# A Bargaining Theory of the "Edwards' Effect" in the 2007-8 Democratic Presidential Primary 

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## Honors Thesis

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#### Abstract

The 2008 Democratic Presidential Primary will be remembered as one of the most competitive races in recent history. Two candidates, Senators Barack Obama (Illinois) and Hillary Clinton (New York), fought an enduring battle to obtain enough delegates/vote-shares to secure the Democratic nomination. Although the Democratic nomination eventually came down to these two contestants, for a while it was a dynamic three-player-race with Senator John Edwards (North Carolina) in the fold. During that time, many people were puzzled by Edwards' insistence on staying in the race even with no reasonable chance of winning. For this honors thesis, I constructed a theoretical model to explain Edwards' rationale for staying in the race. My model shows that if Edwards attained a certain percentage of the vote-shares, he could have pushed the election into a backroom negotiation phase given the right external circumstances. In this phase, he could have become the most pivotal contestant, with his relatively low percentage of vote-shares, ironically giving him the most negotiation power. This would have allowed him to come out of the backroom negotiation with a final prize value that would have exceeded the costs he endured. My paper coins this occurrence as "The Edwards Effect" and explores the conditions for its existence as well as its ramifications.


## I. Introduction

In the 2008 United States Presidential Election cycle, the competitive Democratic primary imparted on an extra layer of intrigue on the road to the White House. The norm in past elections had been for all parties to settle on their de-facto nominees for the General Election well in advance of their respective National Conventions. In the 2008 election cycle, however, the Democratic nominee was determined just two months shy of the National Convention when Senator Hillary Clinton (New York) finally conceded the contest to Senator Barack Obama (Illinois).

Given the 2008 national political landscape, the Democrats had a lot riding on this particular presidential primary. With an economy in recession and an unpopular war in Iraq, public opinion of President George W. Bush had reached historic lows. Some of these negative opinions had trickled down to his party, and public sentiment was in favor of a major ideological/party change in the country's leadership. It was widely believed that whoever won the Democratic Presidential Primary had a great shot at winning the General Election (this was validated on Election Night as Obama cruised to victory to become the first African-American President of the United States). Thus, the stakes were extremely high for all of the Democratic candidates involved.

From the very beginning of the 2008 Democratic primary, only three candidates had a viable chance of winning the nomination - Hillary Clinton, Barack Obama, and John Edwards. Each of the three had strong backing, but no one had the majority of the Democratic Party behind him/her. A back-and-forth battle ensued with no heavily-favored frontrunner emerging from the early primaries and caucuses. Yet
one thing eventually became certain: John Edwards, while having a valuable amount of support, did not have enough backing to win the party's nomination.

The public and media soon wrote Edwards off and saw the primary essentially as a two-player race. However, Edwards himself refused to quit early, and it was not until late January, after more than a year on the campaign trail, that he finally withdrew from the race (Times Online, 2006). This brings up an interesting and perplexing question: why did John Edwards stay in the race even when he knew he could not win the nomination? The Presidential Primary is a time-consuming and strenuous process; it is even tougher for less viable candidates like Edwards, as they have to actively engage in distinctive issues to steal some of the public attention away from the frontrunners and back to them. So, why did John Edwards persist, despite the fact that he clearly did not have the votes or the momentum?

One key motivator behind Edwards' decision to stay in the race is how the Democratic Primary process works-in order to become the party's official nominee, a candidate must obtain the votes of a majority of the delegates at the National Convention. The Democratic Party has two types of delegates, pledged delegates and unpledged/super delegates. The super delegates are high ranking party officials, governors, and congressmen who can vote for any of the candidates at the National Convention; they represent a little less than 20 percent of the total delegate pool. The remaining pledged delegates are selected to the National Convention to vote for a specific candidate. Each candidate receives a percentage of a state's total delegates that is directly proportional to the percentage of votes the candidate received in the
state's primary election/caucus. This means that a candidate does not necessarily have to win every state to win the Party's nomination (Kathleen, 2000).

In past primary cycles, the frontrunner usually obtains enough pledged delegates to make it impossible for his/her competitors to win the party's nomination, so the policy of super delegates has been relatively inconsequential. However, for the 2008 Democratic primary, the two top candidates, Obama and Clinton, possessed roughly the same number of delegates until the very end. As late as the beginning of June 2008, with 2,026 delegates needed to clinch the Democratic nomination, Obama had 1,887 delegates while Clinton had 1,718 (Chozick, 2008). With the results from Michigan and Florida still to be determined in a rules committee hearing (the Democratic National Committee disqualified all of the delegates from those states as punishment for pushing up their primary dates), no one knew just how much longer the race would last.

The lack of an obvious frontrunner made each delegate matter that much more, and this is where the "Edwards Effect" came into play. Although Edwards did not have enough delegates to win the party's nomination, he could have potentially gained enough delegates to push the nomination process into a backroom negotiation phase, thereby forcing the eventual nominee to promise him a "reward" in exchange for dropping out of the race. This reward, or "final prize value," could have been a cabinet position, the vice president post, or even a seat on the Supreme Court.

In this paper, I propose to model a two-stage game that can occur in a close contest involving three players: Edwards, Clinton, and Obama. The first stage describes the primary election process in which each of the three players expands
effort to gain some proportion of the total vote-shares/delegates. Assuming that no one player gains more than fifty percent of the total votes, a second negotiation stage will occur in my model. For this paper, I solve for each player's effort equation in the different scenarios that may occur when no one wins a majority of the total voteshares. I also further explore the most important case of the four scenarios, when all three candidates come out of the backroom negotiation with the same percentage of the final prize value, and see if it can occur in equilibrium.

The results of this specific scenario would show that Edwards could have obtained a prize value that, in the end, would have made up for the cost of his efforts to continue in the race. I argue that this is the reason why Edwards decided to remain in the race for as long as he did even after he realized that he had no chance of winning the nomination. This paper concludes that given a specific set of external circumstances, and by reaching a particular percentage of total vote-shares/delegates in the 2008 Democratic Primary, Senator John Edwards could have stimulated an "Edwards Effect" that would have pushed the primary process into a rare backroom negotiation phase. In this way, he could have transformed the nomination process from a winner-take-all contest into a competition with different prize values for all participants.

This paper has eight additional sections. Section II focuses on existing theoretical and empirical studies that form the basis for my model. Section III provides an in-depth construction of my theoretical model, a three-player, two-stage game. Sections IV and V solve the effort equations for all three players under the different cases of my model. Section VI then explores the constraints that are
necessary to make one particular case the equilibrium solution, and Section VII provides a numerical illustration of the theoretical model. Section VIII provides additional in-depth analyses and ties the theory to its practical applications. Finally, Section IX concludes with a summary of results and points out instances for further investigation.

## II. Literature Review

Most of the existing literature on the United States' Presidential election process focuses directly on the actual election. Additionally, the small number of studies on the U.S. presidential nomination process mainly address specific state primaries and voting patterns. There are a few others, such as Aldrich's model (1980) and Stone, Rapoport, and Atkeson's model (1995), that explore the dynamics of the weeding-out process within a party's primary race. These studies tend to focus on the public perception of the candidates, specifically on their viability in the nomination process and their electability in the General Election. The primary reason for the dearth of literature in my specific topic area is the lack of close nomination races in recent history. With a shortage of data, there are also very few empirical analyses on the topic.

The first stage of my theoretical model assumes that political competition can be modeled as a contest, which is a generally accepted and proven relationship. Konrad (2007) provided a broad survey of the contest literature available today, and the first stage of my model is based largely upon the Tullock contest model that Konrad described. This model, applied by Tullock in 1980, based the probability of a player winning a game on a ratio of his/her own effort over the total of all players' efforts in the game. Although my model uses this same underlying assumption in the
first stage, it builds on Tullock's model by allowing the proportion of vote-shares a player wins for each specific state election to depend on both the player's effort for the current state election, and the vote-shares that the same player had won previously. Specifically, the proportion of vote-shares a player wins in a certain state is a ratio of the sum of the effort that player spends and the vote-shares he/she won previously, over the sum of all the players' efforts and the total amount of vote-shares from previous contests. This way, my model links all of the state elections and will more realistically model the state-by-state election process of the Presidential Primary race. For example, a decent showing in a big state with many delegates up for grabs at the beginning of the primary season could dramatically help a candidate's overall position in the model, while a blown-out victory in a small state at the end of the primary season would not be as effective for that same candidate.

The backroom negotiation/bargaining process described in the second stage of my model is very similar to the government formation process that exists in many Western European countries. Much analysis has been conducted in this field. Most of the existing coalition/government formation literature brings to attention elections, such as those in Belgium, Denmark, and France, where the government is formed by a group of parties. For example, Merlo (1997) portrayed a theoretical model that describes the government formation process in post-war Italy, along with an empirical analysis of his model. Merlo arrived at the conclusion that, although delays in forming a government resulting from the bargaining process can be extremely costly to society, they can be optimal to the players involved due to the stochastic environment. This conclusion involved the assumption that each player's prize value coming out of
the negotiation process may not always be decreasing with respect to time. However, this is different from my model, in which the primary process has a very definitive timeline and usually will not last longer than a country's government formation process. This causes the prize value to decrease with time, since a party needs to have a nominee by the end of its National Convention; any delays in this process will play to the advantage of the other party's nominee, as he/she will have more time to prepare for the General Election.

Today's government-formation model is largely based on the work of Baron and Ferejohn. Baron and Ferejohn (1989) presented a theoretical model that focuses on the bargaining process in legislatures and the formation of agendas. They were able to relate their model in the context of government formation in parliamentary systems by looking at the expected number of seats each party would receive in the final parliamentary government. This model was adapted in my theoretical model as the basis for the second stage, the negotiation phase, of the game. As my model works within the context of the U.S. primary nomination system, the final expected value symbolizes how much influence/power each of the candidates would have in the President's cabinet if the party's nominee is elected President of the United States. However, my model differs from that of Baron and Ferejohn, as it looks at the percentage of vote-shares needed to reach the bargaining phase with different external circumstances. Each set of circumstances can create different relationships among the three final expected values coming out of the backroom negotiation process. Meanwhile, Baron and Ferejohn's model compares the strategies employed by the
different players in the negotiation phase by fixing each player's offering probability to a certain number.

Austen-Smith and Banks (1988) also developed a similar model that dealt with party behavior in a legislature. They created a multi-stage game that involved competition between three parties attempting to get their own legislature agendas across. They found that the game resulted in a noncooperative bargaining game among the three different parties. Similar to my model, the negotiating power that each party comes in with for this model is dependent on the percentage of vote-shares that the party captured in the general election. However, the Austen-Smith and Banks model takes into consideration each party's policy positions in the election process, whereas my model does not, thus simplifying the negotiation stage of the game.

Diermeier and Merlo (2004) used empirical data to test the validity of both Baron and Ferejohn's model and Austen-Smith and Banks' model. Diermeier and Merlo concluded that Baron and Ferejohn's model best fit the empirical data collected from over 313 government formations in 11 multi-party democracies between 1945 and 1997. However, they also concluded that an alternative model that combined the underlying assumptions of both models resulted in an even better fit with the empirical data. As my model attempts to portray a nomination process with less complexity than a government formation process, primary considerations were given to Baron and Ferejohn's assumptions. And for this purpose, Diermeier and Merlo's study was able to confirm the underlying assumptions in, and the validity of, the second stage in my model.

## III. Theoretical Framework

This model involves three main players, who correspond with the last three contestants of the 2008 Democratic Presidential Primary: John Edwards, Hillary Clinton, and Barack Obama. There are two stages in the model, an election stage and a negotiation stage. In order to win the game/contest, a candidate or a group of candidates must obtain a majority of the vote-shares available. During the first stage, or the nomination process, candidates expend effort in a series of state elections in an attempt to obtain a proportion of the total vote-shares. In the following second stage, the candidates go through a negotiation process that results in a team of candidates winning the competition with a majority of the vote-shares. Unlike most models, a minimum of three candidates is required to ensure that no one candidate receives a majority of the vote-shares by the end of the first stage.

## Part A - The Parameters

There are three players: Edwards (e), Clinton (c), and Obama (o)
Each player, $i$, has a certain percentage of votes obtained from previous state elections, $\mathrm{X}_{\mathrm{i}}$. Each player will also exert a certain amount of effort (expenditure) into the current state election, $\mathrm{x}_{\mathrm{i}}$.
$\mathrm{X}_{\mathrm{i}}=$ the percentage of vote-shares player $i$ enters the current state election with
$\mathrm{x}_{\mathrm{i}}=$ the amount of effort player $i$ exerts in the current state election

## Part B - Stage One

In the first stage, each player puts in his/her effort, $\mathrm{x}_{\mathrm{i}}$, which will, at the end of the stage, generate for him/her a certain proportion of the total vote-shares.

The following equation mathematically determines player $i$ 's proportion of the total vote-shares:

$$
\begin{equation*}
p_{i}=\frac{X_{i}+x_{i}}{\sum_{j=1}^{3}\left(X_{j}+x_{j}\right)} \tag{1}
\end{equation*}
$$

Where:

$$
\begin{equation*}
1=\sum_{j=1}^{3} p_{j} \tag{2}
\end{equation*}
$$

Equation 1 shows that the percentage of vote-shares player $i$ comes out of the first stage with is determined by player $i$ 's effort relative to the total efforts of all players. In this stage, each player tries to maximize his/her payoff with respect to his/her effort. For example, Edward's equation would be:

$$
\begin{equation*}
\max \pi_{e}=v_{e}^{*}\left(p_{e}, p_{c}^{*}, p_{o}^{*}\right)-c\left(x_{e}\right) \tag{3}
\end{equation*}
$$

Subject to (1).
In (3), $v_{e}^{*}$ represents the expected final prize value, normalized to 1 , that Edwards should receive under equilibrium conditions at the completion of stage two, the backroom negotiations phase.

## Part C - Stage Two

Each player enters stage two with a certain proportion of the vote-shares, $\mathrm{p}_{\mathrm{i}}$, gained from stage one. Two cases could occur under stage two.

- Case 1: $p_{i}>0.5$ for some $i$.

This is the trivial case. If the proportion of $p_{i}$ is greater than 0.5 , then that player would automatically win 100 percent of the final prize value. None of the other two players would receive any consolation prize, such as power of influence.

- Case 2: $p_{i}<0.5$ for all $i$.

If none of the players has a majority of the votes, then the game would enter a bargaining phase. The game would only end when a player or a group of players captures a majority of the votes. Note that if $\mathrm{p}_{\mathrm{i}}<0.5$ for all $i$, then $\mathrm{p}_{\mathrm{k}}+\mathrm{p}_{1}>.5$ for $i$ $=k, l$.

## Solving Case 2:

## Parameters of interest

1. $\mathrm{p}_{\mathrm{i}}=$ the probability that player $i$ will offer to form an alliance to win the game (Note: This is the same variable as the percentage of votes player $i$ enters stage two with. The higher the proportion of votes $i$ has, the greater the chance player $i$ will offer to form an alliance)
2. $\mathrm{r}_{\mathrm{i}, \mathrm{j}}=$ the probability that player $i$ will offer to form an alliance with player $j$
3. $\delta=$ discounting factor

This number is within the range: $\left[\begin{array}{ll}0 & 1) \text {. The discounting factor portrays the }\end{array}\right.$ attrition effects associated with a long negotiation process. As stage two drags on, each player's expected prize value will decrease as each player expends additional energy, effort, time, and other intangibles that cannot be recovered. For this model, the discounting factor represents the critical loss of time for the eventual Democratic nominee to prepare for the General Election.

## Calculation of expected value, $v$, coming out of the negotiation phase

The expected prize value for each player coming out of negotiation is solved in the same way. Edwards' expected value is solved as an example below:

## Part A.

Edwards can make an offer $\left(p_{e}\right)$ to Clinton $\left(\mathrm{r}_{\mathrm{e}, \mathrm{c}}\right)$ which results in a final expected value of:

Clinton's $=\delta v_{c}$
Obama's $=0$ (as he was not offered to be part of the winning team)

## Part B.

Edwards can make an offer $\left(p_{e}\right)$ to Obama $\left(\mathrm{r}_{\mathrm{e}, \mathrm{O}}\right)$ which results in a final expected value of:
Edwards' $=1-\delta v_{o}$
Clinton's $=0$
Obama's $=\delta v_{o}$

## Part C.

Edwards can receive an offer from Clinton $\left(p_{c}\right)$ with a probability of $\mathrm{r}_{\mathrm{c}, \mathrm{e}}$ with an expected value of:
Edwards' $=\delta v_{e}$

Clinton's $=1-\delta v_{e}$
Obama's $=0$
Part D.
Edwards can receive an offer from Obama ( $p_{o}$ ) with a probability of $\mathrm{r}_{\mathrm{o}, \mathrm{e}}$ with an expected value of:
Edwards' $=\delta v_{e}$
Clinton's $=0$
Obama's $=1-\delta v_{e}$

## Part E.

The total prize value for Edwards will be

$$
\begin{equation*}
v_{e}=p_{e}\left(1-r_{e, c} \delta v_{c}-r_{e, o} \delta v_{o}\right)+p_{c} r_{c, e} \delta v_{e}+p_{o} r_{o, e} \delta v_{e} \tag{4}
\end{equation*}
$$

Equation 4 can be simplified: the probability that Edwards will make an offer to Obama is $\left(1-r_{e, c}\right)$, as Edwards has to make an offer in order to win the game. If Edwards does not offer to form a team with Clinton, then he will need to form a team with Obama.

Now the equation becomes:

$$
\begin{equation*}
v_{e}=p_{e}\left[1-r_{e, c} \delta v_{c}-\left(1-r_{e, c}\right) \delta v_{o}\right]+p_{c} r_{c, e} \delta v_{e}+p_{o} r_{o, e} \delta v_{e} \tag{5}
\end{equation*}
$$

The same logic can be used to find the expected values for Clinton and Obama:

$$
\begin{align*}
& v_{c}=p_{c}\left[1-r_{c, e} \delta v_{e}-\left(1-r_{c, e}\right) \delta v_{o}\right]+p_{e} r_{e, c} \delta v_{c}+p_{o}\left(1-r_{o, e}\right) \delta v_{c}  \tag{6}\\
& v_{o}=p_{o}\left[1-r_{o, e} \delta v_{e}-\left(1-r_{o, e}\right) \delta v_{c}\right]+p_{e}\left(1-r_{e, c}\right) \delta v_{o}+p_{3}\left(1-r_{c, e}\right) \delta v_{o} \tag{7}
\end{align*}
$$

## Finding $p_{i}$ 's value

The expected values can be differentiated into four different cases. Without loss of generality, let $p_{e} \leq p_{c} \leq p_{o}$, with Edwards being the weakest player and Obama being the strongest. From Eraslan (2002), we then know that: $v_{e}^{*} \leq v_{c}^{*} \leq v_{o}^{*}$. Hence, there are 4 possible cases to consider. Below, for each of the 4 cases, the $\mathrm{P}_{\mathrm{i}}$ 's are solved in terms of v 's, r 's and $\delta^{\prime} s$.

Case One: $\quad v_{e}^{*}=v_{c}^{*}=v_{o}^{*}=1 / 3$.
This simplifies equations (5-7) to:

$$
\begin{align*}
& \frac{1}{3}=p_{e}\left[1-\frac{\delta}{3}\right]+\frac{p_{c} r_{c, e} \delta}{3}+\frac{p_{o} r_{o, e} \delta}{3}  \tag{8}\\
& \frac{1}{3}=p_{c}\left[1-\frac{\delta}{3}\right]+\frac{p_{e} r_{e, c} \delta}{3}+\frac{p_{o}\left(1-r_{o, c}\right) \delta}{3}  \tag{9}\\
& \frac{1}{3}=p_{o}\left[1-\frac{\delta}{3}\right]+\frac{p_{e}\left(1-r_{e, c}\right) \delta}{3}+\frac{p_{c}\left(1-r_{c, e}\right) \delta}{3} \tag{10}
\end{align*}
$$

Solving for these equations leads to:

$$
\begin{equation*}
p_{e}=\frac{3-r_{o, e} \delta-2 \delta-r_{c, e} \delta+r_{o, e} \delta^{2}+\delta^{2} r_{c, e}-\delta^{2} r_{c, e} r_{o, e}}{r_{e, c} \delta^{2} r_{o, e}-r_{e, c} \delta^{2} r_{c, e}-r_{o, e} \delta^{2} r_{c, e}-9 \delta+\delta^{2} r_{c, e}+2 \delta^{2}+9} \tag{11}
\end{equation*}
$$

$$
\begin{align*}
& p_{c}=\frac{3-r_{o, e} \delta^{2}+\delta^{2}+r_{e, c} \delta^{2} r_{o, e}-r_{e, c} \delta+r_{o, e} \delta-3 \delta}{r_{e, c} \delta^{2} r_{o, e}-r_{e, c} \delta^{2} r_{c, e}-r_{o, e} \delta^{2} r_{c, e}-9 \delta+\delta^{2} r_{c, e}+2 \delta^{2}+9}  \tag{12}\\
& p_{o}=\frac{r_{e, c} \delta-4 \delta-r_{e, c} \delta^{2} r_{c, e}+\delta^{2}+3+r_{c, e} \delta}{r_{e, c} \delta^{2} r_{o, e}-r_{e, c} \delta^{2} r_{c, e}-r_{o, e} \delta^{2} r_{c, e}-9 \delta+\delta^{2} r_{c, e}+2 \delta^{2}+9} \tag{13}
\end{align*}
$$

Case Two: $v_{e}^{*}<v_{c}^{*}=v_{o}^{*}$
This means that:

$$
r_{c, e}=1, r_{o, e}=1 \text { and } v_{c}^{*}=v_{o}^{*}=v
$$

This simplifies equations (5-7) to:

$$
\begin{align*}
& v_{e}=p_{e}[1-\delta v]+p_{c} \delta v_{e}+p_{o} \delta v_{e}  \tag{14}\\
& v=p_{c}\left[1-\delta v_{e}\right]+p_{e} r_{e, c} \delta v  \tag{15}\\
& v=p_{o}\left[1-\delta v_{e}\right]+p_{e}\left(1-r_{e, c}\right) \delta v \tag{16}
\end{align*}
$$

Solving for these equations leads to:

$$
p_{e}=\frac{v_{e}\left(\delta v_{e}-1+2 \delta v\right)}{\delta v-1+\delta v_{e}}
$$

$$
\begin{equation*}
p_{c}=\frac{v\left(1-\delta v_{e} r_{e, c}+2 r_{e, c} \delta^{2} v v_{e}+\delta^{2} v_{e}^{2} r_{e, c}-\delta v-\delta v_{e}\right)}{\left(\delta v-1+\delta v_{e}\right)\left(\delta v_{e}-1\right)} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
p_{3}=\frac{v\left(1+\delta^{2} v_{e}^{2}-\delta^{2} v_{e}^{2} r_{e, c}+2 \delta^{2} v v_{e}-2 r_{e, c} \delta^{2} v v_{e}-2 \delta v_{e}+r_{e, c} \delta v_{e}-\delta v\right)}{\left(\delta v-1+\delta v_{e}\right)\left(\delta v_{e}-1\right)} \tag{18}
\end{equation*}
$$

Case Three: $v_{e}^{*}=v_{c}^{*}<v_{o}^{*}$

This means that:

$$
r_{e, c}=1, r_{c, e}=1 r_{o, e}=r_{o, c}=\frac{1}{2} \text { and } v_{c}^{*}=v_{e}^{*}=v
$$

This simplifies equations (5-7) to:

$$
\begin{align*}
& v=p_{e}[1-\delta v]+p_{c} \delta v+p_{o} r_{o, e} \delta v  \tag{20}\\
& v=p_{c}[1-\delta v]+p_{e} \delta v+p_{o}\left(1-r_{o, e}\right) \delta v  \tag{21}\\
& v_{o}=p_{o}[1-\delta v] \tag{22}
\end{align*}
$$

Solving for these equations leads to:

$$
\begin{align*}
& p_{e}=\frac{v\left(1-v_{o} r_{o, e} \delta-3 \delta v+2 \delta^{2} v^{2}+\delta^{2} v v_{o}\right)}{(\delta v-1)(2 \delta v-1)}  \tag{23}\\
& p_{c}=\frac{v\left(1-v_{o} r_{o, e} \delta-3 \delta v+2 \delta^{2} v^{2}+\delta^{2} v v_{o}\right)}{(\delta v-1)(2 \delta v-1)}-\frac{\delta v v_{o}}{(\delta v-1)(2 \delta v-1)}  \tag{24}\\
& p_{o}=\frac{v_{o}}{1-\delta v} \tag{25}
\end{align*}
$$

Case Four: $\quad v_{e}^{*}<v_{c}^{*}<v_{o}^{*}$
This means that:

$$
r_{o, e}=1, r_{e, c}=1 \text { and } r_{c, e}=1
$$

This simplifies equations (5-7) to:

$$
\begin{align*}
& v_{e}=p_{e}\left[1-\delta v_{c}\right]+p_{c} \delta v_{e}+p_{o} \delta v_{e}  \tag{26}\\
& v_{c}=p_{c}\left[1-\delta v_{e}\right]+p_{e} \delta v_{c}  \tag{27}\\
& v_{o}=p_{o}\left[1-\delta v_{e}\right] \tag{28}
\end{align*}
$$

Solving for these equations leads to:

$$
\begin{align*}
& p_{e}=\frac{(1-\delta) v_{e}}{1-\delta\left(1-v_{o}\right)}  \tag{29}\\
& p_{c}=1-\frac{(1-\delta) v_{e}}{1-\delta\left(1-v_{o}\right)}-\frac{v_{o}}{1-\delta v_{e}}  \tag{30}\\
& p_{o}=\frac{v_{o}}{1-\delta v_{e}} \tag{31}
\end{align*}
$$

## IV. Equilibrium Effort Solutions for Case One

Among these four cases, this paper will find the necessary conditions for case one, when $v_{e}^{*}=v_{c}^{*}=v_{o}^{*}$, to become the equilibrium solution. This would give validity for the underdog, John Edwards, to stay in the race.

As mentioned previously: $p_{e} \leq p_{c} \leq p_{o}$
Substituting the $P_{i}$ 's with the solutions found for case one from equations 11-13 leads to:

$\frac{r_{e, c} \delta-4 \delta-r_{c, c} \delta^{2} r_{c, e}+\delta^{2}+3+r_{c, e} \delta}{r_{e, c} \delta^{2} r_{o, e}-r_{e, c} \delta^{2} r_{c, e}-r_{0, e} \delta^{2} r_{c, e}-9 \delta+\delta^{2} r_{c, e}+2 \delta^{2}+9}$

## Validation of Case One

To check the validity of the inequalities, assuming all of the candidates enter the final stage with an equal proportion of votes such that $p_{e}=p_{c}=p_{o}=p$

This leads to the following results: $r_{o, e}=0.5, r_{c, e}=0.5$, and $r_{e, c}=0.5$
This will transform the equations into the following:

$$
\begin{aligned}
& \frac{3-0.5 \delta-2 \delta-0.5 \delta+0.5 \delta^{2}+0.5 \delta^{2}-0.25 \delta^{2}}{0.25 \delta^{2}-0.25 \delta^{2}-0.25 \delta^{2}-9 \delta+\delta^{2} 0.5+2 \delta^{2}+9}=\frac{3-0.5 \delta^{2}+\delta^{2}+0.25 \delta^{2}-0.5 \delta+0.5 \delta-3 \delta}{0.25 \delta^{2}-0.25 \delta^{2}-0.25 \delta^{2}-9 \delta+0.5 \delta^{2}+2 \delta^{2}+9} \\
& =\frac{0.5 \delta-4 \delta-0.25 \delta^{2}+\delta^{2}+3+0.5 \delta}{0.25 \delta^{2}-0.25 \delta^{2}-0.25 \delta^{2}-9 \delta+0.5 \delta^{2}+2 \delta^{2}+9}
\end{aligned}
$$

Which simplifies to:

$$
\frac{0.75 \delta^{2}-3 \delta+3}{2.25 \delta^{2}-9 \delta+9}=\frac{0.75 \delta^{2}-3 \delta+3}{2.25 \delta^{2}-9 \delta+9}=\frac{0.75 \delta^{2}-3 \delta+3}{2.25 \delta^{2}-9 \delta+9}
$$

They become the same equation as expected.
Moving on to the first stage of the game:

$$
\text { Assume: } \quad X_{e}, X_{c}=X_{o}=X
$$

For Edwards:

$$
\begin{equation*}
p_{e}=\frac{X_{e}+x_{e}}{2 X+X_{e}+x_{e}+2 x} \tag{32}
\end{equation*}
$$

In order for the candidates to expend positive effort in the bargaining/negotiation phase, the following individual rationality must be true:

$$
\begin{equation*}
\frac{1}{3}-c\left(x_{e}^{*}\right) \geq 0 \Rightarrow x_{e}^{*} \leq \frac{1}{3 c} \tag{33}
\end{equation*}
$$

(Edwards will only exert effort when the cost of his effort is less than or equal to the expected prize value he receives after the bargaining phase)

As

$$
\begin{equation*}
p_{e}=\frac{X_{e}+x_{e}}{2 X+X_{e}+x_{e}+2 x} \text { and } p=\frac{0.75 \delta^{2}-3 \delta+3}{2.25 \delta^{2}-9 \delta+9} \tag{34}
\end{equation*}
$$

Thus:

$$
\frac{0.75 \delta^{2}-3 \delta+3}{2.25 \delta^{2}-9 \delta+9}=\frac{x_{e}+X_{e}}{2 X+x_{e}+X_{e}+2 x}
$$

Solving for $x_{e}^{*}$ :

$$
\begin{gather*}
x_{e}^{*}=X+x-X_{e}  \tag{35}\\
\left(X, x \uparrow \Rightarrow x_{e}^{*} \uparrow \text { and } X_{e} \uparrow \Rightarrow x_{e}^{*} \downarrow\right)
\end{gather*}
$$

Which also means:

$$
\begin{align*}
& x_{e}^{*}= X+x-X_{e} \leq \frac{1}{3 c}  \tag{36}\\
& \quad\left(c \uparrow \Rightarrow x_{e}^{*}, x^{*} \downarrow\right)
\end{align*}
$$

Solving for Obama's and Clinton's efforts will lead to the following:

$$
x^{*}=X_{e}+x_{e}-X
$$

$$
\begin{equation*}
\left(X_{e}, x_{e} \uparrow \Rightarrow x^{*} \uparrow \text { and } X \uparrow \Rightarrow x^{*} \downarrow\right) \tag{37}
\end{equation*}
$$

Which also means:

$$
\begin{gather*}
x^{*}=X_{e}+x_{e}-X \leq \frac{1}{3 c}  \tag{38}\\
\quad\left(c \uparrow \Rightarrow x_{e}^{*}, x^{*} \downarrow\right)
\end{gather*}
$$

The above results show that each candidate's effort will increase as the other candidates' efforts and vote-shares from previous elections increase. This makes intuitive sense, as each player must try harder in order to reach the bargaining phase if the other players were to expend more effort and/or their vote-shares from previous elections were to increase. On the other hand, if a player's vote-shares from previous elections were to increase, that player could exert less effort and still reach the same amount of total vote-shares as before. Hence, his/her effort would go down in equilibrium. As the cost of the effort increases, the amount of effort exerted by all players would decrease, which is demonstrated in (36) and (38). This is also in line
with intuition, a higher cost of effort would cut more deeply into a player's final expected prize value and would lower the incentive for the player to expend the same amount of effort as before.

Next, the following values are used to confirm the results:
Let: $\mathrm{X}_{\mathrm{e}}=\mathrm{X}=0.1$, then $\mathrm{x}_{\mathrm{e}}{ }_{\mathrm{e}}=\mathrm{x}^{*}$
Let: $\mathrm{X}_{\mathrm{e}}=0.1, \mathrm{X}=0.2$, then $\mathrm{x}^{*}{ }_{\mathrm{e}}=\mathrm{x}+0.1$
Edwards would need to make up the difference in the second equation to ensure that everyone arrives at the bargaining table with the same proportion of vote-shares. These results validate the equations gathered for case one.

## Effort functions for the largest vote-shares range in Case One

Below, effort functions are solved for the largest possible vote-shares range that can induce case one's equilibrium. This is the range that Edwards is most interested in, as it shows him the lowest percentage of votes that he needs to obtain the same prize value as Obama or Clinton.

In order to obtain the largest range for $p_{e} \leq p_{c} \leq p_{o}$, the inequalities would become $p_{e}<p_{c} \leq p_{o}$ which would mean: $r_{c, e}=1$ and $r_{o, e}=1$.

The values for $r_{c, e}$ and $r_{o, e}$ represent the notion that Obama and Clinton would always offer to form an alliance with Edwards in the bargaining phase. This makes sense, as the total number of vote-shares for Edwards is the fewest of all the players involved and should command the least amount of value in equilibrium as $v_{e}^{*} \leq v_{c}^{*} \leq v_{o}^{*}$ will
hold. (As explained later, this is also the reason why Edwards could come out of the negotiation phase with a significant portion of the overall prize value.)

With the above information, the inequalities would turn into:

$$
\begin{equation*}
\frac{\delta^{2}-4 \delta+3}{2 \delta^{2}-9 \delta+9} \leq \frac{3+r_{e, c} \delta^{2}-r_{e, c} \delta-2 \delta}{2 \delta^{2}-9 \delta+9} \leq \frac{r_{e, c} \delta-3 \delta-r_{e, c} \delta^{2}+\delta^{2}+3}{2 \delta^{2}-9 \delta+9} \tag{39}
\end{equation*}
$$

To put the model more in line with the actual political situation, Obama and Clinton would enter the bargaining phase with the same percentage of votes which is larger than that of Edwards'.

Therefore: $p_{e}<p_{c}=p_{o}$

This also means that: $r_{e, c}=0.5$ (The person that Edwards offers an alliance to would be truly random, as both Obama and Clinton have the same bargaining power).

Thus the relationship changes to:

$$
\frac{\delta^{2}-4 \delta+3}{2 \delta^{2}-9 \delta+9} \leq \frac{0.5 \delta^{2}-2.5 \delta+3}{2 \delta^{2}-9 \delta+9} \text { where } p_{e}=\frac{\delta^{2}-4 \delta+3}{2 \delta^{2}-9 \delta+9} \text { and } p_{c}=p_{o}=p=\frac{0.5 \delta^{2}-2.5 \delta+3}{2 \delta^{2}-9 \delta+9}
$$

Note: this set of inequalities also demonstrates that as the discounting factor decreases, the range for $\mathrm{p}_{\mathrm{i}}$ 's also decreases.

Now moving on to the first stage of the game:
Assume: $\mathrm{X}_{\mathrm{e}}$ and $\mathrm{X}_{\mathrm{c}}=\mathrm{X}_{\mathrm{o}}=\mathrm{X}$, where $\mathrm{X}_{\mathrm{e}}<\mathrm{X}$
That is, Edwards comes into the last state election with a certain percentage of votes that is less than Obama's and Clinton's, who have the same amount of vote-shares.

Then

$$
\begin{align*}
& p_{e}=\frac{X_{e}+x_{e}}{2 X+X_{e}+x_{e}+2 x}  \tag{40}\\
& p=\frac{X+x}{2 X+X_{e}+x_{e}+2 x} \tag{41}
\end{align*}
$$

Edwards' individual rationality is the same as (33).
As

$$
\begin{equation*}
p_{e}=\frac{X_{e}+x_{e}}{2 X+X_{e}+x_{e}+2 x} \text { and } p_{e}=\frac{\delta^{2}-4 \delta+3}{2 \delta^{2}-9 \delta+9} \tag{42}
\end{equation*}
$$

Solving for $x_{e}^{*}$ leads to:

$$
\begin{gather*}
x_{e}^{*}=\frac{\left(2 X+2 x^{*}\right)}{\frac{2-\delta}{1-\delta}}-X_{e}  \tag{43}\\
\left(X, x \uparrow \Rightarrow x_{e}^{*} \uparrow \text { and } \delta \uparrow, X_{e} \uparrow \Rightarrow x_{e}^{*} \downarrow\right)
\end{gather*}
$$

This also means:

$$
\begin{gather*}
x_{e}^{*}=\frac{\left(2 X+2 x^{*}\right)}{\frac{2-\delta}{1-\delta}}-X_{e} \leq \frac{1}{3 c}  \tag{44}\\
\left(c \uparrow \Rightarrow x_{e}^{*}, x^{*} \downarrow\right)
\end{gather*}
$$

Solving for Obama's and Clinton's amount of effort would give the following equation:

$$
\begin{gather*}
x^{*}=\frac{x_{e}^{*}+X_{e}}{\left(\frac{2-2 \delta}{2-\delta}\right)}-X  \tag{45}\\
\left(\delta, x_{e}, X_{e} \uparrow \Rightarrow x^{*} \uparrow \text { and } X \uparrow \Rightarrow x^{*} \downarrow\right)
\end{gather*}
$$

This also means:

$$
\begin{gather*}
x^{*}=\frac{x_{e}^{*}+X_{e}}{\left(\frac{2 \delta-2}{\delta-2}\right)}-X \leq \frac{1}{3 c}  \tag{46}\\
\quad\left(c \uparrow \Rightarrow x_{e}^{*}, x^{*} \downarrow\right)
\end{gather*}
$$

The intuition behind these results is the same as explained after (38). The only new variable here is the discounting factor. For Edwards, as the discounting factor increases, his effort would decrease in equilibrium. This is because a higher discounting factor increases the range among the $\mathrm{p}_{\mathrm{i}}$ 's, which can induce the equilibrium: $v_{e}^{*}=v_{c}^{*}=v_{o}^{*}=1 / 3$. Therefore, as the discounting factor increases, Edwards would need a smaller proportion of the votes and can lower the amount of effort he expends. The reverse is true for Obama and Clinton, as they would need to expend even more effort to try and attain the higher number of votes needed to bypass the negotiation phase.

## V. Effort solutions for Case 2-4

Using the same analysis that was performed in Section IV, the effort solutions for each of the cases are shown below.

Case Two: $v_{e}<v_{c}=v_{o}=v$

For Edwards:

$$
\begin{gather*}
x_{e}=\frac{(2 X+2 x)}{\left(\frac{(4-\delta) v}{(1-2 v)(1-\delta)}\right)}-X_{e}  \tag{47}\\
\left(X, x \uparrow \Rightarrow x_{e} \uparrow \text { and } v \uparrow, \delta \uparrow, X_{e} \uparrow \Rightarrow x_{e} \downarrow\right)
\end{gather*}
$$

For Clinton and Obama:

$$
\begin{gather*}
x=\frac{x_{e}+X_{e}}{\left(\frac{(1-2 v)(\delta-1)}{v(0.5 \delta-1)}\right)}-X  \tag{48}\\
\left(\delta, x_{e}, v \uparrow \Rightarrow x \uparrow \text { and } X \uparrow \Rightarrow x \downarrow\right)
\end{gather*}
$$

Case Three: $v_{e}=v_{c}=v<v_{o}$
For Edwards and Clinton:

$$
\begin{gather*}
x=\frac{\left(X_{o}+x_{o}\right)}{\left(\frac{1-2 v}{1-\delta v}\right)}-X  \tag{49}\\
\left(X_{o}, x_{o}, v \uparrow \Rightarrow x \uparrow \text { and } X, \delta \uparrow \Rightarrow x \downarrow\right)
\end{gather*}
$$

For Obama:

$$
\begin{gather*}
x_{o}=\frac{2 x+2 X}{\left(\frac{v(2-\delta)}{(1-2 v)}\right)}-X_{o}  \tag{50}\\
\left(x, X, \delta \uparrow \Rightarrow x_{o} \uparrow \text { and } X_{o}, v \uparrow \Rightarrow x_{o} \downarrow\right)
\end{gather*}
$$

Case Four: $v_{e}<v_{c}<v_{o}$
For Edwards:

$$
\begin{gather*}
x_{e}=\frac{\left(X_{c}+X_{o}+x_{c}+x_{o}\right)}{\left(\frac{1-\delta v_{c}-v_{e}}{v_{e}(1-\delta)}\right)}-X_{e}  \tag{51}\\
\left(X_{o}, x_{o}, X_{c}, v_{c}, v_{e}, x_{c} \uparrow \Rightarrow x_{e} \uparrow \text { and } X_{e}, \delta \uparrow \Rightarrow x_{e} \downarrow\right)
\end{gather*}
$$

For Clinton:

$$
\begin{gather*}
x_{c}=\frac{\left(X_{e}+X_{o}+x_{e}+x_{o}\right)}{\left(\frac{1-2 \delta v_{e}-\delta v_{c}+\delta^{2} v_{e}{ }^{2}-v_{c}+2 \delta v_{e} v_{c}+\delta v_{c}^{2}}{v_{c}\left(1-2 \delta v_{e}+\delta^{2} v_{e}-\delta v_{c}\right)}-X_{c}\right.}  \tag{52}\\
\left(X_{e}, X_{o}, x_{e}, x_{o} \uparrow \Rightarrow x_{c} \downarrow, v_{c}, X_{c} \uparrow \Rightarrow x_{c} \uparrow\right)
\end{gather*}
$$

For Obama:

$$
\begin{gather*}
x_{o}=\frac{x_{e}+X_{e}+x_{c}+X_{c}}{\left(\frac{1-\delta v_{e}-v_{o}}{v_{o}}\right)}-X_{o}  \tag{53}\\
\left(X_{c}, x_{c}, X_{e}, x_{e}, v_{o}, v_{e}, \delta \uparrow \Rightarrow x_{o} \uparrow \text { and } X_{o} \uparrow \Rightarrow x_{o} \downarrow\right)
\end{gather*}
$$

## VI. Equilibrium Constraints for Case One

In order for case one to become the equilibrium case, none of the candidates should have an incentive to deviate to any other case. The following constraints must be satisfied for this to become true.

## For Edwards:

$$
\begin{gather*}
\quad \frac{1}{3}-c\left(x_{e}^{*}\right) \geq \max \left\{v_{e}-c\left(x_{e}\right), 0\right\}  \tag{54}\\
\left(\mathrm{V}_{\mathrm{e}}=\right.\text { Edward's Expected Value from Case 2) }
\end{gather*}
$$

This constraint is set to ensure that Edwards does not deviate to case 2. Although the expected value he derives from case 2 would be less than the value he would have received in case one, the reduction in effort would make sense for him to operate under that strategy instead.

For Clinton:

$$
\begin{align*}
& \frac{1}{3}-c\left(x_{c}{ }_{c}^{*}\right) \geq \max \left\{v_{c}-c\left(x_{c}\right), 0\right\}  \tag{55}\\
& \left(\mathrm{V}_{\mathrm{c}}=\right.\text { Clinton's Expected Value from Case 3) }
\end{align*}
$$

Similar to Edwards, this constraint ensures that Clinton does not deviate to Case 3, in which she spends much less effort than she would under Case One.

## For Obama:

$$
\begin{equation*}
\frac{1}{3}-c\left(x_{o}^{*}\right) \geq \max \left\{1-c\left(\overline{x_{o}}\right), 0\right\} \tag{56}
\end{equation*}
$$

$$
(1=\text { Obama's Expected Value from Case } 4)
$$

For Obama, this constraint ensures that he does not have an incentive to bypass the negotiation phase by expending enough effort that he could capture a majority of the votes at the end of stage one.

## VII. Numerical Application of the Theoretical Framework

The following example provides a numerical application of the theoretical framework.

Given a set of external circumstances, it finds the range of the percentage of votes that would lead Edwards to stay in the race.

Parameters:

Let $\delta=\{0.1,0.2,0.9,0.95\}$ and $\mathrm{c}=0.05$

Solving for $\mathrm{x}_{\mathrm{e}}$ and x through the following Matrix equations:
Case One: $\left[\begin{array}{cc}\frac{2-\delta}{1-\delta} & -2 \\ -1 & \frac{2-2 \delta}{2-\delta}\end{array}\right]\left[\begin{array}{l}x_{e}^{*} \\ x^{*}\end{array}\right]=\left[\begin{array}{c}2 X-\left(\frac{2-\delta}{1-\delta}\right) X_{e} \\ X_{e}-\left(\frac{2-2 \delta}{2-\delta}\right) X\end{array}\right]$
Case Two: $\left[\begin{array}{cc}\frac{(4-\delta) v}{(1-2 v)(1-\delta)} & -2 \\ -1 & \frac{(1-2 v)(1-\delta)}{v(1-0.5 \delta)}\end{array}\right]\left[\begin{array}{l}x_{e} \\ x\end{array}\right]=\left[\begin{array}{c}2 X-\left(\frac{(4-\delta) v}{(1-2 v)(1-\delta)}\right) X_{e} \\ X_{e}-\left(\frac{(1-2 v)(1-\delta)}{v(1-0.5 \delta)}\right) X\end{array}\right]$

Case Three: $\left[\begin{array}{cc}\frac{1-2 v}{1-\delta v} & -1 \\ -2 & \frac{v(2-\delta)}{1-2 v}\end{array}\right]\left[\begin{array}{c}x \\ x_{o}\end{array}\right]=\left[\begin{array}{c}X_{o}-\left(\frac{1-2 v}{1-\delta v}\right) X \\ 2 X-\left(\frac{v(2-\delta)}{1-2 v}\right) X_{o}\end{array}\right]$

Case Four: $\quad x_{o}=\frac{x_{e}+X_{e}+x_{c}+X_{c}}{\left(\frac{1-\delta v_{e}-v_{o}}{v_{o}}\right)}-X_{o}$ plug in values $\left(\mathrm{V}_{\mathrm{o}}=1\right.$ and $\mathrm{V}_{\mathrm{e}}$ and $\left.\mathrm{V}_{\mathrm{c}}=0\right)$

Table one and two contain the equilibrium results found for a set of $X_{e}$ and $X$ values:

| When Delta $=0.9$ | $\mathrm{c}=0.05$ |  |  |
| :---: | :---: | :---: | :---: |
| X | X e | x | X e |
| 0.205 | 0.195 | 1.5 | 0.115 |
| 0.21 | 0.19 | 1 | 0.03 |
| 0.211 | 0.189 | 2.002 | 0.213 |
| 0.212 | 0.188 | -2.5 | -0.604 |

Table 1: Case One Results when $\delta=0.9$

| When Delta $=0.95$ | $c=0.05$ |  |  |
| :---: | :---: | :---: | :---: |
| $X$ | $X_{-} e$ | $x$ | $x_{-} \mathrm{e}$ |
| 0.204 | 0.196 | 4.5 | 0.252 |
| 0.205 | 0.195 | 2 | 0.015 |
| 0.206 | 0.194 | -0.5 | 0.194 |

Table 2: Case One Results when $\delta=0.95$

As $\mathrm{X}=0.205$ and $\mathrm{X}_{\mathrm{e}}=0.195$ exist as valid solutions in both of the deltas tested, this particular set is used with other delta values as well. Table 3 lists the $x$ and $x_{e}$ that were obtained when those specific values were used with delta values of 0.2 and 0.1 .

| delta | X | $\mathrm{X}_{-} \mathrm{e}$ | X | $\mathrm{X}_{\mathrm{e}} \mathrm{e}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.205 | 0.195 | 4.358912 | 4.128705 |
| 0.2 | 0.205 | 0.195 | 2 | 1.765 |

Table 3: Case One Results when $\mathrm{X}=0.205 \mathrm{X}_{\mathrm{e}}=0.195$

## Constraints

The above results are checked against constraints (54), (55), and (56) to ensure that they are the equilibrium results.

## For the Edwards' Constraint (54):

Because Edwards could have an incentive to deviate to the strategy given in case two, case two was solved and the following values were gathered. One assumption was made regarding the values v (the expected value coming out of backroom negotiation for Clinton and Obama) and $\mathrm{v}_{\mathrm{e}}$ (the expected value for Edwards).

As $2 v+v_{e}=1$ and $v_{e}<v$, therefore the range for $v$ is: $\frac{1}{3}<v<\frac{1}{2}$

Using this information, the following values were gathered:
When Delta $=0.95$

| When $X e=0.195$ and $X=0.205$ |  |
| :--- | :---: |
| $v=0.33$ | $x_{-} e$ |
| $v=0.4$ | -0.195 |
| $v=0.49$ | -0.195 |

Table 4: Results for Edwards' efforts under Case Two when Delta $=0.95$
When Delta $=0.90$

| When $X e=0.195$ and $X=0.205$ |  |
| :--- | :---: |
| $v=0.33$ | -0.195 |
| $v=0.4$ | -0.195 |
| $v=0.49$ | -0.195 |

Table 5: Results for Edwards' efforts under Case Two when Delta $=0.90$
When Delta $=0.2$

| When $X_{e}=0.195$ and $X=0.205$ |  |
| :--- | :---: |
| $v=0.33$ | $x_{-} e$ |
| $v=0.4$ | -0.195 |
| $v=0.49$ | -0.195 |

Table 6: Results for Edwards' efforts under Case Two when Delta $=0.20$
When Delta $=0.1$

| When $X_{e}=0.195$ and $X=0.205$ |  |
| :--- | :---: |
|  | $x_{-} \mathrm{e}$ |
| $v=0.33$ | -0.195 |
| $v=0.4$ | -0.195 |
| $v=0.49$ | -0.195 |

Table 7: Results for Edwards' efforts under Case Two when Delta $=0.10$

All of the resulting effort values are negative for Edwards, which suggests that

Edwards should not be expending any effort in case 2 . The overall results suggest that Edwards would follow the strategy laid out in Case One.

For Clinton's Constraint (55):

Similar to Edwards' constraint, case three was solved for Clinton and the following values were found. The assumption here is that:

As $2 v+v_{o}=1$ and $v<v_{o}$, therefore the range for v is: $0<v<\frac{1}{3}$
Where v is the expected value for Clinton and Edwards, and $\mathrm{v}_{\mathrm{o}}$ is the expected value for Obama.

When Delta $=0.95$

| When $X=0.195$ and $X \_0=0.205$ |  |
| :--- | :---: |
|  | $X_{-} c$ |
| $v=0.1$ | -0.195 |
| $v=0.2$ | -0.195 |
| $v=0.3$ | -0.195 |

Table 8: Results for Clinton's efforts under Case Three when Delta $=0.95$
When Delta $=0.90$

| When $X=0.195$ and $X \_0=0.205$ |  |
| :--- | :---: |
|  | $x_{\_} c$ |
| $v=0.1$ | -0.195 |
| $v=0.2$ | -0.195 |
| $v=0.3$ | -0.195 |

Table 9: Results for Clinton's efforts under Case Three when Delta $=0.90$
When Delta $=0.2$

| When $X=0.195$ and $X_{0}=0.205$ |  |
| :--- | :---: |
|  | $X_{-} c$ |
| $v=0.1$ | -0.195 |
| $v=0.2$ | -0.195 |
| $v=0.3$ | -0.195 |

Table 10: Results for Clinton's efforts under Case Three when Delta $=0.2$
When Delta $=0.1$

| When $X=0.195$ and $X_{0} 0=0.205$ |  |
| :--- | :---: |
|  | $X_{\_} c$ |
| $v=0.1$ | -0.195 |
| $v=0.2$ | -0.195 |
| $v=0.3$ | -0.195 |

Table 11: Results for Clinton's efforts under Case Three when Delta $=0.1$

All of the resulting effort values are negative for Clinton, which suggests that Clinton should not be expending any effort in case 3. The overall results suggest that Clinton would follow the strategy laid out in Case One.

## For Obama's Constraint (56):

Case four was solved to prove that Obama does not have any incentives to deviate from case one. Since Obama would only expend extra effort to win the race outright, this suggests: $\mathrm{V}_{\mathrm{o}}=1$ while $\mathrm{V}_{\mathrm{c}}=\mathrm{V}_{\mathrm{e}}=0$.

Using these values, case four's results for Obama would arrive at an effort value of infinity. This proves that Obama should follow the strategy laid out in Case 1. By following case four's strategy, the cost of his efforts would overwhelm his expected prize value.

## Vote-shares Range

Now that all of case one's results are proven to be the equilibrium results, stage one is solved using the values. Under each delta value, vote-shares that are needed to arrive at the equilibrium condition are solved and shown in Table 12 below.

| Delta | $X$ | $x_{\_} \mathrm{e}$ | $x_{1}$ | $x_{\_} \mathrm{e}$ | $p_{\_} \mathrm{e}$ | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.205 | 0.195 | 4.3589 | 4.1287 | $32.14 \%$ | $33.93 \%$ |
| 0.2 | 0.205 | 0.195 | 2.0000 | 1.7650 | $30.77 \%$ | $34.62 \%$ |
| 0.9 | 0.205 | 0.195 | 1.5000 | 0.1150 | $8.33 \%$ | $45.83 \%$ |
| 0.95 | 0.205 | 0.195 | 2.0000 | 0.0150 | $4.55 \%$ | $47.73 \%$ |

Table 12: Vote-shares Percentages for different values of Delta
Table 12 shows that when the discounting factor is high (delta is greater than 0.8 ), Edwards could push the game into a negotiation phase with as little as $8.3 \%$ of the total vote-shares, with Obama and Clinton being tied up at $45.83 \%$. These values would change depending on the $X_{e}$ and $X$ coming into the final stage, but it shows
that there is a set of values in which Edwards could push the election into a negotiation stage.

Table 12 also shows that as the discounting factor decreases, the percentage of vote-shares that Edwards would need to push the game into a negotiation phase increases. This hurts Edwards as it would increase the amount of effort he needs to expend. The reason behind this is explained in the next section. However, this example clearly shows that there are cases in real life in which it would be worthwhile for Edwards to stay in the race even if his proportion of the vote-shares is not near the 50 percent mark.

## VIII. In-depth Analysis

The example in the previous section demonstrates that my theoretical model has important real life implications. One significant detail regarding the discounting factor does need to be pointed out; the set of inequalities listed in (39) shows that as the discounting factor decreases, the range of $\mathrm{p}_{\mathrm{i}}$ 's gets narrower. This suggests that if the backroom negotiation process were to be dragged out, it would actually hurt Edwards, which at first glance, might seem counterintuitive. However, when looked at from a different perspective, this result does make sense. The Democratic National Convention is set for late August; if the negotiation drags on to late September or even early October, the loss of valuable campaign time for the General Election would heavily decrease the final expected prize value for all three Democratic candidates-their probability of winning the Presidency would decrease dramatically relative to the time loss. The discounting factor models this loss in the expected value due to a long negotiation process. As the expected value in equilibrium decreases,

Edwards has less of an incentive to remain in the race, which the discounting factor portrays in the theoretical model.

The 2008 Democratic Presidential Primary shows the practical application of the theoretical model. The external circumstances surrounding the race can transform a weak but efficient candidate such as John Edwards into an influential player. Not only was the race intensively fought between two juggernaut candidates in Obama and Clinton, but furthermore, the shroud over the primary race was further thickened by the long-running dispute over the Michigan and Florida delegates. If these two states had been even partly seated in the National Convention, more than 2,026 delegates would have been needed to win the nomination. Thus, Clinton could very well have turned her small delegate deficiency into a virtual tie with Obama. As my theoretical model demonstrates, this would have only benefited Edwards more, as he would have had more leverage in pushing the election into a backroom negotiation/compromise phase.

This apparent confusion in the Democratic Primary was followed closely by the Republicans as well. Amy Chozick, a Wall Street Journal reporter, interviewed Republican pollster Glen Bolger, who stated, "There are not a whole lot of recent models to look at where a severely wounded candidate stays in it." (Chozick, 2008). Although Bolger's comment was directed at Clinton's campaign, it can be applied to Edwards' campaign. Edwards' less-than-stellar performances in some of the early primaries and caucuses severely compromised his shot at winning the nomination race outright. Yet, people in his own campaign still knew he had a shot. The following are
excerpts from an essay written for Politics Magazine by Joe Trippi, Edwards' senior advisor, who describes some of his own regrets at the time:

I had this feeling that if he stayed in the race he would win 300 or so delegates by Super Tuesday and have maybe a one-in-five chance of forcing a brokered convention. That there was a path ahead that would be extremely painful .... I should have told him emphatically that he should stay in. My regret that I did not do so-that I let John Edwards down-grows every day that the fight between Hillary Clinton and Barack Obama continues...

That would mean Edwards, Obama, and Clinton would go into the convention without any of them close to sealing the nomination. You would have had months of Obama and Clinton banging away at each other, with Edwards able to come across to weary Democrats as a welcome, fresh face. You'd have the electability argument begin to play to Edwards' advantage, since he always did well against McCain in polling... (Trippi, 2008)

This is the clearest indication that Edwards' campaign did consider the possibility of forcing the election into a negotiation phase. The reason why Edwards decided not to do so is because of the tremendous effort involved, which Trippi described in his essay as "the extremely painful path."

Trippi believes Edwards commands a strong position in the backroom negotiation phase because of both direct and indirect circumstances playing out in the primary. As Edwards' percentage of votes would have been the smallest, both Obama and Clinton would have expected that winning his endorsement would be the cheapest option in the negotiation phase. In other words, it would have cut into their own expected value the least. Since both Obama and Clinton would have approached Edwards, Edwards' negotiation power, along with his expected value in equilibrium, would have gone up. Ironically, the indirect circumstance that would have contributed to Edwards' powerful position in negotiation is Obama and Clinton holding strong positions such that they would not have dropped out of the race and endorsed

Edwards instead. These two candidates would only win a portion of the prize value if they, themselves, were nominated; this means that they had to be willing to pay out a significant portion of their own expected values to Edwards to get his support.

For practical considerations, even if Edwards dropped out ahead of the National Convention, he would have still laid claim to some of the final prize value through the value of his own endorsement. As my model shows, Edwards could have swung his own vote-shares to either Obama or Clinton and propelled one of them to the final nomination by letting them bypass the negotiation stage. This was validated in reality, as his endorsement was heavily sought after by both Obama and Clinton after Edwards dropped out of the race. When Edwards endorsed Obama in the middle of May, it played a significant role in Clinton's eventual defeat. As BBC News describes it:

Correspondents say the timing of the announcement is significant, on a day when Hillary Clinton has been celebrating a big win in West Virginia which was largely down to her support among poorer white voters - a constituency with which John Edwards is most closely identified. (BBCNews, 2008)

And even if Edwards' delegates alone did not end the primary race for Clinton, his endorsement may have affected the direction his own supporters leaned towards. For example, United Steelerworkers, which had endorsed Edwards originally, quickly followed Edwards' endorsement and endorsed Obama the day after Edwards dropped out. This may have led to additional votes in several of the next state primaries at little to no cost to Obama (United Steelworkers [USW], 2008). These endorsements may have not only helped Obama win the Democratic nomination but also may have served to unite the party. It could also capture additional independent votes for the

General Election, thus increasing Obama's probability of winning the road to the White House.

In the backroom negotiation phase for this endorsement, Obama must have given Edward some sort of promise or guarantee. For example, many analysts believed that Obama would make Edwards' poverty policies a key component of his domestic agenda. Some even believed that Edwards was going to be in line for a cabinet-level position in an Obama administration. There was even evidence suggesting that Edwards was on Obama's shortlist as a Vice President nominee (Associated Press [AP], 2008). However, due to the ensuing extramarital scandal, Edwards' prize value out of this backroom negotiation will, unfortunately, never be known publicly.

Yet, from Clinton's nomination as the Secretary of State to the appointments of Joe Biden and Bill Richardson (two other Democratic Presidential candidates) as the Vice Presidential Nominee and Secretary of Commerce, respectively, there is evidence in the public that supports the hypothesis that Obama's victory in the Democratic Presidential Primary did come at a heavy cost. Many analysts were already casting doubt on Obama's ability to act independently as President, apart from his former rivals. All of these current events show that my theoretical model does have important real-life implications for Edwards, as well as other underdog candidates, in a winner-take-all competition.

Aside from U.S. politics, my theoretical model also has ramifications for elections worldwide. One recent example is the 2008 Cypriot Presidential Election. There were three main candidates, Christofias of AKEL, Kasoulidis of DISY, and the
incumbent Papdopoulos of the DIKO. These three candidates entered Election Day in a virtual tie and the final vote tally was 33.51 percent for Kasoulidis, 33.29 percent for Christofias, and 31.79 pecent for Papadopoulos. According to Cypriot bylaws, a run-off election was called between the two top finishers. Although Papdopoulous was officially out of the race, his endorsement and supporters were widely coveted by both candidates in their run-off campaign; major positions in the government were offered by both candidates, including the Minister of Foreign Affairs as well as the President of the House of Representatives, to the DIKO (AP, 2008). In subsequent campaign speeches, both of the candidates' positions moved closer to that of Papadopoulos. Eventually, with DIKO's endorsement, Christofias won the Presidency of Cyprus. However, analysts are already waiting for the campaign promises made to Papadopoulos to trickle out (Reuters, 2008). This is another real-life example that would follow the expected value/ negotiation power framework proposed in my theoretical model.
IX. Conclusion

I began this paper looking at the reasons underlying Senator John Edwards’ decision to stay in the race even when he had no chance of winning the Democratic Presidential Nomination. My hypothesis responds to the situation in which none of the three major Democratic candidates obtains a majority of the delegates/vote-shares by the National Convention. I argue that this will force a backroom negotiation process in which all three of the candidates will try to form an alliance with one of the two other candidates in order to obtain the delegates/vote-shares that are needed to win. Through this negotiation stage, although Edwards would have come in as the weakest player (as defined by his vote-shares in comparison with Obama and

Clinton), he would have become the most valuable player, as both Clinton and Obama would have tried to negotiate with Edwards to minimize losses to their own expected values. In this situation, Edwards could have obtained a prize value that would compensate him for his level of effort in the state elections process. This prize value coming out of the backroom negotiation could have meant having his own policies on the agenda of the next President of the United States or even a cabinet position in the next Presidential administration.

My theoretical model is based on a combination of a Tullock contest model and Baron and Ferejohn's model of government formation. My model differs from the Tullock contest model in that it connects all of the state elections together in the first stage through additional variables that describe the candidate's own past performances. It also differs from Baron and Ferejohn's model in that it does not fix the proportion of vote-shares/delegates each candidate would enter the negotiation stage with. My model connects the two stages by portraying the probability of a player's decision to offer to form an alliance as dependent on the proportion of voteshares that same player enters the negotiation with. As my model demonstrates, depending on the external circumstances, Edwards could have come into the final stage with an insignificant proportion of the overall vote-shares, but been able to use it to come out of the negotiation phase with a significant percentage of the final prizevalue. My model further tests the external circumstance to ensure that it could be achieved under equilibrium conditions.

One limitation of my model is that it does not address the existence of the "super delegates" in the Democratic Presidential Primary. These delegates include
high-ranking party officials and current and former elected officeholders. The crucial difference between these delegates and the pledged delegates is that the super delegates are free to support any candidate of their choice; the only way for a candidate to win over these delegates is to court them individually. Although the proportion of super delegates pledged to each candidate is usually approximately proportional to the percentage of vote-shares that the candidate gains in the state elections, the candidate could potentially win a nomination by having the support of many super delegates and a less-than-usual number of pledged delegates. This measure was put in place by the Democrats to prevent the nomination of any radical candidates to the General Election due to popular fervor. As such, these super delegates wield a significant amount of power and can change a nomination race at any time. Thus, they should be addressed in any subsequent theoretical model.

Nonetheless, given its limitation, my theoretical model still exposes a critical result in the 2008 Democratic Primary that future campaign strategists should consider. Perhaps, if John Edwards had stayed a little bit longer, he could have demonstrated the "Edwards' Effect" in real life and then, as Trippi writes, "[John Edwards] would hold the key to the convention deadlock. And maybe, just maybe, a brokered convention would have stunned the political world and led to an Edwards' nomination."

## X. Bibliography

Aldrich, John H. (1980). A Dynamic Model of Presidential Nomination Campaigns. American Political Science Review, 74(3), 651-69.

Associated Press. (2008, June 20). Edwards makes Obama's VP list. Retrieved December 9, 2008, from http://politicalticker.blogs.cnn.com/2008/06/20/ap-edwards-makes-obamas-vp-list/

Associated Press. (2008, February 17). Incumbent eliminated from Cyprus presidential election in major Surprise. Retrieved December 9, 2008, from http://www.iht.com/articles/ap/2008/02/17/ news/Cyprus-Election.php.

Austen-Smith, D. \& Banks, J. (1988). Elections, Coalitions, and Legislative Outcomes. American Political Science Review, 82, 405-22.

Baron, D. \& Ferejohn, J. (1989). Bargaining in Legislatures. American Political Science Review, 83(4), 1181-206.

Chozick, A. (2008, May 31). "Tuesday May Not End Democratic Fight." The Wall Street Journal. Retrieved from http://online.wsj.com/.

Diermeier, D. \& Merlo, A.. (2004). An Empirical Investigation of Coalitional Bargaining Procedures. Journal of Public Economics, 88(3), 783-97.

Eraslan, H. (2002). Uniqueness of stationary equilibrium payoffs in the BaronFerejohn model. Journal of Economic Theory, 103(1), 11-30.

Kendall, K. (2000). Communication in the Presidential Primaries: Candidates and the Media. Westport: Praeger.

Konrad, Kai A. (2007). Strategy in Contests - An Introduction. WZB-Markets and Politics Working Paper No. SP II 2007-01.

Merlo, Antionio. (1997). Bargaining over Governments in a Stochastic Environment. Journal of Political Economy, 105(1), 101-31.

Reuters. (2008, February 29). "Communist Christofias wins Cyprus presidential vote." Reuters. Retrieved from http://uk.reuters.com.

Stone, W., Rapoport, R., \& Atkeson, L. (1995). A Simulation Model of Presidential Nomination Choice. American Journal of Political Science, 39(1), 135-61.

Times Online. (2006, December 28). "John Edwards joins race for White House." TimesOnline. Retrieved from http://www.timesonline.co.uk/.

Trippi, Joe. (2008, May). What I Should Have Told John Edwards. Politics, 18-9.
United Steelworkers. (2008, May 15). "United Steelworkers Endorse Senator Barack Obama for President." United Steelworkers. Retrieved from http://legacy.usw.org.


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