CONTEXTUALIZING “EDGEWORTH MARKET GAMES”: MARTIN SHUBIK’S CONTRIBUTION TO THE DEVELOPMENT OF GAME THEORY

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Acknowledgements

I entered Duke intent on studying economics, mathematics, and history. I am both privileged and honored to have done that under the guidance of Professor E. Roy Weintraub, who familiarized me with a field that captured those interests—the history of economics.

I first encountered the work of Martin Shubik in Professor Weintraub's Modern Economic Thought course during my sophomore year. In the two years since, Professor Weintraub educated me in both game theory and in the art of writing economic history. I am thankful for his encouragement and advice during the writing of this paper. I will be forever grateful, however, to Professor Weintraub for introducing me to economic history, a subject with which I hope to have a lifelong fascination.

1. Introduction

Open any current microeconomic theory book and turn to the section concerning general equilibrium analysis; there you will find references to game theory and many of its major tenets. This, however, was not the case a half century ago. While game theory has been one of the most important advances in economics in the last half-century, it was first and foremost a significant *mathematical* development. Not until 1959, with the publication of Martin Shubik's seminal paper, "Edgeworth Market Games," did game theory become firmly established within the discipline of modern economic theory.

This paper shall examine "Edgeworth Market Games," from different perspectives. Contextualizing the Shubik contribution, this paper will narrate some of the personal and social circumstances of Martin Shubik's life.

More specifically, the second section will be a chronicle of Shubik's life through his undergraduate schooling, leading up to his arrival at Princeton University in 1949 as an economics graduate student. Our third section will concern Princeton, the developments in game theory that occurred there, and the various individuals who played a role in Shubik's intellectual development. We will emphasize that while at Princeton, Shubik studied directly under one of the two major first-generation game theorists, Oskar Morgenstern, while his peers in the college residences were fellow second-generation game theorists John Nash and Lloyd Shapley.

Our fourth section will focus on Shubik's work in game theory after leaving Princeton. Following the completion of his Ph.D. in 1953 and a two-year stint there as a research associate in 1955, Shubik left Princeton and accepted a one-year fellowship at the Center for Advanced Study in Behavioral Sciences in Palo Alto. Shubik wrote most of "Edgeworth Market Games" during that fellowship.

In the fifth section, we will analyze "Edgeworth Market Games" itself, concentrating on the paper's structure and results. In the sixth section, we shall discuss the implications of the paper, what it has done to marry game theory with economics, and more specifically, how Shubik discovered that Edgeworth's contract curve was equivalent to the core. In the seventh section, we will detail the story of game theory (more specifically, the core) after "Edgeworth Market Games" and the improvements made by Gerard Debreu and Herbert Scarf in 1963, and by Robert J. Aumann in 1964.

2. Before Princeton

Born in Manhattan in 1926, Martin Shubik was formally educated in London. He received a Harris Scholarship to the University of Toronto, from which he received his B.A. in mathematics in 1947 and his M.A. in political economy in 1949. In 1944, John von Neumann and Oskar Morgenstern set the wheels in motion for game theory to become an independent academic field with the publication of *Theory of Games and Economic Behavior*. In 1948, while still a master's candidate in political economy, Shubik "skim read parts of [*Theory of Games and Economic Behavior*] and did not really understand it…and wrote a relatively bad essay on it"[2] for a course in Economic Theory. But even though he had only dealt with *Theory of Games and Economic Behavior* in a limited capacity, Shubik became convinced that game theory was going to be the next important area of research. Shubik recalled that he believed "that this *had to be* the way to go."[3] He had every intention of studying game theory at Princeton, and even more so, Shubik "wrote to Princeton saying that [he] wanted to study [game theory] with Professor Morgenstern."[4]

3. Princeton

To effectively contextualize "Edgeworth Market Games," we need to analyze the game theory community that developed at Princeton University. This requires a discussion of the first-generation game theorists, John von Neumann and Oskar Morgenstern, Shubik's advisor; and the second-generation game theorists, Shubik, John Nash, and Lloyd Shapley.

Princeton was already established as the leading university in game theory research by the time Shubik and his contemporaries arrived on campus in the late 1940s. But this was not the case a decade earlier. The recruiting of noted academics such as John von Neumann and Oskar Morgenstern bolstered Princeton's reputation by 1950. The Institute for Advanced Studies, founded at Princeton in the early 1930s, was independent from the university. The Institute

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[2] Shubik c.1993, 3. This document is a draft of an autobiographical summary of Shubik's life and work for the original donation of personal materials to the Duke University Rare Book, Manuscript, & Special Collections Library.
[3] Ibid.
[4] Ibid.

needed influential scientists and mathematicians to fill its research posts. While it hired Europe's finest scientists such as Albert Einstein, Kurt Gödel, and Hermann Weyl, the Institute's procurement of von Neumann solidified its position, and Princeton's, at the forefront of academic research. "Weyl insisted, as a condition of his acceptance, that the Institute appoint a bright light from the next generation. Von Neumann, who had just turned thirty, was lured away from the university to become the Institute's youngest professor."[5] The Hungarian-born von Neumann, who came to Princeton in 1930 as a graduate student, became a member of the Institute's second-generation professors, but was poised to establish himself as a first-generation game theorist.

Oskar Morgenstern was an Austrian economist who had served as director of Vienna's Institute for Business Cycle Research from 1931 until 1938.[6] "[He] held that position until Germany's Anschluss of Austria in 1938. At that point, Morgenstern was visiting the U.S., and, finding it more propitious to remain in America, obtained a temporary position at Princeton. He soon became a permanent faculty member…."[7] As Sylvia Nasar indicated in *A Beautiful Mind*, her biography of John Nash: "[Morgenstern] joined the university's economics faculty, but disliked most of his American colleagues. He gravitated to the Institute, where Einstein, von Neumann, and Gödel were working at the time…."[8]

In 1926, von Neumann had been "the first to provide a complete mathematical description of a game and to prove a fundamental result, the min-max theorem."[9] Von Neumann's interest in games and game theory pre-dated his arrival at Princeton. Morgenstern, by contrast, possessed a "fascination with the question of interdependence of economic agents…."[10] "He saw interdependence as the salient feature of all economic decisions, and he was always criticizing other economists for ignoring it."[11] The mathematics of von Neumann and the economics of Morgenstern were separate disciplines, but would revolutionize game theory working in conjunction with one another. The union was formed when "von Neumann and

[5] Nasar 1998, 54
[6] Leonard 1995, 739
[7] Ibid.
[8] Nasar 1998, 84
[9] Ibid., 83
[10] Schotter 1976, viii
[11] Nasar 1998, 84

Morgenstern met in the fall of 1938"[12], when von Neumann "captured the imagination of [Morgenstern, the] distinctly nonmathematical economist…."[13]

Urs Rellstab, in his 1992 paper on the collaboration between von Neumann and Morgenstern, chronicled the academic relationship of von Neumann and Morgenstern, which culminated in the publication of *Theory of Games and Economic Behavior* in 1944. Beginning in April 1940, the two men had intellectual discussions on the many facets of game theory. In May 1941, "VN [von Neumann] advises OM [Morgenstern] to write a paper on 'maxims of behavior'; OM begins immediately."[14] The result of Morgenstern's work was "Quantitative Implications of Maxims of Behavior," which Robert Leonard believed is "impressive…[because it] offers a skeletal social theory that is both methodologically individualistic and cognizant of the importance of social interaction."[15] In September of that same year, "*OM* (not VN…) proposes they write a book together."[16] In November 1941, the draft of the second chapter was completed by mostly by von Neumann, as Rellstab wrote, "VN does the lion's share, characteristic of the rest of the work."[17] The book was finally published in 1944. "It was Morgenstern who crafted the book's provocative introduction and framed the issues in such a way that the book captured the attention of the mathematical and economic community."[18]

As previously mentioned, Martin Shubik encountered the *Theory of Games and Economic Behavior* four years after its initial printing. "I had sat in the library of the University of Toronto and attempted to read *The Theory of Games and Economic Behavior* in 1948 and was convinced that even though I scarcely understood the details of the mathematics, this was the right way to start to mathematize much of economics, political science, and sociology."[19] While many of the students labeled *Theory of Games and Economic Behavior* "the bible"[20], there were some segments of the economics community, especially at Princeton, who scoffed at the notion that game theory was a viable discipline. "Jacob Viner, the chairman of the economics department, heaped scorn on the unpopular Morgenstern by saying that if game theory couldn't

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[12] Leonard 1992, 54
[13] Ibid., 51
[14] Rellstab 1992, 91
[15] Leonard 1992, 55
[16] Rellstab 1992, 91
[17] Ibid., 91
[18] Nasar 1998, 85
[19] Shubik 1992, 151
[20] Nasar 1998, 86

even solve a game like chess, what good was it, since economics was far more complicated than chess?"[21] Shubik's own reminiscences supported this notion of a hostile environment. "When I arrived in Princeton I found that my enthusiasm for the potential of the theory of games was not shared by members of the economics department. Even the Princeton University Press, which as an academic publisher was meant to take reasonable risks with new scholarly enterprises, had required an outside subsidy of $4,000 before it would risk publication [of *TGEB*]."[22]

While *Theory of Games and Economic Behavior* was not universally received, the collaboration of von Neumann's mathematics and Morgenstern's economics was of utmost importance to the development of game theory. The book's ability to transcend two fields of academic study was crucial to spawning the second generation of game theorists at Princeton. As Leonard indicated, "Though directed in its rhetoric toward economic theorists, [*Theory of Games and Economic Behavior*'s] central ideas were equally novel in the mathematics sense. While the authors hoped the impact would be greatest in economic theory, they also surmised correctly that some time would pass before the 'game' idea became common currency."[23] Von Neumann, Morgenstern, and the game theory community did not have to wait long. The successive arrivals of Nash in the fall of 1948 and Shubik and Shapley, both in the fall of 1949, helped push game theory into mainstream economics.

When Shubik arrived at Princeton, "for the most part, those interested in game theory were in the Mathematics Department where Oskar [Morgenstern] had many good friends."[24] The second-generation game theorists also exemplified the meshing of economics and mathematics into game theory research. Shubik studied in the economics department under Morgenstern while Nash and Shapley were in mathematics, but the three second-generation game theorists often worked together. Shubik's recollections revealed why many in the economics department wishing to study game theory often had to venture into the mathematics department:

> "The contrast of attitudes between the economics department and the mathematics department was stamped on my mind soon after arriving at Princeton. The former projected an atmosphere of dull business-as-usual conservatism of a middle league Ph.D. factory; there were some stars but no sense of excitement or challenge. The latter was

[21] Ibid.
[22] Shubik 1992, 151
[23] Leonard 1992, 59
[24] Essays in GT and ME (Shubik 9)

electric with ideas and the sheer joy of the hunt. Psychologically they dwelt on different planets."[25]

While *Theory of Games and Economic Behavior* and the chance to do game theory research with Morgenstern drew Shubik to Princeton, his contemporaries were attracted to the New Jersey campus for different reasons. Nash had come to Princeton at the age of twenty, choosing it over Harvard's graduate mathematics program because "the trouble was that Harvard was offering slightly less money than Princeton."[26] Nash viewed the time spent obtaining a Ph.D. at a top-four school like Princeton (the other three were Harvard, Chicago, and Michigan) as "virtually a prerequisite for eventually landing a good academic appointment."[27] Shapley had entered the Princeton mathematics program at the age of twenty-six, after a year at RAND, the famed research institute in California. He had gone to RAND after graduating from Harvard, which took seven years because of three years in the Army Air Corps that interrupted his studies.[28] At RAND, Shapley was involved in using "game theory applications to solve military problems, and came to Princeton while technically on leave from RAND."[29] Von Neumann, who had been the brightest star of the first generation, pegged Shapley "the brightest young star in game theory research"[30] upon the latter's arrival at Princeton.

To understand the influence of Shubik's fellow second-generation game theorists on Shubik, we must first describe Princeton's Graduate College. The seclusion of the Graduate College was such that it sparked supreme concentration on academics. It was situated halfway between the two miles separating Fine Hall from the Institute of Advanced Studies. "Especially in winter, when it was dark by the time the afternoon seminar ended, it was a good long walk. And once you were there, you didn't feel like going out again."[31] This layout gave Shubik and his dorm mates plenty of time to hold intellectual discussions amongst each other. "The graduate students ate breakfast, lunch, and dinner together…. Women were not allowed in the main dining

[25] Shubik 1992, 153
[26] Nasar 1998, 46
[27] Ibid., 45 and 46
[28] Ibid., 99-100
[29] Ibid., 100
[30] Ibid.
[31] Nasar 1998, 61

hall, and, of course, there were no female students…. Isolation made the real prospects of meeting a girl remote."[32]

Shubik recalled that "Nash, Shapley, and I roomed close to each other at the Graduate College at Princeton and there was considerable interaction between us."[33] Shapley and Shubik shared a room, and lived a down the hallway from Nash. They even shared a bathroom at some point.[34] This interaction to which Shubik referred entailed several forms. Of the more unproductive type, there were, as Nasar described, "the jokes…[that] got totally out of hand…. Shubik recalled: 'Nash's idea of a joke was to unscrew the electric light bulb in the bathroom. There was a glass shade under the bulb, which he filled full of water. We could easily have gotten electrocuted. Did he intend to electrocute me? I'm not sure he didn't intend to."[35]

Of the more productive interactions, there were two examples. The first, and less significant, involved a seminar given by Albert Tucker, who was "actively interested"[36] in game theory. For Shubik and Shapley's first year, the Thursday seminars were "run by Kuhn and Gale while Tucker was at Stanford."[37] Ten years later, Shubik's "Edgeworth Market Games" would be published in Volume IV of the *Contributions to the Theory of Games,* one of whose editors was A. W. Tucker.

The second example was the leisure time at the Graduate College, which involved many applications of game theory. "[These were] the many sessions (often at tea time) devoted to playing games (such as go, chess, and kriegspiel) and to talking informally about paradoxical or pathological properties of games and the possibility of inventing games that illustrated these properties."[38]

Figures such as John von Neumann, whose "intellect staggered"[39] Shubik, exposed the second-generation game theorists to some of the most challenging problems. "The attitude around Fine Hall was that if von Neumann conjectured that a stable-set solution always exists, the betting odds were that it did. Shapley and D. B. Gillies were looking for proofs or

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[32] Ibid., 61-62
[33] Shubik 1992, 155
[34] Nasar 1998, 99-101
[35] Ibid., 101
[36] Shubik 1992, 152
[37] Nasar 1998, 100
[38] Shubik 1992, 158-159
[39] Shubik c.1993, 3

counterexamples….”[40] Shubik, Shapley, and Nash continually discussed many of these academic problems and conjectured ways to solve them. One such talk directly influenced Shubik, and eventually “Edgeworth Market Games.” The seminal idea in “Edgeworth Market Games” was that the core can be thought of as an economic solution concept, and as Shubik wrote, “I was under the impression until I talked to Shapley that it was he who suggested considering it as a solution concept by itself. He pointed out to me that [this] idea came up in our conversations when…I observed that, in essence, the idea of the set of undominated imputations was already in Edgeworth (1881)….”[41] Ironically, a later discussion with Herbert Scarf led to an improvement on “Edgeworth Market Games” by Debreu and Scarf (1963).

The work done by Nash, Shapley, and D. B. Gillies proved crucial to the writing of “Edgeworth Market Games.” “Until the late 1950s the work of developing and extending game theory was done almost exclusively by mathematicians. Several of the concepts that proved to be of greatest importance for economists were the Nash bargaining equilibrium in 1949 (published in 1950), the Shapley value in 1953, and the core by D.B. Gillies in 1953.”[42] We will now analyze the influence of these three concepts on “Edgeworth Market Games.”

In his 1950 *Econometrica* paper entitled “The Bargaining Problem,” Nash showed that “a unique solution existed that maximized the product of the players’ utilities.”[43] Nash theorized that “we idealize the bargaining problem by assuming that the two individuals are highly rational, that each can accurately compare his desires for various things, that they are equal in bargaining skill, and that each has full knowledge of the tastes and preferences of the other.”[44] Nash solved a seventy-year-old bargaining problem, first set forth by “a reclusive Oxford don, Francis Ysidro Edgeworth, in 1881”[45] in *Mathematical Psychics*. Thus, even before Shubik matriculated at Princeton, the game theory community had been exposed to Edgeworth’s 1881 paper, a work that had remained relatively obscure until that time.

Nash’s essay, as described by Shubik, was “important work on equilibrium points in *n*-person games in strategic form.”[46] In some ways it manifested itself in the traits of the games dealt with in “Edgeworth Market Games”: cooperative, *n*-person, non-zero sum games. More

[40] Shubik 1992, 155
[41] Ibid., 156-7
[42] Greenwald 1982, 454
[43] Nasar 1998, 90
[44] Nash, Jr. 1950, 204
[45] Nasar 1998, 88

importantly, however, was the notion of equilibrium. Shubik, as will be discussed in a later section, "had conjectured on the full significance of the core and its convergence to the competitive equilibrium under broad conditions, but I knew that [I] was incapable of proving it."[47] While that result did not appear in "Edgeworth Market Games," Shubik showed that the core converged to the competitive price in a game of transferable utility, demonstrating a link to Nash's essay.

The second crucial development was the Shapley value. Its relationship to the core and to "Edgeworth Market Games" will be discussed in Section VI. Shapley's more important contribution, however, came in his conversations with Shubik. Shubik recalled that he was "helped by discussions with…Lloyd Shapley"[48] in writing "Edgeworth Market Games." As Robert J. Aumann noted in his 1964 paper, the "core as an independent solution concept was developed by Shapley in lectures at Princeton University in the fall of 1953."[49]

D. B. Gillies was another mathematics graduate student at Princeton in the early 1950s. In his 1953 Ph.D. dissertation *Some Theorems on n-Person Games*, Gillies discussed the core, and was the first to consider "the properties of stable sets including their intersection."[50] In his 1982 book *Game Theory in the Social Sciences*, Shubik noted that Shapley had introduced the core as a game theory concept around the time of Gillies's thesis in "Report on an Informal Conference on the Theory of *n*-Person Games to the Department of Mathematics.[51]

Because of the papers of Shapley and Gillies, the core became a major topic of study at Princeton. "In the early 1950s Shapley, Gillies and I had discussed the core of an n-person game in cooperative form."[52] Shubik recounted in his 1992 essay "Game Theory at Princeton, 1949-1955: A Personal Reminiscence" that one of these talks would lead to the connection of the core with Edgeworth's *Mathematical Psychics*.

> "I was under the impression until I talked to Shapley that it was he who suggested considering it [the core] as a solution concept by itself. He pointed out to me that the idea of the core as a solution concept in its own right came up in our conversations when (as I was the only one in the group of us who was meant to know some economics), I observed that, in essence, the idea of the set of undominated imputations was already in Edgeworth

[46] Shubik 1992, 155
[47] Shubik c.1993, 6
[48] Shubik 1992, 157n
[49] Aumann 1964, 47
[50] Shubik 1992, 156
[51] Shubik 1982, 175 and 478
[52] Shubik c.1993, 6

> (1881) in his treatment of the contract curve, along with the idea of the replication of all players in order to study convergence…. Sometime between 1952 and 1959 as we began to better understand what we were saying to each other and how game theory compared with the work of Edgeworth, we understood the core as a separate solution concept."[53]

Shubik was exposed to Edgeworth's *Mathematical Psychics*, to the competitive equilibrium, and to the core via the important papers of Nash, Shapley, and Gillies, which were all crucial to building the foundation upon which he wrote "Edgeworth Market Games." There was one other important piece to this foundation, the influence of Shubik's mentor, Oskar Morgenstern.

Robert Leonard wrote, "it was [*Theory of Games and Economic Behavior*] which shaped the subsequent work of Nash [and] Lloyd Shapley…."[54] As Shubik himself attested, the book was his guiding influence to study game theory at Princeton. Another economic historian, Andrew Schotter, believed that game theory's application to cooperative games was "heavily influenced, at least indirectly, by Morgenstern through the work of Martin Shubik and Lloyd Shapley, the first and foremost authors to pick up the call to arms offered in *The Theory* in a manner consistent with its intent."[55] The work of Morgenstern was very important to Shubik's development as a game theorist, and we now analyze its influence.

In 1967, Shubik edited a volume of essays entitled *Essays in Mathematical Economics in Honor of Oskar Morgenstern*. Concluding the introduction, he wrote: "The advice, guidance, and encouragement which [Morgenstern] has given freely to colleagues and younger economists for nearly forty years…have been instrumental in the development of major contributions to economics…."[56] Surely Shubik must have included himself as one of those "younger economists." There are two Morgenstern influences that manifest themselves in "Edgeworth Market Games."

The first of these influences is Morgenstern's previous writings on Francis Ysidro Edgeworth. In 1927, several months after Edgeworth's death, Morgenstern authored an obituary about Edgeworth for *Zeitschrift fur Vokswirtschaft und Sozialpolitik*. Though Morgenstern did not describe Edgeworth's work in detail, he cited one notable publication. "Although he was not inclined to write books, he has published a few slender volumes, of which we mention only

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[53] Shubik 1992, 156-7
[54] Leonard 1995, 731
[55] Schotter 1992, 104

Mathematical Psychics…. The 'contract curve'…is entirely his own creation."[57] Shubik himself is very unclear about when he first realized the importance of Edgeworth's results and their relation to the core. "Sometime between 1952 and 1959 as we began to better understand…how the game theory compared with the work of Edgeworth, we understood the core as a separate solution concept."[58] Although "Edgeworth Market Games" was written at the Center for Advanced Study in the Behavioral Sciences, Shubik became familiar with Edgeworth and *Mathematical Psychics* at Princeton through Nash and also through Morgenstern.

The second, more general relationship to "Edgeworth Market Games" is Morgenstern's vision for the advancement of cooperative game theory into mainstream economics. "When a new field [game theory] is created out of thin air, one of the ingredients necessary for the field to grow is a strong leader who will encourage others and arrange for opportunities for the best people to find positions and obtain research funding."[59] After the 1944 publication of *Theory of Games and Economic Behavior*, Morgenstern grew into the role of the "strong leader" and helped shape the future of economic research. Schotter wrote: "While the noncooperative theory has become the main tool used today in economics, such an emphasis might have occurred earlier had not attention been focused on the core solution concept in the period up until the late 1970s."[60] The direction of game theory through the 1970s had begun with Morgenstern and his push toward cooperative game theory, but was maintained by Shubik's "Edgeworth Market Games," which combined cooperative game theory with the core.

Shubik was one of only four students in Morgenstern's seminar and decided to study "the behavior of solutions of games under replication"[61] as his thesis. While the thesis, which eventually grew into a 1959 book (Shubik 1959b), dealt with noncooperative equilibrium, Shubik's limitations in mathematics, specifically in measure theory, required him to deal not with a small number of agents but to "[settle] for examining replicated finite sets of agents."[62] Shubik tackled these replicated finite sets of agents in "Edgeworth Market Games." He also dealt with *n* number of players in the paper, and made an important conclusion that the "whole

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[56] Shubik 1967, viii
[57] Schotter 1976, 478
[58] Shubik 1992, 157
[59] Ibid., 103
[60] Ibid., 104
[61] Shubik c.1993, 5
[62] Ibid.

contract curve will remain as a cooperative solution…for any number of traders."[63] Thus it appears that Shubik had become well versed in the economics and the mathematics of *n*-player finite games while doing his thesis under Morgenstern. His familiarity in games with large numbers of players was very important in the writing of "Edgeworth Market Games."

More specifically, however, was the guidance that Shubik received from Morgenstern. Shubik had read Edgeworth's *Mathematical Psychics* at Princeton, which helped him realize a thesis topic, whether Morgenstern suggested the idea to Shubik or Shubik decided on his own.[64] Additionally, Morgenstern's agenda most certainly affected Shubik. "What is noteworthy about…Morgenstern is his eye for what would be of lasting interest in the application of mathematics to economics."[65] Morgenstern's prescience can be seen in Shubik and in the importance of "Edgeworth Market Games." As Shubik himself wrote in an autobiographical summary of his work for Mark Blaug's *Who's Who in Economics*, "My goal has been to reconcile the development in modern mathematical economics with the classical scope of political economy."[66] The importance of "Edgeworth Market Games" lies in the central idea of equating a formal mathematical solution, the core, to a purely economic concept, equilibrium. The paper can be viewed as an extension of Shubik's original attraction to the field and to Morgenstern, as he recounted that *Theory of Games and Economic Behavior* "was the right way to start to mathematize much of economics…."[67]

Investigating the text of "Edgeworth Market Games," we see more direct references to Morgenstern's lifelong mission to make game theory an accepted general economic theory. Shubik commented at the beginning of "Edgeworth Market Games" that "much mathematical work has been done in the investigation of the von Neumann and Morgenstern solution to an n-person game. However very little economic analysis has been applied…."[68] Shubik was driven by a conviction to involve institutional analysis, which was "most certainly a contribution by Morgenstern. It is probably no surprise that the work of Martin Shubik (Morgenstern's most famous student) has consistently been in what he calls 'mathematical institutional economics'."[69] In a 1960 *American Economic Review* article, Shubik remarked that "the study of economics is at

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[63] Shapley and Shubik 1967, 72
[64] Shubik c.1993, 5
[65] Dimand and Dimand 1996, 145-6
[66] Blaug 1999, 781
[67] Shubik 1992, 151
[68] Shubik 1959a, 267

the start of an era which can be characterized by the growth of two new approaches to economics,"[70] one of which is mathematical institutional economics. It is not unreasonable to say that Shubik and "Edgeworth Market Games" helped precipitate this new era.

Shubik completed his Ph.D. at Princeton in 1953 and served as a Research Associate for two years. He described his time at Princeton as "a period of considerable excitement and challenge. New developments were taking place and somehow they seemed to be important even if we did not quite know why."[71] Shubik left Princeton in 1955, however, to pursue further his work in game theory at the Center for Advanced Study in Behavioral Sciences in Palo Alto. A year in California, however, provided the necessary conditions for Shubik to organize his thoughts, and culminated in the writing of "Edgeworth Market Games."

4. Center for Advanced Study in Behavioral Sciences

By the time Shubik arrived at the Center for Advanced Study in Behavioral Sciences (CASBS), an independent research institution on Stanford University's campus, in August 1955, he had published twelve articles. Two of these essays in particular reflected his research interests and can be viewed as crucial to Shubik's development leading up to the writing of "Edgeworth Market Games."

"A Comparison of Treatments of a Duopoly Situation," written with J. P. Mayberry and John Nash, was published in the January 1953 issue of *Econometrica*. The authors considered six solutions to the duopoly problem. The second of those is the Edgeworth contract curve, which they described as a "solution only in the sense that it restricts the possibilities and not in the sense that it determines the outcome uniquely."[72] Shubik later tied the two ideas, contract curve and duopoly, together in "Edgeworth Market Games." In "Edgeworth Market Games," he proved "that as the number of players facing a single monopolist increases, the only undominated imputations approach the single imputation at which the monopolist is totally discriminating and obtains all the gain to be had from trade."[73] This theorem corresponds to the microeconomic idea of the perfectly discriminating monopoly price. But, in the case of a duopoly, Shubik wrote that

[69] Schotter 1992, 103
[70] Shubik 1960, 556
[71] Shubik 1992, 161
[72] Shubik, Mayberry, and Nash 1953, 142
[73] Shubik 1959a, 277

“the core will not contain a single imputation, but there will be an area of indeterminacy between the shares of the monopolists.”[74]

The second essay, “The Role of Game Theory in Economics,” was written at Princeton and published in *Kyklos* in 1953. Shubik offered an explanation of cooperative, noncooperative, and semi-cooperative game theory. Shubik wrote, “game theory uncovers the micro-economic structure of the economy…. By doing so it offers a general framework into which the theory of monopoly, monopolistic competition, pure competition…can all be fitted.”[75] Shubik mentioned Nash’s doctoral thesis at Princeton, specifically that the terms “cooperative” and “noncooperative” were “invented by J. F. Nash Jr.”[76] Touching upon the subject of utility, Shubik wrote that all of game “theory holds for mere orderings on utilities. If we use orderings on utility, then, for instance, Edgeworth’s bilateral-monopoly solution can easily be cast in terms of a cooperative game.”[77] The fact that Shubik mentioned Edgeworth not only indicates that he was already familiar with *Mathematical Psychics* by 1953, but that he had begun to rethink Edgeworth’s work. Shubik wrote in the introduction to “Edgeworth Market Games”: “The specific economic problem which is to be examined here is an extension of a bargaining or bilateral monopoly problem studied by Edgeworth.”[78] The final sentence of “The Role of Game Theory in Economics” is a telling model of Shubik’s attitude toward game theory (shared by the rest of the second-generation game theorists): “Work being done offers the opportunity for the erection of a valuable general theory in great areas of economic study where no such theory previously existed.”[79]

These two essays helped establish a foundation for Shubik to write “Edgeworth Market Games,” in which Shubik acknowledged that “most of the work in this paper was done at The Center for Advanced Study in Behavioral Sciences.”[80] The CASBS “was created by the Ford Foundation to increase knowledge of the principles that govern human behavior.”[81] “The Center brings together each year a select group of scholars from the humanities and the social sciences, providing them with the opportunity and facilities for individual study and informal

[74] Ibid.
[75] Shubik 1953, 23
[76] Ibid., 25
[77] Ibid., 29
[78] Shubik 1959a, 267
[79] Shubik 1953, 32
[80] Shubik 1959a, 267
[81] http://web.ortge.ufl.edu/fyi/v25n11/fyi034.html

association."[82] Fellowships are awarded "each year to scientists and scholars from this country and abroad who show exceptional accomplishment or promise in their respective fields."[83] Combining those who excel in fields such as art history, biology, economics, mathematics, and political science, the CASBS is designed to "lead to the enrichment and development of new skills."[84]

While Shubik was only at the CASBS for a one-year fellowship, that year turned out to be one of the most important of his academic life. Though "Edgeworth Market Games" was not published until 1959, three years after Shubik completed his stint at the CASBS, his year at the Center led him to contemplate the issues discussed in "Edgeworth Market Games." Another prominent game theorist, Howard Raiffa, was also at the CASBS during that year. Shubik acknowledged that when he was writing "Edgeworth Market Games" in 1955, "except for some assistance by Howard Raiffa, I had no one to check my abominal [*sic*] mathematics, and it appeared with several false theorems."[85] Just like his mentor Morgenstern, Shubik's mathematics were limited, a restriction that would not only hinder the influence of "Edgeworth Market Games" but also allow Debreu and Scarf to offer improvements in their 1963 paper.

As Angela M. O'Rand (1992) indicated, the CASBS was one of several out-of-university locations for scholars to further their academic careers. And "the workshops attended and sabbaticals spent at these sites are credited by these authors [Shubik included] for being particularly influential on their own work."[86] Indeed, during his twelve months at the CASBS, Shubik gave at least one seminar in Edgeworth market games. Shubik referenced his notes from this seminar in his 1982 book *Game Theory in the Social Sciences*.[87]

5. Understanding "Edgeworth Market Games"

In "Edgeworth Market Games," Shubik reformulated many theorems from F. Y. Edgeworth's 1881 book entitled *Mathematical Psychics*. "Edgeworth Market Games" is divided into three sections. In the first part, Shubik gave a brief economic introduction to the problem of

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[82] http://www.stanford.edu/dept/soc/researchindex.html
[83] http://web.ortge.ufl.edu/fyi/v24n08/fyi042.html
[84] http://web.ortge.ufl.edu/fyi/v25n11/fyi034.html
[85] Shubik c.1993, 6
[86] O'Rand 1992, 186
[87] Shubik 1982, 175 and 480

the *n*-person game, and clearly stated his objective for this paper. “In order to connect this particular type of game theory with economic analysis we will examine games whose *characteristic functions* are constructed in such a manner that they reflect certain economic aspects of a market.”[88] Shubik described his paper as “an extension of a bargaining or bilateral monopoly problem studied by Edgeworth.”[89] In the second section, Shubik detailed his mathematical model, and showed that the “core of an *n*-person game [is] equivalent to the contract curve derived”[90] by Edgeworth. In the last part, Shubik gave his economic interpretation of these Edgeworth market games.

Shubik made comparisons between the core of an *n*-person game and the contract curve derived by Edgeworth. Edgeworth characterized the market with a two-person non-zero sum game in which each player's preferences were represented by a group of convex curves. In this two-person non-zero sum game, one player has a certain amount of quantity A and none of quantity B while the other player has none of quantity A and a certain amount of B. “Edgeworth selected as a solution to this trading problem a set of distributions which satisfy the conditions that at any of these distributions a player is at least as well off as he would be if he failed to trade.”[91] Thus, if the players are better off not trading, then the problem becomes trivial and there is no sense in analyzing the solutions to the trading problem. Edgeworth defined the solution set to such a game as the set of all possible distributions of goods that left a player with at least as much utility as if the player failed to trade. Assuming that all goods are infinitely divisible, this set of distributions can be drawn as a curve that he called the contract curve. The Edgeworth solution is similar to the Von Neumann-Morgenstern stable set in that it “consists of a set of distributions which do not dominate each other but dominate all other distributions.”[92] We will discuss these differences in the next section.

Though Edgeworth market games do not converge to a single point as the number of players increases to infinity, under certain conditions, the core of these games does approach a single imputation in the limit. Shubik noted that this “single imputation has economic meaning either in the theory of monopoly or the theory of pure competition."[93] In one particular class of

[88] Shubik 1959a, 267
[89] Ibid.
[90] Greenwald 1982, 454
[91] Shubik 1959a, 267
[92] Ibid., 268
[93] Ibid., 271

games, the core approaches a single imputation where a single player will act as a perfectly discriminating monopolist.

The core as a solution has many interesting economic applications, especially for market games. The core of an Edgeworth market game consists of those imputations that are unaffected by any of the possible coalition structures. Shubik later defined the core more clearly, as “the set of imputations that leave no coalition in a position to improve the payoffs of all its members.”[94] We will discuss the further economic ramifications of Shubik’s paper in the next section as well.

As long as an imputation exists within the core then there will be no motivation for any group of players to re-contract. Shubik suggested that the notion of increasing and decreasing returns to scale play an important role in Edgeworth market games of this nature. He further proposed that games with decreasing returns to scale would have an empty core. An empty core indicates that all possible imputations are dominated by some coalition of players. "In general, this states that in an economy with decreasing returns present everywhere, under an optimal allocation of resources no matter how total product is divided there always will be some group in the economy that can form a coalition that is effective against any imputation that is suggested.”[95]

6. Economic Implications of “Edgeworth Market Games”

The effects of Shubik’s paper must be analyzed on two levels. From a narrow viewpoint, “Edgeworth Market Games” not only showed that the core of an *n*-person game was equivalent to Edgeworth’s contract curve, but, it also offered another solution to *n*-person cooperative games. Shubik’s discovery, however, characterized market games in a more specific fashion. For a better understanding of Shubik’s importance, we first need to detail two other solutions to *n*-person games, the Von Neumann-Morgenstern stable set and the Shapley value solution.

The first solution was established by John von Neumann and Oskar Morgenstern in their 1944 *Theory of Games and Economic Behavior,* as “a system of imputations possessing in its entirety some kind of balance and stability….”[96] More clearly defined, the stable set solution is when “no two valuations in the solution dominate each other and every external valuation is

[94] Shubik 1982, 147
[95] Shubik 1959a, 276
[96] von Neumann and Morgenstern 1944, 36

dominated by at least one solution member."[97] Additionally, the final imputation in the Von Neumann-Morgenstern stable set is Pareto efficient. That is, the set of all traders cannot block the final coalition because if the outcome is an imputation, the players get jointly exactly what they would get jointly in the grand coalition."[98] This notion is called internal stability. Recalling that a solution concept arises under a set of arguments about how a game ends, we see that this game ends when stability is achieved.

If we relate this concept to the contract curve, this solution can be reached by moving to the contract curve, making all players better off. There are other imputations in the stable set that are also solutions, in which some players are made worse off and some better off. We arrive at these imputations by moving *along* the contract curve. As Anatol Rapoport wrote in his book *N-Person Game Theory*, "even though the Von Neumann-Morgenstern solutions do not single out unique imputations as *the* outcomes of games played by rational players, this does not in itself make these concepts useless for a normative theory."[99] This passage highlighted the generality of the Von Neumann-Morgenstern stable set, whereas Shubik's core-as-solution concept offered a more specific solution.

The second major solution concept for n-person games is Shapley value. The payoff for each player is merely the expected value to each player based on various coalition strategies.[100] Thus, for an n-person game each player receives an equal portion of the total amount of utility, illustrating the symmetry of the Shapley value solution. While this concept was similar to the core in that the solution is unique, Rapoport outlined its shortcomings. "Its weakness is precisely in that it derives entirely from the characteristic function of the game and not from what is 'behind' the characteristic function, i.e., the strategic structure of the game itself rather than the bargaining positions of the players in the process of coalition formation."[101] Therefore the players appear as equals and their payoffs the same, although their positions to form coalitions might differ. The Shapley value solution concept ignored this idea.

Now that we have analyzed two other n-person game solution concepts, we can effectively discuss Shubik's discovery of the core as a third major solution concept. As discussed earlier, the core is the set of imputations not dominated by any other imputations, where no

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[97] Leonard 1992, 54
[98] Rapoport 1970, 114
[99] Ibid., 96
[100] Ibid., 106-113

imputation in the core dominates any other imputation in the core. As Shubik wrote in "Edgeworth Market Games," "the core of a game consists of those imputations which are unaffected by any type of coalition structure. Following Edgeworth's terminology we may state that if any imputation in the core existed there would be no motivation for any group to recontract."[102] This is really at the heart of the importance of Shubik's discovery. The core gave an excellent interpretation of a market game because it required that all coalitions act in their own best interest. Thus, Shubik showed that an imputation could not be in the core if another coalition (of any size) can do better for itself. The Von Neumann-Morgenstern stable set was a softer solution concept, in that it dealt with cooperation and society, while the core was more ruthless, with groups having the freedom to recontract and players leaving previously held coalitions.[103]

To illustrate the differences, and characteristics, of these solution concepts, we now examine two different games.[104] The first game is a housing market game with three players, one acting as a seller (denoted S) and two acting as potential buyers (denoted B_1 and B_2). Player S will accept no less than \$150,000 for the house, while player B_1 will pay no more than \$175,000 and player B_2 will pay no more than \$200,000. There are eight different coalitions. In four of them, ∅, $\{B_1\}$, $\{B_2\}$, and $\{B_1, B_2\}$, the total payoff is \$0. With the seller by himself, {S}, he keeps the house and the payoff is \$150,000. With the seller joining with B_1, $\{S, B_1\}$, the buyer gets the house and the total payoff is \$175,000. And in the final two coalitions, $\{S, B_2\}$ and $\{S, B_1, B_2\}$, the second buyer purchases the house and the total payoff is \$200,000 in each case. In the final two cases, B_1 is shut out from any of the payoff. Using these restrictions, we use an iterative process to exclude those imputations that can be blocked via a coalition until we arrive at proper solution. Writing the payoff $\{B_1, B_2, S\}$, the imputations in the core will take the form {0, x, 200000-x}, where $0 \leq x \leq 25000$. But the Von Neumann-Morgenstern stable set would have far more imputations, each of the form {x, a, 200000-x-a}, where 'a' is a fixed value between 0 and 25000, inclusive. Thus, in the latter, B_1 does not necessarily receive a \$0 payoff. This illustrates the ruthless and discriminating nature of the core (whereas the Von Neumann-Morgenstern stable set is more cooperative), and the fact that there are fewer imputations in the

[101] Ibid., 113
[102] Shubik 1959a, 276
[103] Discussions with Professor E. Roy Weintraub, 2/3/2000
[104] Ibid.

core than in the Von Neumann-Morgenstern stable set demonstrate the core's superiority as a market game solution.

Continuing the game, we suppose that more and more buyers enter the market each with a maximum purchase price higher than the previous entrant (but all higher than $175,000), converging to the highest price, $200,000. If we get a large number of players, then eventually the payoff will be $200,000 to the seller and $0 to the *n* buyers. That imputation, namely {0, 0, 200000, 0, 0…} will be the only point in the core, and has a significance in the market economy. First, however, note that the Von Neumann-Morgenstern stable set will still contain numerous imputations, while the core will delineate the true solution. Second, notice that as the number of players became large, the core shrunk to one point. This is the competitive equilibrium point that is the subject of every microeconomics supply and demand graph. The core is an effective solution concept for market games because as Shubik discovered if the competitive equilibrium point exists, then it must be in the core. And if the core only contains one point, as it does in this instance, than that point must be the competitive equilibrium. Shubik wrote "in an economy of 'optimum size', i.e., one in which a stage of increasing returns to scale has just given place to constant returns, the core approaches a single point at which every individual is awarded his 'marginal value' productivity. In this economy there is only one imputation that is not dominated by some coalition of players."[105]

In a second game, entitled "Divide the Dollar," there are three players, A, B, and C. There is one dollar to be divided between the three players, and the objective of the game is to determine the imputation (or imputations) that end the game. There are once again eight coalitions, and four of them have no payoff: ∅, {A}, {B}, and {C}. The latter three have no payoff because the other two players can form a coalition to block out the third player. The four other coalitions, {AB}, {AC}, {BC}, and {ABC} all have payoffs of $1. The Von Neumann-Morgenstern stable set includes imputations, once again of the form {a, x, 1-a-x}. The Shapley value is {$^1/_3$, $^1/_3$, $^1/_3$}, which is the most natural interpretation of this game that can be characterized as having 3-person symmetry.

The core, however, is empty. This game illustrates the main weakness of the core as a solution concept. The core is empty because any given coalition can be dominated by another coalition. As Shubik wrote, "under an optimal allocation of resources no matter how total

[105] Shubik 1959a, 276

product is divided there always will be some group in the economy which can form a coalition that is effective against any imputation that is suggested."[106] For instance, {AB} can be dominated by {BC} if player B chooses to leave his coalition with A and recontract with player C. Similarly, {ABC} can be dominated by any of the two person coalitions, if two players choose to combine forces and shut out the third player. Each of the two players would get one-half of the dollar, which was more than the third of the dollar they were getting before. The ineptitude of the core is that it offers no solution to a game such as this, even though it is generally an effective solution for market games.

Analyzing "Edgeworth Market Games" on a larger scale, the paper was an important milestone of game theory's development into the discipline it has since become. Not only was Shubik one of the first economists to publish a significant result in game theory, but he was the first to relate a truly economic concept (the contract curve) to a game theoretic concept (the core). Before Shubik, much of the work being done in game theory was by his peers in the mathematics graduate program, namely Nash and Shapley. Shubik, it seems, did not realize the consequence or the novelty of "Edgeworth Market Games." Rather, it was just a logical progression of his studies at Princeton. "It seemed to me that Edgeworth's work on two sided markets could be regarded as a precursor of a cooperative game approach to markets."[107]

Analysis of *n*-person Edgeworth market games and a careful examination of the core allow for greater clarity in defining abstract terms such as "a world of monopolies". The level of technology available to the players often determines the core of one of these games. The study of Edgeworth market games led to some very interesting conclusions. For example, Shubik pointed out that in "any economy, any imputation of wealth which does not lie in the core implies that some group of individuals is profiting at the expense of another group."[108] Using this conclusion, we can now use an examination of the core of an Edgeworth market game to determine whether or not a coalition will be created in a given situation and whether or not a perfectly discriminating monopoly will arise. Shubik's critical assessment of market games, almost eighty years after Edgeworth wrote the paper, opened the floodgates for analysis in the field of markets and behavior of firms in the market. Much of Shubik's own analysis after his monumental 1959 paper included game theory as a tool for investigating the firm.

[106] Ibid.
[107] Shubik c.1993, 6

The publication of Shubik's paper was the culmination, chronologically and academically, of a decade that saw great changes in game theory. The 1950s began with game theory as primarily a military concept, specifically as "an extension of the military model, particularly under the auspices of the Office of Naval Research (ONR) and the Atomic Energy Commission."[109] Then, game theory moved into a strictly mathematical academic program, manifested especially by the work being done at Princeton. In the mid-1950s, game theory branched out as an economic discipline, indicated by the numerous collaborative articles between Shapley the mathematician and Shubik the economist. With the publication of "Edgeworth Market Games," Shubik propelled game theory into the general discourse, establishing its importance in solving problems other than those of the military or of mathematics. "From that time on, economics has remained by far the largest area of application of game theory."[110] Shubik opened a completely new area of study, that of the core. In the 1960s, "core theory was extensively developed and applied to market economies."[111] Additionally, "most important [in the 1960s] was the forging of a strong, lasting relationship with mathematical economics and economic theory."[112]

Andrew Schotter incorrectly stated that "*Mathematical Psychics* [was] a work that was to be ignored in the Anglo-American literature until the early 1960's,"[113] thereby not only ignoring Shubik's "Edgeworth Market Games" but also Nash's "The Bargaining Problem." In spite of his oversight, Schotter alluded to the overall importance of "Edgeworth Market Games." Schotter wrote: "Edgeworth's conjecture was to become the central theorem in the game theoretical literature on the core of an economy."[114] Shubik's "Edgeworth Market Games" was the direct cause of the game theory community's attention to the core: the paper is cited in at least twenty-one different economic papers (not including papers by Shubik) in the first twenty-five years after its publication.[115] Shubik's accomplishment was the demonstration of the core as a true economic solution. This concept has far-reaching ramifications. After "Edgeworth Market Games," many problems were reconsidered in a game-theoretic fashion. His notion of relating

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[108] Shubik 1959a, 277
[109] Mirowski 1991, 230
[110] Eatwell, Milgate, and Newman 1987, 467
[111] Ibid., 471
[112] Ibid.
[113] Schotter 1976, viii
[114] Ibid.
[115] Social Science Citation Index

contracting, recontracting, and coalitions to economics not only applied to market economies, but also to a diverse number of non-economic disciplines.

7. Limitations and Criticisms of "Edgeworth Market Games"

While "Edgeworth Market Games" was certainly a very crucial paper for the mainstreaming of game theory, its limitations, due to Shubik's own mathematical inadequacies, prevented the paper from being fully recognized as the singular cause of the economic community's focus on the core in the 1960s and 1970s. While Shubik raised certain issues, his deficiencies in mathematics did not allow for him to reach concrete conclusions. The theorems in "Edgeworth Market Games" were improved by Gerard Debreu and Herbert Scarf and then polished by Robert J. Aumann.

With the publication of their 1963 paper, "A Limit Theorem on the Core of an Economy," Debreu and Scarf completed the movement of game theory into economics by generalizing the core. "The 'core' was…shown by Debreu and Scarf in 1963 to converge to a Walrasian general equilibrium, thus demonstrating that neoclassical theory and the game-theoretic program were perfectly compatible."[116]

As stated earlier, Shubik had postulated that the core would converge to the competitive equilibrium, but knew that he "was incapable of proving it."[117] While Shubik showed that the core converged to the competitive price in transferable utility (TU) games, "some years later [after the publication of "Edgeworth Market Games"], on a walk from Columbia University to downtown New York after Herbert Scarf had given a paper on an economy with a single dynamically unstable equilibrium point, [Shubik] suggested to him that the core could be regarded as a combinatoric test for stability and I conjectured that the convergence of the core was probably true for NTU games."[118] The proof for NTU, or nontransferable utility, games had eluded Shubik, even though he had "recognized that the treatment in Edgeworth was of a game without transferable utility…as it was much easier to consider the game in…transferable utility (TU) form."[119]

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[116] Mirowski 1992, 115
[117] Shubik c.1993, 6
[118] Shubik 1992, 157
[119] Ibid.

In their paper, Debreu and Scarf showed that the core always contained the competitive (Walrasian) equilibrium point under the premise of nontransferable utility (*i.e.*, that the TU assumption does not have to hold). Debreu and Scarf believed that the assumption of TU had "remained foreign to the mainstream of economic thought"[120] and thus limited the effectiveness of the core as a true economic concept. Thus, they proved the existence of the competitive equilibrium in the case of players trading goods for goods, in contrast to Shubik, whom had effectively monetized the problem by assigning utility to the commodities and making them equally exchangeable. The authors successfully showed that the improvement on Shubik's proof is "widely applicable and, thereby, [obtain] a further considerable simplification of the study of the core."[121]

In "Edgeworth Market Games," Shubik simply replicated the game. He added more consumers to each type of player, but kept constant both the two commodities being owned and the two *types* of traders. Whereas Shubik's proof could not be generalized to complex trading, Debreu and Scarf achieved their "further considerable simplification" by using "an arbitrary number of consumers and an arbitrary number of commodities. Economies consisting of *r* consumers of each type were considered…."[122] Though Shubik improved the notion of price equilibrium by showing that the core was a more discriminating solution than the Von Neumann-Morgenstern stable set, Debreu and Scarf further improved upon Shubik. They generalized the notion of the core, and included it in any type of economy, one with *n* players, *m* commodities, and *r* consumers of each type.

Although Shubik recognized this general case, he could not prove it himself, thereby lessening the economic significance of his paper. Aumann (1964) improved on Debreu and Scarf's paper, further pushing Shubik and "Edgeworth Market Games" from the spotlight. In "Markets with a Continuum of Traders" Aumann explained that "the influence of an individual participant on the economy cannot be mathematically negligible, as long as there are only finitely many participants. Thus *a mathematical model appropriate to the intuitive notion of perfect competition must contain infinitely many participants*."[123] This was a change from both Shubik's paper and from Debreu and Scarf's paper, who assumed a finite number of traders.

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[120] Debreu and Scarf 1963, 236
[121] Ibid., 237
[122] Ibid.
[123] Aumann 1964, 39

Aumann concluded, therefore, "in the continuous model—but in no finite model—the two concepts [competitive equilibrium and core] are essentially equivalent."[124]

The purpose of Aumann's paper was to examine the "extreme case in which perfect competition does obtain, i.e. there are no individually significant traders."[125] After he proved this idea, Aumann spent the last portion of the paper discussing the "literature" that had preceded his paper and explaining their shortcomings. In dealing with the two other papers (Shubik 1959a, Debreu and Scarf 1963), Aumann emphasized that "the trouble is that the core and the set of equilibrium allocations are subsets of a space whose dimension varies with the number of traders."[126] But Aumann countered that his model has "no problem working in spaces of varying dimension, because we start with a space of *infinite* dimension and remain in this space throughout the investigation."[127]

Another weakness of Shubik and Debreu and Scarf was their use of finitely many types of traders "involves the further assumption that there are 'many' traders of each type; in fact the number of traders of each type must be very large compared to the number of types in order for the model to be applicable. This seems far from economic reality…. The continuous model allows *all* traders to have different initial bundles and different preferences."[128] In his book *Introduction to Contemporary Microeconomics*, Vivian Walsh (1970) explained that Edgeworth's work "lay virtually ignored until the work of Shubick [*sic*] (1958) and the elegant developments of Scarf (1962) and of Scarf and Debreu (1963) and of Aumann (1964)."[129] Though unknowingly, Walsh provided the major reason as to why Shubik's paper is not viewed with more importance. While Shubik was the first to *recognize* that Edgeworth's contract curve was equivalent to the core, it was Debreu and Scarf and then Aumann who *improved* upon this notion. Each paper generalized the conditions under which the core could exist, and thus elevated the importance of the core as an economic solution concept.

Moving away from the improvements that were made on "Edgeworth Market Games," we focus on another reason as to why Shubik's paper is not treated with more economic significance. "Edgeworth Market Games" dealt with cooperative games at a time when game

[124] Ibid.
[125] Ibid., 41
[126] Ibid., 48
[127] Ibid., 48-49
[128] Ibid., 48
[129] Walsh 1970, 173

theory was moving toward noncooperative games. Shubik himself explained why game theory was headed toward the noncooperative side. It was difficult to "interest the community of economists in cooperative game theory [because] the representation of a game by a characteristic function entailed the implicit or explicit assumption of the existence of a magic substance or 'utility pill' with a constant marginal utility to all traders."[130]

As Philip Mirowski wrote of the standard history of game theory, after *Theory of Games and Economic Behavior*, "the next landmark in this history is John Nash's 1951 paper on a solution concept for a class of non-zero-sum games dubbed 'non-cooperative.'"[131] So even before Shubik wrote "Edgeworth Market Games," game theory had already begun to move toward the noncooperative games. In 1952, Shubik unknowingly forecasted the importance of Nash's work in game theory. On a train ride to Princeton from New York with John von Neumann, "I recall suggesting that I thought that Nash's non-cooperative equilibrium solution theory might be of considerable value in applications to economics."[132] While von Neumann disagreed and felt that cooperative games "made more social sense,"[133] the larger game theory community did not share his opinion. As Sylvia Nasar pointed out, "Nash's concept of equilibrium from strategic games is one of the basic paradigms in social sciences and biology."[134]

Shubik's thesis at Princeton eventually became a 1959 book entitled *Strategy and Market Structure* that applied Nash's noncooperative equilibrium.[135] As Shubik wrote: "Although my interest in cooperative game theory was, and remains high, it seemed to me that there was much to be done especially in oligopoly theory, utilizing noncooperative theory. I decided that this would be the main concept employed in my thesis."[136] Ironically, Shubik's thesis concentrated on noncooperative games while "Edgeworth Market Games," the paper that remains his most important to the development of game theory, incorporated cooperative games.

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[130] Shubik 1992, 157
[131] Mirowski 1992, 115
[132] Shubik 1992, 155
[133] Ibid.
[134] Nasar 1998, 98
[135] Shubik c.1993, 5

8. Conclusion

In this paper, we explained the conditions influencing Martin Shubik when he wrote "Edgeworth Market Games," an important paper in the development of game theory that is often neglected in the general history of economics. The purpose of our paper was to provide an historical narrative of "Edgeworth Market Games" that discussed not only the content of Shubik's paper, but also its economic significance, its shortcomings, and its place in economic history.

While some of the material contained in this paper was not new to the history of economics, the synthesis of this information with new material is significant. In addition to supplying a thorough chronicle of Shubik's years at Princeton, we offered an account of the influence of three second-generation game theorists and new insights into Shubik's year at the CASBS to give a detailed narrative with which to understand "Edgeworth Market Games."

Our analysis of the improvements by subsequent economists on Shubik's discovery that the contract curve was equivalent to the core explained why "Edgeworth Market Games" has been pushed from the forefront. Before our contextualization, there had not been a paper that concentrated solely on "Edgeworth Market Games," a perspective that is unique to the history of economics.

[136] Ibid.

References

Aumann, Robert J. 1964. Markets with a Continuum of Traders. *Econometrica* 32 (January-April): 39-50.

Blaug, Mark, ed. 1999. *Who's Who in Economics*. Cambridge, MA: The MIT Press. Cambridge: Cambridge University Press.

Debreu, Gerard and Herbert Scarf. 1963. A Limit Theorem on the Core of an Economy. *International Economic Review* 4 (October): 235-246.Leonard, Robert J. 1992.

Dimand, Mary Ann and Robert W. Dimand. 1996. *The History of Game Theory, Volume 1: From the beginning to 1945*. London: Routledge.

Eatwell, John, Murray Milgate, and Peter Newman, eds. 1987. *The New Palgrave: A Dictionary of Economics*. London: Macmillan Press Ltd.

Edgeworth, F. Y. 1881. *Mathematical Psychics*. London: C. Kegan Paul & Co.

Gillies, D. B. 1953. *Some Theorems on N-Person Games*, Ph.D. diss. Department of Mathematics, Princeton University.

Greenwald, Douglas, ed. 1982. *The McGraw-Hill Encyclopedia of Economics*. New York: McGraw-Hill.

————. 1983. *The McGraw-Hill Dictionary of Modern Economics*. New York: McGraw-Hill.

Leonard, Robert J. 1992. Creating a Context for Game Theory. In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 29-76.

————. 1995. From Parlor Games to Social Science: von Neumann, Morgenstern, and the Creation of Game Theory 1928-1944. *Journal of Economic Literature*, vol. XXXIII (June): 730-761.

McKinsey, J. C. C. 1952. *Introduction to the Theory of Games*. New York: McGraw-Hill Book Company, Inc.

Mirowski, Philip. 1991. When Games Grow Deadly Serious: The Military Influence upon the Evolution of Game Theory. In *Economics and National Security*, edited by Craufurd Goodwin. Durham, NC: Duke University Press: 227-255.

————. 1992. What were von Neumann and Morgenstern Trying to Accomplish? In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 113-147.

Morgenstern, Oskar. 1927. Francis Y. Edgeworth. *Zeitschrift für Vokswirtschaft und Sozialpolitik*, vol. 5, no. 10-12 (October-December): 261-290.

Nasar, Sylvia. 1998. *A Beautiful Mind*. New York: Simon & Schuster.

Nash, Jr., John F. 1950. The Bargaining Problem. *Econometrica* 18, 2 (April): 155-162.

O'Rand, Angela M. 1992. Mathematizing Social Science in the 1950s: The Early Development and Diffusion of Game Theory. In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 177-204.

Rapoport, Anatol. 1970. *N-Person Game Theory*. Ann Arbor, MI: University of Michigan Press.

Rellstab, Urs. 1992. New Insights into the Collaboration between John von Neumann and Oskar Morgenstern on the *Theory of Games and Economic Behavior*. In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 77-94.

Schotter, Andrew, ed. 1976. *Selected Economic Writings of Oskar Morgenstern*. New York: New York University Press.

————. 1992. Oskar Morgenstern's Contribution to the Development of the Theory of Games. In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 95-112.

Shapley, L. S. and Martin Shubik. 1967. Concepts and Theories of Pure Competition. In *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, edited by Martin Shubik. Princeton: Princeton University Press: 63-79.

Shubik, Martin, J. P. Mayberry, and J. F. Nash. 1953. A Comparison of Treatments of a Duopoly Situation. *Econometrica* 21 (January): 141-154.

Shubik, Martin. 1953. The Role of Game Theory in Economics. *Kyklos* 7, 2: 21-34.

————. 1959a. Edgeworth Market Games. In *Contributions to the Theory of Games*, vol. 4, edited by A. W. Tucker and R. D. Luce. Princeton: Princeton University Press: 267-278.

————. 1959b. *Strategy and Market Structure*. New York: Wiley.

————. 1960. Game Theory as an Approach to the Firm. *American Economic Review* 50, 2: 556-559.

————. 1981. Oskar Morgenstern. In *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern*. Mannheim: Bibliographisches Institut: 9-10.

————. 1982. *Game Theory in the Social Sciences*. Cambridge, MA: The MIT Press.

————. 1992. Game Theory at Princeton, 1949-1955: A Personal Reminiscence. In *Toward a History of Game Theory*, edited by E. Roy Weintraub. Durham, NC: Duke University Press: 151-164.

————. c.1993. Draft of an autobiographical summary of Shubik's life and work. In Martin Shubik Papers, Rare Book, Manuscript, & Special Collections Library, Duke University, Durham, North Carolina.

Söderqvist, Thomas, ed. 1997. *The Historiography of Contemporary Science and Technology*. Australia: Harwood Academic Publishers: vii-ix.

Stanford University: Stanford Research Centers website http://www.stanford.edu/dept/soc/researchindex.html

Telser, Lester G. 1988. *Theories of Competition*. New York: North-Holland.

University of Florida: Office of Research, Technology and Graduate Education website. http://web.ortge.ufl.edu/fyi/v25n11/fyi034.html

————. http://web.ortge.ufl.edu/fyi/v24n08/fyi042.html

Varian, Hal R. 1978. *Microeconomic Analysis*. New York: W.W. Norton & Company.

von Neumann, John and Oskar Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

Walsh, Vivian Charles. 1970. *Introduction to Contemporary Microeconomics*. New York: McGraw-Hill.

Weintraub, E. Roy. 1991. *Stabilizing Dynamics: Constructing Economic Knowledge*. Cambridge: Cambridge University Press.