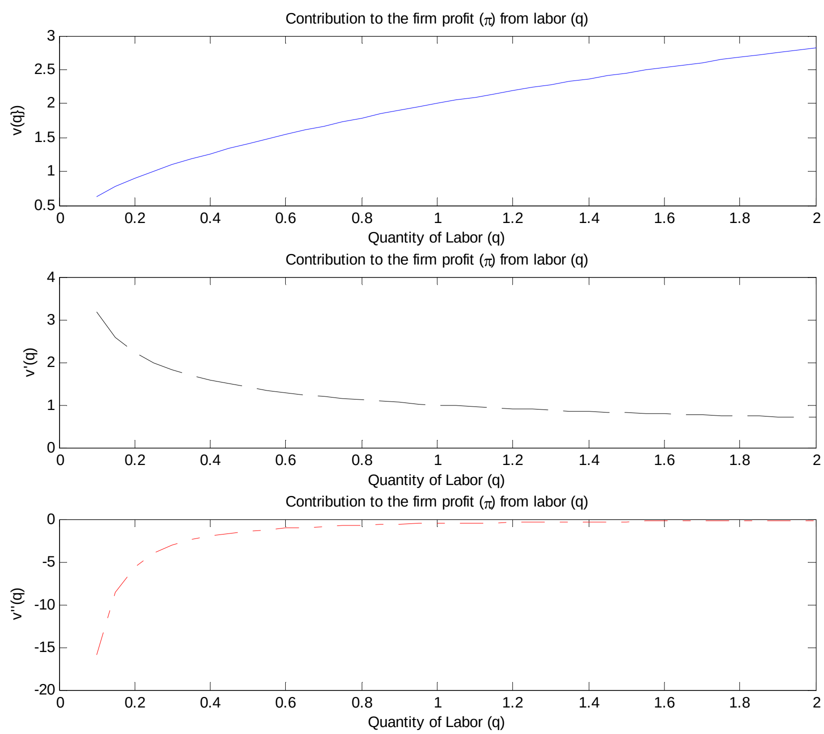


Figure 2. Behavior of the production function

Definition 2. The probability an employee becomes ill, α, is a function of effort defined as $\alpha = \alpha_0 e^{-k\varepsilon}$ where ε is the amount of effort exerted, α_0 is the probability of illness when no effort is exerted and k is the effectiveness of effort. Moreover, we expect $\alpha(0) = \alpha_0$, $\alpha' < 0$ and $\alpha'' > 0$. Note that $\alpha = \alpha_0 e^{-k\varepsilon}$ satisfies these requirements.

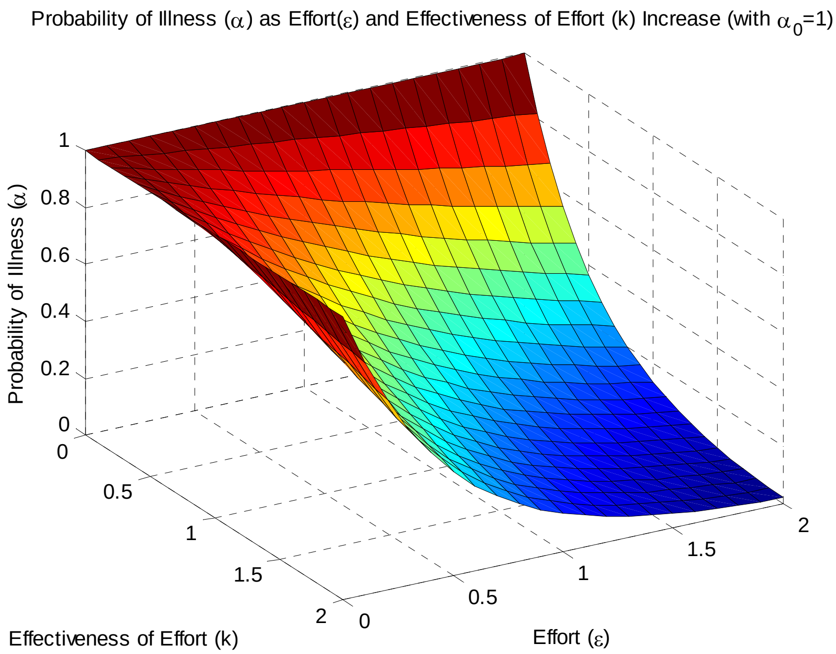


Figure 3. Behavior of the probability of illness function

Definition 3. The cost of exerting effort to reduce the probability of illness, d, is a function of effort defined as $d = d_0\varepsilon$ where ε is the amount of effort exerted and d_0 is the marginal cost of effort. Moreover, we expect $d(0) = 0$, $d' > 0$. Note that $d = d_0\varepsilon$ satisfies these requirements.

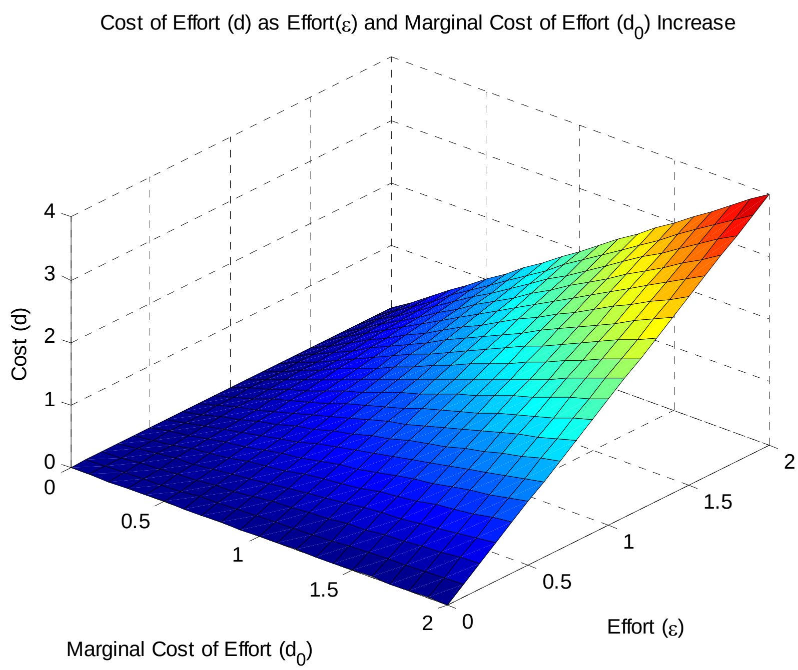


Figure 4. Behavior of the cost of effort function

4 Modeling

The Basic Model

There exist workplaces where the probability of becoming sick cannot really be affected by effort and where employers, perhaps because they closely observe their workers or perhaps because the employees are actually self-employed, will not have to worry about employees concealing their state of health. For instance, active duty military personal provide a good example: they are closely monitored by their commanding officers and staying healthy is part of the job description. Another example would be a self-employed fitness instructor already in peak physical condition, who has little to gain from lying to herself about her true state of health.

In developing this first case, the approach is to imagine that the employer and the employee treat the probability of illness as exogenous; that it is fixed and unchangeable. Further, one supposes that the firm knows the state of health of the employee when he or she comes to work and assigns work accordingly.

This situation is solved mathematically in the appendix but it can also be explained economically. Employees will be paid precisely an amount equal to their total cost of production when sick and when healthy (i.e. $p_s = c_s \cdot q_s$ and $p_h = c_h \cdot q_h$) and will be asked to produce until the amount they are paid per unit is equal to the cost of production for that additional unit (i.e. until the marginal output from labor equals the marginal cost of labor). Though this is solved in the appendix and expressed mathematically to be $v'(q_h) = c_h$ and $v'(q_s) = c_s$, what it means economically is that employers in this case reap the largest possible profit they can in the face of exogenous probability of illness. Thus this is termed the "First Best", as it is the best the firm can do, in the case of exogenous probability of illness.

Indeed, the expected profit in this case $\pi = \frac{1}{c_h} + \alpha \cdot \left[\frac{1}{c_s} - \frac{1}{c_h} \right]$ is the most profit the employer can earn and the expected utility, $U_0 = 0$, is the least the employee can earn.

Introducing "Hidden Information" into the Basic Model

Though the First Best may be appropriate to a limited number of occupations, it is legitimate to wonder whether employees might be able to come to work and pretend to be sick (or healthy) in order to increase their utility. Indeed, in an average office environment, employees are not closely monitored by their managers and their work is largely independent. Whether the probability of illness can be reduced through effort is a good question, but certainly there are some jobs where effort to stay healthy plays a small or nonexistent role even if employees can lie about their true state of health, especially those jobs in which the probability of becoming sick is tied to work tasks, or alternately in professions where efforts to remain healthy are incidentally required for employment.

This introduces the possibility of lying into the model. This possibility is termed "Hidden Information" since it introduces a new challenge to the employer. Now the firm cannot determine whether employees are truly sick or truly healthy.

This changes the nature of the relationship between workers and the firm considerably. When the employer chooses effort, the only consideration is how much profit can be produced given the constraint that no utility (U_s, U_h and U_0) can be less than zero. The possibility of lying introduces a new constraint and that is that the utility from lying U_{lying} must be smaller than he utility from being honest. The utility from lying can be thought of as the benefit from pretending to be sick or healthy realized by an employee.

For a sick employee, $U_{lying} = p_h - c_s \cdot q_h = U_h + c_h \cdot q_h - c_s \cdot q_h = U_h - q_h(c_s - c_h)$

For a healthy employee, $U_{lying} = p_s - c_h \cdot q_s = U_s + c_s \cdot q_s - c_s \cdot q_s = U_s + q_s(c_s - c_h)$

These equations immediately make it clear who will have an incentive to lie in the early cases. Since $U_s = U_h = 0$ in the First Best, if the employer naively chooses to pay every employee as if there was no hidden information then, lying workers would receive,

For a sick employee, $U_{lying} = -q_h(c_s - c_h)$ which is strictly less than U_s

For a healthy employee, $U_{lying} = q_s(c_s - c_h)$ which is strictly greater than U_h

Notice, sick workers do worse when they lie, so they will want stay honest. Healthy employees, however, will want to lie since $U_{lying} > U_h$. To counteract this incentive to lie, the employer offers $U_h > p_s - c_h \cdot q_s$ since this makes $U_{lying} < U_h$.

Thus, only one new constraint is introduced, namely that $U_h > p_s - c_h \cdot q_s$. However, this constraint fundamentally changes the outcome for the employee. Since now $U_h > 0$ and $U_s \geq 0$, it is immediately clear that $U_0 > 0$ since it is a weighted average of these utilities.

One can think of non-zero utility as a bonus over the utility offered in the First Best, which implies the first substantive conclusion of the model.

Conclusion 1. In the case of hidden information and exogenous probability of illness, employees are overcompensated.

With regard to this finding, it is important to note that sick (s) and healthy (h) denote the state of illness of the same employee. This means that when one analyzes how U_h and U_s are treated, he is not seeing the quantities that employees and employers really care about, which are the expected values, U_0 and π. It is not so important that the employee is overcompensated when healthy or when sick. We care instead that the employee is now overcompensated on average because of the introduction of hidden information.

This second case is also solved mathematically in the appendix, but its strategic implications are equally important, for the employer reduces the payment to sick employees in a clever way. The employer is tasked with making it unattractive for healthy workers to feign illness. Since the employer wants $U_h > p_s - c_h \cdot q_s$, the firm faces two options. The firm can either pay more for work from healthy employees or demand less work from sick employees. Ultimately, the employer does both.

One can explain why the employer chooses to increase pay to healthy workers by considering the new constraint carefully. Since $U_s = 0$ in the First Best, the employer cannot reduce the utility from working while sick any further than it already has, so $p_s = c_s \cdot q_s$ since the employer has no incentive to manipulate U_s. This, however, changes the compensation package for healthy workers because now $U_h > q_s \cdot (c_s - c_h)$. This can be further rewritten as $p_h - c_h \cdot q_h > q_s \cdot (c_s - c_h)$. Or with some adjustment, $p_h > q_s \cdot (c_s - c_h) + c_h \cdot q_h > c_h \cdot q_h$.

Though the employer does pay more to employees when healthy than in the First Best, the employer also reduces q_s in order to minimize this bonus. In fact, the firm could pay no bonus at all if the firm chooses $q_s = 0$. However, the firm will never reduce q_s this much. To see why, imagine if the employer did choose $q_s = 0$. Then $U_h = 0$ and $v'(q_h) = c_h$ and $\pi = \frac{1}{c_h} - \alpha \cdot \left(\frac{1}{c_h}\right)$. However, the employer can do better. Instead the employer chooses $v'(q_h) = c_h$ and $v'(q_s) = \frac{(1-\alpha)\cdot[c_s - c_h]}{\alpha} + c_s$. By doing so, the firm finds that profit increases to $\pi = \frac{1}{c_h} - \alpha \cdot \left(\frac{1}{c_h}\right) + \frac{\alpha^2}{(1-\alpha)\cdot c_h + c_s}$. Where the last term is the difference between the bonus paid to healthy workers and the excess output from sick workers. Thus the employer manipulates q_s such that $U_h > \frac{\alpha^2(c_s - c_h)}{(c_s - (1-\alpha)\cdot c_h)^2}$ where $q_s^* = \frac{\alpha^2}{(c_s - (1-\alpha)\cdot c_h)^2}$. The net result is this: the employees when sick receive less work, but no reduction in compensation, while employees when healthy receive a bonus. This bonus to healthy workers and reduction in quantity of labor demanded from sick workers combine to reduce firm profit in the hidden information case, to something strictly less than or equal to the profit in the First Best (i.e. $\pi \le \pi_{FB}$).

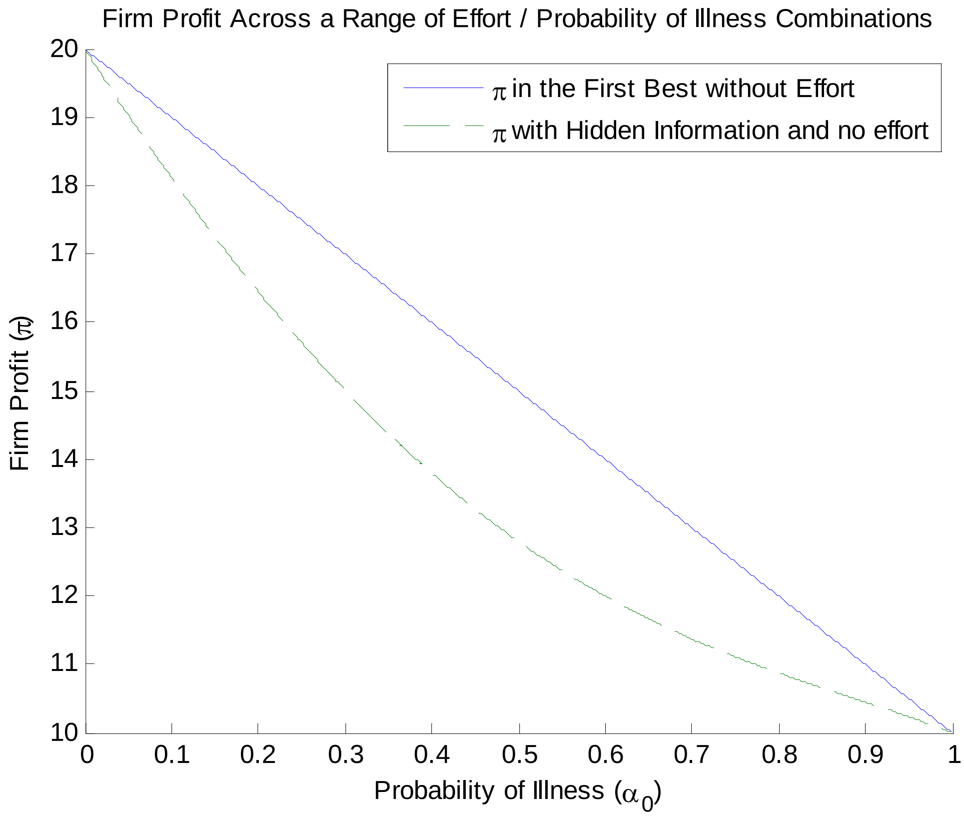


Figure 5. Profit in the case of Hidden Information is strictly less than in the First Best ($\pi \le \pi_{FB}$)

The probability of illness affects profit in the ways one expects. As the probability of illness limits to one ($\alpha \to 1$) or limits to zero ($\alpha \to 0$), the difference in compensation between sick and healthy workers falls to zero.

An important and not entirely intuitive outcome arises as the difference in the marginal cost of production between the healthy and sick state grows to infinity $(c_s - c_h) \to \infty$. In this case, the employer eventually asks for no work from sick employees and thus reduces the bonus for healthy employees to zero. This may seem counterintuitive. However, this means that if when illness strikes it is incapacitating, and this is known to the employer, then the profit from getting any work at all from a sick worker goes to zero. Thus, the employer will ask for no labor from sick workers and will just send them home. Sending sick workers home allows the employer to pay healthy workers $q_s(c_s - c_h) = 0 \cdot (c_s - c_h) = 0$ (i.e. no additional compensation). This also increases the firm profit in this case such that it is closer to that in the First Best. In fact, $\lim_{c_s - c_h \to \infty} \pi(p_s, q_s, p_h, q_h) = \pi_{FB}$. This interesting observation deserves its own remark.

Conclusion 2. Firms prefer severe illness over moderate illness in cases involving hidden information. Of course, they prefer light illnesses as well.

These conclusions simply summarizes the fact that $\lim_{c_s - c_h \to \infty} \pi(p_s, q_s, p_h, q_h) = \pi_{FB}$ and that $\lim_{c_s - c_h \to 0} \pi(p_s, q_s, p_h, q_h) = \pi_{FB}$.

The Effort Model

Until this point, only the very limited number of occupations in which the likelihood of illness cannot be changed through effort have been considered. However, the number of such jobs is small. Rather, the possibility that employees could exert effort to reduce the probability of illness is now introduced. As before, the model is built up from a simple framework to more complicated cases.

Employer Chooses Effort

In the most basic case, there is no hidden information and the employer decides how much effort a worker will exert to reduce his or her probability of becoming ill. Because there is no hidden information, the employer can observe the true state of health of the employee when he or she comes to work and assign that worker to the appropriate task.

When the employer chooses effort, the only consideration is how much profit can be produced given the constraint that no utility (U_s, U_h and U_0) can be less than zero.

Since the employer chooses, one can deduce that the maximum profit is achieved when $U_0 = 0$. Why? It was already shown that in the case where no effort is exerted, the employer will give $U_0 = 0$ as compensation. This establishes a baseline profit, π_{FB}, which is the minimum the firm will make in profit from the work of an employee. Now, the employer will only choose to increase U_s or U_h if the resulting profit, $\pi_{FB(\varepsilon)}$, is greater than this baseline. But the only way for the firm to improve this profit is if the employee exerts effort, which reduces U_0. Thus, the employer will increase U_s or U_h only to just exactly offset the cost of effort and so $U_0 = 0$.

Already, one can see that the optimal effort, ε^*, will depend on the effectiveness of effort, the marginal cost of effort, the initial probability of illness and somehow on the difference in the marginal cost of labor for a sick worker or a healthy worker. If effort becomes more effective at reducing the probability of illness, then the same amount of effort will suddenly yield a greater reduction in the probability of becoming ill, inducing a positive wealth effect. The net result should thus actually be a reduction in effort. If the cost of effort rises one certainly expects the firm to scale back the amount of effort it demands, since each unit of effort must be offset with a corresponding increase in payment, the firm will have the entire burden of the higher marginal cost passed on from the employee. So the amount of effort expected should be reduced. If the initial probability of illness is low, then effort will do less to reduce it in comparison to its initial value than if the initial probability of illness is high. So a lower initial probability of illness should lead to a lower effort. Finally, if the difference between the marginal cost of labor for healthy and sick workers is small, there is little incentive for the employer to choose the healthy state over the unhealthy one. Alternately, if healthy workers yield a big return over sick workers, one expects the company will want to induce much more health. So effort should increase as the disparity between marginal costs grows.

Beginning with the constraints and fundamental equations, one finds that

$$\varepsilon^* = \max\left\{\frac{1}{k}\cdot\ln\left(\frac{\alpha_0\cdot k}{d_0}\cdot\left[\frac{1}{c_h}-\frac{1}{c_s}\right]\right),\quad 0\right\}$$

A result derived fully in the appendix. This implies that there is a point, or threshold inherent probability of illness, below which the employer will not have the employee exert any effort. That value is readily taken from ε^* and is

$$\alpha_0 \geq \left[\frac{k}{d_0}\right]^{-1} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]^{-1}$$

Which we have written this way to emphasize two very important quantities which appear throughout the paper. $\frac{k}{d_0}$ is the "cost effectiveness" of exerting effort to reduce the probability of illness, as it is the direct ratio of the marginal effectiveness of effort over the marginal cost of effort. The other quantity, $\frac{1}{c_h} - \frac{1}{c_s}$, is the absolute severity of illness. This term is a measure of the difference in the difficulty of working when sick and when healthy, and it influencing employee and employer decisions under a wide array of circumstances. Now, for probabilities of illness below value listed above, the employer will simply select the First Best without effort.

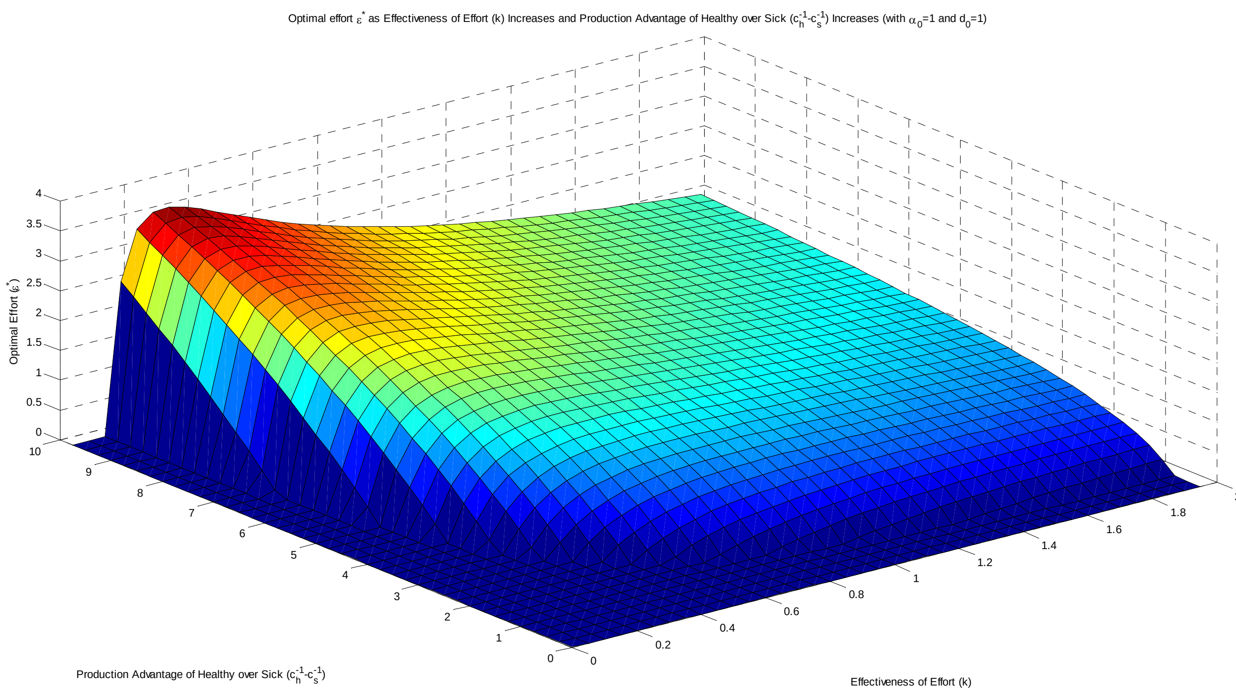


Figure 6. Optimal Efforts over a range of effort effectiveness and production advantage

The optimal effort derived from the equations behaves just as economic intuition indicates it should. But the question remains: How do firms distribute compensation? After all, just because $U_0 = 0$ does not mean that $U_s = 0$ or $U_h = 0$. In fact, since effort reduces U_0, for positive effort, one of these must be positive.

Let $\alpha^* = \alpha(\varepsilon^*)$ and $d^* = d(\varepsilon^*)$Note that $0 = \alpha^* \cdot U_s + (1-\alpha^*) \cdot U_h - d^*$ implies $d^* = \alpha^* \cdot U_s + (1-\alpha^*) \cdot U_h$. Then the question is, given that $U_s \geq 0$, $U_h \geq 0$ and $d^* = \alpha^* \cdot U_s + (1-\alpha^*) \cdot U_h$ where $\{\alpha^*, d^*\}$ are fixed, how many solutions exist? The answer is infinitely many. The bounds are established by looking at the extremes, in which $U_s = 0$ or $U_h = 0$.

Suppose $U_s = 0$. Then $d^* = (1-\alpha^*) \cdot U_h$ and $\dfrac{d^*}{1-\alpha^*} = U_h$.

Suppose $U_h = 0$. Then $d^* = \alpha^* \cdot U_s$ and $\dfrac{d^*}{\alpha^*} = U_s$.

Any combination of payments which satisfies $d^* = \alpha^* \cdot U_s + (1-\alpha^*) \cdot U_h$ at ε^* will yield the same profit for the firm and it will be the maximum. Thus,

$$U_s \in \left[0, \frac{d^*}{\alpha^*}\right] \text{ and } U_h \in \left[\frac{d^*}{1-\alpha^*}, 0\right]$$

There is an underlying reason for this. The contract is signed before the state of illness is known and since the firm chooses the amount of effort exerted, as long as it can make the expected value of signing the contract at least zero, an employee will sign the contract. Though employees pay a fixed cost before they discover they are ill, and therefore could sometimes receive negative overall utility, employees treat effort to remain healthy as a sunk cost. As long as U_s and U_h are each individually at or above zero, then no matter whether the employee transitions into the sick or healthy state, he or she will still want to fulfill the contract, since the act of completing the work has utility greater than or equal to zero. So though the worker utility will vary depending on whether the employee becomes sick or remains healthy, the firm and the employee expect that $U_0 = 0$.

Since the employer can always achieve at least π_{FB} it is clear that $\pi_{FB} \leq \pi_{FB(\varepsilon)}$. The firm does not care which employees, either sick or healthy, receive bonuses to compensation and it does not really make a difference, because expected compensation for employees between this case and the First Best are the same, namely zero. Thus, this case in which employers choose effort with perfect information forms a new First Best for the employer, the First Best with an endogenous probability of illness. This deserves a conclusion, though it may be a rather obvious one:

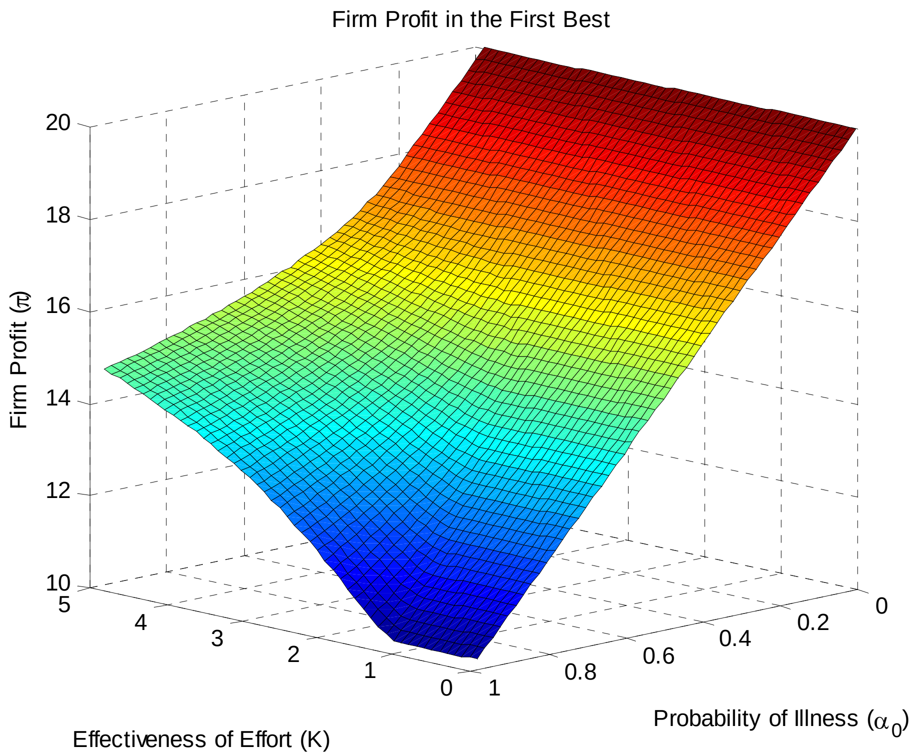


Figure 7. Firm profit in the new First Best. Notice that when the effectiveness of effort is low, $\pi_{FB} \approx \pi_{FB(\varepsilon)}$.

Conclusion 3. Firms prefer the ability to exert effort to reduce the probability of illness, *ceterus paribus.*

This follows from the fact that $\pi_{FB} \leq \pi_{FB(\varepsilon)}$.

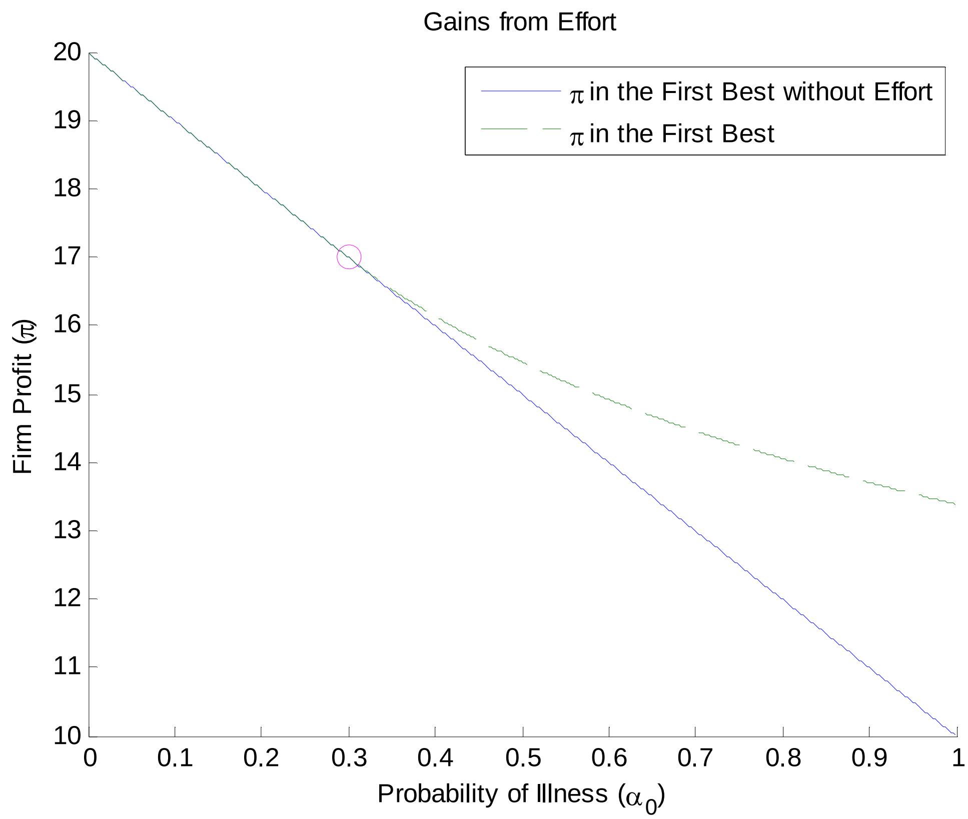


Figure 8. The firm does progressively better compared to the First Best with no effort as the inherent probability of illness rises. Notice $\pi_{FB} \leq \pi_{FB(\varepsilon)}$.

Employee Chooses Effort

With this new case in hand, it seems only reasonable to adjust the model further. Though some occupations give employers control of how much effort workers will exert, a far greater number offer employees the opportunity to select the level of effort themselves. This situation in which employees choose their own level of effort is termed 'hidden action' since the employer cannot directly control or directly observe the level of effort employees choose.

Thus there is a new case to consider. Leaving out hidden information for now, this case allows for hidden action by the employee –- the employer cannot choose or observe the actual effort the employee will exert to remain healthy.

This case is the most general yet and describes a broad range of occupations in which illness can be observed readily because employees are highly supervised. Service jobs, jobs in retail and at restaurants all seem like perfect examples. Professional athletes also provide an excellent model occupation, as the illnesses that matter are injuries, and though the firm cannot know if the employee took every precaution against one, it will be obvious when one has occurred and thus avoid the problem of misrepresenting the true state of health.

In these situations the employee now decides how much effort he or she will exert to reduce his or her probability of becoming ill. For simplicity, let the employer still observe the true state of health of the employee when he or she comes to work and assign that worker to the appropriate task.

When the employee chooses effort, he seeks to maximize his expected utility U_0. This situation is markedly different from when the employer chooses effort. Now, the employer will set effort such that $\frac{\partial U_0}{\partial \varepsilon} = 0$ since doing so will cause the employee to adopt the effort proposed by the employer. Thus, $U_0 = \alpha \cdot U_s + (1-\alpha) \cdot U_h - d$ implies $0 = \alpha' \cdot U_s - \alpha' \cdot U_h - d'$. Then $\frac{d'}{\alpha'} = U_s - U_h$ and this implies that, $\frac{d_0}{k\alpha} = U_h - U_s$ and so $U_s + \frac{d_0}{k\alpha} = U_h$.

Since the employer will choose incentives which minimize U_0, but $U_0 > 0$ (if $\varepsilon \neq 0$) (See Appendix B), the utility when sick will be set to zero, since giving such compensation yields the minimum value of U_s and the minimum value of U_h. Since they are simultaneously minimized when $U_s = 0$ this clearly minimizes U_0, maximizing π and so this will be the distribution of payments. (See Appendix B for why minimizing U_0 maximizes π). Solving for optimal effort in this case reveals that

$$\varepsilon^* = \max\left\{\frac{1}{2k}\cdot\ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h}-\frac{1}{c_s}\right]\right),\quad 0\right\}$$

Which is a value less than or equal to $\varepsilon^*_{FB(\varepsilon)}$. See Appendix A for the derivation of this result and Appendix B for proof the $\varepsilon^* \le \varepsilon^*_{FB(\varepsilon)}$

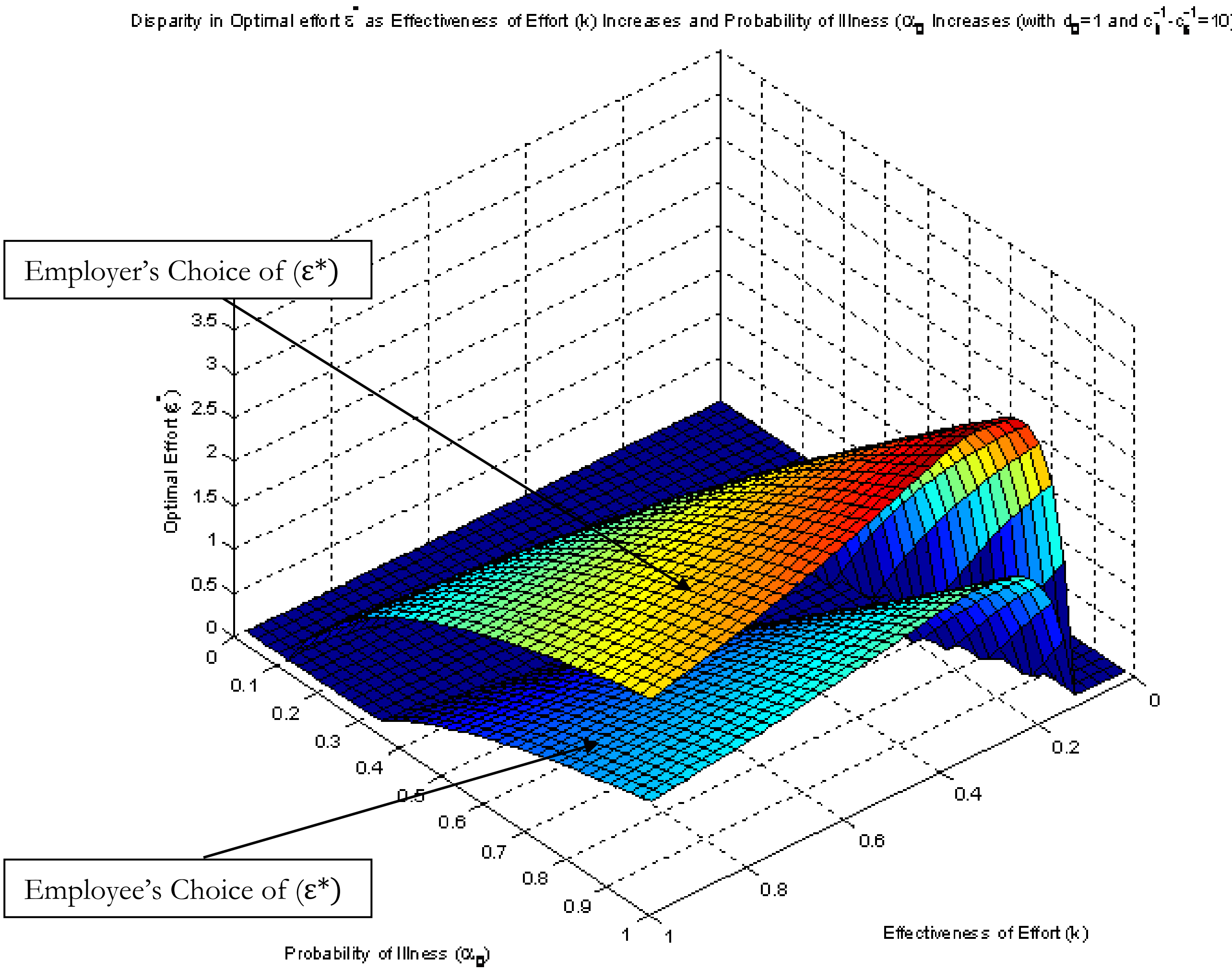


Figure 9. Contrasting the choices of effort when the employer selects effort vs. when the employee selects effort

Now, it is not precisely clear how the bonuses will be distributed, but their derivation is found in Appendix B. Namely,

$$U_h = \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s} \right]} \text{ and } U_s = 0$$

So in this case, healthy workers are rewarded with a bonus while sick workers, though not penalized, receive no additional compensation beyond their base sick pay.

Now, importantly, the optimal effort in this case, which is very similar to the case where the employer chooses the effort, differs from it in a few remarkable ways. Here are a few of the important findings for this case.

Conclusion 4. The optimal effort when the employee selects the effort will always be less than in the case when the employer chooses effort.

This makes sense. Since effort is more costly to the employer because the firm must now offer greater compensation than in the case where the employer chose effort, the amount of effort exerted will always be less than in the first-best. Mathematically, $\varepsilon^* \leq \varepsilon^*_{FB(\varepsilon)}$.

This further implies that firm profit will always be less in this case than in the first best. However, since the employer cannot force $U_0 = 0$, the fact that profit is strictly less makes perfect sense. The bonus must be large enough to attract the employee to offer the necessary amount of effort himself, rather than just big enough to cover the base compensation.

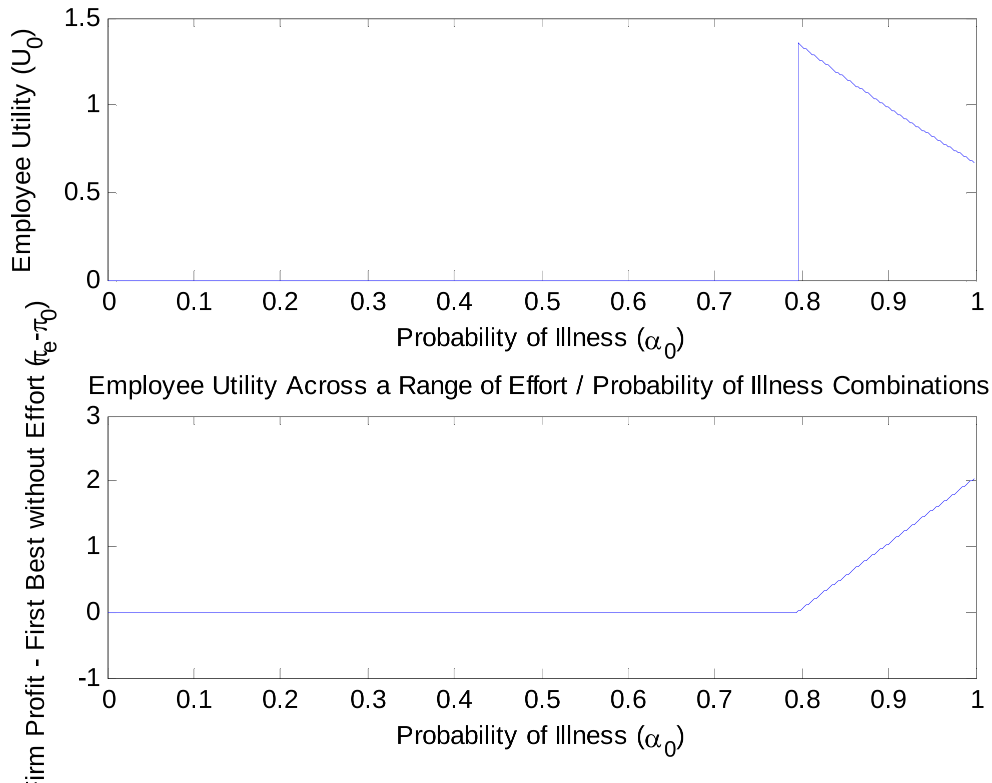


Figure 10. Though firm profit continues to increase relative to the First Best without effort, as the inherent probability of illness increases, the utility the employee receives diminishes.

Conclusion 5. When employees select their own non-zero effort, they are overcompensated, but their compensation diminishes as the inherent probability of illness increases.

This follows from the fact proven in Appendix B that $U_h > 0$ whenever employees select nonzero effort, save for the boundary at which $\alpha_0 = 1$ could, in an extremely unlikely case, potentially have $U_h = 0$. Both this overpayment and diminishing return make sense. The employer makes additional profit on every unit of effort exerted, but employees will not exert effort unless they receive a bonus to increase their efforts as well. In the case in which the employer chooses effort, the employer keeps all the additional profits from effort. In this case, at least some of the additional gains from effort have to be passed to the employee in order to induce him or her to pay the additional cost to exert effort. However, as α_0 increases, the premium the employee receives for effort falls because as the probability of illness increases, the employee's own expected utility falls and the employer does not pay to cover the shortfall, so the employee willing works harder to achieve the same bonus.

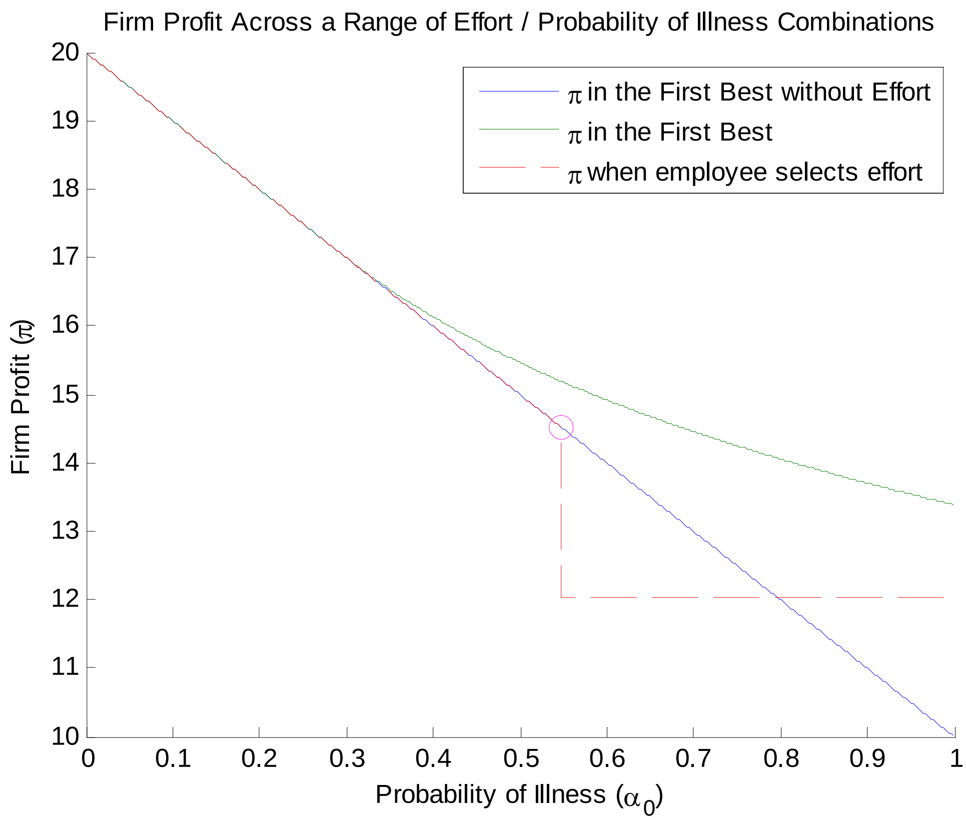


Figure 11. The dotted red line shows firm profit if the firm compensates the employee for effort whenever the employee is *willing* to exert effort, with the discontinuity representing the point at which the employee will work to reduce the probability of illness when offered a bonus. This is *not* the optimal solution however, as the first best with no effort offers the firm greater profit.

Conclusion 6. Firms will only pay employees U_h (to select nonzero effort) if the intrinsic probability of illness α_0 is greater than a threshold. Namely,

$$\alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}.$$

This result is derived fully in Appendix B and it is shown there that if this condition is not met the firm actually does worse paying for non-zero effort.

This means one should take care when talking about ε^*. It is the effort that the firm will seek should the firm seek to incentivize effort, a fact explored in Appendix B. For if the employer chooses to give a bonus to reduce the probability of illness, the employer will only receive an increase in profit related to $\alpha = \dfrac{d_0}{k \cdot U_h}$. This quantity can be adjusted by the employer, but the employer is not bound to it. The employer can choose $U_h = 0$, a case in which the employee will then choose to exert no effort. Thus, if the employer seeks any effort at all the employer will seek effort only if it yields more profit than cost, which occurs only for values of intrinsic probability of illness where $\alpha_0 \geq \sqrt{\dfrac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \dfrac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}$. Otherwise the employer will do better seeking $\varepsilon = 0$.

Conclusion 7. When employees select their own effort, firms receive the same profit in the face of different inherent probabilities of illness α_0 for all nonzero efforts regardless of the specific values of other exogenous variables. Also, profit in the case where employees select effort is always less than profit in the First Best with Effort.

This result is derived fully in Appendix B and is truly remarkable. What this finding means is that if the employer wants any effort at all from its employee, the employer must pay a fixed premium which offsets a portion of the profits which would have been gained from a favorable inherent probability of illness α_0. That surplus must be paid to the employee to induce cooperation. This means that the employer only comes away with a portion of the new total output when effort is improved. However, as the inherent probability of illness rises, the portion of this premium that the employee gets to keep diminishes. On the other hand, when the employer first offers compensation, the employee receives the entire surplus. This makes sense. When the probability of illness is reduced, employees must exert proportionally more effort but will also gain a greater benefit.

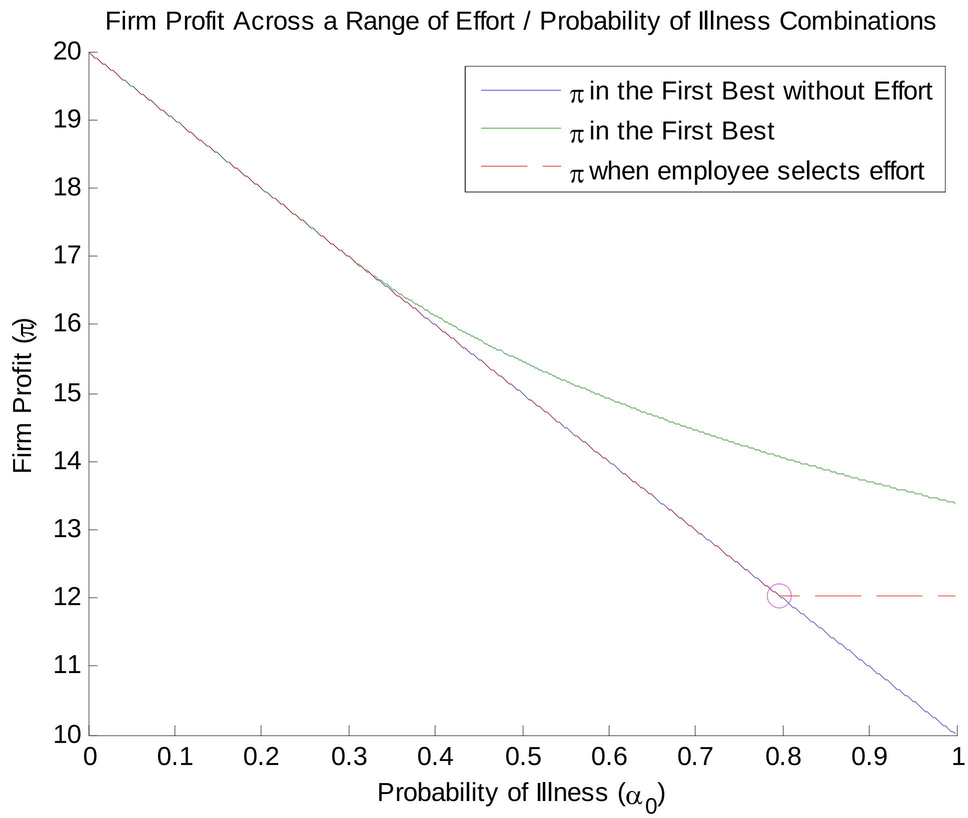


Figure 12. The dotted red line shows firm profit in the case when the employee selects effort. The firm will receive profit equivalent to the first best for most inherent probabilities of illness. However, the point at which the two diverge (circled) is the threshold point outlined in conclusion 7. At this point, it is more profitable for the firm to offer a bonus to the employee to select effort than to pay nothing.

The case in which employees select their own effort presents a list of serious considerations for the employer, but perhaps most important of all is the threshold point

$$\alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}$$

This threshold indicates a profound shift in the behavior of firms when they cannot compel effort. We seek to explore this relationship in more depth. We make the argument that $d_0 \propto c_h$. This is a reasonable assumption: the cost of production when healthy and the cost of effort (which we presume takes place when healthy) both apply to an individual in a healthy state attempting to accomplish work (either reducing their probability of illness or producing a product for the firm). So we make the substitution that $d_0 = c_h \cdot y$ Further, we also note that $c_s \propto c_h$ allowing us to simplify even more by making the substitution that

$c_s = c_h \cdot z$. This simply acknowledges that c_s is "a certain number of times more" than c_h. We find that after such substitutions, and with some rearrangement that

$$\frac{c_h^2 \cdot y \cdot z}{k \cdot (z-1)} \leq 2 - \alpha_0 - 2 \cdot \sqrt{1-\alpha_0}$$

Which can be back-substituted to become

$$\left[\frac{k}{d_0}\right]^{-1} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]^{-1} \leq 2 - \alpha_0 - 2 \cdot \sqrt{1-\alpha_0}$$

Now, to understand how the threshold behaves, we note that we can break the equation into two parts. Namely, we can set $\beta = \frac{k}{d_0}$ which we can think of as the "Cost Effectiveness" of effort, as it is the ratio of the effectiveness of effort to the cost of effort in reducing the probability of illness. The other term $\left[\frac{1}{c_h} - \frac{1}{c_s}\right]$, which has appeared before in this paper, can be thought of as the "Severity of Illness" as it reflects the difference in the marginal profits the firm receives from sick and healthy workers. We can set it to its own variable $\eta = \left[\frac{1}{c_h} - \frac{1}{c_s}\right]$ and rewrite the equation as:

$$\frac{1}{\beta \cdot \eta} \leq 2 - \alpha_0 - 2 \cdot \sqrt{1-\alpha_0}$$

Which is an invertible function (since $1 \geq \alpha_0 \geq 0$ it is $1{:}1$ and onto), and can be rewritten as

$$\alpha_0 \geq 2\sqrt{\frac{1}{\beta \cdot \eta}} - \frac{1}{\beta \cdot \eta}$$

Figure 13 shows how the threshold point behaves when β and/or η are large.

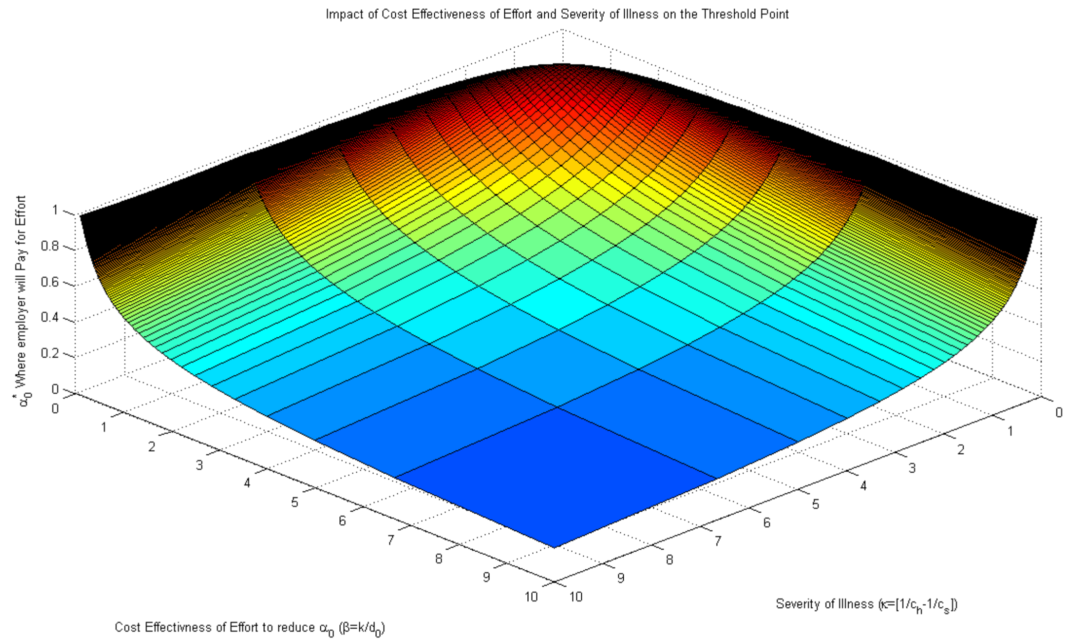


Figure 13. Behavior of the Threshold Point as $\beta \rightarrow \infty$ and as $\eta \rightarrow \infty$ both make sense, as they take the threshold point $\rightarrow 0$.

However, the behavior of real interest in this function occurs when the severity of illness is small, $c_s \approx c_h$. For those points where the severity is small, even as the cost effectiveness of effort becomes large, potentially $\beta >> \eta$, the threshold for which the employer will first choose to give compensation in exchange for effort to reduce the probability of becoming ill remains high, over a large region.

This strongly implies the following conclusion.

Conclusion 8. When employees select their own effort, firms will not give incentives for illnesses which have a low severity of illness, even if there is a high ratio of effectiveness of effort to cost.

This means that if illnesses do not significantly affect work performance, employers will not give incentives for employees, since the employee will instead keep the extra compensation while taking the risk of becoming ill more often. Thus, common illnesses, even with relatively high rates of transmission, like the cold or flu, which may not significantly hinder worker performance, will not receive attention from employers, even if they are easily prevented.

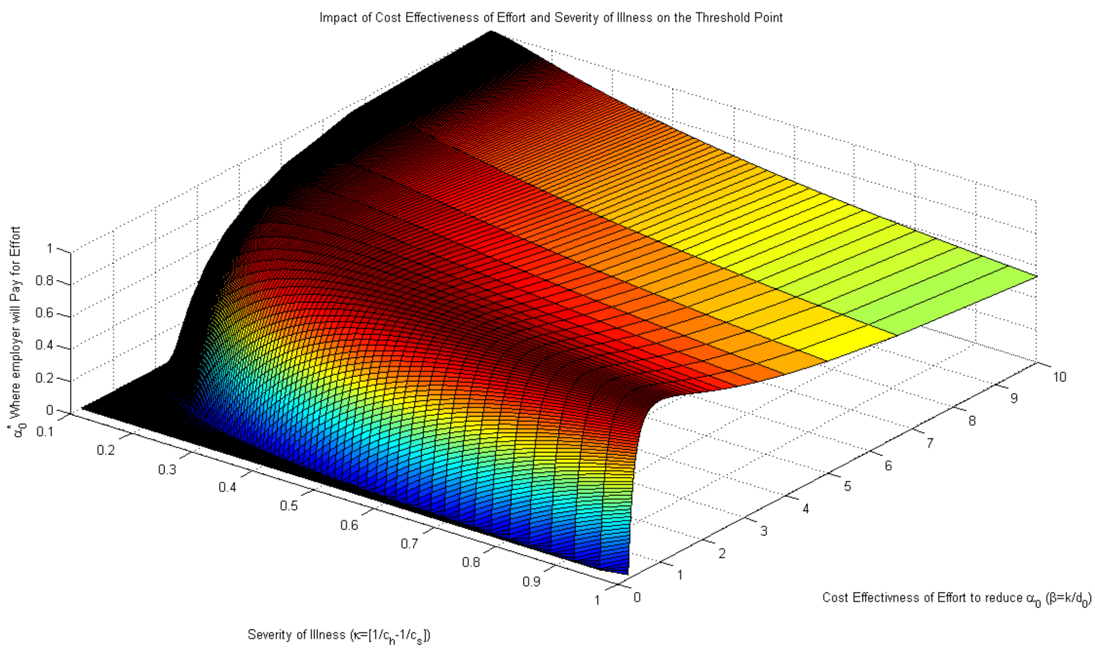


Figure 14. The behavior of the threshold probability when the severity of illness is small, i.e. $c_s \approx c_h$. Notice that as the severity of illness falls, the threshold point rises to nearly 1.

Employer Chooses Effort, but Employees can Lie

The next case returns to the possibility that employees might misrepresent their state of health. In this case, there is no hidden action but there is hidden information—the employer can choose or observe the actual effort the employee will exert to remain healthy, but cannot tell whether the employee actually is sick or healthy once that outcome is realized.

When the employer chooses effort, the only consideration is how much profit can be produced given the constraint that no utility (U_s, U_h and U_0) can be less than zero. The possibility of lying introduces a new constraint and that is that the utility from lying U_{lying} be less than zero. How the employer behaves in the case was previously derived, but the important point is that lying adds the additional constraint that $U_h > q_s \cdot (c_s - c_h)$.

Now in the case where the employer chose the effort, no fixed value for compensation emerged for employees. Indeed, a spectrum of possible combinations of sick and healthy utility emerged. Any combination of payments which satisfies $d = \alpha \cdot U_s + (1-\alpha) \cdot U_h$ at ε^* yielded the same expected maximum profit for the firm. Thus,

$$U_s \in \left[0, \frac{d}{\alpha}\right] \text{ and } U_h \in \left[0, \frac{d}{1-\alpha}\right]$$

Then now that lying is introduced, an important question is, can the employer set the payments such that $U_h \geq p_s - c_h \cdot q_s$ while satisfying any of the values in this spectrum?

Well, suppose $U_s = 0$, in this case U_h attains its maximum value and

$$U_h = \frac{d}{1-\alpha} \text{ (The maximum healthy utility)}$$

Then if the constraint on utility for healthy employees is imposed $\frac{d}{1-\alpha} > p_s - c_h q_s$ which is equivalent to $\frac{d}{1-\alpha} > (c_s - c_h) \cdot q_s$. Since $q_s = \frac{1}{c_s^2}$ in this case, the inequality implies that

$$\frac{d}{1-\alpha} > \frac{c_s - c_h}{c_s^2}$$

If this inequality holds, the employer makes the same profit he would have made had he set the effort without hidden information, since the firm has already chosen a combination of utilities that gives incentives for healthy workers to work as healthy workers without changing the optimal quantities of labor or compensation.

This situation will not always arise though, since there are at least the cases where $d = 0$ since there are cases where $\varepsilon = 0$. In such instances, the inequality is clearly not satisfied.

This is the difficult part. In these cases, the employer must offer an even greater difference than optimal between U_s and U_h. This will induce the firm to choose for employees to exert supra-optimal effort while simultaneously readjusting the quantity of labor demanded from sick workers.

The firm will wish to readjust quantity of labor provided by sick workers down while readjusting effort up. To see this, we note that one must readjust q_s downward at least in some cases, because $U_h = p_s - c_h \cdot q_s = (c_s - c_h) \cdot q_s$ which implies $p_h = (c_s - c_h) \cdot q_s + c_h \cdot q_h$ and so

$$\pi = \alpha \cdot [v(q_s) - c_s \cdot q_s] + (1-\alpha) \cdot [v(q_h) - (c_s - c_h) \cdot q_s - c_h \cdot q_h]$$

Now, this value of q_s depends on α, but α is also itself determined by effort. To figure out this new q_s, note that the equality,

$$U_h = (c_s - c_h) \cdot q_s = \frac{d}{1-\alpha}$$

continues to hold. So via substitution,

$$q_s = \frac{d}{(1-\alpha) \cdot (c_s - c_h)}$$

Then substituting back into the profit function yields

$$\pi = \alpha \cdot \left[2 \cdot \sqrt{\frac{d}{(1-\alpha) \cdot (c_s - c_h)}} - c_s \cdot \frac{d}{(1-\alpha) \cdot (c_s - c_h)} \right] + (1-\alpha) \cdot \left[\frac{1}{c_h} - (c_s - c_h) \cdot \frac{d}{(1-\alpha) \cdot (c_s - c_h)} \right]$$

This simplifies to,

$$\pi = -\alpha \cdot 2 \cdot \sqrt{\frac{d}{(1-\alpha) \cdot (c_s - c_h)}} + \frac{c_s \cdot d}{(1-\alpha) \cdot (c_s - c_h)} + \frac{\alpha}{c_h} - \frac{d \cdot c_h}{c_h - c_s}$$

Using this equation for profit, the optimal effort ε^* can be found numerically which maximizes firm profit. A plot of one such set of maximizations is shown in Figure 15.

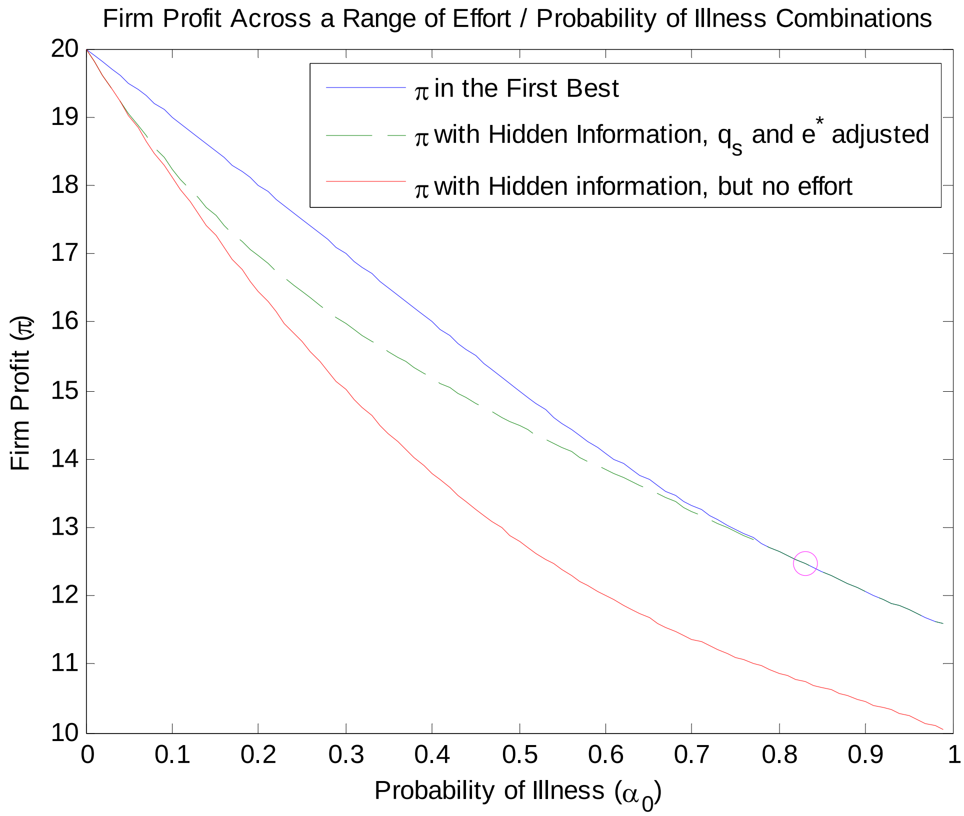


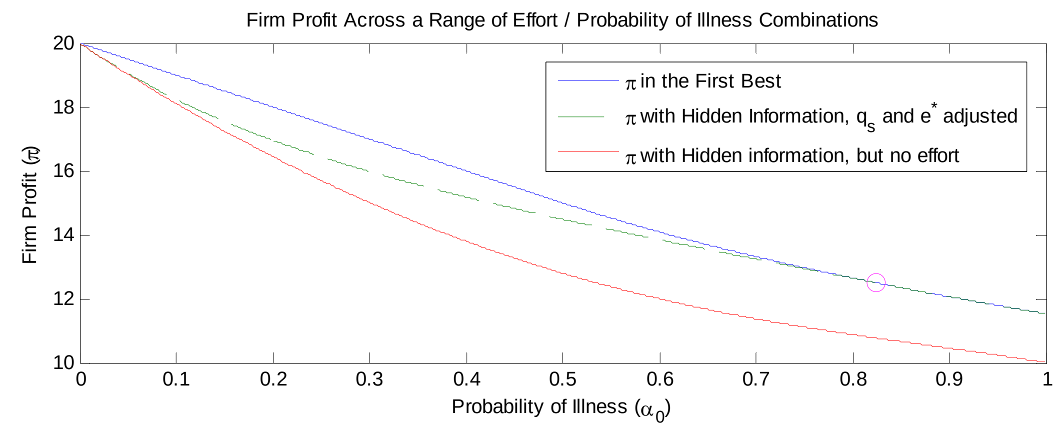
Figure 15. The firm makes profit very close to the profit in the hidden information with no effort case at low inherent probabilities of illness (graph upper left), since very little effort would be demanded. At high levels of inherent probability of illness (graph lower right), once the threshold point (denoted by a circle on the plot above) is crossed, the employer sees no reduction in profit from the first best with effort due to the possibility of hidden information.

Graphing the profit function yields good insight into its behavior, and leads to an important conclusion.

Conclusion 9. Firm profit is strictly higher for hidden information in the case where effort can be exerted and the employer selects it. At low inherent probabilities of illness α_0, where the employer would normally incentivize small amounts of effort, profit is reduced to the case of lying without effort. Alternately, when the firm would already pay a large premium for effort, at high values of α_0, the firm will see no loss at all due to hidden information.

> When the firm seeks a large amount of effort, it will pay an associated premium in order to make sure the constraint $U_0 = 0$ is maintained. As this premium for effort grows because the employer is willing to pay for it anyway, the necessary amount by which the firm must "overpay" healthy workers to insure they do not lie shrinks. This allows the firm to recover the bulk of their extra payment to healthy workers through increased effort by all workers to remain healthy. Once the intrinsic probability of illness is great enough, the firm recovers the First Best entirely.

The graph of the optimal effort ε^* also grants interesting insight. At the threshold point, shown on the graph of the profit function in Figure 16 as well, the effort in this case and in the First Best become and remain equal. However, the relationship of effort in the first best and the case of hidden information leads confirms another conclusion.



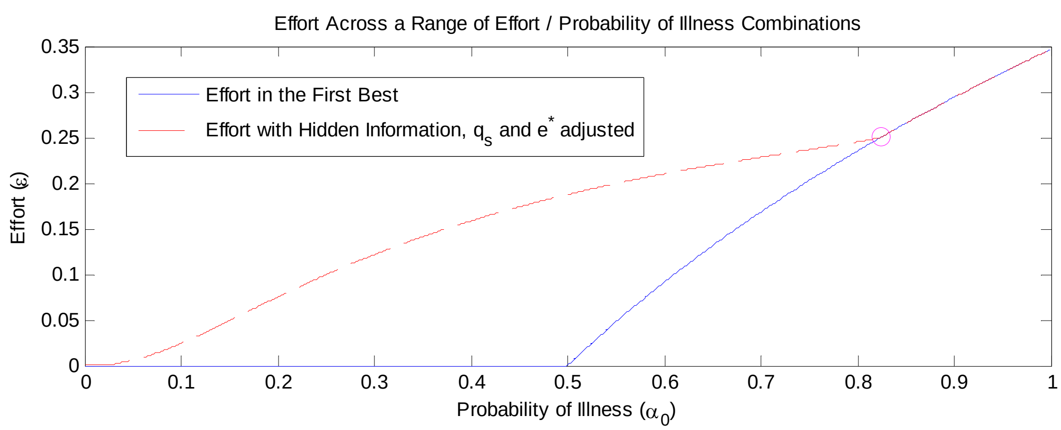


Figure 16. A plot of the effort in the case of hidden information. The effort demanded from employees in the case of hidden information is is greater than or equal to that demanded in the First Best.

Conclusion 10. In cases in which firms can determine effort but cannot determine whether employee's report their state of health honestly, the employer will force excess effort.

Firms are already paying healthy workers a bonus not to misrepresent their true state of health. In the case where the employer selects the level of effort, the firm can recoup at least some of this condition by driving the expected utility for the employee to zero by setting effort more highly.

Thus, employees do not receive excess compensation on average ($U_0 = 0$) because the firm directs that they exert effort which reduces their compensation to zero. This means that in the case where employees can misrepresent their true state of health, the employee comes out no better off than in the First Best, while the firm comes out with less profit, making it obvious that the possibility of lying helps neither the firm nor the employee in the case where the employer selects effort and the employee can lie. This is in contrast to the case where no effort can be exerted, wherein employees are then overcompensated on average.

Employee Chooses Effort and Employee Can Lie

In the final case this paper considers, we introduce a workplace with both hidden action and hidden information, in which the employee selects his or her own level of effort in response to a salary bonus and then may lie when reporting his or her true state of health.

This case could accurately describe jobs in which the majority of the work performed is done independently, without direct supervision to ensure that safety precautions are followed and where the injuries which can subsequently afflict employees are difficult to verify. Companies which come to mind are UPS and FedEx which both have drivers out in the field, lifting heavy packages without direct supervision, with the possibility of back injury, a notoriously difficult injury to diagnose.

These professions present a challenging analysis. It must be considered both that employees are free to select their own level of effort, while also free to subsequently misreport their true state of health.

Proceeding as straightforwardly as possible, one notes that the employer will undoubtedly set $U_s = 0$ by reasoning presented in the Appendix and a previous section. Then any non-zero U_h selected by the employer will conform to the constrain that

$$U_h = -\frac{d'}{\alpha'} \text{ which implies } U_h = \frac{d_0}{k \cdot \alpha}$$

Proceeding similarly to the analysis employed when the employer selected effort in the face of Hidden information, note that if the constraint on utility for healthy employees is imposed $\frac{d_0}{k \cdot \alpha} > p_s - c_h q_s$ which is equivalent to $\frac{d_0}{k \cdot \alpha} > (c_s - c_h) \cdot q_s$.

We would like to presume, as in the previous case, that $q_s = \frac{1}{c_s^2}$, and that the inequality implies that $\frac{d_0}{k \cdot \alpha} > \frac{c_s - c_h}{c_s^2}$. However, this inequality does not hold in general.

Since

$$U_h = \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

At the threshold point where the employer would generally select for the employee to exert effort, for the case in which the employee selects effort, we see that the inequality would be

$$\frac{(c_s - c_h)}{c_s^2} < \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

which does not hold in all circumstances, since it requires that $\frac{(c_s - c_h) \cdot c_h}{c_s^3} < \frac{d_0}{k}$ which may not always be the case. Further, we expect that $q_s < \frac{1}{c_s^2}$ given the behavior of the previous case. Thus the employer will need to actually offer a U_h above the optimal U_h derived in the case where employees select their own effort and profit will be reduced from the case where employee's select effort without lying.

Conclusion 11. Unlike the previous case involving effort and hidden information in the case where employees select effort and can falsely report, the employer cannot achieve the same profit it achieves in the case where it can observe the employee's true state of health.

This also means that firm profit is strictly lower in this case than in the case where the employee selects effort but his state of illness is verifiable. See Figure 17.

The firm will wish to readjust the quantity of labor provided by sick workers down while readjusting effort up. To see this, we note that one must readjust q_s downward at least in some cases. The profit function can be simplified, given that $q_h = \frac{1}{c_h^2}$ and $U_s = 0$. In that case

$$\pi = \alpha \cdot \left[v(q_s) - c_s \cdot q_s\right] + (1-\alpha) \cdot \left[\frac{1}{c_h} - U_h\right]$$

Now, this value of q_s depends on α, but α is also determined by effort. To figure out this new q_s, note that the equality,

$$U_h = (c_s - c_h) \cdot q_s = \frac{d_0}{k \cdot \alpha}$$

continues to hold. So via arithmetic,

$$q_s = \frac{d_0}{\alpha \cdot (c_s - c_h)}$$

Then substituting back into the profit function yields

$$\pi = \alpha \cdot \left[2 \cdot \sqrt{\frac{d_0}{\alpha \cdot (c_s - c_h)}} - c_s \cdot \frac{d_0}{\alpha \cdot (c_s - c_h)}\right] + (1-\alpha) \cdot \left[\frac{1}{c_h} - \frac{d_0}{k \cdot \alpha}\right]$$

Using this equation for profit, the optimal effort ε^* can be found numerically which maximizes firm profit. A plot of one such set of maximizations is shown in Figure 17.

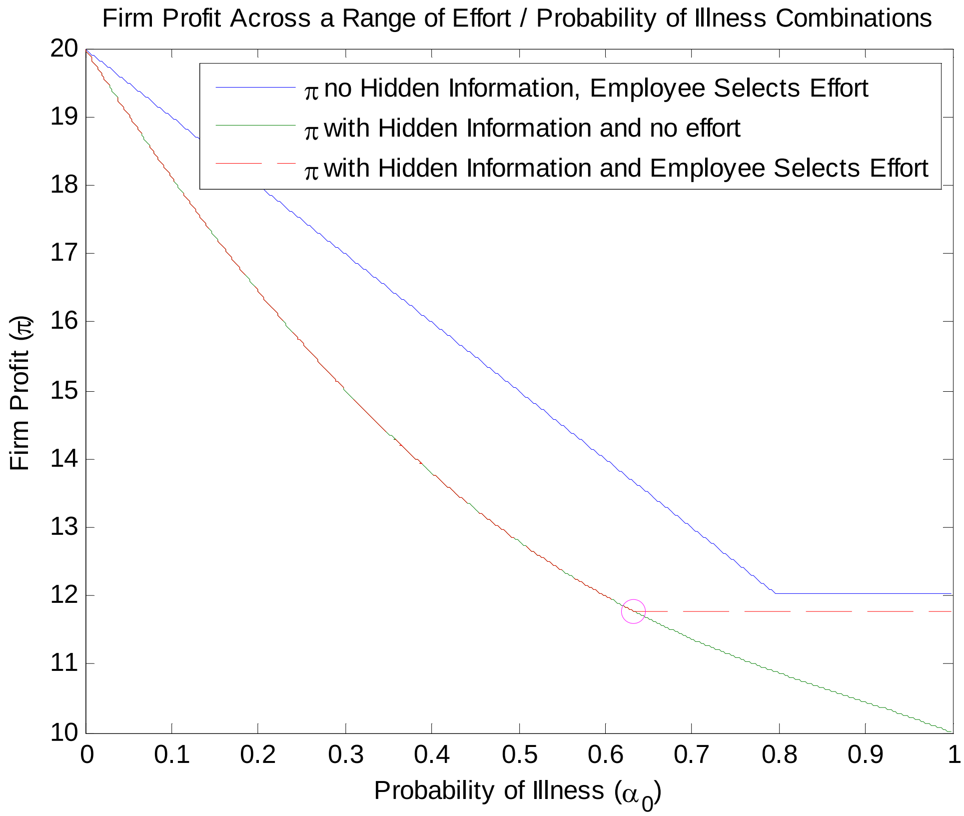


Figure 17. The employer will not be able to incentivize effort, and thus can achieve only the profit above the case of hidden information with no effort, until the probability of illness reaches a threshold (circled) at which the utility bonus the firm wishes to offer to healthy workers is strictly larger than the bonus needed to prevent lying. Then the case behaves similarly to the case in which the employee selects his or her own effort (though at a lower profit).

This case indicates that just like in the previous case in which the employee selected his or her own effort, the employer has a threshold probability of illness, based on the severity of the illness and the cost effectiveness of effort. Though profit for which the employer recovers a profit function similar to that in the case in which there is no possibility of hidden information, that difference seems to be, in general, small compared to the profit which the employer recovers.

Conclusion 12. Once employers are committed to incentivizing employees to exert effort to reduce their probability of illness, the fact that they can misreport makes only a small difference to the employer.

> This means that, though hidden information really hurts the firm's bottom line when the firm would not incentivize effort to reduce illness to begin with, in an industry where an employer would expect to pay employees to make an effort to reduce their probability of illness, the possibility of false reporting does not make a significant difference.

This means that if an employer in the case where an employee's true state of health could be observed knows that it will offer an incentive for effort, the firm will not need to spend time and money to actually verify that employees are sick or healthy, since their bonuses are already sufficient to incentivize honest reporting.

Summary of Key Findings

Figure 18 summarizes the most general findings of the present study. In the situation in which employers have perfect information about the probability of illness of their employees, but there is no possibility of changing the probability of illness, profit's decline linearly as the probability of illness increases. When the probability of illness still cannot be changed, but employees can hide whether they are sick or healthy, employer's cannot achieve the same profit, forced to the convex curve below the first best without effort, and employees are overcompensated. If we consider professions in which steps can be taken to reduce the probability of illness, the employer-employee relationship changes significantly.

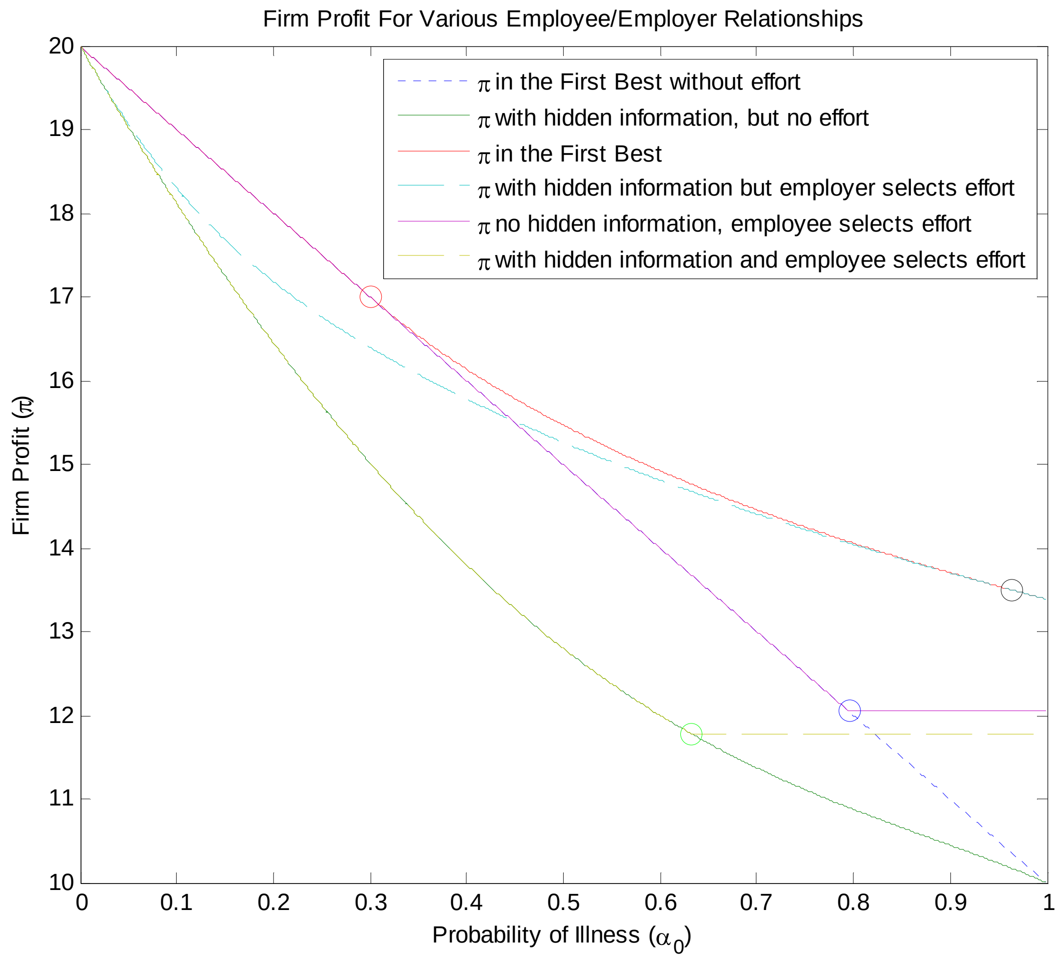


Figure 18. Expected profit for the firm across the range of employer-employee relationships considered in this model.

With perfect information, able to set the amount of effort exerted by employees and to know their true state of illness or health, (labeled "First Best" in Figure 18), employer profits significantly improve over situations in which no effort may be exerted. When the situation changes, and employees select their own effort, though the employer still observes whether employees are sick or healthy, profit is forced to the First Best without effort until a threshold is crossed, at which point employers experience a profit floor. Whenever employees are exerting effort in this case, however, they are overcompensated. When employer's select employee effort, even with the possibility of misreporting, we find that they can recover the First Best as the probability of illness rises. Finally, when employer's cannot select effort and cannot observe the true state of health of their employees, we find that they nevertheless experience a profit floor similar to that observed in the case where employee's choose effort, meaning that for occupations in which effort would normally be incentivized, employer's lose little profit in the face of the possibility of misreporting.

5 Discussion

The purpose of this study was twofold: to develop and evaluate a microeconomic model which accounted for sources of information asymmetry in the employee-employer relationship; and to shed light on the optimal decisions for firms facing various constraints on information in the face of employee illness. The study was motivated by the number and variation in sick-pay incentives packages offered by employers, which cannot be adequately described by prevailing theories. We find that the possibility of misrepresenting one's true state of health and the possibility of reducing the probability of illness through effort both significantly influence and explain incentives firms offer across types of employment, lending insight into the nature and limitations of these incentives.

The model developed here offers an analytical tool for developing tests for variation in actual employee-employer behavior. We find that professions which entail high degrees of supervision and the possibility of aligned incentives between the employer and the employee are the least likely to require health-related bonuses and are unlikely to exhibit high-degrees of absence, since no incentives are required to maintain honest reporting and practices which reduce the probability of illness. This case does not merely describe those who are self-employed but also those employees strongly invested in the success of the company or already receiving performance-based incentives are likely to meet the criteria of these cases. Thus, high ranking executives, managers and administrators are also likely to require the least sick-pay and to take the fewest false sick-days, a finding consistent with common sense and which confirms previous literature. Other types of employment might also be classified under this model: professions in which effort is likely to be unobserved and yet in which misreporting is difficult abound.

Moreover, we find that there exist distinct incentive-regions within occupations, specifically those in which employees select their own effort. These regions lead us to further subdivide those occupations into two distinct categories: jobs which have a significant probability of debilitating illness and jobs which generally contend with illnesses which have lower probabilities of milder ailments. We postulate that employment contracts in professions of the first type will involve significant bonuses for remaining healthy and few restrictions on reporting, while professions of the second type will likely exhibit a great deal of variation in sick-pay incentives and ad-hoc situational approaches to sick pay.

How sports franchises respond to the possibility of player injuries offers a good example of the first type of occupation. Unlike in many other professions, a severe injury is likely to increase the cost of work astronomically, since injured players simply cannot play. In such instances, the analysis of the threshold point in the case where employees choose effort implies a remarkable result. Figure 13 shows that, as the severity of illness $\eta \to \infty$, the threshold probability for which the employer will begin to offer incentives for remaining healthy goes to zero. This means that sports teams in, for instance, the National Basketball Association (the leading professional basketball league in the country), will offer significant bonuses for remaining free of injuries, but will not need to monitor effort, since the incentives are such that a player will exert the effort without supervision as long as a sufficient bonus, posited to be proportional to the severity of the potential injury or illness and inversely proportional to the cost effectiveness of effort, is offered.

For professions of the second type, with lower probabilities of milder ailments, surprisingly, the findings of this thesis suggests that these professions—such as office work, which allow employees great latitude in making effort decisions and which have employers who are unlikely to investigate whether employees are actually sick—will require the most-varied health-related bonuses and will also suffer the most abuse due to improper incentives. All things being equal, the model predicts industries in which illness is easily observed (demanding a high degree of employee supervision) and effort is set by the employer (most likely to be trade and service jobs) to exhibit fewer days of sick leave, controlling for confounding factors, than professions with more independence, since such professions do not allow firms to reduce the probability of illness as much as they would like.

The notion that service professions and professions exhibiting less independence would actually have fewer days of sick leave seems counter intuitive. After all, service occupations usually exhibit lower rates of job satisfaction than other professions. Yet, published figures on professions and their average rates of sick leave from the British Health and Safety Executive support this assertion, as shown in Table 1.

Profession	Average Days of Sick Leave (Per worker)	Nearest Case Described in Thesis
Process, plant and machine operatives	1.21	Verifiable Illness/ Employer Effort
Professional occupations	0.87	Unverifiable Illness/ Employee Effort
Associate professional and technical occupations	1.12	Unverifiable Illness/ Employee Effort
Administrative and secretarial occupations	1.06	Unverifiable Illness/ Employee Effort
Personal service occupations	1.35	Unverifiable Illness/ Employee Effort
Skilled trades occupations	0.85	Unverifiable Illness/ Employer Effort
Sales and customer service occupations	0.65	Unverifiable Illness/ Employer Effort
Elementary occupations	0.92	Unverifiable Illness/ Employer Effort
Managers and senior officials	0.77	Unverifiable Illness/ Employer Effort
All occupations (illness ascribed to the current or most recent job)	0.96	

Table 1. Estimated days (full-day equivalent) off work and associated average days lost per (full-time equivalent) worker due to a self-reported illness caused or made worse by current or most recent job, by occupational major and sub-major group (Source: HSE).

The data compiled by the HSE, though only a first look at the relationship between data and the model's predictions, reveals surprising agreement between the model and actual behavior in the real world. The HSE, taken at a high level of confidence, reveals that the mean of the occupations which have employee-selected effort and unverifiable illness have 1.1 days of sick leave per worker, whereas for employees with employer effort and unverifiable illness, the average number is .80 days of sick leave, indicating that employees in those professions take fewer absences attributable to their work. One might imagine that the case presented by the HSE of verifiable illness and employer effort, skewed to 1.21 days per worker, is influenced more by nature of the work itself (Process, plant and machine operatives) which makes it difficult to compare with other professions which pose inherently lower risks to employee safety and health.

In addition to these predictions, the model also proposes that new key variables be introduced and strongly considered in future analyses of employment incentives, variables which might allow for future empirical evaluation. One is the cost effectiveness of effort, presented here as $\beta = k/d_0$, which strongly influences both employee decisions and employer incentives. Most importantly, in situations involving limited information, the term often acts counter-intuitively. Increasing β encourages employees to exert greater effort, while simultaneously reducing employer incentives. Thus, it turns out that in many cases it is employers that prefer sick-pay packages for illnesses which exhibit high cost-effectiveness of effort (β) while employees favor compensation for illnesses which exhibit low cost-effectiveness.

The other term $\eta = \left[\frac{1}{c_h} - \frac{1}{c_s}\right]$, which can be thought of as the "severity of illness," reflects the difference in the marginal profits the firm receives from sick and healthy workers. One novel finding relating to the severity of illness already mentioned in the results section revealed that the severity of illness strongly influences the behavior of firms and employees when employees select their own effort. When η is sufficiently small, employers are highly unlikely to give incentives for employees to reduce their probability of illness. This means that firms will not seek to reduce the probability of some illnesses in the workplace. This finding lends substantial support to the data presented in the American Productivity Audit. Firms can be complicit in allowing employees to come to work sick more often if their productivity is not significantly hindered. Yet, over time the sum of this lost productive time can become quite substantial.

Nevertheless, the behavior of firms in response to illnesses of low severity was not studied in detail in this paper because one of the major limitations of our model is that it treats firms as having a *single* employee, eliminating the possibility of (and need for) accounting for the negative externality to healthy employees of working next to sick coworkers. Future work on this model might involve treating a workplace with multiple employees whose decisions are influenced by possible illness in their coworkers. Since such a model would have proven significantly more complex, it was not feasible to consider this sort of extension here.

There were two other major limitations on the model presented here. Both involve consideration of work as a multi-stage game. In particular, the "one-shot" employment contract presented in this paper does not fully account for the fact that employment contracts are usually negotiated well in advance, to account for the possibility of illness over the duration of a long employment contract. The introduction of time into the model would also account for the insurance-like nature of many sick-pay policies. In addition, it would take into account the importance employers place on the "time" at which illness is reported—if reported in advance, employers can sometimes recoup at least some of their losses through substitute employees or by diverting other employees to complete some of the absent worker's tasks.

6 Conclusion

We sought to understand how employers should develop incentives packages which reduce the probability of illness and also mitigate the loss from an employee intentionally misrepresenting his or her true state of health. Ultimately, we found that the possibility of misrepresenting one's true state of health and the possibility of reducing the probability of illness through effort both significantly influence and explain the incentives firms offer across many types of employment, lending insight into the nature and limitations of these incentives.

Future empirical work will involve testing many of the key findings in this paper against actual employment contracts. Considering whether the key variables presented here actually significantly influence the nature of these contracts is only one of many possible extensions. Future theoretical work could seek to overcome the limitations of this model, revising the it into a multi-stage situation in which employment contracts are negotiated at different points over the course of employment. Introducing multiple interacting employees would also prove fruitful.

Nevertheless, the present study sets out a novel perspective on employee-employer interactions in response to the possibility of illness and could serve as a good point of departure for new insights into the problem of selecting optimal sick pay policies.

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8 Appendix A (Cases)

First Case. Exogenous Probability of Illness, No Hidden Information

Begin with the profit maximization problem for the employer.

$$\max_{(p_h, q_h, p_s, q_s)} \quad \pi = \alpha_0 \cdot [v(q_s) - p_s] + (1-\alpha_0) \cdot [v(q_h) - p_h]$$

Subject to

i. $p_s - c_s \cdot q_s \geq 0$

ii. $p_h - c_h \cdot q_h \geq 0$

Then the employer will seek to offer $U_0 = 0$ and since $U_s \geq 0$ and $U_h \geq 0$ the payments are $p_h = c_h \cdot q_h$ and $p_s = c_s \cdot q_s$. Then, substitution yields,

$$\pi = \alpha \cdot [v(q_s) - c_s \cdot q_s] + (1-\alpha) \cdot [v(q_h) - c_h \cdot q_h]$$

Then maximizing with respect to q_s gives,

$$\frac{\partial \pi}{\partial q_s} = 0 = \alpha \cdot [v'(q_s) - c_s] = v'(q_s) - c_s$$

This implies $v'(q_s) = c_s$ which means since $v'(q_s) = \frac{1}{\sqrt{q_s}}$ that $q_s^* = \frac{1}{c_s^2}$

The same operation on q_h gives,

$$\frac{\partial \pi}{\partial q_h} = 0 = (1-\alpha_0) \cdot [v'(q_h) - c_h]$$

This implies $v'(q_h) = c_h$ which means since $v'(q_h) = \frac{1}{\sqrt{q_h}}$ that $q_h^* = \frac{1}{c_h^2}$

So the optimal quantities are $q_h^* = \frac{1}{c_h^2}$ and $q_s^* = \frac{1}{c_s^2}$.

This further implies that $\pi = \alpha_0 \cdot \frac{1}{c_s} + (1-\alpha) \cdot \frac{1}{c_h} = \frac{1}{c_h} - \alpha_0 \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]$

Second Case. Hidden Information , Exogenous Probability of Illness

Begin with the profit maximization problem for the employer.

$$\max_{(p_h, q_h, p_s, q_s)} \quad \pi = \alpha_0 \cdot [v(q_s) - p_s] + (1-\alpha_0) \cdot [v(q_h) - p_h]$$

Subject to

i. $p_s - c_s \cdot q_s \geq 0$

ii. $p_h - c_h \cdot q_h \geq 0$

iii. $p_s - c_s \cdot q_s \geq p_h - c_s \cdot q_h$

iv. $p_h - c_h \cdot q_h \geq p_s - c_h \cdot q_s$

Then the smallest possible payment to sick workers is $p_s = c_s \cdot q_s$ and since sick workers will not cheat because they have higher costs of effort this constraint binds. Then to find a binding constraint on p_h simply reduce it to the smallest possible value in (iv) which yields,

$$p_h = c_s \cdot q_s - c_h \cdot q_s + c_h \cdot q_h$$

Then substitution gives:

$$\pi = \alpha_0 \cdot [v(q_s) - c_s \cdot q_s] + (1-\alpha_0) \cdot [v(q_h) - c_s \cdot q_s - c_h \cdot q_h + c_h \cdot q_s]$$

Then solving for the profit maximizing value of q_h gives,

$$\frac{\partial \pi}{\partial q_h} = 0 = (1-\alpha_0) \cdot [v'(q_h) - c_h]$$

This implies $v'(q_h) = c_h$ which means since $v'(q_h) = \frac{1}{\sqrt{q_h}}$ that $q_h^* = \frac{1}{c_h^2}$

The same operation on q_s yields,

$$\frac{\partial \pi}{\partial q_s} = 0 = \alpha_0 \cdot [v'(q_s) - c_s] + (1-\alpha_0) \cdot [c_h - c_s]$$

And so

$$v'(q_s) = \frac{(1-\alpha_0) \cdot [c_s - c_h]}{\alpha_0} + c_s$$

With some rearrangement looks like,

$$q_s^* = \frac{\alpha_0^2}{(c_s - (1-\alpha) \cdot c_h)^2}$$

So the optimal quantities are $q_s^* = \dfrac{\alpha_0^2}{\left(c_s - (1-\alpha_0)\cdot c_h\right)^2}$ and $q_h^* = \dfrac{1}{c_h^2}$

Further, $U_h = \dfrac{\alpha_0^2 \cdot (c_s - c_h)}{\left(c_s - (1-\alpha_0)\cdot c_h\right)^2}$ and $U_s = 0$ and $U_0 = \dfrac{\alpha_0^2 (1-\alpha_0)(c_s - c_h)}{\left(c_s - (1-\alpha)\cdot c_h\right)^2}$

Finally,

$$\pi = \frac{1}{c_h} - \alpha_0 \cdot \left(\frac{1}{c_h}\right) + \frac{\alpha^2}{(1-\alpha)\cdot c_h + c_s}$$

Third Case: Introducing Effort

Begin with the profit maximization problem for the employer.

$$\max_{(p_h, q_h, p_s, q_s)} \quad \pi = \alpha(\varepsilon)\cdot\left[v(q_s) - p_s\right] + (1-\alpha(\varepsilon))\cdot\left[v(q_h) - p_h\right]$$

Subject to

i. $U_0 \geq 0$
ii. $U_s \geq 0$
iii. $U_h \geq 0$
iv. $\varepsilon, p_h, q_h, p_s, q_s \geq 0$

$U_0 = 0$ is a binding constraint yields,

$$U_0 = 0 = \alpha(\varepsilon)\cdot U_s + (1-\alpha(\varepsilon))\cdot U_h - d(\varepsilon)$$
$$\Rightarrow U_h = \frac{d(\varepsilon) - \alpha(\varepsilon)\cdot U_s}{1-\alpha(\varepsilon)}$$

Then substitution into π gives,

$$\begin{aligned}
\pi &= \alpha(\varepsilon)\cdot\left[v(q_s) - U_s - c_s\cdot q_s\right] + (1-\alpha(\varepsilon))\cdot\left[v(q_h) - U_h - c_h\cdot q_h\right] \\
&= \alpha(\varepsilon)\cdot\left[v(q_s) - U_s - c_s\cdot q_s\right] + (1-\alpha(e))\cdot\left[v(q_h) - \frac{d(\varepsilon)}{1-\alpha(\varepsilon)} + \frac{\alpha(\varepsilon)\cdot U_s}{1-\alpha(\varepsilon)} - c_h\cdot q_h\right] \\
&= \alpha(\varepsilon)\cdot\left[v(q_s) - c_s\cdot q_s\right] - \alpha(\varepsilon)\cdot U_s + (1-\alpha(\varepsilon))\cdot\left[v(q_h) - c_h\cdot q_h\right] + \alpha(\varepsilon)\cdot U_s - d(\varepsilon) \\
&= \alpha(\varepsilon)\cdot\left[v(q_s) - c_s\cdot q_s\right] + (1-\alpha(\varepsilon))\cdot\left[v(q_h) - c_h\cdot q_h\right] - d(\varepsilon)
\end{aligned}$$

With dependence only on q_h, q_s, e. Then solving for the profit maximizing values yields,

$$\frac{\partial \pi}{\partial q_s} = 0 = \alpha(\varepsilon)\cdot\left[v'(q_s) - c_s\right] = v'(q_s) - c_s$$

This implies $v'(q_s) = c_s$ which means since $v'(q_s) = \frac{1}{\sqrt{q_s}}$ that $q_s^* = \frac{1}{c_s^2}$

The same operation on q_h gives,

$$\frac{\partial \pi}{\partial q_h} = 0 = (1 - \alpha(\varepsilon)) \cdot [v'(q_h) - c_h]$$

This implies $v'(q_h) = c_h$ which means since $v'(q_h) = \frac{1}{\sqrt{q_h}}$ that $q_h^* = \frac{1}{c_h^2}$

Now substituting into π for q_h and q_s and taking a derivative with respect to e yields,

$$\begin{aligned}\frac{\partial \pi}{\partial \varepsilon} = 0 &= \alpha'(\varepsilon) \cdot \left[\frac{1}{c_s}\right] - \alpha'(\varepsilon) \cdot \left[\frac{1}{c_h}\right] - d'(\varepsilon) \\ &= \alpha'(\varepsilon) \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - d'(\varepsilon) \\ &= -k \cdot \alpha(\varepsilon) \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - d_0\end{aligned}$$

Finally, rearranging and solving for ε,

$$\varepsilon = \frac{1}{k} \cdot \ln\left(\frac{\alpha_0 \cdot k}{d_0} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right)$$

Effort must be greater than zero by constraint, so the final form for effort is,

$$\varepsilon^* = \max\left\{\frac{1}{k} \cdot \ln\left(\frac{\alpha_0 \cdot k}{d_0} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right), \quad 0\right\}$$

Fourth Case: Employee Chooses Effort

Begin with the profit maximization problem for the employer.

$$\max_{(p_h, q_h, p_s, q_s)} \quad \pi = \alpha(\varepsilon) \cdot [v(q_s) - p_s] + (1 - \alpha(\varepsilon)) \cdot [v(q_h) - p_h]$$

Subject to

i. $U_0 \geq 0$
ii. $U_s \geq 0$
iii. $U_h \geq 0$
iv. $\varepsilon, p_h, q_h, p_s, q_s \geq 0$

With the additional constraint that

v. $\dfrac{\partial U_0}{\partial \varepsilon} \geq 0$

Then, given $U_0 = \alpha \cdot U_s + (1-\alpha) \cdot U_h - d$ implies $0 \leq \alpha' \cdot U_s - \alpha' \cdot U_h - d'$ then

$\dfrac{d'}{\alpha'} \leq U_s - U_h$ and this implies that, $\dfrac{d_0}{k\alpha} \geq U_h - U_s$ and so,

$$U_s + \frac{d_0}{k\alpha} \leq U_h$$

Now from earlier solutions, it is clear that $v'(q_s) = \dfrac{1}{\sqrt{q_s}}$ and that $q_s^* = \dfrac{1}{c_s^2}$. Additionally, we

know that $v'(q_h) = \dfrac{1}{\sqrt{q_h}}$ that $q_h^* = \dfrac{1}{c_h^2}$

Thus, the profit function becomes $\pi = \alpha \cdot \left[\dfrac{1}{c_s}\right] + (1-\alpha) \cdot \left[\dfrac{1}{c_h}\right] - d - U_0$ which is the same as

$$\pi = \alpha \cdot \left[\frac{1}{c_s}\right] + (1-\alpha) \cdot \left[\frac{1}{c_h}\right] - \alpha \cdot U_s - (1-\alpha) \cdot U_h$$

Which reduces to (since $U_s = 0$),

$$\pi = \alpha \cdot \left[\frac{1}{c_s}\right] + (1-\alpha) \cdot \left[\frac{1}{c_h}\right] - (1-\alpha) \cdot U_h$$

Additionally, since $U_s = 0$ then $U_s + \dfrac{d_0}{k\alpha} \leq U_h$ becomes $U_h \geq \dfrac{d_0}{k\alpha}$ (i.e. $\dfrac{d'}{\alpha'} \leq U_s - U_h$

becomes $-\dfrac{d'}{\alpha'} \geq U_h$) And so the profit function is further reduced to

$$\pi = \alpha \cdot \left[\frac{1}{c_s}\right] + (1-\alpha) \cdot \left[\frac{1}{c_h}\right]_s - (1-\alpha) \cdot \left[-\frac{d'}{\alpha'}\right]$$

Now, maximizing the profit function with respect to effort, we take the derivative which yields,

$$\frac{\partial \pi}{\partial \varepsilon} = \alpha' \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] + \left(\frac{d'}{\alpha'}\right)' - \left(\alpha \cdot \frac{d'}{\alpha'}\right)'$$

Which becomes:

$$\frac{\partial \pi}{\partial \varepsilon} = \alpha' \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] + \left(\frac{\alpha' d'' - d'\alpha''}{\alpha'^2}\right) - \alpha' \cdot \frac{d'}{\alpha'} - \alpha \cdot \left(\frac{\alpha' d'' - d'\alpha''}{\alpha'^2}\right)$$

$$\frac{\partial \pi}{\partial \varepsilon} = \alpha' \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] + (1-\alpha) \cdot \left(\frac{\alpha' d'' - d'\alpha''}{\alpha'^2}\right) - d'$$

Recalling that $\alpha = \alpha_0 e^{-k\varepsilon}$ and $\alpha' = -k\alpha_0 e^{-k\varepsilon}$ and $\alpha'' = k^2 \alpha_0 e^{-k\varepsilon}$ and $d = d_0 \varepsilon$ and $d' = d_0$ and $d'' = 0$

$$\frac{\partial \pi}{\partial \varepsilon} = -k\alpha \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] + (1-\alpha) \cdot \left(\frac{0 - d_0 k^2 \alpha}{k^2 \alpha^2}\right) - d_0$$

Which reduces to

$$\frac{\partial \pi}{\partial \varepsilon} = -k\alpha \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - \frac{d_0 k^2 \alpha}{k^2 \alpha^2} + \alpha \cdot \frac{d_0 k^2 \alpha}{k^2 \alpha^2} - d_0$$

$$\frac{\partial \pi}{\partial \varepsilon} = -k\alpha \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - \frac{d_0}{\alpha}$$

So,

$$0 = -k\alpha^2 \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - d_0$$

Which implies,

$$\alpha = \sqrt{\frac{d_0}{k \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}}$$

And so we solve noting,

$$\alpha_0 e^{-k\varepsilon} = \sqrt{\frac{d_0}{k \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}} \text{ which implies } e^{-k\varepsilon} = \sqrt{\frac{d_0}{\alpha_0^2 k \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}}$$

And so

$$\varepsilon = -\frac{1}{k}\ln\left(\sqrt{\frac{d_0}{\alpha_0^2 k \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}}\right) \text{ and so } \varepsilon = -\frac{1}{2k}\ln\left(\frac{d_0}{\alpha_0^2 k \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}\right)$$

Which implies

$$\varepsilon^* = \frac{1}{2k}\ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right)$$

Effort must be greater than zero by constraint, so the final form for effort is,

$$\varepsilon^* = \max\left\{\frac{1}{2k}\ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right), \quad 0\right\}$$

Profit in this case can be written explicitly as,

$$\pi_\varepsilon = \begin{cases} \pi_{FB} & \text{if} \quad \alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)} \\ 2 \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \cdot \left(\frac{k}{d_0} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right]\right)^{-1/2} - \frac{d_0}{k} + \frac{1}{c_h} & \text{if} \quad \alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)} \end{cases}$$

Where only π_{FB} and the threshold value depend on α_0. (See Appendix B).

9 Appendix B (Additional Considerations)

Why Minimizing Expected Utility Maximizes Firm Profit

Let U_0 be the expected utility for an employee and let π be the expected profit for the firm. Then by equation,

$$U_0 = \alpha \cdot U_s + (1-\alpha) \cdot U_h - d$$

and

$$\pi = \alpha \cdot [v(q_s) - p_s] + (1-\alpha) \cdot [v(q_h) - p_h]$$

With some rearrangement, this can be rewritten as

$$\pi = \alpha \cdot [v(q_s) - U_s - c_s \cdot q_s] + (1-\alpha) \cdot [v(q_h) - U_h - c_h \cdot q_h]$$

And this can be rewritten as

$$\pi = \alpha \cdot [v(q_s) - c_s \cdot q_s] + (1-\alpha) \cdot [v(q_h) - c_h \cdot q_h] - \alpha \cdot U_s - (1-\alpha) \cdot U_h$$

Now, U_s and U_h can be replaced with U_0 and d,

$$\pi = \alpha \cdot [v(q_s) - c_s \cdot q_s] + (1-\alpha) \cdot [v(q_h) - c_h \cdot q_h] - d - U_0$$

This can be further simplified to,

$$\pi = \pi_0 - U_0$$

Where π_0 would be the profit for the firm if $U_0 = 0$.This result makes clear that any non-zero expected utility for the employee directly reduces firm profit, so π is maximized when U_0 is minimized.

The Threshold where the Employer selects Nonzero Effort in the First Best

The firm will not have the employee exert any effort if the quantity $\frac{\alpha_0 \cdot k}{d_0} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right] < 1$ which implies $\alpha_0 < \frac{d_0}{k}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]^{-1}$. Thus, the firm will only have the employee exert effort if $\alpha_0 \geq \frac{d_0}{k}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]^{-1}$.

Why Employee Selected Effort is Less than or Equal to Employer Selected Effort

Let $\varepsilon^*_{FB(\varepsilon)} = \max\left\{\frac{1}{k} \cdot \ln\left(\frac{\alpha_0 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right), \quad 0\right\}$ be the effort the employer selects and let $\varepsilon^* = \max\left\{\frac{1}{2k} \cdot \ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right), \quad 0\right\}$ be the effort the employee selects.

First consider the interior of the natural logarithms. We assert that

$$\frac{\alpha_0 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right] \geq \frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]$$

And proceed to prove it. Since the above implies, that $\frac{\alpha_0 k}{d_0} \geq \frac{\alpha_0^2 k}{d_0}$ which implies $\alpha_0 k \geq \alpha_0^2 k$ which implies $\alpha_0 \geq \alpha_0^2$ which implies $1 \geq \alpha_0$. Since $\alpha_0 \in [0,1]$, this inequality holds, so this is proven. Since $\ln$ is a monotonically increasing function we can state that

$$\ln\left(\frac{\alpha_0 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right) \geq \ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right)$$

This also means that, without loss of generality, if $\varepsilon^*_{FB(\varepsilon)} = 0$ then $\varepsilon^* = 0$ so those cases in which one or both equal zero are accounted for. Finally, since $\frac{1}{k} \geq \frac{1}{2k}$ we can multiply appropriately to find

$$\frac{1}{k} \cdot \ln\left(\frac{\alpha_0 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right) \geq \frac{1}{2k} \cdot \ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right)$$

Which is what we sought to show.

More On the Effort when Employee Selects Effort: Why the Employer will not always compensate for ε*

We ask, given a certain bonus, how much effort will an employee choose to exert?

Knowing the constraint that if $U_s = 0$ then $U_s + \frac{d_0}{k\alpha} = U_h$ becomes $U_h = \frac{d_0}{k\alpha}$ (i.e. $\frac{d'}{\alpha'} = U_s - U_h$ becomes $-\frac{d'}{\alpha'} = U_h$). Now $U_h = \frac{d_0}{k\alpha}$ implies $\alpha = \frac{d_0}{k \cdot U_h}$ and $\alpha_0 \cdot e^{-k\cdot\varepsilon} = \frac{d_0}{k \cdot U_h}$ thus $-k \cdot \varepsilon = \ln\left(\frac{d_0}{\alpha_0 \cdot k \cdot U_h}\right)$ and so $\varepsilon = \frac{1}{k}\ln\left(\frac{\alpha_0 \cdot k \cdot U_h}{d_0}\right)$

Via substitution, we can now discover how the employer will choose the optimal payment. Note that,

$$\pi = \frac{d_0}{k \cdot U_h} \cdot \left[\frac{1}{c_s}\right] + \left(1 - \frac{d_0}{k \cdot U_h}\right) \cdot \left[\frac{2}{c_h} - p_h\right]$$

Thus,

$$\pi = \frac{d_0}{k \cdot U_h} \cdot \left[\frac{1}{c_s}\right] + \left(1 - \frac{d_0}{k \cdot U_h}\right) \cdot \left[\frac{2}{c_h} - U_h - \frac{1}{c_h}\right]$$

$$\pi = \frac{d_0}{k \cdot c_s \cdot U_h} + \left(1 - \frac{d_0}{k \cdot U_h}\right) \cdot \left[\frac{1}{c_h} - U_h\right]$$

$$\pi = \frac{d_0}{k \cdot c_s \cdot U_h} + \frac{1}{c_h} - U_h - \frac{d_0}{k \cdot c_h \cdot U_h} + \frac{d_0}{k}$$

$$\pi = \frac{d_0 \cdot (c_s - c_h)}{k \cdot c_s \cdot c_h \cdot U_h} - U_h + \left[\frac{1}{c_h} + \frac{d_0}{k}\right]$$

$$\pi = \frac{d_0}{k \cdot U_h} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] - U_h + \left[\frac{1}{c_h} + \frac{d_0}{k}\right]$$

$$\pi = \frac{d_0}{k} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \cdot \frac{1}{U_h} - U_h + \left[\frac{1}{c_h} + \frac{d_0}{k}\right]$$

$$\frac{\partial \pi}{\partial U_h} = 0 = \frac{d_0}{k} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \cdot - \frac{1}{U_h^2} - 1$$

$$U_h^2 = -\frac{d_0}{k} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right]$$

$$U_h = \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

This gives a good check on ε^*, for if $\varepsilon = \frac{1}{k} \ln\left(\frac{\alpha_0 \cdot k \cdot U_h}{d_0}\right)$ and $U_h = \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$

Then

$$\varepsilon = \frac{1}{k} \ln\left(\frac{\alpha_0 \cdot k}{d_0} \cdot \sqrt{\frac{d_0}{k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}\right)$$

$$\varepsilon = \frac{1}{k}\ln\left(\sqrt{\frac{k^2 \cdot \alpha_0^2 \cdot d_0}{d_0^2 \cdot k} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]}\right)$$

$$\varepsilon = \frac{1}{2 \cdot k}\ln\left(\frac{\alpha_0^2 \cdot k}{d_0} \cdot \left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right)$$

Which is precisely the value for optimal effort ε derived in Appendix A. But the question remains open: when will the employer actually choose to give a bonus which leads to ε? When is the choice of this effort profitable? Precisely when the profit is greater than the profit available in the first best without effort:

$$\pi_{FB} \leq \pi_{\varepsilon}$$

$$\alpha_0 \cdot \left[\frac{1}{c_s}\right] + (1-\alpha_0) \cdot \left[\frac{1}{c_h}\right] \leq \alpha^* \cdot \left[\frac{1}{c_s}\right] + (1-\alpha^*) \cdot \left[\frac{1}{c_h}\right] - \frac{d_0}{k \cdot \alpha^*} + \frac{d_0}{k} + d^*$$

$$\alpha_0 \cdot \left[\frac{1}{c_s}\right] + \left[\frac{1}{c_h}\right] - \alpha_0 \cdot \left[\frac{1}{c_h}\right] \leq 2 \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \cdot \left(\frac{k}{d_0} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right]\right)^{-1/2} - \frac{d_0}{k} + \frac{1}{c_h}$$

$$\alpha_0 \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \leq 2 \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right] \cdot \left(\frac{k}{d_0} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right]\right)^{-1/2} - \frac{d_0}{k}$$

$$\left(\frac{k}{d_0} \cdot \left[\frac{1}{c_s} - \frac{1}{c_h}\right]\right)^{1/2} \leq \frac{2 \cdot k \cdot (c_h - c_s)}{k \cdot \alpha_0 \cdot (c_s - c_s) - c_h \cdot c_s \cdot d_0}$$

This can be solved to show that this implies that

$$\alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}$$

Thus, it is only profitable to select ε^* if $\alpha_0 \geq \sqrt{\frac{4 \cdot c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}} - \frac{c_h \cdot c_s \cdot d_0}{k \cdot (c_s - c_h)}$ since otherwise, the employer does worse when selecting nonzero ε^* regardless of the combination of c_s and c_h.

More On the Effort when Employee Selects Effort: Whether the constraint that $U_0 \geq 0$ will ever be binding

We know the equations

$$U_0 = \alpha \cdot U_s + (1 - \alpha) \cdot U_h - d$$
$$\pi = \alpha \cdot (v(q_s) - U_s - c_s \cdot q_s) + (1 - \alpha) \cdot (v(q_h) - U_h - c_h \cdot q_h)$$

By which the employer and employee will seek to maximize. In the case where the employee selects effort, we know that $\varepsilon^* = \frac{1}{k} \ln\left(\frac{k \cdot \alpha_0 \cdot U_h}{d_0}\right)$ and further that $U_h = \sqrt{\frac{d_0}{k}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$ and thus that $\varepsilon^* = \max\left\{\frac{1}{2k} \cdot \ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right), \quad 0\right\}$.

The question is whether the constraint that $U_0 \geq 0$ must be considered. We begin by noting that $0 \leq U_0 = \alpha \cdot U_s + (1 - \alpha) \cdot U_h - d$ reduces in the case where the employee selects effort to $0 \leq (1 - \alpha) \cdot U_h - d$ which implies $\frac{d}{1 - \alpha} \leq U_h$ where d and α are functions of effort.

From the initial constraints on the problem we happen to already know that $U_h = \frac{d_0}{k \cdot \alpha}$ and so we substitute here to find that

$$\frac{d}{1 - \frac{d_0}{k \cdot U_h}} \leq U_h$$

Which with rearrangement becomes

$$d \le U_h - \frac{d_0}{k}$$

Which further reduces to

$$\frac{d_0}{2k} \cdot \ln\left(\frac{\alpha_0^2 k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]\right) + \frac{d_0}{k} \le \sqrt{\frac{d_0}{k}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

Which becomes

$$\ln\left(\alpha_0 \cdot \sqrt{\frac{k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}\right) + 1 \le \frac{k}{d_0}\sqrt{\frac{d_0}{k}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

Which simplifies to become

$$\ln\left(\alpha_0 \cdot \sqrt{\frac{k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}\right) + 1 \le \sqrt{\frac{k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$$

We make the replacement $Z = \sqrt{\frac{k}{d_0}\left[\frac{1}{c_h} - \frac{1}{c_s}\right]}$ and note that this becomes $\ln(\alpha_0 \cdot Z) + 1 \le Z$

And since $\ln(Z) + 1 \le Z$ always holds, and $\ln(Z)$ is monotonically increasing, $\ln(\alpha_0 \cdot Z) + 1 \le Z$ also holds for all values of α_0 and so $U_0 \ge 0$ is not a constraint on the employer's choice of U_h (In other words, the employer always selects a U_h large enough to satisfy this constraint without explicitly accounting for this constraint).

Further, $\ln(\alpha_0 \cdot Z) + 1 = Z$ precisely when $\alpha_0 \cdot Z = 1$ which implies that the only inherent probability of illness for which an employee selects a non-zero effort could occur when $\alpha_0 = 1$ and also when $Z = 1$. In practice, this will rarely occur.

10 Appendix C (Methods of Numerical Optimization and Figure Generation)

Figures were generated in MATLAB. The figures were generated with the following fixed parameters: $d_0 = 6$, $k = 2$, $c_s = .1$, $c_h = .05$ unless otherwise noted.

For cases in which optimal effort and quantity of effort cannot be solved directly, MATLAB's "`fminsearch`" was employed to compute $-\pi$ using the Nelder-Mead downhill simplex method.