Regret in First-Price Sealed Bid Auctions

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Introduction

Most auction literature assumes one specific form of utility function (see Vickrey 1961, Myerson 1981, Milgrom and Weber 1982) – one that is solely dependent on the expected "profit" from the bid.[2] Though some competitive bidding models might allow for different risk preferences, the sole determinant of bidders' outcome utilities remains the outcome's profit. In "The Effect of Regret on Optimal Bidding in Auctions" (1987), Richard Engelbrecht-Wiggans began to explore a bidding model with the bidders' utilities determined by elements other than solely profit; in Engelbrecht-Wiggans' model, the bidders' utilities are affected by regret from money left on the table.

In second price auctions, optimal bidding would involve no regret.[3] If the bidder wins, he or she pays the second highest price, and thus the least necessary to be the winner (leaving no money on the table). If the bidder loses, it is because another bidder is willing to pay more than the bidder's valuation (meaning no possible surplus was lost). Unlike second price auctions, in first-price sealed bid auctions, the bidders can experience regret.

There are two possible ways to regret the outcome of a bid in a first-price sealed bid auction.[4] If the bidder wins the auction, unless the winning bid is the same amount as the next highest bidder, there is money left on the table from overbidding. If the bidder does not win the auction, he or she may have also left money on the table – if the winning bid is between the bidder's valuation and his or her own bid, the bidder missed out on a potential surplus. Regret, if included in a bidder's prior utility function, could alter the Nash equilibrium.

Regret and Information

Engelbrecht-Wiggans hypothesizes that money left on the table from a winning bid in excess of the second bid is more apparent than foregone surplus. His conclusion is that incorporating regret into the utility function would bias bids downward to avoid regret from money left on the table from excess bidding. Engelbrecht-Wiggans

[2] Defining profit as the difference between the bidder's valuation of the good and the bid.
[3] In a second-price auction, the highest bid wins the auction and pays the amount of the second highest bid. In this type of auction, the optimal bid is the bidder's valuation.

conclusion is based on the assumption that there is a better sense of money left on the table when winning. His argument would follow logically if information were an important element of regret.

According to Richard Larrick and Terry Boles, "Regret theory (Bell 1983) posits that learning about the outcome of a foregone alternative creates the possibility of experiencing regret." In "Risk Premiums for Decision Regret," David Bell defines "regret premium," the amount one is willing to forego in order to avoid feedback on foregone risky alternatives (Larrick and Boles). According to these authors, information feedback causes regret to have a greater (negative) effect on utility.

Engelbrecht-Wiggans' intuition follows from Bell's principle of regret influenced by information feedback. The influence, then, of regret in auctions on bidders' utilities would be affected by the amount of information that the bidders would receive after the auction (given the knowledge before the auction that they will receive this information).

Experimental Design

My experiment tests the effects of regret on bidding, using information feedback as the varying condition. In the experiment, the subjects participate in an auction with five competing bidders.

1) Subjects were recruited by asking if they would like to participate in an auction to earn $1 with the possibility of winning more money.

2) In total, 48 people were recruited to participate in 8 separate auctions (6 people per auction). Each subject had to give an email address in order to give the participants feedback.

3) Subjects were given auction instruction sheets. These instructions assigned the participant a resale value, which was to be kept private.[5] There were 24 resale values randomly drawn from a uniform distribution between $0 and $200 (rounded to the

[4] In this type of auction, the bidders have known private valuations and make one bid unknown to the other bidders. The highest bid wins the auction and pays the amount of the bid.

nearest cent). Each resale value was used twice (once under each experimental condition).

4) The participants were given limited information about the resale values. Each participant knew that his or her resale value was randomly determined from a uniform distribution between zero and an unknown amount. The participant also learned that he or she was competing against 5 other participants in the auction. From this information, the participant was to consider a bid, where the auction winner would receive the difference between his or her resale value and his or her bid.

5) In addition to the above information, the participants were instructed that they would receive feedback about the outcome of the auction. On half of the instructions, participants learned that they would receive feedback about the winning price of the auction. On the other half, participants learned that only the winner would be given the results of the auction and that winner would learn the second highest bid.

6) After receiving all of the above information, subjects, then, were prompted to write down their bid on the instructions page.

7) The subjects were also asked a comprehension question, "Your payoff if you win?" The subjects had to subtract their bid from their resale value and write down the answer on the instructions page.

8) The participants' instruction pages with their written bids and responses to the comprehension question were collected. The auctions were completed at a separate site through random grouping of participants under each condition. The winning participants were paid, and the feedback was given in accordance with the instructions.

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[5] The resale values served as valuations for the item to be bid upon (which was a poker chip).

Equilibrium Predictions

Assuming risk neutrality and utility functions based solely on expected profit, the standard RNNE (risk neutral Nash equilibrium) solution would be expected. This solution would involve each participant bidding:[6]

$$(1)\quad B_i = V_{\min} + \left(\frac{n-1}{n}\right)\cdot(V_i - V_{\min})$$

Where:
$B_i =$ The bid of bidder i
$V_i =$ The assigned valuation of bidder i
$V_{\min} =$ The minimum valuation of the uniform distribution of values
$n =$ The number of bidders participating in the auction

Each participant is instructed that $V_{\min} = 0$.

So, the risk neutral Nash equilibrium bid from each participant would be:

$$(2)\quad B_i = \left(\frac{n-1}{n}\right)\cdot V_i = \frac{5}{6}\cdot V$$

Equilibrium Predictions for my experiment instead were based on the following alternative model:

$$u(B_i) = u(V_i - B_i)\cdot p(B_i) + a_1(E[B_i - B_{i-1}])\cdot p(B_i) + a_2(E[V_i - B_1])\cdot q(B_i)$$

Where:

$B_i =$ The bid of bidder i
$V_i =$ The assigned valuation of bidder i
$p(B_i) =$ The probability of winning for a bid of B_i
$a_1 =$ The regret (magnitude function) from a winning bid; negative utility function
$B_{i-1} =$ The next highest bid after B_i
$a_2 =$ The regret (magnitude function) from a losing bid; negative utility function
$B_1 =$ The highest bid
$q(B_i) =$ The probability of $B_i < B_1 < V_i$ for a bid of B_i ; $q(B_i) \leq 1 - p(B_i)$

and

[6] The equations used for equilibrium predictions come from Vickrey 1961.

$$\frac{d\alpha_1}{dB_i} = k_1$$
$$\frac{d\alpha_2}{dB_i} = k_2$$
$$\frac{dp(B_i)}{dB_i} = k_3$$
$$\frac{dq(B_i)}{dB_i} = k_4$$

Where k_1, k_3 >0 and k_2, k_4 <0.

The difference in predictions from this model is that in the model that does not incorporate regret the α_1 and α_2 pieces have no effect. In the experimental auction described above, it means that the feedback given to the auction participants does not affect their bid. In the alternative model, however, the α_1 and α_2 pieces have an effect dependent on the feedback.

In the case of giving all of the participants feedback about the winning bid, this should induce a higher bid. At the RNNE bid level, the subject feels (greater) regret from bids between his or her valuation and his or her bid. As the bidder increases the bid (toward his or her valuation), the probability of this regret occurring decreases. The subject can improve his or her utility by increasing the bid from the RNNE level.

In the case of giving only the winner feedback about the second highest bid, this should induce a lower bid. At the RNNE bid level, the winning subject feels increasing regret the greater the difference between their bid and the next highest. As the bidder decreases their bid (toward the second highest), the magnitude of the regret decreases. The subject can improve their utility by decreasing their bid from the RNNE level.

In previous experimental auction studies (see Cox, Roberson, and Smith 1982), subjects tend to make bids that are higher than the RNNE level. A common explanation is that bidders are not risk neutral – if the bidders were risk averse, they would put a premium on probability of profit over total profit, and this would induce higher bids (with a lower expected profit).[7]

The manipulation in the auction is based on regret and evaluating behavior under the two feedback conditions. The design of the experiment was not intended to demonstrate deviation from RNNE. Instead, the experiment hypothesizes that subjects under the (winner only) second bid feedback condition will bid less than subjects under the winning bid feedback condition.

Results[8]

The experimental results were not consistent with the equilibrium predictions. Subjects under the (winner only) second bid feedback condition bid more than subjects under the winning bid feedback condition. On average, the first condition yielded bids $5.72 higher; in 14 of 24 cases, the bid in the first condition was higher than the second condition. A matched pairs t-test for difference of means was, however, insignificant.[9]

Discussion

There are many possible reasons for experimental results that are the opposite of the predicted results: incorrect intuition or flaws with the experimental design. I believe that the design of the experiment did not accurately induce utility functions based on the model.

The insignificance (and non-intuitive direction) of the results suggests that participants ignored the experimental condition (information feedback). Subjects were rewarded based on their decision in the auction game (which was the same regardless of information condition). There was no financial incentive to understand the experimental condition beyond understanding the auction game. Let us assume that participants place a value on their time and associate a cost with expending energy reading. By this logic, subjects would skim over or actively choose to ignore the information condition.

Let us further assume that subjects dislike (or place a cost) on doing calculations like subtraction. This might induce some bids that involve round numbers or easy bids to subtract from the resale value. The results indicate this trend – 15 of the 48 bids would give the participant (if he or she were the winner) a round dollar figure and 21 of the 48

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[7] See Extensions section for comments on this explanation.
[8] The experimental procedure described above was completed using 48 Duke University undergraduate students as subjects.
[9] T-stat for the difference in means was .49, which corresponds to a 32 percent chance of results with an equal or greater (than the observed) difference in means given that the true difference is zero.

bids were a round dollar figure. One would only expect 1 in 100 bids and potential winnings to have this characteristic.[10]

The model for the utility of winnings is likely inappropriate for the bidders. The model ignores that participants likely place a cost on the collection of winnings; this would bias their bids lower (winnings higher). Having already received a small payoff might also make the bidders more risk seeking. In addition, the participants might not have trusted the experimenter to make a future payment. This belief would severely diminish regret from losing; it would also induce a risk seeking attitude.[11] Furthermore, some bids indicated a total misunderstanding of the auction or a bid that could not reasonably be maximizing for utility based primarily on expected profit.[12]

The round number bias, the costs associated with collection, the risk attitudes, and the understanding of the auction should balance out and not affect the difference of the bids under the two conditions. These influences on bidders' utilities, nonetheless, likely dominated the utility maximization, making the information condition insignificant.

Extensions

1) *Potential uses of regret in auctions*

If regret dependent on information feedback plays a role in utilities of bidders, then not all auctions return the same price to the bid taker. A first-price sealed bid auction where the bid taker informs all bidders after the auction about the winning price should yield the highest price for the bid taker.

A second price or English auction would not create bidder regret after the auction (for a good with a known private value). In first-price auctions, inducing regret for losing and thus biasing the bids toward winning (higher bidding) would be more profitable,

[10] A skilled bidder, if they were considering bidding a round number, would realize that there is a very high return on increasing the bid by 1 penny. As an example, one auction resulted in a tie with winning bids of $150.00 (half payment to each winner). Here a 1 penny increase would nearly double the bidder's return.
[11] The described attitude of risk seeking might be more accurately named "loss-loving" or "winning-averse." See Extensions on risk aversion.
[12] I define these bids as any bid less than half of the assigned resale value. 10 of the 24 pairs had at least one bid that would fall into this category.

according to the equilibrium predictions of this paper's model, for the bid taker. Most online auctions, however, are English style auctions.[13]

2) *Risk Aversion versus Loss Aversion (Rabin, Thaler)*

Though a risk-averse model of preferences would fit most experimental auction data, it is not necessarily an appropriate explanation. For most experimental stakes, if bidders' deviations from risk neutral predictions were explained by risk aversion, it would have absurd implications about their behavior in higher stakes auctions.

A more appropriate explanation for sacrificing expected profit in order to increase the probability of winning would be loss aversion. Loss aversion might be demonstrated in a Dutch auction.[14] Consider a bidder who is making a decision when the current price is between his or her valuation and the RNNE level. The bidder might think that he or she owns the current surplus (in that he or she can bid and take the surplus). Alternatively, the bidder can "gamble" the current surplus and bid at a later point (risking losing the surplus by another participant bidding). This gamble has a positive expectation up until the RNNE bid. Instead, bidders in experimental Dutch auctions tend to bid above RNNE. I believe that this preference for winning (over expected profit) is more accurately defined as loss aversion rather than as risk aversion.

In Cox, Roberson, and Smith (1982), the authors reject a risk neutral hypothesis in favor of a risk averse model for experimental results; in these results, subjects' bids in experimental auctions tended to exceed the risk neutral Nash equilibrium bid. As example, the paper presents results of subjects in a first price auction with 6 people and a \$14.50 resale value; the theoretical RNNE result is for these players to bid \$12.10.[15] Instead, the experimenters observed an average bid of \$13.22. For a risk averse function of $u(X) = X^a$, the Cox, Roberson, and Smith data has $a = 0.14$. For the same risk preferences, these bidders would prefer \$100 to a 50% chance at \$14,000.

[13] A typical explanation for the prevalence of this type of auction would be familiarity with the rules and procedures.
[14] In this type of auction, there is an arbitrarily high starting price that declines until a bid stops the auction. This bid is the winning bid and the execution price. The optimal bid in this type of auction should be identical to a first-price sealed bid auction.
[15] This result can be calculated using equation 1 given $V_{\min} = \$0.10$ for the uniform distribution of valuations.

3) *Future Experiment Changes*

In gathering experimental data, there are tradeoffs between ease of gathering information and quality of information gathered. In this case, the strictest experimental conditions were sacrificed in order to gather responses in a quick and efficient manner, while providing the least inconvenience to the experimental subjects.

If I were to run the experiment again, I would have a less casual setting for gathering the data. I would like to assemble 6 people at one time in an isolated room and read aloud the directions (and answer questions) to make sure that everyone understands the auction. This setting would give the participants greater assurances about the future payoff; I would also like to pay the winner in cash on the spot. By making these adjustments, I think that the subjects' bids would be more in line with their preferences for probability and return than the experimental results presented in this paper.

The most important adjustment would be to make sure that all of the participants hear and understand the information condition. This might involve highlighting the condition on the instructions page, or it could go as far as a comprehension question.[16] The next improvement that I would like to make to improve the effect of the information condition would be to extend the time of the auction. If participants were forced to spend two minutes thinking about their bid (after reading the directions), more thought would likely be given to the information condition. These changes would be important to improving the quality of the data and to observing an effect of the information condition (in the correct direction).

[16] e.g. What information will you learn after the auction?

Results

Resale Values	RNNE Prediction	a_1 Condition	a_2 Condition	a_2 - a_1
15.28	12.73	5.28	13.28	8.00
22.06	18.38	17.06	18.06	1.00
25.61	21.34	20.00	23.00	3.00
39.48	32.90	30.32	9.99	-20.33
45.89	38.24	40.89	30.00	-10.89
47.44	39.53	43.00	31.27	-11.73
72.35	60.29	20.00	50.00	30.00
73.14	60.95	70.15	23.14	-47.01
74.24	61.86	54.24	40.00	-14.24
92.13	76.77	36.13	72.13	36.00
98.83	82.35	5.00	90.02	85.02
100.97	84.14	95.00	0.97	-94.03
122.56	102.13	102.56	71.00	-31.56
133.15	110.95	123.15	73.15	-50.00
145.1	120.91	123.43	140.10	16.67
145.23	121.02	134.60	45.00	-89.6
147.68	123.06	100.00	137.70	37.70
149.37	124.47	141.00	101.37	-39.63
165.08	137.56	155.00	130.00	-25.00
166.87	139.05	97.50	60.00	-37.50
169.59	141.32	150.00	100.00	-50.00
180.75	150.62	120.55	100.43	-20.12
182.35	151.95	150.00	150.00	0.00
190.36	158.63	1.00	187.98	186.98

Appendix Calculations

1) $q(B_i)$

$$q(Bi) = \Pr[\, l \cdot Vi < \max(d \cdot Vj) < Vi]$$

where: $Bi = l \cdot Vi$

$Bj = d \cdot Vj$

$$q(Bi) = \int_0^1 \Pr[\max(d \cdot Vj) \in (\tfrac{l \cdot Vi}{d}, \tfrac{Vi}{d})] \cdot dVi = \int_0^1 [(\tfrac{Vi}{d})^{n-1} - (\tfrac{l \cdot Vi}{d})^{n-1}] \cdot dVi$$

$$q(Bi) = \frac{(1-l)^{n-1}}{d^{n-1}} \cdot \int_0^1 (Vi)^{n-1} \cdot dVi$$

$$q(Bi) = \frac{(1-l)^{n-1}}{n \cdot d^{n-1}}$$

2) $E[B_i - B_{i-1}]$

Solve for: $E[V_i - V_{i-1}]$

with: $Bi = l \cdot Vi$

$$E[V_{i-1}] = \int_0^{Vi} V_{i-1} \cdot (\tfrac{n-1}{Vi}) \cdot (\tfrac{Vi-1}{Vi})^{n-2} \cdot dV_{i-1}$$

$$E[V_{i-1}] = (\tfrac{n-1}{n}) \cdot V_i^{1-n} \cdot V_{i-1}^{n} \Big|_0^{Vi}$$

$$E[V_{i-1}] = (\tfrac{n-1}{n}) \cdot V_i$$

$$E[V_i - V_{i-1}] = \tfrac{Vi}{n}$$

$$E[B_{i-1}] = l \cdot (\tfrac{n-1}{n}) \cdot V_i$$

for $l = (\tfrac{n-1}{n})$, $E[B_{i-1}] = (\tfrac{n-1}{n})^2 \cdot V_i$

$$E[B_i - B_{i-1}] = (\tfrac{n-1}{n^2}) \cdot V_i$$

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