THE SWAPS MARKET:

A CASE STUDY DETAILING MARKET EVOLUTION, CURRENT MARKET DYNAMICS AND PRICING ISSUES VIA THE BINOMIAL TREE MODEL

Joshua Schiffrin[1]
Duke University
Durham, NC
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[1] Joshua Schiffrin graduated Summe Cum Laude from Duke in May 2001 with a BS in Economics and Mathematics. After graduation he will work as a derivatives analyst at Goldman Sachs in the Fixed Income Division.

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This thesis is dedicated to my mother, who always is there.

Preface[2]:

Financial derivatives currently are widely used by corporations, money managers and hedge funds. Investment banks make markets in a wide array of derivatives and can price and hedge nearly any option, future or combination thereof. However, nearly three decades ago, the market for such innovative and custom-tailored products was virtually nonexistent. It is only recently that the rapid growth in financial derivatives has occurred; the fact that the products today are so widespread and sophisticated should not obscure their relative nascence in financial markets. In particular, the market for interest rate and cross currency swaps has grown tremendously.

Studying the swaps markets will allow an excellent analysis of the growth and current sophistication of financial derivatives. Discussing the basic structure of a swap and how various swaps work is a necessary first step in the analysis. Following these preliminaries, understanding the early history of swaps and why they were introduced allows insight into innovation in financial markets. Examining the structure of current swaps markets including the various uses of swaps, how swap rates are set and how the swaps markets function will illustrate the importance and complexity of current derivatives markets. It will also be important to discuss the wide array of custom-tailored derivatives introduced via the swaps markets, including more exotic swaps, options on swaps known as swaptions, as well as interest rate caps and floors. Each of these products, or combinations thereof, can be used to transfer risk in a variety of ways. Finally, exploring the pricing of several such fixed income derivatives via the binomial model will permit a discussion of option valuation in the fixed income world and result in a more concrete overall analysis.

Preliminaries

A swap is a contractual agreement in which two parties, known as counterparties, agree to make periodic payments to each other (Marshall 3). Two parties can agree to swap payments of nearly anything and the concept of a swap is not limited to financial markets. For example, consider two Duke students who have received their housing assignments for the full four years

[2] Note: All exhibits referenced in the text are located in Appendix A, beginning on page 37.

at Duke upon arriving freshman year. Student A is given what is considered excellent housing for the first two years, but poor housing for the last two years. On the other hand, Student B is given what is considered poor housing for the first two years and excellent housing for the last two years. Now, let us assume that Student A wants to save his good housing years for the end of his Duke career, while Student B wants to start out immediately with good housing. They discuss their respective situations and decide to swap housing assignments. Both student A and student B are now better off than before they swapped – overall utility has increased! Keeping this example in mind, we move to discussing the swaps commonly used in the financial marketplace.

In financial markets, there are primarily two types of swaps: fixed-for-floating rate swaps known as *interest rate swaps* and swaps of different currencies known as *cross currency swaps*. In an interest rate swap, one party will agree to make floating-rate payments based on some floating index, say 3 month Libor and the other party will agree to make fixed-rate payments. The payments are calculated based on a notional amount that is not exchanged in an interest rate swap. Exhibit 1 illustrates, via a diagram, the basic schematics of an interest rate swap. The diagram shows that the notional payments need not be exchanged in an interest rate swap. Additionally, in an interest rate swap, only the net differential of the fixed and floating rate payments would need to be exchanged. Exhibit 2 illustrates the exchange of cash flows for a 5 year swap with payments made annually. Note that the first payment is made a period after the initiation of the swap agreement as Hull describes is convention (Hull 122). The floating rate is the rate observed at the beginning of the period. Therefore, the first floating rate payment is always known at the initiation of the swap. These basic diagrams lend a basic understanding of what is often termed, “plain vanilla” interest rate swaps (Hull 121).

The main difference between an interest rate swap and a cross currency swap is that the notional principals are denominated in different currencies in a currency swap. As a result of two different currencies being involved, the notional principals are most often actually exchanged at the inception and ending of the swap and thus the term ‘notional’ here is somewhat of a misnomer (Marshall 79). The notional principals are exchanged at inception and at the ending of the swap at the spot exchange rate at the inception of the swap. As John Marshall notes, “while this may initially seem strange, it is logical once one realizes that the difference between the forward and spot exchange rates has already been accounted for in the interest rates on the swap" (Marshall 80). Cross currency swaps typically have interest payments where one party is paying a fixed rate on one currency and the other party is paying a floating rate on another currency. They can also be structured whereby both parties pay a fixed rate on different currencies or both parties pay a floating rate on different currencies. The structure of a currency swap is determined by the needs of the counterparties involved. For example, Exhibit 3 illustrates the exchanges that would take place in a fixed for floating dollar/euro cross currency swap. Party B is swapping dollars for euros and then making floating rate payments in euro, whereas Party A is swapping euros for dollars and will make fixed rate payments in dollars. Currency swaps clearly can allow a great deal of flexibility in managing foreign exchange operations – thus first gaining a basic understanding of the structure of a currency swap is certainly beneficial.

Interest rate swaps and cross currency swaps are the two most common swaps structures in the financial marketplace. However, the basic achievement of these types of swaps is no different than that of the aforementioned hypothetical housing swap conducted by the two Duke students. An interest rate swap can allow a counterparty to switch from making floating rate payments to fixed rate payments and a cross currency swap can allow a counterparty to switch from making fixed rate payments in one currency to making floating rate payments in another currency. This is analogous to allowing a student to switch from receiving good housing in earlier years to receiving good housing in later years of university study. The key in all three examples is in making sure there is another party present who wants to do the exact opposite operation. In the housing example that required a student who wants good housing now rather than later. In the interest rate swap example that requires a party who wants to switch from making fixed rate payments to making floating rate payments. In the cross currency swap example, a party who wants to switch to making fixed payments in the opposite currency is required.

As will be discussed, in the early days of swaps in the financial marketplace, matching parties one by one was actually the way business was conducted. This tedious process meant actually searching for a counterparty to take the other side of the trade. For standard trades this was not difficult – the odds are seemingly good that there exists a corporation who may want to swap from fixed to floating payments for 5 years on a notional of 100 million dollars. However, for more complex trades, say a corporation wanting to pay fixed and receive floating for the first five years of a swap and then receive fixed and pay floating for the final five years on an amortizing notional amount, finding a counterparty who wants the exact opposite structure is clearly difficult. Returning to the simple housing example, a student wishing to swap every other month of housing with a semester abroad his third year would probably have difficulty finding someone to swap with as the likelihood of finding somewhat with exact opposite desires would be slim. Additionally, another problem arises if a student who has swapped with another student decides to leave Duke after his first year, effectively defaulting on the agreement. In the financial marketplace, this would be akin to a company in a swap going out of business. The frustration of matching counterparties and the threat of default by a counterparty are two issues that shaped the early evolution of the swaps markets. Looking at the origins and early history of the swaps markets allows a full exploration of how the swaps markets evolved into what they are today and how the issues of matching counterparties and the possibility of default were handled.

The Origins and The Early Development of the Swaps Market

The first swap that gained notoriety in the financial marketplace was the now famous cross currency swap between The World Bank and IBM (Ludwig 13), however some believe that cross currency swaps were taking place as early as 1979 (Marshall 5). The first interest rate swaps followed the first currency swaps and are believed to have taken place in late 1981 in London (Marshall 6). The conditions that led to cross currency swaps first taking place are fascinating. In Great Britain, during the 1970's, the British government sought to encourage domestic investment by taxing foreign-exchange transactions on its own currency (Marshall 6). This control made it difficult for British multinational companies to transact in the foreign exchange market. As a result, British companies often engaged in what was known as back-to-back loans. Exhibit 4 illustrates the back-to-back loan procedure. A back-to-back loan involves two companies located in separate countries. Each company will borrow money in their domestic financial marketplace and then lend this borrowed money to the other firm. By this simple exchange, each company can access the capital markets in the other country without involving foreign exchange transactions (Marshall 6-8).

The currency swap is merely a simple extension of the back-to-back loan concept. The back-to-back loan suffered two problems that caused currency swaps to be introduced. Firstly, such back-to-back loans required one counterparty to find another counterparty with mirror image needs – often an exhaustive task for a corporation. Secondly, as these back-to-back loans were two different loans, they were evidenced by two loan agreements, completely separate from one another (Marshall 5). These are the very two problems discussed earlier to which currency swaps provided the solution. The currency swap would operate in a manner such that two counterparties would be brought together by a central matchmaker, or swap dealer (often an investment house) thus eliminating the exhaustive search costs to the corporation of finding another counterparty. Secondly, the transaction would now be witnessed by one document that would spell out the circumstances and provisions under the possibility of default by one counterparty (Marshall 5).

The World Bank and IBM currency swap in 1981 showcased the benefits of currency swaps in the financial markets. The World Bank desired to borrow Swiss francs, but the size of its borrowing could not be absorbed by the small Swiss francs debt capital markets. The World Bank, however, could borrow cheaply in dollars. By completing a currency swap with IBM, brokered by Solomon Brothers, the World Bank was able to obtain the Swiss franc financing it needed without actually borrowing in the Swiss franc markets (Ludwig 13-14). Thus, although swaps originated as a response to circumvent foreign exchange controls in Great Britain, it was their cost-reducing and risk-transferring measures that led to their popularity and widespread usefulness (Marshall 6). Indeed, with the exchange rate volatility of the late 1970's and early 1980's after the collapse of the Bretton Woods agreement, swaps were clearly excellent structures through which foreign exchange risks could be controlled (Marhsall 5). Additionally, interest rate differentials around the world, particularly as Ludwig notes, the "much higher yields in New Zealand and Australia" helped boost the currency swap market (Ludwig 14).

As interest rate swaps are merely cross currency swaps with both legs of the swap denominated in a single currency, it was not long before financial market participants realized that they could convert fixed rate to floating rate borrowing via an interest rate swap. Such a realization came at a time when the U.S. markets were undergoing a period of extremely high interest rates. Banks did not want to lend at such high fixed rates for long periods of time and investors did not want to miss out on even higher rates that they felt were soon to come (Ludwig 14). Thus, the interest rate swap market provided a way for banks and investors to manage their interest rate risk. The first major interest rate swap is often considered to be Sallie Mae's fixed for floating transaction in 1982 (Marshall 6).

It is interesting to note that the innovation of swaps was entirely solution driven – that is, they were brought about to codify and simplify the tedious back-to-back loans that were painstakingly being employed by corporations. Once these first swaps were conducted, their ability to transfer risk was recognized and thus growth exploded (Marshall 4). Having an investment house, known as a swaps dealer, match up counterparties essentially made each swaps desk its own miniature swaps exchange. It is interesting to contrast the introduction of swaps to the introduction of index futures, another financial derivative introduced in the United States in the early 1980's. Whereas index futures were subject to extreme regulation by the government (Schiffrin, Paper 1, 2-4) swaps were completely unregulated. This lack of regulation in the swaps market is primarily the result of the way swaps were initially conducted – in an over the counter market wherein the investment houses and the counterparties themselves were the only ones who knew a swap had even been conducted. The lack of regulation in the swaps market continues today and for nearly their entire history swaps have been considered to be "off-balance sheet transactions" (Marshall 20, Ludwig 30). Though swaps have not been regulated by governmental entities, standardization of swaps did begin to take place in the mid-1980's. The International Swap Dealers Association (now known as the International Swaps and Derivatives Association or ISDA) established a set of standard terms for interest rate swaps that made negotiating a swap much easier (Arditti 256). Additionally the Basle Accord in 1992 followed up on the 1988 Basle agreement that focused on monitoring credit/default risk in financial markets (Coopers 139, Marshall 21). This standardization helped fuel the swaps markets by setting up protocol for the way in which swaps would be transacted.

As noted, swaps transactions were completely matched (each trade had two counterparties that were mirror images of each other) by swaps dealers for most of the 1980s. In an interest rate swap, the dealer would merely extract a bid-ask spread by receiving fixed interest rate payments at a higher interest than it was paying fixed interest rate payments while paying and receiving the same floating rate (Arditti 248). Such a transaction is outlined in Exhibit 5. The dealer merely stands in the middle of the trade, effectively collecting a fee in return for matching the two counterparties. The only risk to the swaps dealer was that one of the two counterparties would default, but with the standardization of swaps agreement via ISDA, such provisions were often accounted for (Ludwig 26-27).

The swaps market grew exponentially during the 1980's. As Exhibit 6 shows with partial data from the1980's, the swaps market had grown from practically nothing to a trillion dollar market. Noting that the 1980's were a time extremely volatile interest rates in the United States, absolute interest rate differentials around the world were large and foreign exchange rates were fluctuating greatly made swaps a risk management vehicle of choice (Coopers IX). The popularity of swaps continued in the 1990's and further innovation and sophistication was introduced. Indeed, today's swaps market is vastly more complicated than that in the 1980s. Looking at the structure and mechanics of today's swaps markets will provide illustrate how derivatives markets have evolved and how they are now essential in current financial markets.

The Current Swaps Market

The swaps market has continued its amazing growth during the 1990's. The main reason for this growth is due to the way investment banks have handled their swaps books. As noted, throughout most of the 1980's investment banks ran completely matched books, actively seeking out counterparties to take on the opposite side of a trade and effectively serving merely as a middleman. However as the market for swaps became more widespread, swap dealers began "warehousing swaps" (Marshall 13, Hull 128). In warehousing swaps, dealers would conduct the swap directly with the counterparty without finding another counterparty to immediately take the opposite side of the trade. The swaps dealer could hedge out the risk of the swap in the financial marketplace. The treasury market was the initial vehicle of choice to make these hedges as it is extremely liquid (Marshall 15). For instance, if a company wants to receive fixed and pay floating in a swap for 5 years, the swaps dealer can hedge out this risk in the financial markets; this is illustrated in Exhibit 7 (which is adapted from Marshall 52-54). The dealer will go long a 5 year Treasury bond to hedge its risk that fixed rates fall and will short a three month treasury bill and roll over this short position every three months to hedge the risk that short term floating rates fall (Marshall 52-54). Eventually, the dealer will take off this hedge when a counterparty is available to take the other side of the trade (Ludwig 48-49). In the meantime however, the lack of a matching counterparty does not preclude Party A from swapping with the swaps dealer. It should be noted that the hedge illustrated in Exhibit 7 is not perfect. The dealer is receiving floating rate payments of 3 month Libor and is effectively paying short floating Treasury rates. The risk to the dealer is that the correlation between 3 month Libor and the treasury bills (this risk is often known as basis risk) is not perfect. The dealer is exposed if 3 month Libor falls more than the 3 month treasury bill. Now if a dealer wishes to take a position on the future of rates, it could leave its swaps book to benefit from a rise or fall in rates, depending on its particular view.

Swap dealers, by warehousing swaps, were effectively making markets in swaps and as swaps served a great deal of purposes for a variety of end-users, the market grew quickly. Exploring some of the basic uses of swaps will enable a more detailed analysis of the current swaps market. A very basic use of swaps is to transform a liability. As corporations can sometimes borrow more cheaply in either fixed rate or floating rate markets, the swaps market can be used to maneuver the cheaper borrowing into a corporation's needs (Ludwig 77-79). An example is clearly necessary. Assume that company A can borrow comparatively more cheaply in the fixed rate markets than in the floating rate markets. Also assume that company A has floating rate assets and desires that its assets match its liabilities. Company A could issue fixed rate debt and then swap with a swaps dealer to synthetically convert the debt to floating rate debt. This trade is illustrated in Exhibit 8. This use of swaps – synthetically converting a liability – is quite widespread. A second use of swaps that is extensive is that it can enable borrowing in a foreign market (Coopers IX). This use of swaps was recognized early on and was the principle for which the World Bank-IBM cross currency swap was based. Today, such an example might occur when an American company with a poor foreign presence wishes to expand internationally. Exhibit 9 illustrates the process. Such a company can borrow much more cheaply in American markets than in Japanese markets. Thus by borrowing in U.S. dollars and then synthetically converting such debt to Japanese yen (to finance its Japanese operations), this company has effectively reduced its borrowing costs via the swap. This reduction in borrowing costs assumes that the fixed swap rate it will pay the swap dealer is less than the rate it could have achieved outright by borrowing fixed rate debt directly in the Japanese capital markets. As this is often the case, synthetically allowing borrowing in foreign markets is clearly a very useful application of swaps (Coopers IX). A third way in which swaps are extremely useful is in adjusting the floating/fixed mix of their debt portfolio. If a corporation decides it wants more of its debt to be floating, thereby thinking rates will fall, it can engage in a swap with an investment bank to increase the floating rate portion of their debt portfolio. Similarly, in times of rate uncertainty, cautious customers can increase the fixed rate portion of their debt by swapping to pay fixed in an interest rate swap. Before the advent of swaps, such adjustments within a debt portfolio could only have been done through refinancing (Ludwig 85). Since refinancing is not always an option, interest rate swaps became the vehicle of choice in structuring or restructuring the floating/fix mix of a debt portfolio.

It is thus apparent that swaps have a variety of uses in the financial markets. In fact, swaps have became related to nearly every other market in the fixed income world. This is first seen in the fact that swaps are quoted as a spread above U.S. treasuries. The quote a swaps dealer gives is for the fixed rate in a swap – where a dealer will pay fixed versus receiving 3 month Libor for a designated amount of time (Ludwig 37-38). This fixed rate is quoted as a spread above a comparable treasury bond with the same maturity. Thus a 5 year swap would be quoted as a spread above the 5 year treasury bond. A dealer will effectively give two sided prices for swaps: one price is where it will pay fixed rate payments to a counterparty and the other side is where it will receive fixed rate payments from a counterparty (Ludwig 35). As Exhibit 10 illustrates, naturally, a dealer will pay fixed at a lower rate than it will receive fixed. Thus, given the rates in Exhibit 10, a corporation with good credit quality could transact to pay fixed in a swap and receive floating from a swaps dealer for 5 years at a rate of 6.69%, 49 basis points above the comparative treasury rate. The spread above treasuries is known as the swap spread. As the treasury curve is a riskless rate, the swap rate in the United States will always be above the corresponding treasury rate, thus making for a positive swap spread (Ludwig 36-38). The amount the swap rate is above the treasury rate, the swap spread, constantly changes. Exhibit 11 gives a graphical illustration of the swap spread. Swap spreads are moved by a variety of factors – for shorter maturities they are mainly driven by expectations of future floating rates as will be detailed when actual pricing mechanisms are discussed, whereas longer maturity spreads are mainly driven by corporate debt issuance (Ludwig 11). For example, if there is an imbalance and lots of corporate customers want to pay fixed in a swap and very few customers want to receive fixed in a 30 year swap, swap spreads will widen.

Swaps are additionally linked to the Eurodollar futures market. Since the eurodollar futures contract is a contract on the future of 3 month Libor interest rates, as eurodollar futures rates rise, 3 month Libor will be expected to rise as well. This implies that floating rates will be higher in the future and since quite often, one of the legs in an interest rate swap is based on 3 month Libor, swap rates will rise as well. The exact relationship between Eurodollar futures and swaps will be explored in the actual pricing of a swap, but for now it is sufficient to note that the Eurodollar futures market is intertwined with the swap market (Arditti 262-263).

Thus, it is readily apparent that swaps play an integral role in many fixed income markets. The swaps markets interrelationship with the treasury market and futures markets has made monitoring the swaps market and swap spreads, in particular, vital in the fixed income world. With the U.S. Treasury debt rapidly shrinking, swaps will most likely play an even more important role in years to come. This is a result of the importance of benchmarking in financial markets. As noted with respect to the S&P 500 index futures contract (Schiffrin, Paper 1, 10), the importance of finding a benchmark in financial markets can not be understated. With treasury bonds becoming more and more scarce, liquidity may eventually become quite reduced. Indeed, many corporate bond deals have recently been priced in relationship to swaps as opposed to treasuries as had previously been the case (http://biz.yahoo.com/rf/001114/n14438738.html). With its variety of uses and applications throughout the fixed income world, making a case for swaps as a new benchmark fixed income instrument would certainly be viable, if the treasury debt continues to be reduced as expected and a new benchmark is necessary.

Further Innovations in the Swaps Markets

The preceding analysis has focused only on plain-vanilla swaps – that is swaps for a set amount of time for a fixed notional value, starting immediately at quoted market prices. However, as the swap market is solely an over the counter market, various more complicated swap structures have been easy to introduce (Ludwig 21-23). These included combining swaps with options, embedding options in swaps and more complicated variants of the basic swap structure. Describing and analyzing these structures permits a window into the exceedingly complicated world of financial derivatives and showcases the amazing variety of possibilities that corporations and other end-users as well as speculators now have to choose from among.

One of the most successful and widely used products in swaps markets today is swaptions. A swaption is the option to enter into a swap (Ludwig 141). A swaption is like any other option product, with an exercise date and strike price but in a swaption the underlying security is a swap itself. Nearly all swaptions are European and a swaption to pay fixed in a swap is known as a payer swaption while a swaption to receive fixed is known as a receiver swaption (Arditti 297). A payer swaption is comparable to a put a bond – if rates rise, the swaption will be exercised whereas a receiver swaption is comparable to a call on a bond – if rates fall, the swaption will be exercised. Swaptions provide flexibility to corporations who may know that they might need to swap sometime in the future and would like the right but not the obligation to do so. Like other option products, swaptions require a premium, typically quoted in basis points; thus a swaption that costs 30 basis points on a 100 million notional swap would cost the buyer of the option $300,000 (Ludwig 141). Additionally, there also exist swaptions that are 'Bermudan': these swaptions are exercisable at several dates. The market for swaptions has grown considerably and the flexibility options provide led to even embedding options in swaps.

Embedding options in swaps, for a fee often priced into the swap itself, allow more flexibility in the terms of a swap. A callable swap contains an option that allows the fixed rate payer the right but not the obligation to terminate the swap. Such an option would be exercised if rates fall (Arditti 297). A puttable swap contains an option that the floating rate payer the right but not the obligation to terminate the swap. A puttable swap termination option would be exercised if rates rose (Arditti 297-298). Companies that want to buy insurance can make use of these embedded option products (Ludwig 183). Embedding simple and call put options in swaps can enable a corporation to create any payoff structure it desires merely by combining options. Embedded options in swaps are somewhat similar to interest rate caps and floors, other option-like products.

A cap allows a company to fix the highest interest rate it will pay on floating rate debt. Consider a company who is paying floating rate interest on a 5 year loan at 3 month Libor which is currently at 8%. The company can buy a cap for a premium from an investment bank that will limit its exposure to up to rises in 3 month Libor up to 9%. If rates rise above 9%, the investment bank will make up the difference between 3 month Libor and 9%. The cap will be applied on each interest payment, where the floating rate is set at the beginning of the period and the payment is made at the end of the period as is custom. Exhibit 13 illustrates this example. A cap is thus effectively a series of options on interest rates, a view that is more precisely seen when actual pricing is discussed (Ludwig 154). Caps are another means of buying insurance if one is worried about rising rates. For a fee, a corporation can set the highest interest rate it is willing to pay on floating rate debt.

An interest rate floor is the mirror image of a cap. It sets the lowest level that a corporation will pay on floating rate debt. Whereas a user who is concerned about rising rates would employ a cap, a user who is concerned about falling rates would employ a floor. As would be expected, the cost of both caps and floors increase as the length of time covered increases. The two structures can be combined – in fact, corporations often sell floors (giving up the upside of falling rates) to lower the cost of buying expensive caps (Ludwig 157). Such a structure results in an interest rate collar, whereby the floating rate paid will fluctuate between two preset levels (Arditti 134-135). By selling the floor, the corporation has set the lowest rate it will pay on debt, and by buying a cap, it has set the highest rate it will pay. The strike prices of the cap and the floor can be set at any level, depending on the preference of the buyer. Exhibit 14 illustrates a collar with a cap (bought) struck at 10% and a floor (sold) that is struck at 8%. The strike prices of the cap and the floor can be structured such that the proceeds from selling the floor exactly equals the cost of the cap. Such a collar is known as a zero-cost collar and is a popular product due to its lack of an upfront premium (Arditti 135-136). Caps and floors can be used in conjunction with interest rate swaps to limit the interest rate risk of the floating payer.

The sophistication of caps and floors has continued with further options embedded in these structures creating even more complicated payoff structures. One of the most popular of these structures is the knockout cap. A knockout cap limits the interest rate exposure above a specified strike rate. However, a second barrier is specified that will cause the cap to cease to exist if rates increase above this level. For example, a knockout cap with a strike rate of 9% and a barrier of 12% will result in the cap seller paying the cap buyer nothing if rates are below 9%, the difference between the interest rate and 9% if rates are between 9% and 12% and nothing if rates are above 12%. Since some of the upside is sacrificed, a knockout cap is similar to an interest rate collar, but often times since so much upside is sacrificed (all scenarios where rates are above 12% in this example give no protection to the cap buyer) the price of a knockout cap is much less expensive. A knockout cap can be viewed as buying a cap and selling a cap with a higher strike price. Such a product illustrates that a corporation can achieve nearly any payoff structure it desires via caps and floors.

Stand alone swap options, embedded options in swaps and options on interest rates combined with swaps are clearly all very effective risk transferring products. However, variations of the basic swap structure itself have proved to be very successful as well. The most basic variation of a swap is a forward starting swap, also known as a delayed starting swap. A forward starting swap is a forward contract on a swap, thus at the end of the forward period a swap will be initiated. Such a swap would be useful to a user who knows that a swap will be necessary at some point in the future and finds current rates attractive. Rather than take the risk that rates might be different when the swap is needed, a forward starting swap can be transacted (Arditti 293). The fixed rate on a forward starting swap is merely the current swap rate plus the carry over the duration of the forward time period (Ludwig 175-176). Amortizing and accrediting swaps are other variations of plain-vanilla swaps. In an amortizing swap, the notional value of the swap increases over the length of the swap, whereas in an accrediting swap, the notional value decreases. Banks often utilize amortizing swaps whereas real estate market participants find accrediting swap useful (Luwdig 176-177).

A basis swap is another kind of swap, in which the two counterparties each pay interest based on a floating rate. An example of a basis swap would be a swap in which a party pays 3 month Libor versus receiving 6 month Libor. Such swaps can be employed by relative value hedge funds that feel one interest rate may increase or decrease relative to another (Arditti 292). As Exhibit 15 illustrates, a basis swap is merely the combination or net of two plain vanilla swaps. Additionally, zero coupon swaps can be employed in which only one fixed payment is made – that being at the final swap payment (Marshall 65). Swap variations are not limited to interest rate swaps. An example of a variation of a cross-currency swap is a differential swap in which a floating interest rate in a domestic currency is exchanged for a floating rate in a foreign currency with both interest rates being applied to the same domestic principal (Hull 142). Additionally, a currency swap can be structured so that the spot exchange rate on which the notional values will be exchanged can be set after the swap has begun. The exchange rate can be an average over the rate setting time period, or even the most favorable rate to a client – of course, the terms of the agreement would affect how such a structure is priced. Finally, any swap can be done off-market. If a company wanting to pay fixed in a swap has an abundance of cash now and wants to reduce the fixed rate it will pay over the duration of the swap, it can give an upfront fee to a swap dealer in return for a lower fixed rate (Marshall 60-62). Similarly, a company can structure a swap to receive cash up front, in return for paying a higher fixed rate (or receiving a lower fixed rate).

The plain vanilla swap structure can be modified or varied, embedded with options, or even combined with an option to produce nearly any payout function desired. As the swap market is an over-the-counter market with each transaction being custom tailored, it is easy to see that how the complexity of the market can have grown so rapidly in such a short period of time – after all, very few corporations have the exact same borrowing needs and very few hedge funds or money managers have exactly the same view on interest rates or currency exchange rates. The swap market can be used to tailor products to the needs of corporations and allow hedge funds and money mangers to express these views; the wide variety of products related to swaps is testament to the complexity of today's derivative markets.

One potential outgrowth of having such complexity in the financial markets is that it makes pricing of these instruments quite difficult. Each variation of a swap must be priced differently and options of any form must be accurately priced. For example, a callable swap must have greater value than a non-callable swap, all else equal. Clearly a discussion of pricing in the swaps market is necessary. By first looking at how plain-vanilla interest rate swaps are priced will allow an introduction into the basic pricing mechanism in the swaps market. Moving on to analyze how swaps with embedded options, caps, swaptions and are priced will take the preceding descriptive analysis of such securities to a more concrete level. Additionally, such pricing will allow application of the binomial tree option valuation model in a variety of different situations, now in the fixed income world. A detailed explanation of pricing is indeed necessary for an analysis of swaps, and interest rate derivatives in general, to be complete.

Swap Pricing

The pricing of swaps is based on the concept of present value. In an interest rate swap (only the pricing of interest rate swaps will be considered here) the initial net present value of the swap is zero (Hull 133). This logic makes intuitive sense in that neither party would want to enter into the swap if one side currently has an "advantage" at the beginning of the swap. The issue in swaps pricing then is how to equate the present value of the floating rate interest payments to the present value of fixed rate interest payments. Rephrasing the issue, to price the swap one must set the fixed interest rate so that the present value of the payments based on this fixed rate are equal to the present value of payments based on the floating rate (Marshall 146).

The floating rate in interest rate swaps is most often 3 month Libor (though it could be indexed and thus priced off of any floating rate). The fixed rate that will be determined is the swap mid-rate and the swaps dealer can add or subtract a few basis points depending on whether it will be the fixed rate payer or receiver in the swap. In order to get the present value of all the floating rate payments one must use floating rates for time periods way out into the future. Assuming the floating leg of the swap is based on 3 month Libor, if one views the Eurodollar futures quotes as unbiased estimates of 3 month Libor, then one effectively has a mechanism for viewing what the market believes floating rates will be in the future. The Eurodollar market implies 3 month Libor out in the future – a quote of 92 on a March 2002 Eurodollar contract implies that 3 month Libor will be 8% (100-92=8) for three month deposits beginning in March 2002. (In actuality, Eurodollar futures quotes require a convexity correction to be more closely considered to be unbiased estimates of 3 month Libor. Hull discusses why such a convexity correction is needed on page 108 and Arditti explains the concept via a more precise hedging argument on pages 269-270.) As the Eurodollar market is extremely liquid out to 2 years (with contracts trading out to five years) pricing short dated swaps using Eurodollar futures is common. For longer dated swaps, issuance of corporate debt and supply and demand factors take a more prominent role, as noted earlier (Ludwig 11).

In considering the pricing of a short dated swap, one must look at the forward term structure of interest rates. If interest rates are expected to rise in the future, the fixed rate in the swap would have to be set such that in early payments the fixed rate payer is paying the net difference, while in the later payments, the fixed rate payer is receiving the net difference. This is more easily seen in a diagram, as shown in Exhibit 16. The fixed rate should be set so that the present value of Area A equals the present value of Area B.

In a very steeply upward sloping yield curve, the fixed rate set will be dramatically higher than the initial floating rate. If the term structure of interest rates is constant, the fixed rate will very nearly equal the floating rate, as here interest rates are not expected to move at all in the future. Of course, what markets are expected to do is not what they always end up doing, and as swap payments take place, the swap will have positive value to one party and negative value to another party. If rates rise higher than expected, the fixed rate payer will benefit, if rates rise less than expected, the floating rate payer will benefit. Now that the methodology of swaps pricing has been discussed, it will be useful to detail the pricing of a simple short-dated swap.

Consider the pricing of a 2 year swap. The notional of the swap considered is 100 million dollars and assume today's date is December 20, 2000 [though data from market close December 13, 2000 will be used]. This assumption is made because December 20, 2000 is the day that the December Eurodollar contract expires. Swap payments will be made every six months with the swap payment dates coinciding with the expiration of a Eurodollar futures contract. Thus, the reset dates of the swap (the days in which the floating rate of the swap for the payment period is set will coincide with the Eurodollar futures contract expiration dates). Valuing a swap where the resets dates are the same as Eurodollar expiration dates will simplify the pricing of the swap. Exhibit 17, using current data from The Wall Street Journal displays 3 month Libor and Eurodollar futures contracts data. As the exhibit indicates, from the Eurodollar quotes, one can estimate what implied 3 month Libor will be in the future. Additionally, the exhibit also shows the number of days in each of the periods considered. The number of days in a period is vital because the floating leg of the swap is based on an actual/360 daycount convention. From this Eurodollar data, discount factors for each of the 3 month periods can be calculated and from these discount factors, the fixed swap rate can be determined. The discount factors will be used to discount the fixed rate payments at the appropriate rates for when the fixed rate payments will be made. This process is outlined with the calculations included in Exhibit 18.

In essence, the procedure in Exhibit 18 views the fixed rate portion of the swap as a fixed interest rate paying bond and attempts to value this bond. This fixed interest rate of the bond results in the swap having net present value of zero and this is the "par swap rate" (Arditti 267). Exhibit 19 illustrates this procedure. The two year rate determined is the rate at which the NPV of the swap is zero at initiation. The value of 6.133% determined makes intuitive sense as the market appears to believe that rates will drop, followed by a rise in the future and 6.133% appears to be in the middle of that area. The expected swap payments, shown in Exhibit 20 show that indeed the net present value of the swap at initiation is zero. It is important to recall that the first swap payment is made six months after the swaps trade date. Thus, the first payment is always known as 3 month Libor at the time of the trade is known and the fixed swap rate is specified. The payment must nevertheless be discounted as it will not take place until six months from the trade date. In this swap, if the Eurodollar future quotes actually turn out to be 3 month Libor (i.e. 3 month Libor in June 2001 is 5.94% and so forth for the subsequent reset dates), the floating payer will make the net payment in the first period and the fixed payer will make the net payment in the final three periods. This makes sense as the market is pricing in a sharp drop in interest rates. Comparing the actual two year swap rate of 6.17% to the rate of 6.133% determined above shows great accuracy! Recalling the assumption that the swap was valued assuming the data was from December 20, 2000 and not the actual date from which the data was obtained (at market close of December 8, 2000) as well as the fact that no convexity correction was made helps explain this small error.

This concrete example of swaps pricing exhibits a couple of important facts described earlier in terms of how the swaps market operates. With the 2 year treasury bond trading with a yield to maturity of 5.47, a two year swap rate of 6.133 results in a swap spread of 66.3 basis points. One can now see why the U.S. swap spread must be a positive number as the swap curve is built using Libor forwards. There is of course the possibility, albeit remote that a bank will default whereas the U.S. Treasury rate is taken to be riskless. Additionally, one can now see that Eurodollar futures can easily be used to hedge a swaps book until a matching counterparty can be found. A dealer can effectively estimate how a certain basis point change in a Eurodollar futures contract and go long or short an appropriate number of contracts to insulate himself from such a risk (Arditti 267-268). Thus, it is now easy to see how liquidity in the Eurodollar market would enable dealers to "warehouse" swaps. Though the U.S. Treasury market can be used to hedge, basis risk will result as the swap rate is not built from treasury rates, but from the forward Libor curve.

Finally, though the swap has a starting net present value of zero, it is unlikely that the Libor forwards will actually be the floating rates on the reset dates. Thus, marking the swap to market will result in either the fixed rate payer or the floating rate payer having a negative value associated with the swap on his books. If rates move drastically, the negative value of the swap with respect to one counterparty can affect that counterparty's financial status greatly. As swaps have been off-balance sheet items until very recently, as noted earlier, such losses have been able to be hidden from the public.

Overall, the pricing of even plain vanilla swaps is clearly not simple. Even with assuming swap payments coincided with Eurodollar future expirations and ignoring convexity issues, pricing even a two year swap was certainly tedious. However, the analysis was definitely useful, exhibiting the elements that go into equating the present values of the fixed and floating rate payments of the swap. Though swaps are often priced using software programs, it is still useful to understand the elements that go into such pricing as hedging is based on these financial underpinnings. Moving on now to using the binomial tree methodology to price more complicated fixed income derivatives will expand the pricing analysis of swaps and swaptions.

Applying the Binomial Model to Fixed Income Securities

The binomial model for pricing options considers the possible paths of the price of the underlying security during the life of the option (Hull 201). From these paths the value of the option can be valued by working backward through the tree as at expiration, the value of the option is known with certainty. The binomial tree model can be used to value European, American and exotic stock options (see Schiffrin, Paper 3). However, modifications to the binomial model as used to value stock options need to be made in order to translate the binomial model to the fixed income world. The basic reason that the stock valuation can not be directly used is that fixed income securities have both a price and a yield and thus the price function of a fixed income security differs from the price function of an equity (Fabozzi 178). One problem that results is that in a fixed income security, as the maturity date approaches, the price of the bond is known with more certainty. This is best illustrated via a zero-coupon bond whose price will be 100 at maturity – as there is no uncertainty about this price, it does not make sense to apply the same volatility throughout as towards the end of the life of the bond, the price volatility will clearly decrease (Fabozzi 179-180). A second problem with applying the binomial model to the price function of fixed income securities is that the lognormal assumption on which the model is based does not work for bonds – this is a result of the fact that bonds have a minimum and maximum price, with the implicit assumption made that interest rates are always positive (Fabozzi 180). Assuming a lognormal price function would indicate that there is some probability, however small, that the bond will reach any positive price – an assumption that is not valid. The model used to value fixed income securities will have to make adjustments for these factors – a *binomial model based on yield to maturity* will prove sufficient. By assuming that bond yields are lognormally distributed, the possibility of negative interest rates is immediately eliminated. Additionally, as the bond prices will be calculated from the yield distribution, the prices will reflect the decrease in volatility as the bond gets closer to maturity (Fabozzi 182).

Thus, the two problems encountered are rectified and a binomial model based on yield can be applied to valuing first simple fixed income securities and then towards valuing more complicated securities. In all examples, the probability that interest rates will rise will be assumed to equal the probability interest rates will fall at a node: thus the simplifying assumption is made that the variable p=.5.

The first fixed income security that will be valued is a simple call option on the risk free rate. Such a call option can be likened to a call on a treasury bond. Exhibit 21 illustrates the binomial tree model being used to price a call with strike rate of 9% for a 5 period European call option. The spot risk free rate at the start of the period is 10%. It is important to realize that such an option is in the money if rates fall and out of the money if rates rise. For example, if at expiry (period t=5) the risk free rate is 5.49%, the option will be 351.19 basis points in the money. If rates rise above 10%, the option will be out of the money. Additionally, the option price tree was computed by discounting each node at the risk free rate at the corresponding node of the interest rate tree. Though this makes constructing the model more difficult, this avoids the problem of discounting at one rate throughout while assuming rates are constantly changing in the model itself. The value of the hypothetical call turns out to be 43.62 basis point – on a bond with a $1000 face value, this is $4.36.

Having proven successful in valuing a stand alone bond option, it is now worthwhile to see whether the binomial model can price an embedded option. Consider an option for the issuer to call the preceding bond at par at any time after period two. For this example we'll assume that a 10% rate corresponds to a price of par. The issuer in this case would be considered to be the U.S. government (the issuer of risk free bonds). Exhibit 22 illustrates the binomial tree model for such a bond. As the diagram illustrates, such an embedded option in a bond merely allows the holder of the option the right to buy back the bond at any time, whereas the earlier call only permitted exercise at the end of the last period only. The shaded in cells indicate where the bond would be called – it is the increase in value of these cells that lead to the slightly higher price of the call option. The price of this embedded bond option is 46.55 basis points, equivalent to $4.65 on a bond with $1000 fact value. This price is higher than the previous European call option, as was expected.

Now that simple bond options have been considered, an interest rate cap can be priced. So as to keep the example simple and to focus mainly on the pricing methodology, assume that the cap will be on the risk free rate itself. This could be considered somewhat analogous to the U.S. government capping its exposure to floating rate debt. Though this example does not replicate real-world conditions in that the government rarely issues such floating rate debt, the pricing mechanism works the same way as if we were dealing with a corporation issuing risk debt. Assume that the government issues floating rate debt with a duration of 5 periods with the spot rate at 10% as before. Now let's assume that the government wants to cap its interest rate payments at, conveniently, 10%, the spot rate of issuance. If rates rise above 10%, the government will receive the difference between the higher rate and 10%. Exhibit 23 illustrates the cap pricing. Essentially in pricing the cap, one is pricing a portfolio of options. This is due to the fact that the cap pays off every period. In the fifth period, for example, if the interest rate is 18.22%, the cap payment would be 822 basis points and if the interest rate is 13.50%, the cap payment would be 350 basis points. This cap payment would be made at the end of period 5. To find the present value of these possible cap payments, one must work back through the tree, remembering to view the cap payment of period 5 occurring at the end of that period. This process is somewhat tedious in that each payment throughout the tree must be discounted by the corresponding rate of the node on the interest rate tree. As Exhibit 23 shows, the present value of the cap payments of period 5 is 53.08 basis points. A similar process must be done for the cap payment for the rates observed at the other periods. Thus, for a five period cap, one is essentially valuing 4 options (not five options because the rate at the cap's initiation is known and it is not above 10% - it *is 10%*) and this is represented in Exhibit 23. The sum of these present values is the price of the cap and as the exhibit details, this value is 248.22 basis points, or $24.82 on a $1000 face value bond. Looking at this figure another way, by viewing the 248.22 basis in yearly payments or in other words as an annuity, this would be equivalent to 65 basis points a year over 5 years. Thus, effectively, each year the government would pay 65 basis points to protect themselves from a rise in interest rates above 10%. As the entire cap payment is made up front, the 248.22 basis points is the price that would be quoted. Finally, if the volatility of the underlying was increased from .15 to .16 the cap price would increase to 265.81 basis points (not exhibited), indicating that the cap is quite sensitive to even a slight change in volatility.

This cap at first glance seems expensive. As noted earlier, a knockout structure can be employed to make the cap less expensive. Assume the bond issuer is concerned that rates may rise, but doubts they will rise above 12%. By employing a knockout cap with a barrier of 12%, the can buyer can implement this view. Though the cap pricing model was tedious to first implement, it is not hard to modify once it is in place to value a more exotic structure. Exhibit 24 illustrates the cap pricing of a knockout cap with a strike price of 10% and a barrier of 12% - equivalent to the prior cap with only the barrier added. The price of such a cap is $10.50 on a bond with $1000 face value. This cap is indeed cheaper, but the bond issuer has sacrificed in return for the lower price. In period 5, if interest rates have risen to 18.22%, the issuer is unprotected. Thus, the price of the cap is lowered because the possible value of cap payments has been reduced. It is essential to note that the interest rate tree is only being constructed once a period, meaning that at the end of five periods, there are only 5 possible interest rates. A more accurate model would decrease the time between levels of the tree and thus trace out more possible paths. However, the trees composed are sufficient in that they lend an extremely transparent view into the pricing process for more complicated derivatives.

The binomial model can thus be effectively used to value interest rate caps and even more exotic cap structures. The final application of the binomial model will be in pricing a swaption and such pricing is quite simple and is based almost entirely on the first bond option considered (that of a simple call on a bond). Exhibit 25 illustrates the pricing of a European payer swaption with a strike of 10% that can be exercised at the end of 5 periods. We now assume that the swap rate will follow the interest rate path previously followed by the risk free rate and that for discounting purposes the risk free rate is always 2% below the swap rate at each node. Thus, the simplifying assumption being made here is that the correlation between the swap rate and the risk free rate is perfect and thus the swap spread is constant. The option would thus have a value of 822 basis points if the swap rate rose to 18.22% after 5 periods (one would much rather pay a 10% fixed rate than a 18.22% fixed rate). Once this relationship is specified, valuing the swaption is no different from valuing the earlier call on a bond, except that here the underlying interest rate is the swap rate. The price of the option as Exhibit 25 shows is $79.99. From this model, it is not hard to see that a payer swaption is analogous to a put on a bond. Comparing Exhibit 25 to Exhibit 21 makes such a relationship easier to view.

Pricing interest rate options using the binomial model indeed enables amazing transparency into the way these options are valued. The binomial model enabled one to view a cap as a portfolio of options and a swaption as analogous to a put on a bond. Additionally, though it is more tedious to price these options in the fixed income world as we are not employing the unrealistic assumption that the rate used to discount the payments is constant, rather the interest rate is taken to be stochastic, once the model has been built, it is not hard to modify it to value any payoff structure. The binomial model has been successfully applied and has certainly permitted a more complete and concrete analysis of the interest rate derivatives markets.

Final Analysis and Conclusions

In conclusion, it is quite clear that the swaps market has become quite complex during its relatively brief existence. Initially, in order for a transaction to be completed, two counterparties had to be matched up with the swaps dealer serving as an intermediary. As the market for swaps grew, dealers began warehousing swaps, using Eurodollar futures and U.S. Treasury bonds as a hedge until a matching counterparty could be found. As was demonstrated, the mechanics of the pricing of interest rate swaps focuses on the Eurodollar futures market, thus the swaps market and Eurodollar futures market are effectively intertwined. The ability to warehouse swaps certainly helped to fuel the remarkable growth of the market.

As market participants realized the usefulness and flexibility of the basic swap structure, more complicated swaps began to be completed and a market for options embedded in swaps, stand alone swap options and other interest rate derivatives such as caps and floors related to the swaps market grew. The binomial tree option valuation model is an excellent tool that can be used to value these securities. The model is incredibly transparent and can be used to value nearly any payout function. The examples presented showcased the binomial model and enabled a more thorough understanding of the structure and pricing mechanics of these complex derivatives.

As the swaps market enters its third decade, its importance continues to grow. The swaps markets' interrelationships with other fixed income markets has made monitoring swap rates and swap spreads essential. The ability to create a swap structure that can benefit corporations and can allow market participants the ability to express nearly any view on interest rates has been a driving force behind the swaps markets continued growth. Indeed with the continuing reduction of U.S. Treasury debt, it would not be surprising for swap rates to become a benchmark in the fixed income markets.

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* Website references in the text refer to specific websites and thus there is no need to describe them in further detail as part of this 'work cited' page.

Appendix A:

Tables and Charts

Exhibit 1: Schematics of an Interest Rate Swap

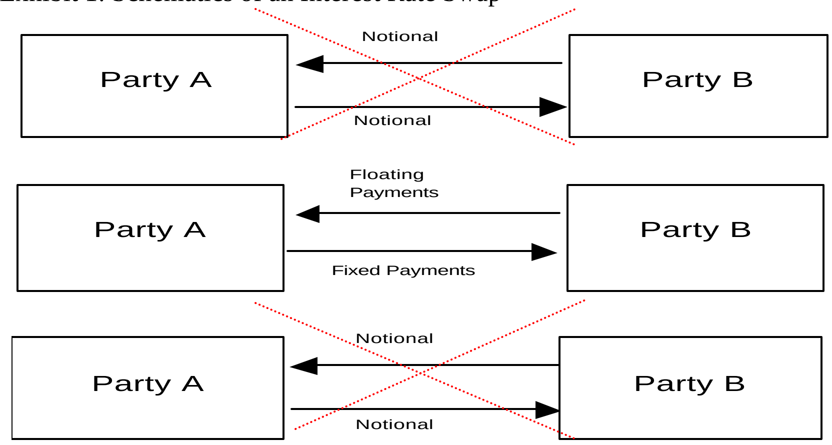


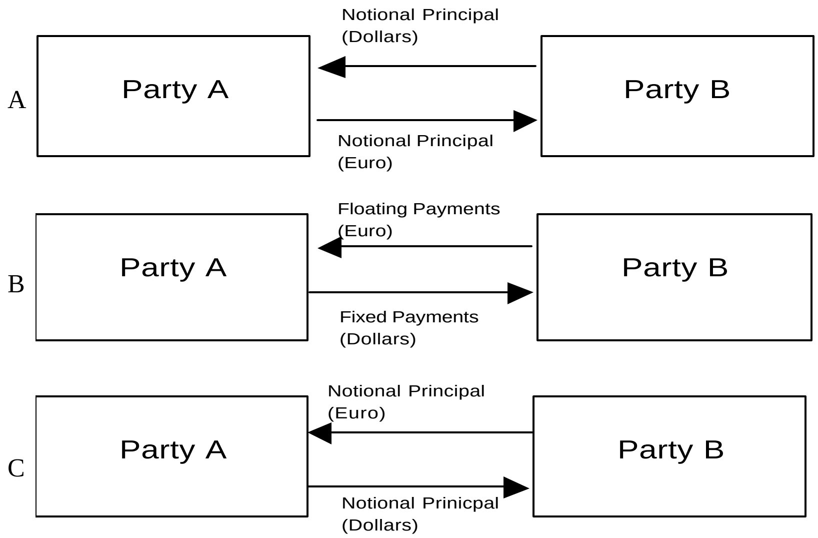
Exhibit 2

Five Year Swap - Payments Made Annually (From View of Floating Payor)

Notional: 100 million   Fixed Rate: 9%

Year	Floating Rate	Fixed Rate	Floating Payment	Fixed Payment	Net Difference
0	6%	9%		**in millions**	
1	7%	9%	-6	9	3
2	8%	9%	-7	9	2
3	9%	9%	-8	9	1
4	10%	9%	-9	9	0
5			-10	9	-1

Exhibit 3: Schematics of a Cross Currency Swap



Exchanges A and C are made at the spot dollar/euro exchange rate at the swap's inception

Exhibit 4: An Exchange of Borrowings

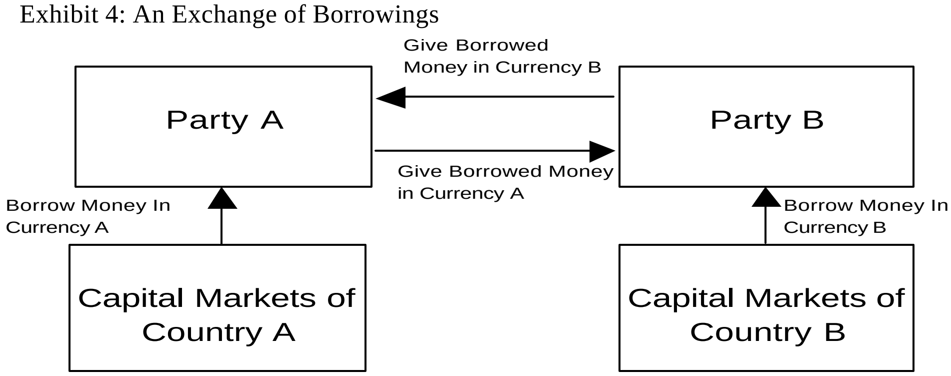


Exhibit 5 Dealer Pays Fixed 6.50% Dealer Receives Fixed 6.54%

The dealer will make a spread of 4 basis points on the transaction`.

Exhibit 6

New Swaps (All Figures in Billions)

Year	All Swaps	Interest Rate Swaps
1981	3	
1982	10	
1983	45	40
1984	70	60
1985	100	80
1986		
1987		683
1988		1010
1989		1503
1990	1477	1264
1991	923	762

Source: Coopers pg X

Exhibit 7 Party A wants to receive fixed and pay floating for five years. The swaps dealer hedges this risk in the financial markets by buying a five year Treasury bond and shorting three month Treasury bill rolled over every sixth month.

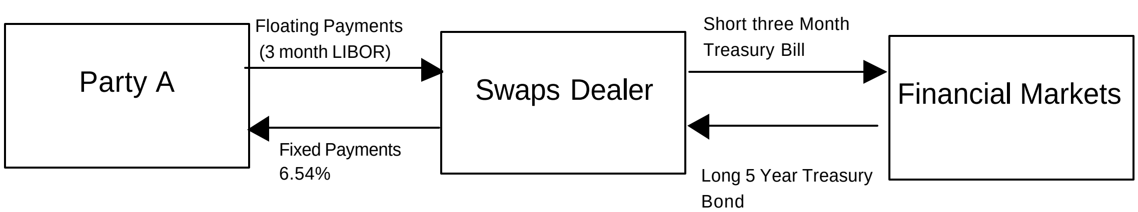


Exhibit 8 - Converting a Fixed Rate Liability to a Floating Rate Liability

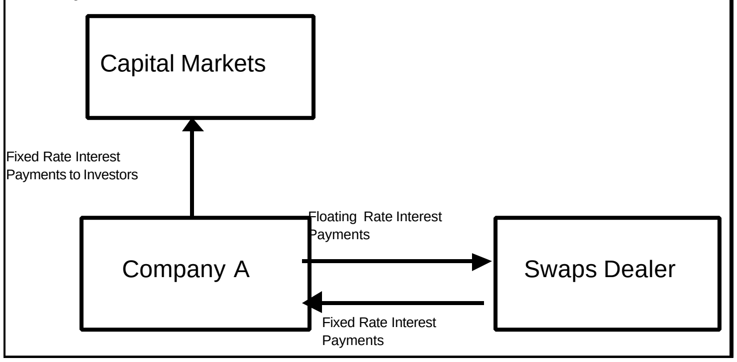


Exhibit 9 - Converting US Dollar debt to Japanese Yen

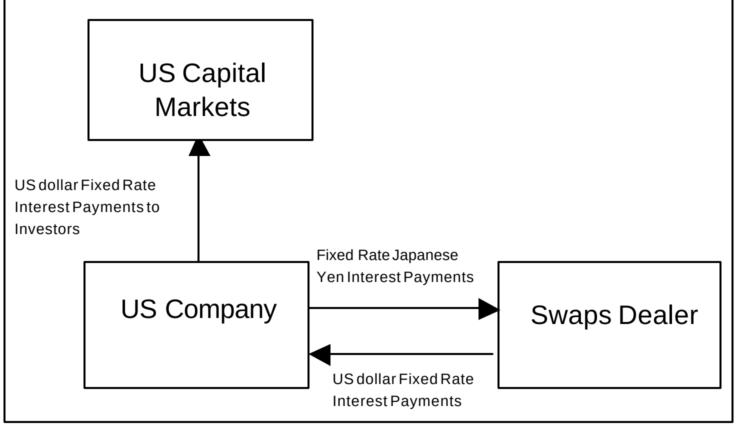


Exhibit 10 - Indicative Swaps Pricing Schedule

Maturity	Treasury Note	(Swap Spread) Pays/Receives Fixed	Swap Rate
2	6.00%	38/42	6.38/6.42%
5	6.20%	45/49	6.65/6.69%
10	6.30%	60/64	6.90/6.94%
30	6.40%	70/76	7.10/7.16%

Basis Points

Exhibit 11- The Swap Spread

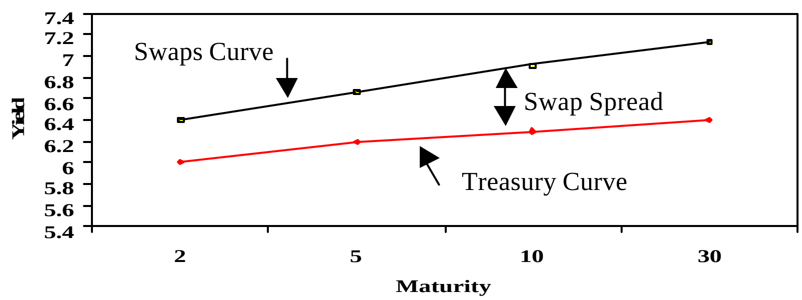
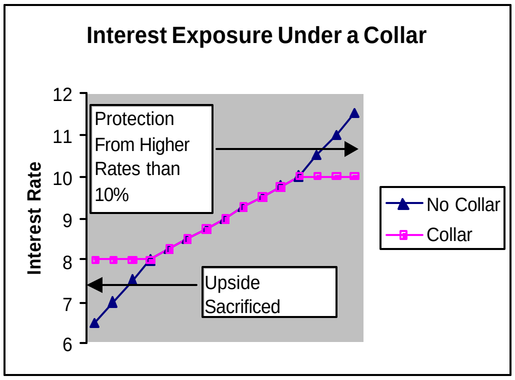


Exhibit 13 - A Simple Cap

5 Year Debt - Interest Payments Annually on 3 month Libor
Cap with 9% strike

Year	3 month Libor	Payment
0	8.00%	
1	8.25%	0
2	8.00%	0
3	9.25%	0
4	9.10%	25 basis points
5		10 basis points

Exhibit 14



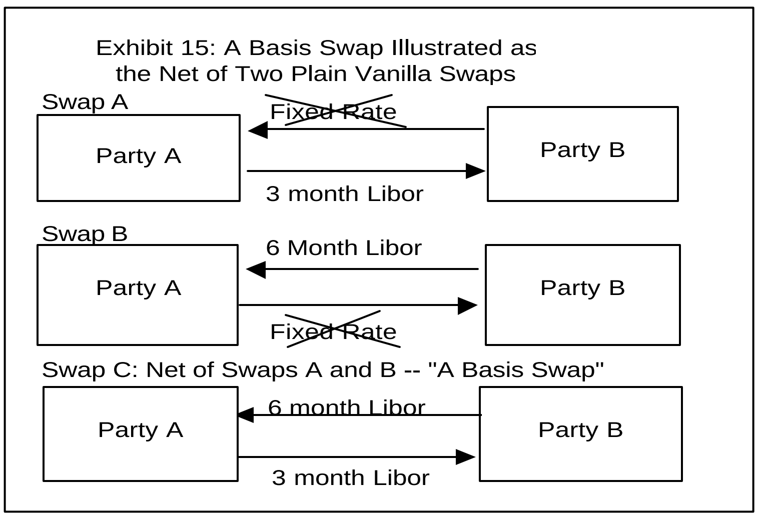


Exhibit 16: The NPV of a Swap at Initiation

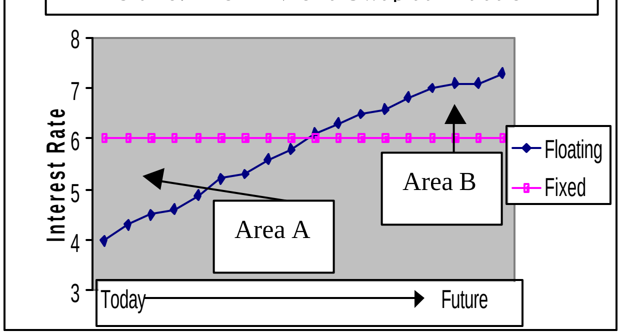


Exhibit 17	Price	Implied 3 month Libor	Contract Expiration	Days in Period
3 month Libor (6.57%)	Swap Start Date: 12/20/00		-	91
March '01 Eurodollar	93.85	6.15	03/21/01	91
June '01 Eurodollar	94.06	5.94	06/20/01	91
September '01 Eurodollar	94.16	5.84	09/19/01	91
December '01 Eurodollar	94.10	5.90	12/19/01	91
March '02 Eurodollar	94.16	5.84	03/20/02	91
June '02 Eurodollar	94.11	5.89	06/19/02	91
September '02 Eurodollar	94.06	5.94	09/18/02	93
	Swap End Date: 12/20/02			

Exhibit 18

To calculate the discount factor between dates X and Y we use the following formula

DF = 1/(1+R(x,y)*Days(x,y)/360) where R(x,y) denotes the implied (or actual if we have it) interest rate between dates X and Y and Days(x,y) denotes the number of days between X and Y.

Since the payments in a swap are made semiannually, we need to have a (zero-coupon) sixth month discount factor, a one year discount factor, a one and a half year discount factor and a two year discount factor to value a two year swap.

Since the discount factors we will compute will be 3 month factors, to find the factors we need (as above) we will need to combine these factors.

First computing all the discount factors we will need...

Between 12/21/00 and 3/21/01 DF = 1/(1+.0657*91/360) = .98366

Between 3/22/01 and 6/20/01 DF = 1/(1+.0615*91/360) = .98469

Between 6/21/01 abd 9/19/01 DF = 1/(1+.0594*91/360) = .985207

Between 9/20/01 and 12/19/01DF = 1/(1+.0584*91/360) = .98545

Between 12/20/01 and 3/20/02 DF = 1/(1+.0590*91/360) = .98531

Between 3/21/01 and 6/19/02 DF = 1/(1+.0584*91/360) = .98545

Between 6/20/02 and 9/18/02 DF = 1/(1+.0589*91/360) = .985329

Between 9/18/02 and 12/20/02 DF = 1/(1+.0594*91/360) = .98507

Now we have the initial 3 month discount factor - it is .98366 - to find the (zero-coupon) sixth month discount factor

Sixth month discount factor = (DF between 12/21/00 and 3/21/01)*(DF between 3/22/01 and 6/20/01)
= .98366*.98469
=**.9686**

Similarly, 9 month discount factor = (DF between 12/21/00 and 6/20/01)*(DF between 6/21/01 and 9/20/01)
= .9686*.985207
= .95427

Now, the **One year discount factor** = (DF between 12/21/00 and 9/20/01)*(DF between 9/21/01and 12/19/01)
= .95427*.98545= **.94038**

By this method we calculate the 1.5 year discount factor to be .91308 and the 2 year discount factor to be .886256

Exhibit 19 - Determining the Swap Rate

With discount factors: 6 month (.9686) 1 year (.94083) 1.5 year (.91308) and 2 year (.886256)

Price of 2 Year Par Bond = PV(Coupon at t=.5) + PV(Coupon at t=1) + PV(Coupon at t=1.5)
+ PV(Principal + Coupon at t=2)

Solving for the two year swap rate, denoted as X

$$100 = 100*(X/2)*.9686+100*(X/2)*.94083+100*(X/2)*.91308+100*(1+X/2)*.886256$$

Two year Swap Rate = X = .06133 or 6.133%.

Exhibit 20	Discounted Fixed Payments	Discounted Floating Payments
At .5 Years	100,000,000*.06133/2*.9686 = 2970212	100,000,000*.0657(current 3m Libor)*181/360*.9686 = 3199528
At 1 Year	100,000,000*.06133/2*.94038 = 2993675	100,000,000*.0594(implied 3m Libor in June 2001)*181/360*.94038 = 2808445
At 1.5Years	100,000,000*.06133/2*.91038 = 2799960	100,000,000*.0590(implied 3m Libor in December 2001)*181/360*.91038 = 2700541
At 2 Years	100,000,000*.06133/2*.886256 = 2717704	100,000,000*.0589(implied 3m Libor in June 2001)*183/360*.886256 = 2653524
	Sum=PV(Fixed Payments) = 11,371,551	Sum=PV(Floating Payments) = 11,362,038

Thus, the PV (Fixed Payments) approximately equal the PV(Floating Payments). The small error is due to rounding.

Exhibit 21: Valuation of a Simple Call Option on the Risk Free Rate

	Spot Risk Free Rate	Volatility	Time Periods	Strike Rate	P
	10%	0.15	5	9%	0.5
	Up	1.162			
	Down	0.861			
		Risk Free Rate Paths			
Period					
1	10.00%				
2	8.61%	11.62%			
3	7.41%	10.00%	13.50%		
4	6.38%	8.61%	11.62%	15.68%	
5	5.49%	7.41%	10.00%	13.50%	18.22%
		Price of Option (Basis Points)			
5	351.19	159.18	0.00	0.00	0.00
4	239.42	73.03	0.00	0.00	
3	145.07	33.04	0.00		
2	81.71	14.71			
1	43.62 ←	Price			

On a $1000 bond, 43.62 basis points is equivalent to (1000*.004362) = $4.362.

Exhibit 22: Valuation of an Embedded Call Option on the Risk Free Rate

	Spot Risk Free Rate	Volatility	Time Periods	Strike Rate	P
	10%	0.15	5	9%	0.5
	Up	1.162			
	Down	0.861			
		Risk Free Rate Paths			
Period					
1	10.00%				
2	8.61%	11.62%			
3	7.41%	10.00%	13.50%		
4	6.38%	8.61%	11.62%	15.68%	
5	5.49%	7.41%	10.00%	13.50%	18.22%
		Price of Option (Basis Points)			
5	351.19	159.18	0.00	0.00	0.00
4	262.37	73.03	0.00	0.00	
3	159.18	33.04	0.00		
2	88.18	14.71			
1	46.55 ←	Price			

On a $1000 bond, 46.55 basis points is equivalent to (1000*.004655) = $4.65.

Exhibit 23: Valuation of a Cap on the Risk-Free Rate

Spot Risk Free Rate	Volatility	Time Periods	Strike Rate	P
10%	0.15	5	10%	0.5

Up	1.162
Down	0.861

Risk Free Rate Paths

Period					
1	10.00%				
2	8.61%	11.62%			
3	7.41%	10.00%	13.50%		
4	6.38%	8.61%	11.62%	15.68%	
5	5.49%	7.41%	10.00%	13.50%	18.22%

Valuing the Cap Payment of Period 5

5	0.00	0.00	0.00	349.86	822.12	Cap Payment
4	0.00	0.00	137.22	428.99		
3	0.00	55.62	215.71			
2	23.20	107.11				
1	53.08					

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

Valuing the Cap Payment of Period 4

Period					
4	0.00	0.00	161.83	568.31	Cap Payment
3	0.00	65.60	277.96		
2	27.36	135.56			
1	66.34				

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

Valuing the Cap Payment of Period 3

3	0.00	0.00	349.86	Cap Payment
2	0.00	137.22		
1	55.62			

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

Valuing the Cap Payment of Period 2

2	0	162	Cap Payment
1	73.22		

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

The Cap Payment of Period 1 is known to be zero as the current spot rate is 10%. Thus adding up the present values of the expected cap payments for the other 4 periods yields (53.08+66.34+55.62+73.22)= **248.22** basis points or $24.82 on a bond with $1000 face value.

This 248.22 basis points can be considered to be **65.47** basis points per period using annuity formulas.

Exhibit 24: Valuation of a Knock-Out Cap on the Risk-Free Rate

Spot Risk Free	Volatility	ime Period	Strike Rate	P	Barrier
10%	0.15	5	10%	0.5	12%

Up	1.162
Down	0.861

Risk Free Rate Paths

Period					
1	10.00%				
2	8.61%	11.62%			
3	7.41%	10.00%	13.50%		
4	6.38%	8.61%	11.62%	15.68%	
5	5.49%	7.41%	10.00%	13.50%	18.22%

Valuing the Cap Payment of Period 5

5	0.00	0.00	0.00	0.00	0.00	Cap Payment
4	0.00	0.00	0.00	0.00		
3	0.00	0.00	0.00			
2	0.00	0.00				
1	0.00					

Working Backwards Through the Tree to Find the Present

Valuing the Cap Payment of Period 4

Period					
4	0.00	0.00	161.83	0.00	Cap Payment
3	0.00	65.60	63.34		
2	27.36	51.39			
1	32.23				

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

Valuing the Cap Payment of Period 3

3	0.00	0.00	0.00	Cap Payment
2	0.00	0.00		
1	0.00			

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

Valuing the Cap Payment of Period 2

2	0	161.85	Cap Payment
1	73.22		

Working Backwards Through the Tree to Find the Present Value of the Possible Cap Payments Now

The cap price here is (32.23+73.22) = 105.45 basis points, or $10.50 on a bond with $1000 face value.

Exhibit 25: Valuation of a European Payer Swaption

Spot Swap Rate	Volatility	Time Periods	Strike Rate	P
10%	0.15	5	10%	0.5

Up	1.162
Down	0.861

Swap Rate

Period					
1	10.00%				
2	8.61%	11.62%			
3	7.41%	10.00%	13.50%		
4	6.38%	8.61%	11.62%	15.68%	
5	5.49%	7.41%	10.00%	13.50%	18.22%

Price of Option (Basis Points)

5	-	-	-	349.86	822.12
4	-	-	152.66	491.01	
3	-	67.70	275.63		
2	30.44	149.81			
1	79.93				

On a $1000 bond, 43.62 basis points is equivalent to (1000*.007993) = $79.9