

Real Estate Investment Trusts and Seasonal Volatility:

A Periodic GARCH Model

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Abstract

Real Estate Investment Trusts (REITs) are correlated with the stock and real estate markets; both of which exhibit seasonal fluctuations. In this study, key factors that influence the seasonal volatility of Equity REIT (EREIT) returns are analyzed, and EREIT seasonal volatility is modeled using a periodic generalized autoregressive conditional heteroskedasticity (P-GARCH) model. EREIT returns are found to exhibit more pronounced seasonal volatility patterns than the general stock market. EREITs show increased overall volatility in April, June, September, October and December along with a greater sensitivity to news arrival in the summer months. The results provide a more complete description of EREIT volatility patterns that can be used as a framework for those involved in trading EREITs or EREIT options. The study also establishes the P-GARCH model as a useful tool for modeling EREIT volatility.

1. Introduction

Real Estate Investment Trusts (REITs) are publicly traded investment companies that invest in real estate properties and mortgages. REITs are generally classified into two broad categories; Equity REITS (EREITs) invest at least 75% of their total assets in income producing real estate properties, and mortgage REITs (MREITs) invest at least 75% of their total assets in residential or commercial mortgages. REITs are a unique asset because of their dependence on the stock, bond and real estate markets. Most previous research has focused on EREITs because they make up 85% of all publicly traded REITs. From 1992 to 2003 EREITs have grown in market capitalization from just under \$11 billion to over \$175 billion, as a result they are becoming a more common part of an investors portfolio and are more frequently traded. It is therefore important that their price movements be understood and that a comprehensive picture of the return and volatility structure of EREITs be available.

In this study, key factors that influence the seasonal volatility of EREIT returns are analyzed, and EREIT seasonality is modeled using a periodic generalized autoregressive conditional heteroskedasticity (P-GARCH) model. The P-GARCH model accommodates a seasonal structure in the volatility of EREITs that brings the residual series closer to normality. The results of this study provide a more complete description of EREIT volatility and interactions of EREITs with other markets; this could be useful in options trading and for those investing in EREITs or adding them to a portfolio. The study also establishes the P-GARCH model as a useful tool for modeling EREIT volatility processes.

2. Past Research

Past research on EREITs provides insight into the complex interactions between EREITs and the stock, real estate and interest rate markets, and the possible sources of EREIT

seasonality. Most studies have focused on the influence of stock market and bond market factors on EREIT returns. Peterson and Hsieh (1997) examine EREIT returns using the five-factor model of Fama and French (1993). Fama and French found that the average returns on U.S. common stock can be explained by size (price x shares outstanding), book-to-market ratio, the term premium of interest rates, the default risk premium of interest rates, and a market factor. Peterson and Hsieh argued that because EREIT shares trade on the major stock exchanges just like common stocks, this model could also be applied to EREITs. The two interest rate factors in the model are used to capture the portion of EREIT cash flows that stem from bond-type products such as the long-term fixed leases often held by EREITs or the mortgages held by the underlying real estate properties. They found that this five-factor model predicts EREIT returns in a similar manner to common stock returns, and the three stock market factors in the model were particularly significant for EREITs.

Chan, Herdershott, and Sanders (1990) used a multi-factor CAPM to model EREIT returns. They used five macroeconomic factors that were pre-specified by Chen, Roll, and Ross (1986), including (a) industrial production, (b) expected inflation, (c) unexpected inflation, (d) risk structure of interest rates and (e) term structure of interest rates. The risk and term structure of interest rates and unexpected inflation were found to be significant in explaining EREIT returns. However, EREIT returns were approximately 60% less sensitive to these factors than stock returns. This implies that EREIT returns are explained by slightly different factors than stock returns.

These studies provide evidence that while the performance of EREITs can be explained by general factors that influence the performance of stocks and bonds, EREIT returns may be less sensitive to these factors. In addition, other factors, such as activity in the real estate market,

likely play a role in EREIT return series. For example, Young and Geltner et al. (1996) examined the relationship between EREIT returns and the National Council of Real Estate Investment Fiduciaries (NCREIF) property index, a commonly used indicator of the quarterly performance of commercial property in the United States. They found that lagged values of the NCREIF index were significant in explaining EREIT returns. Gyourko and Keim (1992) found that EREITs were correlated with small-cap stocks returns¹, with a correlation coefficient of $\rho=0.82$, and EREITs were also positively correlated with the National Association of Realtors' (NAR) existing home price appreciation rate ($\rho=0.41$).

Previous research on seasonality in REIT returns is limited, with no studies on the seasonal volatility structure of EREITs. Friday and Peterson (1997) examined return seasonality in EREITs. They concluded that EREIT returns exhibit the “January effect” that is common in stock returns, in which the January returns are significantly higher than other months on average. They postulated that the January effect in EREITs and most other stocks is caused primarily by pressure to sell toward the end of the year by investors who have seen losses throughout the year. This increased selling artificially deflates end of the year prices, only for prices to rebound in January.

While research on the seasonality of EREITs is sparse, evidence for seasonality in the stock and real estate markets is well documented. Chinloy (1999) found that the housing real estate market displays pronounced seasonal returns, with returns to housing markets higher in the summer months. Young and Geltner et al. (1996) investigated the NCREIF property index and found slight quarterly seasonality in the returns of commercial real estate. This finding of seasonal returns in the NCREIF property index is also supported by Graff (1998).

¹ Higher correlation of EREITs with small-cap stocks is most likely due to the small to mid size capitalization of most EREITs as stated by Gyourko and Keim (1992).

Returns and volatility of stocks and the stock market have been shown to follow several seasonal patterns. Hamilton and Lin (1996) determined that stock market volatility could be characterized by the stage of the general business cycle, in which higher volatility is more common during economic recessions. The seasonal volatility of stock returns on a monthly basis was analyzed by Beller and Nofsinger (1998). Using a GARCH-M model with seasonal intercepts in the volatility equation, they found that stock volatility is significantly higher in January and October, similar results were also found by Glosten et al. (1993).

Given the seasonal fluctuations found in the returns and/or volatility of stocks and real estate, it is likely that EREITs also experience seasonal volatility, given their unique status as assets connected to both the real estate and stock markets. The next two sections of this paper will describe the GARCH and Periodic GARCH models, followed by a section describing the data, and then the estimation of the models along with an interpretation of results and a concluding section. The models and interpretation will focus on addressing the issue of seasonal volatility in EREITs using a Periodic GARCH model in order to gain a better understanding of the patterns and structure of EREIT volatility and provide a basis for further research.

3. GARCH Model Overview

The Autoregressive Conditional Heteroskedasticity (ARCH) process was first introduced by Engle (1982). This process addresses the issue of heteroskedasticity and volatility clustering frequently found in financial markets by specifying the conditional variance as a function of the past squared errors, allowing volatility to evolve over time.

The ARCH(q) model can be given by

$$\begin{aligned}
R_t &= \delta + \varepsilon_t \\
\varepsilon_t | \Omega_{t-1} &\sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2
\end{aligned} \tag{1}$$

where R_t is an observable stationary discrete time stochastic process, Ω_{t-1} is the information set at time t-1, δ is a constant, ε_t is a random error, and σ_t^2 is the conditional variance of ε_t .

This model was later generalized to the form most commonly used today, the Generalized ARCH, or GARCH model proposed by Bollerslev (1986). The GARCH(p,q) variance specification is given by

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \tag{2}$$

This model allows for the conditional variance to be linearly dependent on the past behavior of the squared residuals and a moving average of the past conditional variances². The lagged squared error terms imply that if past errors have been large in absolute value, they are likely to be large in the present, leading to volatility clustering.

The GARCH model holds several advantages over the ARCH model and many others when fitting financial data. The addition of the lagged conditional variances is important because the β_i coefficient allows for a smooth process, which evolves over a long time period. GARCH also lets volatility depend on lagged conditional variances and squared errors that are farther in the past without the need for a large number of coefficients. By comparison, ARCH models, which include a limited number of lags in the conditional variance, are classified as more short memory models (Elyasiani 1998). Lamoureux and Lastrapes (1990) provide an explanation to the economic theory behind the presence of ARCH effects. They argue that

² For a more detailed analysis of ARCH and GARCH models see Hamilton (1994) pp. 655-677 or Greene (2003) pp. 238-247.

ARCH effects can be thought of as the appearance of clustering in trading volume on the micro level. According to Bollerslev et al. (1992), GARCH effects are due to volatility clustering which results from macro level variables such as dividend yield, margin requirement, money supply, business cycle and information patters. Two plausible explanations for volatility clustering are provided by Engle et al. (1990): the news arrival process and market dynamics in response to the news.

Previous empirical evidence of GARCH effects in the return series of EREITs is provided by Devaney (2001). He employed a four-factor Arbitrage Pricing Theory (APT) model with a GARCH in the mean (GARCH-M) process to model the returns of EREITs as measured by the monthly levels of the NAREIT index. The GARCH-M specification is used to investigate the presence of a risk premium that changes over time in the mean equation. While Devaney did not find evidence for a GARCH-M model in EREITs, he did find GARCH effects at the 1% confidence level. The evidence from this study gives reason to explore more GARCH models and applications to explain EREIT returns.

The GARCH(1,1) model has been shown to sufficiently fit most economic time series data (Bollerslev 1987) and it will be used in this study to provide a rich description of the volatility process. The GARCH model will also lead to a method of detecting and measuring the presence of seasonal volatility, which will be described in the next section. Bollerslev et al. gives the following description of GARCH models:

The GARCH specification does not arise directly out of any economic theory, but as in the traditional autoregressive and moving average time series analogue, it provides a close parsimonious approximation to the form of heteroskedasticity typically encountered with economic time series data. (Bollerslev et al., 1988, p.119)

4. P-GARCH models

The P-GARCH model was first introduced by Bollerslev and Ghysels (1996) as a means of better characterizing periodic or seasonal patterns in financial market volatility. This model is similar to the GARCH model but now includes seasonally varying autoregressive coefficients.

The class of P-GARCH(p,q) processes can be defined as

$$\sigma_t^2 = \omega_{s(t)} + \sum_{i=1}^q \alpha_{is(t)} \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_{js(t)} \sigma_{t-j}^2 \quad (3)$$

where $s(t)$ is the stage of the period cycle at time t . When estimating this model, the conditional variance, σ_t^2 , must be positive in order for a plausible fit to be obtained. According to

Bollerslev and Ghysels, conditions may need to be placed on the $\omega_{s(t)}$, $\alpha_{is(t)}$, and $\beta_{js(t)}$ parameters for a positive variance to result. These conditions can be formulated on a case-by-case basis according to Nelson and Cao (1992) who suggest the condition of restricting the two seasonal coefficients to be non-negative, with the seasonal intercept strictly positive.

The effects of seasonal volatility are generally limited to variation in the intercept parameter, $\omega_{s(t)}$, but the P-GARCH model allows for a more versatile structure in which all the conditional variance parameters can vary with each season or period. The autoregressive coefficient, $\alpha_{is(t)}$, can be interpreted as quantifying the direct impact of news arrival. This is due to the fact that $\alpha_{is(t)}$ is the coefficient on the lagged error term. The error term shows that there is not perfect information available and therefore when news arrives, the model will not be able to perfectly predict this new information. The magnitude of the $\alpha_{is(t)}$ term then illustrates the extent to which the news arrival, or error term, affects the volatility. A larger $\alpha_{is(t)}$ implies a greater sensitivity to news arrival.

The $\beta_{js(t)}$ coefficient describes the smooth long-term volatility process development. Since $\beta_{js(t)}$ measures a long-term effect, it is acceptable to restrict the seasonal variation to the $\omega_{s(t)}$ and $\alpha_{is(t)}$ parameters while keeping $\beta_{js(t)} \equiv \beta_j$ constant when searching for a relatively short term period process such as the monthly seasonal estimations to be made in this study (Bollerslev and Ghysels 1996). The improvement that the P-GARCH and other GARCH family specifications provide for a model, if correctly applied, can be seen by not only looking at the statistical significance of the seasonal coefficients, but also by looking at selection criteria, reduction of serial correlation in the residuals and squared residuals and reduced kurtosis and skewness in the model, a sign that the specification brings the series closer to normality (Glosten et al. 1993, and Bollerslev and Ghysels 1996).

5. Data

The EREIT series analyzed in this paper are two of the most comprehensive and commonly referred to EREIT indices: the Wilshire REIT index and the National Association of Real Estate Investment Trusts (NAREIT) EREIT index. The principle analyses in this paper will use the Wilshire index, the NAREIT index has been included as a means for comparison and to ensure that the patterns found in the Wilshire index can be generalized to other EREIT indices.

The NAREIT EREIT index is a value-weighted index of all publicly traded EREITs, which currently totals 150 assets. Monthly levels of the NAREIT index are available from February 1972 to the end of 2002. However, daily returns of the index were only measured starting in 1999, providing 1043 daily observations. This relatively small time period of daily data for the NAREIT index is not ideal for gathering seasonal data because only a small number of seasons, or months in this case, are observed. However, this series still provides some reliable results that can be compared with the Wilshire Index results. The Wilshire REIT index is a

value-weighted index consisting of a comprehensive cross-section of all publicly traded EREITs that have a book value of at least \$100 million (currently 106 EREITs). Daily data from this index is available starting on February 1, 1996³. This gives a very complete data set of 1804 observations ending December 31, 2002 and covering many months in order to provide a rich analysis of any seasonal patterns that might be present.

Returns, r_t , are calculated as the daily percentage change in price, P_t , defined by:

$r_t = 100 * \log(P_t / P_{t-1})$. The excess returns, R_t , are then calculated as the daily return minus the daily return on the 3-month T-Bill. The data are then fit to their appropriate ARMA model in order to remove serial correlation in the residuals and to allow for more reliable GARCH estimates. Both excess return series are found to have an AR(1) structure, which is common when working with index data. Daily data is used as the primary means of analysis because it enables a more complete description of the volatility patterns that may be occurring in certain months, and daily observations provide a considerably larger data set. Table 1 contains the summary statistics for the daily excess returns.

Table 1
Summary statistics on EREIT excess returns

	Wilshire Index	NAREIT index
Observations	1804	1043
Mean	0.0386	0.0046
Std. Dev.	0.731	0.721
Skewness	-0.086	0.281
Kurtosis	9.786	7.857
Jarque-Bera statistic	3463.92**	1038.85**
ARCH LM test	216.23**	120.97**
ADF unit root test	-37.43**	-27.02**

Significance levels are **=1%, and *=5%

³ Data for the Wilshire Index is available at <http://www.wilshire.com> or through the Center for Research in Security Prices (CRSP), ticker symbol REI.

Both of these series exhibit excess kurtosis and skewness which leads to the rejection of normality with the Jarque-Bera test. The presence of ARCH effects are seen in the residuals by the ARCH LM test, which rejects the null hypothesis of no ARCH effects at the 1% level for both series. The Ljung-Box Q(30) test for serial correlation in the residuals finds no correlation in the first 30 lags at the 1% confidence level. However, the large values of the Q(30) test of the squared residuals suggest that volatility clustering is present in the return series. It is also important to note that the Augmented Dickey Fuller (ADF) test rejects the existence of a unit root in both series, providing evidence that the data series are stationary.

6. Estimation and Empirical Results

All of the models here are estimated using the Log Likelihood Object in EViews v4.1 which uses the Marquardt Algorithm. Bollerslev and Ghysels (1996) provide a derivation of the log likelihood function for the P-GARCH model which they determine to be the sum of the log likelihoods for each of the seasonal cycles. Due to the extreme non-normality of the residuals for both excess return series, the Quasi Maximum Likelihood Estimators are actually estimated. This non-normality will not be a problem because Bollerslev and Ghysels (1996) state that the results of previous studies indicate that for moderately large sample sizes the Quasi Maximum Likelihood Estimators are comparable to the Maximum Likelihood Estimators.

Model 1: GARCH(1,1)

The first model estimated is a GARCH(1,1) model. This specification is given by

$$\begin{aligned}
 R_t &= \delta + \varepsilon_t \\
 \varepsilon_t | \Omega_{t-1} &\sim N(0, \sigma_t^2) \\
 \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2
 \end{aligned} \tag{4}$$

The sum of the coefficients in the conditional variance equation, $(\alpha + \beta)$, must be less than unity for the process to be stationary. This sum also indicates the level of persistence in the volatility

shocks. A sum close to unity is favorable for providing evidence of a persistent volatility process (Bollerslev 1986).

The results of this analysis are reported in table 2. These results show that all coefficients are highly significant and the volatility process is persistent and stationary in both series. The excess kurtosis is also considerably reduced from the original series. The Q(30) test for the squared standardized residuals fails to reject the null hypothesis of no serial correlation of up to 30th order at the 1% level, indicating that the model has adequately fit the dependence structure in the volatility. However, the Jarque-Bera test still rejects normality at the 1% level, implying that there is room for a more dynamic model to improve the specification and normalize the standard error series. When the ARCH LM test is applied to these results, it rejects the null hypothesis of no ARCH effects, indicating that the ARCH structure of both series is represented by the GARCH(1,1) model. These results provide strong evidence of ARCH and GARCH structures in both series and show that the GARCH(1,1) model can reduce the excess kurtosis.

Table 2
Model 1: GARCH(1,1) models

	Wilshire Index	NAREIT Index
δ	0.059***	0.010***
ω	0.028***	0.062***
α	0.213***	0.186***
β	0.745***	0.691***
Log Likelihood	-1688	-1031
Akaike information criteria (AIC)	2.076	2.064
Schwartz information criteria (SIC)	2.041	2.183
Kurtosis	6.527	6.594
Skewness	-0.16	-.058
Jarque-Bera	942.62***	561.98***

***Significance at the 1% level

Model 2: P-GARCH(1,1) with seasonal $\omega_{s(t)}$

The next model estimated is the P-GARCH(1,1) specification with α and β held constant. This model is represented by

$$\begin{aligned} R_t &= \delta + \varepsilon_t \\ \varepsilon_t | \Omega_{t-1} &\sim N(0, \sigma_t^2) \\ \sigma_t^2 &= \omega_{s(t)} + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \quad (5)$$

The seasonal function, $s(t)$, is assigned to the months of the year, with $1 \leq s(t) \leq 12$ where month 1 is January. It was not necessary to apply any restrictions on the parameters for this model, as all of the coefficients are positive. Table 3 shows results from this model, along with estimates from the S&P 500, which will be used to detect sources of EREIT volatility patterns.

Table 3
Model 2: P-GARCH(1,1) with monthly intercepts

	Wilshire Index	NAREIT index	S&P 500
δ	0.053***	0.012	0.056**
ω_1 (Jan)	0.016**	0.076***	0.036**
ω_2 (Feb)	0.027***	0.075***	0.064***
ω_3 (Mar)	0.038***	0.066***	0.026
ω_4 (Apr)	0.054***	0.150***	0.058***
ω_5 (May)	0.017***	0.076***	0.003
ω_6 (June)	0.054***	0.118***	0.024**
ω_7 (July)	0.029***	0.090***	0.060***
ω_8 (Aug)	0.032***	0.074***	0.043*
ω_9 (Sep)	0.063***	0.235***	0.027
ω_{10} (Oct)	0.070***	0.177***	0.062***
ω_{11} (Nov)	0.014*	0.047**	0.015
ω_{12} (Dec)	0.087***	0.181***	0.057***
α	0.186***	0.196***	0.081***
β	0.731***	0.572***	0.895***
Log Likelihood	-1665	-1012	-2829
AIC	1.964	1.991	3.158
SIC	1.973	2.122	3.204
Kurtosis	5.487	4.983	4.602
Skewness	-0.148	0.157	-0.363
Jarque-Bera	471.43***	175.24***	232.31***

Significance levels: ***=1%, **=5%, and *=10%

This specification provides statistically significant estimates at the 1% level for nearly all intercepts of both series. A noteworthy improvement of this model is the considerable decrease in excess kurtosis and the Jarque-Bera test statistic of both index return series. For example, the kurtosis of the Wilshire Index is reduced from 6.527 in the GARCH(1,1) model to 5.487 in this model. Also, serial correlation is not found at up the 30th order in the squared residuals at the 1% confidence level according to the Ljung-Box Q(30) test.

Both series display consistent patterns of markedly increased volatility in terms of the seasonal intercept for the months of April, June, September, October and December. Note that the Wilshire and the NAREIT series have similar volatility patterns in the parameter estimates, but the magnitude of the parameters differ. The larger magnitude in the volatility of the NAREIT index is most likely due to the shorter time period modeled. Several hypotheses can be presented for the increase in volatility for these specific months. The past research on REITs already presented shows their connection with variables from several markets, including the stock, bond and real estate markets. It is possible that the seasonal volatility patterns of EREITs are simply in line with the seasonal patterns of the overall stock market.

To test this hypothesis, the same P-GARCH model is applied to the excess returns of the S&P 500 over the same sample period as the Wilshire Index (2/1/1996 – 12/31/2002). These results can be seen in table 3. The S&P 500 presents similar increases in volatility for the months of April, October and December, while other months such as February and August are also both significant and higher. This suggests that some of the seasonal volatility of EREITs can be attributed to the volatility fluctuations of the general stock market.

The commonly observed increased volatility in April and December is likely the effect of year-end selling for tax purposes and then tax payment time in April. Glosten et al. (1993)

provide several explanations for why market volatility is higher in certain months. They speculate that the fourth quarter, and December in particular, is an important holiday season for consumer sales. Investors then begin to anticipate and predict the key consumer spending figures, and this level of uncertainty about the state of the economy creates higher volatility levels in the stock market. Glosten et al. notes that the higher volatility in October is due to the inclusion of October 1987 in their sample period. However, this is not a valid explanation for higher October volatility in this study because the sample period is from February 1996 forward.

As previous research has found EREITs to be correlated with interest rates, it is appropriate to investigate possible seasonality in short-term interest rates that could affect EREIT volatility patterns. Interest rate seasonality has been shown to be minor, if it exists at all. This is not surprising because the implicit seasonal policy of the Federal Reserve has been to reduce, and possibly eliminate, seasonal fluctuations in interest rates. Lawler (1979) found that the 3-month T-Bill rates tend to be lower in February and then reach their peak from July to September. Nonparametric tests of interest rate seasonality were performed by Sharp (1988), in which he found evidence for seasonal variation in the short T-Bill rate from 1952-1985. Indications that the short interest rate tended to be higher in July through September were also found in this study. However, Barth and Bennett (1975) found no evidence for a systematic month-to-month pattern in interest rates. The presence of seasonal volatility in the short-term nominal interest rate was found by Jaditz (2000). He used monthly intercepts in the variance equation to determine that a small degree of seasonal volatility is present in nominal interest rates, with February being the most volatile month and December being the most stable. These studies provide evidence for possible seasonality in both the mean and variance of short-term interest rates. However, the overall volatility levels of short T-Bills are very small compared to

EREITs and other equities, and the variation from month to month in short T-Bill volatility appears to be small, if it does indeed exist, and most likely is not a significant factor in the seasonal fluctuations of EREIT volatility.

These results suggest that some of the seasonality of EREITs may be a result of the underlying market volatility, but they also provide evidence that EREITs show signs of a unique and more pronounced volatility pattern. The seasonality in EREITs is very well defined in terms of the statistical significance of the $\omega_{s(t)}$ parameters while the S&P 500 displays a less pronounced pattern, as evidenced by the fewer number of significant intercept terms. The more definite pattern visible in EREITs could be due to their correlation with the real estate market. For example, the volatility increase of EREITs in June might be in response to the start of increased activity in real estate as the summer months begin, as shown by Chinloy (1999). Also, the lower levels of volatility seen in the winter months of January, February and March could be the result of the decreased winter activity in real estate construction and home sales. This model is a major improvement over the standard GARCH(1,1) model, but the P-GARCH process is still being partially restricted by imposing the condition of $\alpha_{s(t)} \equiv \alpha$. In the next model, this condition will be eliminated and a more dynamic volatility process will be estimated.

Model 3: P-GARCH(1,1) with seasonal $\omega_{s(t)}$ and $\alpha_{s(t)}$

By allowing the autoregressive parameter, $\alpha_{s(t)}$, to vary, a more flexible and realistic model is formed. This P-GARCH model is given by

$$\begin{aligned}
 R_t &= \delta + \varepsilon_t \\
 \varepsilon_t | \Omega_{t-1} &\sim N(0, \sigma_t^2) \\
 \sigma_t^2 &= \omega_{s(t)} + \alpha_{s(t)} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2
 \end{aligned} \tag{6}$$

When this model is estimated with no restrictions placed on the coefficients, the Wilshire Index model converges and produces positive variance estimates. However, several of the intercept coefficients for the NAREIT index were negative, resulting in the possibility of an unrealistic negative variance. To correct for this problem, the absolute value of the coefficients are estimated, leading to positive values for all significant coefficients (Bester 1999). This procedure is not fully theoretically justified but the results will still be presented here. Monte Carlo tests were run on simulated data to ensure the usefulness of estimating the absolute value of the coefficients. The results from these simulations show that, in general, the models converge and tend to produce reliable estimates for some of the coefficients, but there is a tendency for many of the coefficients of a 12 period model, such as the one used here, to not be statistically different from zero. For example, 15 of the 25 parameter simulation estimates were, on average, not statistically different from 0 at the 5% confidence level. A detailed summary of the results and Monte Carlo procedures from these simulation tests are available in the appendix in tables 5 and 6.

The results from the model 3 P-GARCH(1,1) estimation can be seen below in table 4. Nearly all of the coefficients for the Wilshire Index are statistically significant at the 1% and 5% levels. However the NAREIT Index parameters, specifically the $\alpha_{s(t)}$ parameters, are not as consistent in their significance. This could be due to the different estimation technique used, or the relatively smaller sample size of the NAREIT index which prevents this dynamic P-GARCH process from being evident. There is no evidence of serial correlation in the first 30 lags of the squared residuals at the 1% level by the Q(30) test. This model is able to further reduce the excess kurtosis in both models, even though the null hypothesis of normality is still rejected. Glosten et al. (1993) note that properly specified GARCH family models should be able to

reduce the excess skewness as well as kurtosis. The skewness of the Wilshire residual series for this model show a decrease from both the original GARCH(1,1) model and the P-GARCH(1,1) of model 2, signifying that this dynamic P-GARCH model is a more appropriate specification.

Table 4
Model 3: P-GARCH(1,1) with seasonal intercepts and autoregressive coefficients

	Wilshire Index	NAREIT Index	S&P 500
δ	0.059***	0.008	0.053**
ω_1 (Jan)	0.022***	0.047***	0.073**
ω_2 (Feb)	0.024***	0.067***	0.085**
ω_3 (Mar)	0.032***	0.032	7.94E-06
ω_4 (Apr)	0.030***	0.087*	0.069**
ω_5 (May)	0.013***	0.068***	0.005
ω_6 (June)	0.037**	0.109***	0.040*
ω_7 (July)	0.005	0.029	0.048
ω_8 (Aug)	0.003	0.065***	9.95E-06
ω_9 (Sep)	0.009	0.184***	0.019
ω_{10} (Oct)	0.036***	0.098**	0.076***
ω_{11} (Nov)	0.017*	0.025	0.007
ω_{12} (Dec)	0.060***	0.177***	0.079***
α_1 (Jan)	0.005	0.073**	0.032
α_2 (Feb)	0.088*	0.039	0.047
α_3 (Mar)	0.066**	0.180**	0.108***
α_4 (Apr)	0.145***	0.188***	0.056*
α_5 (May)	0.042**	0.025	0.071***
α_6 (June)	0.161**	-3.39E-05	0.064**
α_7 (July)	0.178***	0.292***	0.044*
α_8 (Aug)	0.223***	-0.026	0.131***
α_9 (Sep)	0.215***	-0.064	0.084*
α_{10} (Oct)	0.115***	0.158**	0.062
α_{11} (Nov)	0.039*	0.136**	0.084***
α_{12} (Dec)	0.133***	1.47E-05	0.040*
β	0.813***	0.711***	0.902***
Log Likelihood	-1647	-1004	-2821
AIC	1.857	1.977	3.162
SIC	1.936	2.101	3.242
Kurtosis	5.04	4.873	4.443
Skewness	-0.047	0.224	-0.316
Jarque-Bera	313.51***	161.29***	186.33***

Significance levels: ***=1%, **=5%, and *=10%

The estimates of β for both series are different than in the simple P-GARCH model, indicating that allowing for the seasonal impact of news arrival changes the smooth evolution of the volatility, in an ideal model β would remain relatively stable. For two months of the Wilshire Index estimates, the sum of $\alpha_{s(t)} + \beta$ exceeds unity, however this does not mean that the process is not covariance stationary. For a P-GARCH(1,1) model the condition that $(\alpha_{s(t)} + \beta) < 1$ for all $\alpha_{s(t)}$ is a sufficient but not necessary condition for the process to be stationary. Bollerslev and Ghysels (1996) show that a P-GARCH(1,1) model is stationary if the geometric mean of all the $\alpha_{s(t)} + \beta$ terms is less than unity⁴. The geometric mean of these terms in the Wilshire estimates is 0.928 so the process is indeed covariance stationary. It should be noted that both the Akaike Information Criteria (AIC) and the Schwartz Information Criteria (SIC) of the Wilshire Index appear to slightly favor this new model over both models 1 and 2, suggesting the importance for a less restrictive model of the periodic structure of EREIT volatility. The AIC and SIC are model selection criteria similar to an adjusted R^2 , but they both place a larger penalty on the loss of degrees of freedom that occurs when a model is expanded. For the AIC and SIC, a smaller number is associated with a better model. The SIC shows a smaller improvement than the AIC; this could be a result of expanding the model because the definition of the SIC favors simpler models⁵.

A closer look at the results of this model shows the same seasonal patterns in the $\omega_{s(t)}$ parameter as model 2 and also reveals a new pattern for the autoregressive parameter, $\alpha_{s(t)}$. A majority of the following analysis will be limited to the Wilshire Index results, as they are more

⁴ The geometric mean of the sequence $\{\alpha_{s(t)} + \beta\}_{s(t)=1}^{12}$ is defined by $\left(\prod_{s(t)=1}^{12} (\alpha_{s(t)} + \beta) \right)^{\frac{1}{12}}$

⁵ For a more detailed description of AIC and SIC see Greene (2003) pp. 159-160.

reliable and no restrictions are needed for the process to be well defined, and the NAREIT index was originally included only as means for comparison. The autoregressive coefficients are consistently larger for the summer months of June, July, August and into September. This indicates that EREITs have a greater sensitivity to news arrival during the summer.

Once again, a similar model was estimated for the S&P 500 in order to investigate the source of the seasonal patterns of EREITs. The findings of the S&P 500 model are similar to those from model 2 and the results can be seen in table 4. The S&P 500 has slightly larger autoregressive coefficients in the summer months, specifically August and September. However, the seasonality in the $\alpha_{s(t)}$ parameter is not as pronounced and consistently different from zero as in the Wilshire Index, indicating other seasonal factors at work in EREIT returns. Aside from the effects of the general stock market, the values of $\alpha_{s(t)}$ may be higher in the Wilshire Index as a result of EREITs interaction with the real estate market. The increased activity of the real estate market in the summer months would make it more sensitive to any news arrival that could potentially help or hurt this crucial real estate season. This increased news sensitivity in the real estate market could cause the direct impact of news arrival relating to real estate to have a larger impact on EREIT volatility, thereby increasing $\alpha_{s(t)}$ for these months.

To ensure the reliability of using Log Likelihood estimation in Eviews as the estimation technique for P-GARCH(1,1) models, Monte Carlo simulations were run on simulated data using the same P-GARCH models and procedures as used in models 2 and 3. All of the Monte Carlo simulations were performed using a sample size of 1800, which is comparable to the sample size of the Wilshire Index. However, 3600 observations were actually simulated and the first 1800 observations discarded to avoid start-up problems (Bollerslev and Ghysels 1996). The appropriate P-GARCH(1,1) models with periodic cycles of lengths 6 and 12 were simulated and

then tested using the Log Likelihood estimation procedure in EViews. The simulation results demonstrate that this method produces reliable coefficient estimates for model 2. The procedure for the non-restricted model 3 was reliable for a cycle of length 6 but lost some of its accuracy for 12 phases. However, 17 of the 25 parameters were still estimated to be, on average, statistically different from 0 at the 5% confidence level, and these coefficients were also relatively close to their true value. These results provide evidence for the validity of the Log Likelihood estimation procedure for these P-GARCH(1,1) models. The exact simulation results have been included in the Appendix in tables 7-9.

7. Conclusion

The estimation of these three models illustrates the presence of a P-GARCH structure which allows for greater flexibility in modeling the seasonal volatility patterns of EREITs. Model 1 shows that a GARCH(1,1) specification is an appropriate model to explain the volatility structure, but does not address the issue of seasonality. Using model 1 as a benchmark, the two P-GARCH models provide a richer dynamic description of the periodic volatility in EREIT returns. Importantly, not only are most of the parameters for all the models significant and positive, but also each model reduces the amount of excess kurtosis, thereby bringing the residual series closer to normality. It should also be noted that all models estimated, except the NAREIT index in model 3, are found to fit the P-GARCH specification and have positive variance without the need for parameter restrictions.

In summary, the study finds that EREITs display seasonal volatility patterns in which their overall return volatility is higher during the months of April, June, September, October, and December. The hypotheses investigated here show that some of the EREIT seasonality may be attributed to the volatility patterns of the primary stock markets that EREIT shares trade in, such

as the S&P 500. However, the seasonal volatility patterns of EREIT returns are more pronounced and persistent than the seasonal patterns of the major stock markets, perhaps a result of EREITs connection with the highly seasonal real estate market. The sensitivity of EREIT returns to news arrival is found to be higher during the summer months, possibly as a result of the importance of these months in the real estate market.

The problems encountered in estimating these models are not major but could be resolved or improved in future research. The restrictions applied when estimating the NAREIT index estimates in model 3 are not fully theoretically justified and could be eliminated with a more advanced estimation technique, or possibly by analyzing a longer sample range. Also, the inconsistency of the parameter estimates for the NAREIT Index in model 3 could potentially be alleviated with a larger and higher quality data set.

Overall, the findings are promising for the application of P-GARCH models in identifying seasonal volatility patterns of EREITs. The results also provide a more complete description of EREIT volatility patterns that can be used as a framework for those involved in trading EREITs or EREIT options. Potential directions for future research include the development of more sophisticated P-GARCH models and a more in-depth interpretation and explanation of the seasonal volatility patterns of EREITs.

Appendix

Results from Monte Carlo simulations testing the effectiveness of the method of estimating the absolute value of the coefficients when using the Log Likelihood procedure, as used for the NAREIT index in model 3. A total of 3600 observations were simulated and then the last 1800 used to obtain estimates for the coefficients in Eviews. 25 simulations were performed for each variation of the models. The numbers presented in the tables below represent the actual parameter value used in the simulation, the averaged estimated value of this parameter along with the average statistical significance of each parameter, and then the average standard error for each parameter.

Table 5
P-GARCH(1,1) Simulated Data and Estimates
Performed by estimating the absolute value of each parameter

	True Value	Simulation Estimates (Average)	Standard Error (Average)
ω_1	1.7	2.197***	0.815
ω_2	1	1.250**	0.528
ω_3	2.1	2.565**	0.949
ω_4	1.5	1.828**	0.690
ω_5	0.9	1.012***	0.336
ω_6	2.3	2.821***	0.861
α_1	0.09	0.068	0.041
α_2	0.13	0.097**	0.040
α_3	0.15	0.141***	0.043
α_4	0.1	0.085*	0.036
α_5	0.06	0.056	0.036
α_6	0.11	0.065	0.043
β	0.8	0.796***	0.040
log likelihood		-5025	
AIC		5.589	
SIC		5.629	
MSE		8.824	

Average Significance Levels: *=10%, **=5%, ***=1%

Table 6

P-GARCH(1,1) Simulated Data and Estimates

Performed by estimating the absolute value of each parameter

	True Value	Simulation Estimates (Average)	Standard Error (Average)
ω_1	1.7	2.530*	1.346
ω_2	1	1.372	0.890
ω_3	2.1	2.244*	1.175
ω_4	1.5	1.93**	1.040
ω_5	0.9	1.074	0.496
ω_6	2.3	2.821**	1.080
ω_7	2	2.312*	1.224
ω_8	1.8	2.355**	1.193
ω_9	1	1.467	0.907
ω_{10}	1.3	1.703**	0.715
ω_{11}	2.2	2.991*	1.231
ω_{12}	2.5	3.485*	1.787
α_1	0.09	0.062	0.052
α_2	0.13	0.11*	0.062
α_3	0.15	0.144**	0.063
α_4	0.1	0.089**	0.055
α_5	0.06	0.039	0.050
α_6	0.11	0.089**	0.068
α_7	0.09	0.076	0.060
α_8	0.14	0.108	0.057
α_9	0.12	0.105**	0.057
α_{10}	0.09	0.064	0.053
α_{11}	0.13	0.088	0.059
α_{12}	0.16	0.134**	0.055
β	0.8	0.794***	0.040
log likelihood		-5166	
AIC		5.763	
SIC		5.839	
MSE		21.940	

Average Significance Levels: *=10%, **=5%, ***=1%

The remaining tables present the results from testing the effectiveness of the Log Likelihood estimation with no parameter restrictions. This method was used for all the models except the NAREIT index in model 3. The same procedure as describe above was used. Overall, the procedure provides a very reliable and accurate estimation technique.

Table 7
P-GARCH(1,1) Simulated Data and Estimates
No parameter restrictions

	True Value	Simulation Estimates (Average)	Standard Error (Average)
ω_1	1.7	1.777*	0.904
ω_2	1	1.081*	0.511
ω_3	2.1	2.216**	0.700
ω_4	1.5	1.710**	0.745
ω_5	0.9	1.115**	0.499
ω_6	2.3	2.227***	0.714
ω_7	2	2.373**	0.880
ω_8	1.8	2.030**	0.818
ω_9	1	1.108*	0.561
ω_{10}	1.3	1.478***	0.532
ω_{11}	2.2	2.121***	0.731
ω_{12}	2.5	3.092***	1.072
α	0.13	0.121***	0.022
β	0.8	0.802***	0.034
log likelihood		-5324	
AIC		5.921	
SIC		5.964	
MSE		8.682	

Average Significance Levels: *=10%, **=5%, ***=1%

Table 8
P-GARCH(1,1) Simulated Data and Estimates
No parameter restrictions

	True Value	Simulation Estimates (Average)	Standard Error (Average)
ω_1	1.7	2.005**	0.733
ω_2	1	1.162**	0.508
ω_3	2.1	2.579**	0.942
ω_4	1.5	1.661**	0.644
ω_5	0.9	1.008***	0.337
ω_6	2.3	2.422***	0.761
α_1	0.09	0.075	0.040
α_2	0.13	0.121***	0.039
α_3	0.15	0.139**	0.042
α_4	0.1	0.094*	0.037
α_5	0.06	0.047	0.037
α_6	0.11	0.109**	0.041
β	0.8	0.798***	0.038
log likelihood		-5039	
AIC		5.604	
SIC		5.644	
MSE		8.86	

Average Significance Levels: *=10%, **=5%, ***=1%

Table 9
P-GARCH(1,1) Simulated Data and Estimates
No parameter restrictions

	True Value	Simulation Estimates (Average)	Standard Error (Average)
ω_1	1.7	2.176**	1.251
ω_2	1	1.336**	0.791
ω_3	2.1	2.305**	1.113
ω_4	1.5	1.967**	1.023
ω_5	0.9	1.085**	0.531
ω_6	2.3	2.872***	1.047
ω_7	2	2.131**	1.021
ω_8	1.8	1.926**	1.071
ω_9	1	1.132**	0.770
ω_{10}	1.3	1.503**	0.705
ω_{11}	2.2	2.501**	1.118
ω_{12}	2.5	3.515**	1.740
α_1	0.09	0.079*	0.052
α_2	0.13	0.106*	5.145
α_3	0.15	0.136**	0.063
α_4	0.1	0.076*	0.051
α_5	0.06	0.041	0.061
α_6	0.11	0.06	0.067
α_7	0.09	0.074*	0.065
α_8	0.14	0.138**	0.058
α_9	0.12	0.118**	0.055
α_{10}	0.09	0.075*	0.053
α_{11}	0.13	0.111*	0.064
α_{12}	0.16	0.137**	0.059
β	0.8	0.797***	0.037
Log likelihood		-5167	
AIC		5.759	
SIC		5.836	
MSE		19.928	

Average Significance Levels: *=10%, **=5%, ***=1%

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