Supplemental Appendices for Identifiability, State Repression, and the Onset of Ethnic Conflict

Christine S. Mele* David A. Siegel†

Equilibrium

In the text, we signed the derivatives for p and m for cases A and B directly in terms of the identifiability σ. Since our analysis below relies on these derivatives, we are able to elide further discussion of internal and external identifiability here. In addition to simplifying things, this also allows us to mirror the analysis of M&S with respect to the last three stages of our game, even as we consider a different region of the parameter space.

We start with the last decision of the game, which is customary. We assume that indifference consistently implies a lack of action to avoid knife-edged cases. R chooses A if and only if R is strictly better off by doing so. If R chooses A, the probability that this results in a reduction in repression is $p(m(\phi_i, \sigma), \sigma)$. It receives $(1 - \phi_i + e)$ if it is successful and $(1 - \phi_i)$ if not. R's expected utility for each choice (i.e., at the fourth stage), with c the realized value of the cost, is

$$U_4 = \begin{cases} p(m(\phi_i, \sigma), \sigma)(1 - \phi_i + e) + (1 - p(m(\phi_i, \sigma), \sigma))(1 - \phi_i) - d(|\sigma - \sigma_0|, \delta) - c & \text{for } A, \\ (1 - \phi_i) - d(|\sigma - \sigma_0|, \delta) & \text{for } \sim A. \end{cases}$$

Thus, R chooses A in equilibrium if and only if $c < ep(m(\phi_i, \sigma), \sigma)$.

Turn now to the third decision. Let F_c be the cdf of c, $p^{A|\epsilon} = Pr(A|P_\epsilon)$ be G's expectation that R will choose A conditional on G's choosing P_ϵ, and $p^{A|0} = Pr(A|P_0)$ be G's expectation that R will choose A conditional on G's choosing P_0. We can use R's equilibrium conflict decision to write[1]

$$\begin{aligned} p^{A|\epsilon} &= F_c[ep(m(\phi_0 + \epsilon, \sigma), \sigma)], \\ p^{A|0} &= F_c[ep(m(\phi_0, \sigma), \sigma)]. \end{aligned}$$

G's expected utility depends on the probabilities that R will attack, $p^{A|\epsilon}$ and $p^{A|0}$, and on the corresponding probabilities of success were it to do so, $p(m(\phi_0 + \epsilon, \sigma), \sigma)$ and

*Technical Delivery Lead and Data Scientist at Waggl, Inc; E-mail: Christine.mele@gmail.com

†Associate Professor, Department of Political Science, Duke University; E-mail: david.siegel@duke.edu.

[1] We have not yet employed the assumption that c is uniformly distributed, and will not need to for comparative statics related to R's conflict decision (Proposition 1 below).

$p(m(\phi_0,\sigma),\sigma)$. We write it as (leaving off the $h(\delta)$ from each to save space)

$$V_3 = \begin{cases} p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)(\phi_0+\epsilon-e)+(1-p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma))(\phi_0+\epsilon)-\kappa & \text{for } P_\epsilon, \\ p^{A|0}p(m(\phi_0,\sigma),\sigma)(\phi_0-e)+(1-p^{A|0}p(m(\phi_0,\sigma),\sigma))(\phi_0) & \text{for } P_0. \end{cases}$$

In equilibrium, G chooses P_ϵ if and only if the first line in V_3 exceeds the second. Thus, G chooses P_ϵ in equilibrium if and only if $\kappa < \epsilon + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}$.

Turn now to the second decision. Let F_κ be the cdf of κ, and $p^\epsilon = Pr(\epsilon)$ be R's expectation that G will choose P_ϵ. Using G's second equilibrium decision produces[2]

$$p^\epsilon = F_\kappa[\epsilon + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}].$$

The probabilities of action p^ϵ, $p^{A|\epsilon}$, and $p^{A|0}$ allow us to expand the ϕ_f in R's utility. All that additionally is needed is the expected cost of engaging in anti-state operations in the fourth stage. This is $E[c]$. Since A is chosen whenever $c < ep(m(\phi_i,\sigma),\sigma)$, we can write this expected cost as $p^\epsilon \int_0^{ep(m(\phi_0+\epsilon,\sigma),\sigma)} cdc + (1-p^\epsilon)\int_0^{ep(m(\phi_0,\sigma),\sigma)} cdc$. Evaluating this yields the expected cost $\frac{e^2}{2}[p^\epsilon(p(m(\phi_0+\epsilon,\sigma)))^2 + (1-p^\epsilon)(p(m(\phi_0,\sigma)))^2]$. With this, we get R's expected utility at the second decision of the game:

$$\begin{aligned} U_2 &= 1-\phi_0-\epsilon p^\epsilon + e[p^{A|\epsilon}p^\epsilon p(m(\phi_0+\epsilon,\sigma),\sigma) + p^{A|0}(1-p^\epsilon)p(m(\phi_0,\sigma),\sigma)] \\ &\quad -d(|\sigma-\sigma_0|,\delta) - \frac{e^2}{2}[p^\epsilon(p(m(\phi_0+\epsilon,\sigma)))^2 + (1-p^\epsilon)(p(m(\phi_0,\sigma)))^2]. \end{aligned}$$

R's equilibrium level of identifiability, $\sigma^*(\delta)$, maximizes U_2 for a given value of δ. Continuity of all functions and compactness of the domain ensure that this maximum exists. We discuss equilibrium identifiability responses of the minority group after first discussing the later two decisions.

Finally, consider the first decision. We can again expand the ϕ_f, and again we need an expected cost, this time the expected cost of increasing repression in the third stage. This is $E[\kappa]$. Since P_ϵ is chosen whenever $\kappa < \epsilon + e(p(m(\phi_0,\sigma),\sigma)p^{A|0} - p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon})$, we can write this expected cost as $\int_0^{\epsilon+e(p(m(\phi_0,\sigma),\sigma)p^{A|0}-p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon})} \kappa d\kappa$. Evaluating this yields the expected cost $\frac{1}{2}[\epsilon + e(p(m(\phi_0,\sigma),\sigma)p^{A|0} - p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon})]^2$. With this, we get G's expected utility at the first decision of the game:

$$\begin{aligned} V_1 &= p^\epsilon[p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)(\phi_0+\epsilon-e) + (1-p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma))(\phi_0+\epsilon)] \\ &\quad +(1-p^\epsilon)[p^{A|0}p(m(\phi_0,\sigma),\sigma)(\phi_0-e) + (1-p^{A|0}p(m(\phi_0,\sigma),\sigma))(\phi_0)] - h(\delta) \\ &\quad -\frac{1}{2}[\epsilon + e(p(m(\phi_0,\sigma),\sigma)p^{A|0} - p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon})]^2. \end{aligned}$$

G's equilibrium level of δ maximizes V_1. Continuity of all functions and compactness of the domain ensure that this maximum exists. We discuss equilibrium choices for the government after first discussing the later three decisions.

[2]We have not yet employed the assumption that κ is uniformly distributed, and will not need to for comparative statics related to G's and R's second decisions (Propositions 1 and 2 below).

The probabilities of action p^{ϵ}, $p^{A|\epsilon}$, and $p^{A|0}$ are not just necessary for the computation of equilibria. The also allow us to clarify our comparative statics. Rather than considering variation in the size of the parameter subspace corresponding to each equilibrium outcome, we instead compute the effect of each parameter on these probabilities of action. This entails no loss of rigor; these probabilities vary identically to the sizes of the relevant subspaces, and our analysis produces identical intuition (and empirical hypotheses).

Proofs for all Propositions

To begin, we note that the only place that σ_0 occurs is in the cost term in R's utility function, and that the assumptions on the cost term directly imply that the equilibrium value of σ is increasing in σ_0. This last result implies that the sign of all comparative statics with respect to σ_0 can be found by differentiating with respect to σ. Note also that in these proofs we keep the distributions of c, F_c, and κ, F_κ, general until such time as we need to assume a uniform distribution. Finally, to keep notation simple, we call the case in which the state must be discriminate in countering operations case A, and the case in which it may be indiscriminate in doing so case B.

Proof of Proposition 1

1.

$$\begin{aligned}\frac{dp^{A|\epsilon}}{de} &= F_c'[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]p(m(\phi_0+\epsilon,\sigma),\sigma)\\ &= f_c[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]p(m(\phi_0+\epsilon,\sigma),\sigma) > 0\end{aligned}$$

$$\begin{aligned}\frac{dp^{A|0}}{de} &= F_c'[p(m(\phi_0,\sigma),\sigma)(e)]p(m(\phi_0,\sigma),\sigma)\\ &= f_c[p(m(\phi_0,\sigma),\sigma)(e)]p(m(\phi_0,\sigma),\sigma) > 0\end{aligned}$$

2.

$$\begin{aligned}\frac{dp^{A|\epsilon}}{d\epsilon} &= eF_c'[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon}\\ &= ef_c[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]\frac{\partial p(m,\sigma)}{\partial m}\frac{dm(\phi_0+\epsilon,\sigma)}{d\epsilon}\end{aligned}$$

Since success is increasing in mobilization in all cases, the first derivative on the RHS is positive. As $\frac{dm(\phi_0+\epsilon)}{d\epsilon} > 0$, $\frac{dp^{A|\epsilon}}{d\epsilon} > 0$.

$$\frac{\partial p^{A|0}}{\partial \epsilon} = 0$$

3.

$$\begin{aligned}\frac{dp^{A|\epsilon}}{d\phi_0} &= eF_c'[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\phi_0}\\ &= ef_c[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]\frac{\partial p(m,\sigma)}{\partial m}\frac{dm(\phi_0+\epsilon,\sigma)}{d\phi_0}\end{aligned}$$

The same logic again implies that $\frac{dp^{A|\epsilon}}{d\phi_0} > 0$.

$$\begin{aligned}\frac{dp^{A|o}}{d\phi_0} &= eF_c'[p(m(\phi_0,\sigma),\sigma)(e)]\frac{dp(m(\phi_0,\sigma),\sigma)}{d\phi_0}\\ &= ef_c[p(m(\phi_0,\sigma),\sigma)(e)]\frac{\partial p(m,\sigma)}{\partial m}\frac{dm(\phi_0,\sigma)}{d\phi_0}\end{aligned}$$

The same logic again implies that $\frac{dp^{A|0}}{d\phi_0} > 0$.

4.

$$\frac{dp^{A|\epsilon}}{d\sigma} = ef_c[p(m(\phi_0+\epsilon,\sigma),\sigma)(e)]\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma}$$

Cases A and B are distinguished by the effect of identifiability, σ, on p. For case A, $\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma} < 0$, which implies that $\frac{dp^{A|\epsilon}}{d\sigma} < 0$. For case B, $\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma} > 0$, which implies that $\frac{dp^{A|\epsilon}}{d\sigma} > 0$. As noted above, this provides comparative statics relative to σ_0 in each case.

$$\frac{dp^{A|0}}{d\sigma} = ef_c[p(m(\phi_0,\sigma),\sigma)(e)]\frac{dp(m(\phi_0,\sigma),\sigma)}{d\sigma}$$

The same logic implies that in case A, $\frac{dp^{A|0}}{d\sigma} < 0$, and in case B, $\frac{dp^{A|0}}{d\sigma} > 0$.

Proof of Proposition 2

1.

$$\begin{aligned}\frac{dp^{\epsilon}}{de} &= F_k'[(\epsilon)+ep(m(\phi_0,\sigma),\sigma)p^{A|0}-ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}]\\ &\quad (p(m(\phi_0,\sigma),\sigma)p^{A|0}+p(m(\phi_0,\sigma),\sigma)e\frac{dp^{A|0}}{de}-p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}\\ &\quad -p(m(\phi_0+\epsilon,\sigma),\sigma)e\frac{dp^{A|\epsilon}}{de})\\ &= f_k[(\epsilon)+ep(m(\phi_0,\sigma),\sigma)p^{A|0}-ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}]\\ &\quad [p(m(\phi_0,\sigma),\sigma)p^{A|0}-p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}\\ &\quad +e(p(m(\phi_0,\sigma),\sigma)\frac{dp^{A|0}}{de}-p(m(\phi_0+\epsilon,\sigma),\sigma)\frac{dp^{A|\epsilon}}{de})]\end{aligned}$$

Since the cost function for c is uniform, we know that $f_c = 1$ and $p^{A|\cdot} = ep(\cdot)$. Therefore we can simplify to:

$$\frac{dp^{\epsilon}}{de} = f_k[(\epsilon) + p(m(\phi_0,\sigma),\sigma)p^{A|0}(e) - p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}(e)] \\ [2e(p^2(m(\phi_0,\sigma),\sigma) - p^2(m(\phi_0+\epsilon,\sigma),\sigma))]$$

We know that $(p^2(m(\phi_0,\sigma),\sigma) - p^2(m(\phi_0+\epsilon,\sigma),\sigma)) < 0$. From this we know that $\frac{dp^{\epsilon}}{de} < 0$.

2.

$$\frac{dp^{\epsilon}}{d\epsilon} = f_k[(\epsilon) + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}] \\ (1 - e(\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon}p^{A|\epsilon} + \frac{dp^{A|\epsilon}}{d\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)))$$

Again using the uniformity of the cost function, we can simplify this to:

$$\frac{dp^{\epsilon}}{d\epsilon} = f_k[(\epsilon) + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}] \\ (1 - 2e^2\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma))$$

Since $\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon} > 0$, whether the derivative is positive or negative depends on the sign of $1 - 2e^2\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)$. This is positive when the marginal direct utility benefit to increasing repression of 1 is greater than the marginal indirect cost of increased repression arising from increased successful attacks by R: $2e^2\frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)$. It is negative when the reverse is true.

3.

$$\frac{dp^{\epsilon}}{d\phi_0} = f_k[(\epsilon) + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}] \\ (\frac{dp(m(\phi_0,\sigma),\sigma)}{d\phi_0}p^{A|0}e + \frac{dp^{A|0}}{d\phi_0}p(m(\phi_0,\sigma),\sigma)e - \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\phi_0}p^{A|\epsilon}e \\ -\frac{dp^{A|\epsilon}}{d\phi_0}p(m(\phi_0+\epsilon,\sigma),\sigma)e)$$

Again using the uniformity of the cost function, we can simplify to:

$$\frac{dp^{\epsilon}}{d\phi_0} = f_k[(\epsilon) + ep(m(\phi_0,\sigma),\sigma)p^{A|0} + ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}] \\ [2e^2\left(\frac{dp(m(\phi_0,\sigma),\sigma)}{d\phi_0}p(m(\phi_0,\sigma),\sigma) - \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\phi_0}p(m(\phi_0+\epsilon,\sigma),\sigma)\right)]$$

We know that $p(m(\phi_0+\epsilon,\sigma),\sigma) > p(m(\phi_0,\sigma),\sigma)$. We also know, since $\frac{\partial^2 p}{\partial\phi_0\partial\epsilon} < 0$, that $\frac{dp(m(\phi_0,\sigma),\sigma)}{d\phi_0} > \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\phi_0}$, with both terms positive. Consequently we cannot sign this comparative static. The likelihood of increasing repression is increasing in ϕ_0 whenever $\frac{dp(m(\phi_0,\sigma),\sigma)}{d\phi_0}p(m(\phi_0,\sigma),\sigma) > \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\phi_0}p(m(\phi_0+\epsilon,\sigma),\sigma)$ and decreasing when the inequality is reversed.

If instead $\frac{\partial^2 p}{\partial\phi_0\partial\epsilon} > 0$, so that R sees increasing returns to mobilization from repression, the sign of the comparative static is always negative, implying that the state is less likely to increase repression the higher is its initial level. This affects no subsequent results.

4. Once again using the uniformity of the cost function, we can simplify this derivative (which is basically the same as the previous one, explaining the skipped step below) to:

$$\begin{aligned}\frac{dp^\epsilon}{d\sigma} &= f_k[(\epsilon) + ep(m(\phi_0,\sigma),\sigma)p^{A|0} - ep(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon}] \\ &\quad [2e^2\left(\frac{dp(m(\phi_0,\sigma),\sigma)}{d\sigma}p(m(\phi_0,\sigma),\sigma) - \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma}p(m(\phi_0+\epsilon,\sigma),\sigma)\right)]\end{aligned}$$

In case B, $\frac{\partial^2 p}{\partial\epsilon\partial\sigma} > 0$, which implies that $\frac{dp(m(\phi_0,\sigma),\sigma)}{d\sigma} < \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma}$, with both terms positive. This implies the second, negative, term in brackets is greater than the first, so that the derivative is negative in case B. In case A, $\frac{\partial^2 p}{\partial\epsilon\partial\sigma} < 0$ and the signs on the derivatives within the large parentheses are negative. This implies that the second, positive, term within these parentheses exceeds the first, negative, one, and so the comparative static is positive for Case A.

If instead $\frac{\partial^2 p}{\partial\sigma\partial\epsilon} < 0$ in case B or $\frac{\partial^2 p}{\partial\sigma\partial\epsilon} > 0$ in case A, so that increasing repression leads to a lessened absolute effect of identifiability on the chance of operational success rather than a strengthened absolute effect, then our results still go through as long as $\frac{dp(m(\phi_0,\sigma),\sigma)}{d\sigma}p(m(\phi_0,\sigma),\sigma) - \frac{dp(m(\phi_0+\epsilon,\sigma),\sigma)}{d\sigma}p(m(\phi_0+\epsilon,\sigma),\sigma)$ is negative, in case B, or positive, in case A. If these weaker conditions fail to hold, then $\frac{dp^\epsilon}{d\sigma} > 0$ in case B, implying that the state is more likely to increase repression the higher is the minority group's initial level of identifiability, and $\frac{dp^\epsilon}{d\sigma} < 0$ in case A, implying that the state is less likely to increase repression the higher is the minority group's initial level of identifiability. When either of these is the case, it makes possible differentiation in case A or assimilation in case B, but only when the additional conditions shown in Proposition 3 hold.

Proof of Proposition 3

Recall that

$$\begin{aligned}U_2 &= 1-\phi_0-\epsilon p^\epsilon + e[p^{A|\epsilon}p^\epsilon p(m(\phi_0+\epsilon,\sigma),\sigma) + p^{A|0}(1-p^\epsilon)p(m(\phi_0,\sigma),\sigma)] \\ &\quad -d(|\sigma-\sigma_0|,\delta) - \frac{e^2}{2}[p^\epsilon p^2(m(\phi_0+\epsilon,\sigma)) + (1-p^\epsilon)p^2(m(\phi_0,\sigma))].\end{aligned}$$

As the cost term, $d(\cdot)$, is arbitrary and increasing in magnitude in both directions away from σ_0, rather than produce an equation that implicitly defines the equilibrium value of σ, we focus on whether R increases or decreases identifiability in equilibrium. To do this we drop the $d(\cdot)$ term in U_2 and compute the derivative of U_2. This indicates whether the minority group's expected utility, absent cost, is decreasing or increasing in σ, and thus whether the minority group will assimilate or differentiate, respectively, in equilibrium.

Before computing the derivative, we can simplify U_2 using previous results. Dropping the cost to alter identifiability, plugging in for $p^{A|\cdot}$ and p^{ϵ} (the latter in reverse), making use of the uniformity assumptions on c and κ, and doing some algebra yields:

$$\begin{aligned} U_2 &= 1-\phi_0+p^{\epsilon}[-\epsilon+\frac{e^2}{2}(p^2(m(\phi_0+\epsilon,\sigma))-p^2(m(\phi_0,\sigma)))]+\frac{e^2}{2}p^2(m(\phi_0,\sigma)) \\ &= 1-\phi_0-\frac{(p^{\epsilon})^2}{2}-\frac{\epsilon p^{\epsilon}}{2}+\frac{e^2p^2(m(\phi_0,\sigma))}{2}. \end{aligned}$$

Its derivative is

$$\frac{dU_2}{d\sigma} = -(p^{\epsilon}+\frac{\epsilon}{2})\frac{dp^{\epsilon}}{d\sigma}+e^2p(m(\phi_0,\sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}.$$

The second term is negative for case A and positive for case B by Proposition 1. The first term is positive for case B and negative from case A by Proposition 2. This implies that $\frac{dU_2}{d\sigma} < 0$ in case A and > 0 in case B always, so that R assimilates in the former and differentiates in the latter in equilibrium.

The sign of the second term, which corresponds to the change in the expected benefit of operational capacity with increasing identifiability, is fixed by Proposition 1. However, the sign of the first term, which corresponds to the state's increasing repression more or less often with increasing identifiability, could switch if our assumption on $\frac{\partial^2 p}{\partial\sigma\partial\epsilon}$ in each case fails to hold. Should this happen, whether or not our results hold depends on the trade-off between the first and second terms. Thus, should $\frac{dp^{\epsilon}}{d\sigma} > 0$ in case B, implying that the state is more likely to increase repression the higher is the minority group's initial level of identifiability, and $\frac{dp^{\epsilon}}{d\sigma} < 0$ in case A, implying that the state is less likely to increase repression the higher is the minority group's initial level of identifiability, we can have assimilation in case B and differentiation in case A if managing the state's choice of repression is the dominant incentive underlying the minority group's action. In other words, assimilation (differentiation) in case B (A) would occur if repression were to increase (decrease) in identifiability and avoiding (seeking out) this increase (decrease) was a greater driver of R's identifiability choice than the decrease in operational capacity this choice would occasion.

Proof of Proposition 4

Recall that

$$\begin{aligned} V_1 &= p^\epsilon[p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma)(\phi_0+\epsilon-e)+(1-p^{A|\epsilon}p(m(\phi_0+\epsilon,\sigma),\sigma))(\phi_0+\epsilon)] \\ &\quad +(1-p^\epsilon)[p^{A|0}p(m(\phi_0,\sigma),\sigma)(\phi_0-e)+(1-p^{A|0}p(m(\phi_0,\sigma),\sigma))(\phi_0)]-h(\delta) \\ &\quad -\frac{1}{2}[\epsilon+e(p(m(\phi_0,\sigma),\sigma)p^{A|0}-p(m(\phi_0+\epsilon,\sigma),\sigma)p^{A|\epsilon})]^2. \end{aligned}$$

In much the same manner as we simplified U_2, we may simplify V_1. This yields

$$V_1 = \phi_0 - e^2(p(m(\phi_0,\sigma^*(\delta))))^2+\frac{1}{2}(p^\epsilon)^2-h(\delta).$$

We note that δ enters into the model in only two ways, via its: (i) associated cost term, $h(\delta)$; and (ii) effect on the equilibrium value of identifiability, $\sigma^*(\delta)$, which itself feeds into the three probabilities of action. With respect to (ii), δ affects $\sigma^*(\delta)$ only via the cost term $d(|\sigma-\sigma_0|,\delta)$, and we have assumed in this regard that the marginal cost of increasing σ is increasing in δ. This implies that the magnitude of the equilibrium change in σ is decreasing in δ. As with $\sigma^*(\delta)$ itself, we choose not to solve for the equilibrium value of δ directly, as it is of limited interest. Rather, we focus on whether the government wants to hinder or facilitate each of assimilation and differentiation by increasing δ past 1. To do this we drop the $h(\cdot)$ term and compute the derivative of V_1 with respect to σ. If this is positive, then government wants to facilitate differentiation and limit assimilation. It will choose an optimal level of $\delta>1$ in this case whenever the minority group chooses to assimilate, and choose $\delta=1$ when the minority group chooses to differentiate. If it is negative, the reverse is true.

Dropping the cost term, the derivative of V_1 with respect to σ is

$$\frac{dV_1}{d\sigma} = -2e^2p(m(\phi_0,\sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}+p^\epsilon\frac{dp^\epsilon}{d\sigma}.$$

The first term is positive for case A and negative for case B by Proposition 1. The second term is positive for case A and negative for case B by Proposition 2. This implies that $\frac{dV_1}{d\sigma}>0$ in case A and <0 in case B always, so that G makes it more difficult for R to assimilate in the former and more difficult for R to differentiate in the latter in equilibrium.[3]

The sign of the first term, which corresponds to the change in the expected cost to the government due to operational capacity arising from increasing identifiability, is fixed by Proposition 1. However, the sign of the second term, which corresponds to the state's increasing repression more or less often with increasing identifiability, could switch if our assumption on $\frac{\partial^2 p}{\partial\sigma\partial\epsilon}$ in each case fails to hold. Should this happen, whether or not our results hold depends on the trade-off between the first and second terms. Thus, should $\frac{dp^\epsilon}{d\sigma}>0$ in

[3] The fact that in both cases the government wants to increase the difficulty of the equilibrium action of the minority group is the reason we can use only one choice variable for G's decision here.

case B, implying that the state is more likely to increase repression the higher is the minority group's initial level of identifiability, and $\frac{dp^\epsilon}{d\sigma} < 0$ in case A, implying that the state is less likely to increase repression the higher is the minority group's initial level of identifiability, we can have the state disadvantaging assimilation in case B and differentiation in case A if managing the state's choice of repression is the dominant incentive underlying the minority group's action. In other words, disadvantaging assimilation (differentiation) in case B (A) would occur if repression were to increase (decrease) in identifiability and avoiding (seeking out) this increase (decrease) was a greater driver of R's identifiability choice than the decrease in operational capacity this choice would occasion.

Note the ranges of parameters that cause an inversion of the results in Propositions 3 and 4 are not identical. In case A, Proposition 3 fails to hold when $-(p^\epsilon + \frac{\epsilon}{2})\frac{dp^\epsilon}{d\sigma} > -e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}$ and Proposition 4 fails to hold when $-p^\epsilon \frac{dp^\epsilon}{d\sigma} > -2e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}$, with all terms positive. The LHS of the first of these inequalities is greater than the LHS of the second, while the RHS of the second is greater than the RHS of the first. Thus, the first will hold more often than the second, suggesting that, in Case A, there may be a range of parameter values in which R will try to differentiate and G will not try to make this more difficult. Similarly, in case B, Proposition 3 fails to hold when $(p^\epsilon + \frac{\epsilon}{2})\frac{dp^\epsilon}{d\sigma} > e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}$ and Proposition 4 fails to hold when $p^\epsilon \frac{dp^\epsilon}{d\sigma} > 2e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma}$, with all terms positive, and the same ordering holds. This suggests that, in Case B, there may be a range of parameter values in which R will try to assimilate and G will not try to make this more difficult.

Proof of Proposition 5

The proof of Proposition 5 begins at this point in the proof of Proposition 4:

$$\frac{dV_1}{d\sigma} = -2e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0, \sigma))}{d\sigma} + p^\epsilon \frac{dp^\epsilon}{d\sigma}.$$

The difference is now the signs of each of the terms. Like in Case A, the first term is positive in the setting of M&S. The second term, by Proposition 2 of M&S, is negative, however, suggesting that the sign of the derivative is indeterminate. Putting this result together with the result in Proposition 3 of M&S, that R may either assimilate or differentiate in equilibrium, depending on the parameters of the model, implies that there are four different possibilities to consider: (i) G hinders R's assimilation attempt, (ii) G facilitates R's assimilation attempt, (iii) G hinders R's differentiation attempt, and (iv) G facilitates R's differentiation attempt.

The proof of Proposition 3 in M&S indicates that assimilation happens when

$$e^2 p(m(\phi_0, \sigma))\frac{dp(m(\phi_0, \sigma))}{d\sigma} < (p^\epsilon + \frac{\epsilon}{2})\frac{dp^\epsilon}{d\sigma},$$

and differentiation when the reverse is true. Consider first the case of assimilation, and so possibility (i) and (ii). G prefers a greater level of identifiability of R, implying that it will attempt to hinder R's attempt at assimilation, when $-2e^2 p(m(\phi_0,\sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma} > -p^\epsilon \frac{dp^\epsilon}{d\sigma}$, and prefers a lower level, implying that it will facilitate R's assimilation attempt, when the reverse is true.

The condition for facilitation of assimilation can be transformed into $e^2 p(m(\phi_0,\sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma} > \frac{1}{2}p^\epsilon \frac{dp^\epsilon}{d\sigma}$. Together with the condition for assimilation, we then have $(p^\epsilon + \frac{\epsilon}{2})\frac{dp^\epsilon}{d\sigma} > \frac{1}{2}p^\epsilon \frac{dp^\epsilon}{d\sigma}$ or $(p^\epsilon + \epsilon)\frac{dp^\epsilon}{d\sigma} > 0$. But this is impossible because the LHS is negative, so G always hinders R's assimilation when R attempts to assimilate.

When differentiation happens instead, hindrance occurs when $e^2 p(m(\phi_0,\sigma))\frac{dp(m(\phi_0,\sigma))}{d\sigma} > \frac{1}{2}p^\epsilon \frac{dp^\epsilon}{d\sigma}$ and facilitation when the reverse is true. Both are consistent with the condition for differentiation.

From M&S, using uniformity of the cost distributions allows us to write the differentiation condition as

$$\frac{dp(m(\phi_0,\sigma),\sigma)/d\sigma}{dp(m(\phi_0+\epsilon,\sigma),\sigma)/d\sigma} Z > \frac{p(m(\phi_0+\epsilon,\sigma)}{p(m(\phi_0,\sigma)},$$

where

$$Z = \frac{3\epsilon + 2e^2(p^2(m(\phi_0,\sigma)) - p^2(m(\phi_0+\epsilon,\sigma))) - 1}{3\epsilon + 2e^2(p^2(m(\phi_0,\sigma)) - p^2(m(\phi_0+\epsilon,\sigma)))}.$$

We can do the same with the condition for hindrance under differentiation to get:

$$\frac{dp(m(\phi_0,\sigma),\sigma)/d\sigma}{dp(m(\phi_0+\epsilon,\sigma),\sigma)/d\sigma} Y > \frac{p(m(\phi_0+\epsilon,\sigma)}{p(m(\phi_0,\sigma)},$$

where

$$Y = \frac{\epsilon + e^2(p^2(m(\phi_0,\sigma)) - p^2(m(\phi_0+\epsilon,\sigma))) - 1}{\epsilon + e^2(p^2(m(\phi_0,\sigma)) - p^2(m(\phi_0+\epsilon,\sigma)))}.$$

These conditions are similar, suggesting that typically when differentiation is attempted, the state will usually attempt to hinder it. However, because Y and Z are generically different and $Y \le Z$ under the conditions of M&S, this is not always true and there can be cases in which the government actually attempts to facilitate R's attempt at differentiation. This occurs when

$$Y < \frac{p(m(\phi_0+\epsilon,\sigma)dp(m(\phi_0+\epsilon,\sigma),\sigma)/d\sigma}{p(m(\phi_0,\sigma)dp(m(\phi_0,\sigma),\sigma)/d\sigma} < Z,$$

which is likely to be true when R is close to indifferent between differentiation and assimilation. At this point, G prefers to help R differentiate near both sides of the indifference condition.