On-Line Appendix

for

“A Strategic Theory of Policy Diffusion Via Intergovernmental Competition”

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Part I

Derivation of Comparative Statics for the Lottery Competition Model and a Continuous Time Model of States' Utilities

We begin this part of the appendix by deriving comparative statics for our strategic theory of policy diffusion specialized to the case of lottery adoptions for a two-state (*r* and *s*) system. In particular, we determine the marginal effect of six parameters on the probability that one of the states, say *s*, adopts a lottery (π_s): *s*'s cost of adoption (c_s), *r*'s cost of adoption (c_r), and four variables reflecting a state's conditional revenue gain from adopting a lottery: $\delta_s(0)$ [*s*'s revenue gain as a monopoly provider], $\delta_s(1)$ [*s*'s revenue gain under competition], $\delta_r(0)$ [*r*'s revenue gain as a monopoly provider], and $\delta_r(1)$ [*r*'s revenue gain under competition]. To derive comparative statics, we begin by separating the problem into two distinct scenarios from the point of view of one of the states, say *s*. The first is that *s*'s neighbor, *r*, has previously adopted; the second is that *r* has not adopted.

Consider the first scenario, in which *r* has adopted a lottery in a previous period. Since we have assumed that *r* cannot "disadopt," the only decision belongs to *s*, and *s*'s choice is fairly simple. Given equation (3), which writes π_s as a function of Δ_s, specifying $\pi_s(1)$ requires only that we derive $\Delta_s(1)$.

When *r* has already adopted, the change in utility for adopting, $\Delta_s(1)$, takes a particularly simple form:

$$\Delta_s(1) = [b_s^A(1) - c_s] - b_s^{\sim A}(1) = b_s^A(1) - b_s^{\sim A}(1) - c_s = \delta_s(1) - c_s. \qquad \text{(A1)}$$

Comparative statics are easy to compute in this case, and yield an unsurprising result. Since $\Delta_s(1) = \delta_s(1) - c_s$, *s*'s utility gain from adopting [$\Delta_s(1)$] is increasing in its revenue gain from adopting [$\delta_s(1)$] and decreasing in its cost of adoption (c_s). It follows from equation (3) that *s*'s probability of adopting is also increasing in $\delta_s(1)$ and decreasing in c_s.

The problem is more complex when *r* has not yet adopted. Now *s* must not only worry about its own gains and losses; it must anticipate *r*'s adoption decision, since the states' decisions are coupled. Formally, we can write

$$\begin{aligned}
\Delta_s(0) &= [\pi_r(0)\, b_s^A(1)] + [(1 - \pi_r(0))\, b_s^A(0)] - c_s - [\pi_r(0)\, b_s^{\sim A}(1)] - [(1 - \pi_r(0))\, b_s^{\sim A}(0)] \\
&= b_s^A(0) - b_s^{\sim A}(0) - c_s + \{\pi_r(0)[b_s^A(1) - b_s^{\sim A}(1) - (b_s^A(0) - b_s^{\sim A}(0))]\} \\
&= \delta_s(0) + \pi_r(0)[\delta_s(1) - \delta_s(0)] - c_s \\
&= [1 - \pi_r(0)]\, \delta_s(0) + \pi_r(0)\delta_s(1) - c_s \qquad \text{(A2)}
\end{aligned}$$

A similar equation may be written for $\Delta_r(0)$, with *r* and *s* subscripts flipped. Due to the dependence of $\pi_s(0)$ on $\pi_r(0)$, and vice versa, when trying to discern how the probability of adoption varies with the revenue parameters (b_i^A and $b_i^{\sim A}$) it is necessary to understand not only the direct effect of parameter variation on the probability of adoption, but also the indirect effect arising from a change in the other state's probability of adoption. This reflects the strategic nature of the interaction.

Simple inspection of equation (A2) indicates that the direct effect of increasing each of $\delta_s(1)$ and $\delta_s(0)$ is to increase $\Delta_s(0)$, which in turn—by equation (3)—increases $\pi_s(0)$, while the direct effect of increasing c_s is to decrease this probability. But this does not take into account changes in the equilibrium value of $\pi_r(0)$. As $\delta_s(1)$ - $\delta_s(0)$ is multiplied by $\pi_r(0)$ in equation (A2), there are four cases to consider: (*i*) $\delta_s(1) - \delta_s(0) > 0$, $\delta_r(1) - \delta_r(0) > 0$; (*ii*) $\delta_s(1) - \delta_s(0) < 0$, $\delta_r(1) - \delta_r(0) < 0$; (*iii*) $\delta_s(1) - \delta_s(0) > 0$, $\delta_r(1) - \delta_r(0) < 0$; and (*iv*) $\delta_s(1) - \delta_s(0) < 0$, $\delta_r(1) - \delta_r(0) > 0$.[1] We analyze each in order.

In case (*i*), an increase in $\pi_s(0)$ increases $\pi_r(0)$, which increases $\pi_s(0)$, and so on. Formally, this means that the game played by the two states has strategic complementarities, which implies that equilibrium values of $\pi_s(0)$ and $\pi_r(0)$ are both increasing in any parameter that increases either one. In fact, increasing each of $\delta_s(1)$, $\delta_s(0)$, $\delta_r(1)$, and $\delta_r(0)$, and decreasing either c_s and c_r increases both $\pi_s(0)$ and $\pi_r(0)$ in equilibrium.[2]

In case (*ii*), an increase in $\pi_s(0)$ decreases $\pi_r(0)$, which increases $\pi_s(0)$, and so on. By the same logic as in the previous case, an increase in $\delta_s(1)$ or $\delta_s(0)$, or a decrease in c_s, increases equilibrium values of $\pi_s(0)$ and decreases equilibrium values of $\pi_r(0)$. Similarly, increasing $\delta_r(1)$ or $\delta_r(0)$, or decreasing c_r, decreases equilibrium values of $\pi_s(0)$ while increasing equilibrium values of $\pi_r(0)$.

This same logic does not hold in the remaining two cases. For example, in case (*iii*), an increase in $\pi_s(0)$ decreases $\pi_r(0)$, which decreases $\pi_s(0)$, which increases $\pi_r(0)$, and so on. To derive comparative statics for these cases, we directly calculate them by differentiating the Logit Equilibrium correspondence for state *s*, making sure to keep track of the implicit dependence of the equilibrium value of $\pi_r(0)$ on $\delta_s(1)$, $\delta_s(0)$ and c_s, and the same for $\pi_s(0)$ on $\delta_r(1)$, $\delta_r(0)$ and c_r. After some algebra, this procedure yields the following set of comparative statics:

$$\frac{\partial \pi_s(0)}{\partial \delta_s(0)} = \frac{\lambda \pi_s(1-\pi_s)(1-\pi_r)}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]},$$

$$\frac{\partial \pi_s(0)}{\partial \delta_s(1)} = \frac{\lambda \pi_s(1-\pi_s)\pi_r}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]},$$

$$\frac{\partial \pi_s(0)}{\partial c_s} = \frac{-\lambda \pi_s(1-\pi_s)}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]},$$

$$\frac{\partial \pi_s(0)}{\partial \delta_r(0)} = \frac{\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)](1-\pi_s)}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]},$$

$$\frac{\partial \pi_s(0)}{\partial \delta_r(1)} = \frac{\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)]\pi_s}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]},$$

$$\frac{\partial \pi_s(0)}{\partial c_r} = \frac{-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)]}{1-\lambda^2 \pi_s(1-\pi_s)\pi_r(1-\pi_r)[\delta_s(1)-\delta_s(0)][\delta_r(1)-\delta_r(0)]}.$$

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[1] We ignore knife-edge scenarios in which $\delta_i(1) - \delta_i(0) = 0$ for state *i*.

[2] This follows from Theorem 4 of Ashworth and Bueno de Mesquita (2006), as for each state, *i*, *i*'s Logit Equilibrium correspondence is increasing in $\delta_i(1)$, $\delta_i(0)$, $-c_i$, and $\pi_i(0)$. (Technically, we know only that the least and the greatest fixed points of the correspondence vary according to the comparative statics detailed here.)

In cases (*iii*) and (*iv*), the shared denominator of the six derivatives is always positive, implying that the sign of the numerator provides the sign of the respective comparative static. We see that in both cases $\pi_s(0)$ is increasing in $\delta_s(1)$ and $\delta_s(0)$, and decreasing in c_s. In case (*iii*), $\pi_s(0)$ is increasing in $\delta_r(1)$ and $\delta_r(0)$, and decreasing in c_r; while in case (*iv*), $\pi_s(0)$ is decreasing in $\delta_r(1)$ and $\delta_r(0)$, and increasing in c_r. This analysis yields the summary results on page 14 of the paper.

To facilitate analysis in the paper we assume adoption decisions by governments that occur in discrete intervals. However, in general, governments' decision-making processes occur in continuous time. In the remainder of this part of the appendix, we briefly specify continuous-time utilities for governments, intended to be a jumping-off point for further work.

In this continuous time model, the utility each government derives from all actions is continuously decreasing according to a factor, ρ_s. Given this notation, we can state that if *s* is rational, it will adopt at time *t* only if adopting makes its discounted expected utility looking forward from time *t*, $U_{s,t}$, greater than its discounted utility from not adopting, where

$$U_{s,t} = \int_t^\infty e^{-\rho_s t} \left(X^*_{s,t}[b^A_{s,t}(X^*_{L_{s,t},t}) - c_{s,t}] + (1 - X^*_{s,t}) b^{\sim A}_{s,t}(X^*_{L_{s,t},t}) \right) dt \,, \tag{A3}$$

starred actions indicate equilibrium values, and the dependence of $X^*_{L_{s,t},t}$ on $X^*_{s,t}$ and of the expected utility on the beliefs of *s* about $b^A_{s,t}$, $b^{\sim A}_{s,t}$, and $c_{s,t}$ are implicit. Together each state's expected utility, along with the paper's Assumptions 1 and 2, define a very general, spatially-explicit decision problem for a set of governments seeking to decide on policy rationally.

Reference

Ashworth, Scott, and Ethan Bueno de Mesquita. 2006. "Monotone Comparative Statics for Models of Politics." *American Journal of Political Science* 50(1):214-31.

Part II

Derivation of Spatial Expectations

This section offers a more technical version of the theoretical defense for the three expectations about competition presented in the section, "Expectations from the Lottery Competition Model."

Additional Technical Assumptions

Footnote 5 notes that additional technical assumptions are needed to derive the expectations that form the basis for an empirical test of our theory in the context of state lottery adoptions. In particular, we assume that monopoly lottery provision does not yield too high a revenue when lottery adoption is sufficiently likely. Formally, for the Offensive Competition Expectation and the Anticipatory Competition Expectation to hold, the following conditions must be met:

(B1) $1 - (v_1 + v_2)\,\pi_r(0)) > (\delta_s(0) - \delta_S(1))v_3\lambda\pi_r(0)\pi_s(0)(1 - \pi_r(0))$, and
(B2) $\mu_1 < (\delta_s(0) - \delta_S(1))\lambda(1 - \pi_r(0))(\mu_2 - (\mu_2 + \mu_3)\pi_s(0))$,

where v_1, v_2, and v_3 ($0 \leq v_i \leq 1$) represent differences in the magnitude of the shifts in parameters $\delta_s(0)$, $\delta_s(1)$ and $\delta_r(1)$, respectively, with increases in the size of *r*'s population living within *D* miles of *s* relative to the total population of *s*;[3] and the μ_1, μ_2, and μ_3 ($0 \leq \mu_i \leq 1$) represent differences in the magnitude of the shifts in parameters $\delta_s(1)$, $\delta_r(0)$ and $\delta_r(1)$, respectively, with increases in the size of *s*'s population living within *D* miles of *r* relative to the total population of *s*. (These six shifts in magnitude cannot be calculated without specifying a model of individual consumer behavior, though we can determine relative magnitudes in a few cases.) If $\pi_s(0)$ and $\pi_r(0)$ are small, as they are in the empirical case of lottery adoptions by American states, then conditions (B1) and (B2) hold as long as $\delta_s(0) - \delta_s(1)$ is positive and neither this difference nor the level of rationality, λ, is too big or too small. When the gains to monopoly lottery provision are too great or states are too strongly rational (λ very high), anticipation of the other state's preemptive adoption to prevent one's likely poaching diminishes too greatly a state's desire to poach. When states are too weakly rational (λ very low), they fail to anticipate the other state's likely adoption and adopt more often than is optimal. In this sense, a test of the Anticipatory Competition Hypothesis amounts to a test of the assumption that states consider other states' equilibrium behavior in formulating their own actions. Conditions (B1) and (B2) are also more likely to hold the larger are the shifts in the two terms reflecting revenue gains from a monopoly lottery adoption (μ_2 and v_1) relative to the shifts in the four terms reflecting revenue gains from an adoption under competition (μ_1, μ_3, v_2 and v_3).

Spatial Expectations

Given conditions (B1) and (B2), we turn to an examination of two regions of interest: the portion of *s* that is within *D* miles of *r* (to be called *s*'s *border region*), and the portion of *r* that is within *D* miles of *s* (to be called *r*'s *border region*). Increasing the proportion of *s*'s population in its own border region has two effects on revenue. First, it should decrease $b_s^{\sim A}(1)$ since more

[3] Recall that *D* is the maximum distance someone would be willing to travel to play the lottery.

interested individuals of s should play r's lottery, decreasing s's revenue. Second, it should decrease $b_s^A(1)$ by increasing the importance of s's border region to the overall revenue of s, thereby leading s to increase its competitive effort. At the extreme, if no one lived in s's border region, s could behave as a monopoly lottery producer, earning maximal revenues. In contrast, the proportion of s's population in its own border region has no effect on either $b_s^{\sim A}(0)$ or $b_s^A(0)$, because, absent r's adoption, the locations of the interested individuals in s are irrelevant to their consumption choices within s.

We claim that $\delta_s(1)$ [$= b_s^A(1) - b_s^{\sim A}(1)$], s's revenue gain from a lottery adoption under competition, is increasing in the proportion of s's population in its own border region, as in no equilibrium would the decrease in $b_s^A(1)$ arising from an increase in the proportion of residents in this border region exceed the decrease in $b_s^{\sim A}(1)$. To see this, consider the effect on the equilibrium level of competition of moving one interested individual in s from the center of the state to its border region when r has a lottery. There are two possibilities: either the person is moved to s's side of the cutline, or to r's side.[4]

If the person is moved to r's side of the cutline, then there is no incentive for r to increase its level of competition unless s would. But consider s's choice. If s were not to adopt, the expected value of $b_s^{\sim A}(1)$ would decrease by the potential loss of revenue derived from this individual. The marginal increase in competition arising from this person's movement cannot decrease total revenue from the lottery more than this revenue loss absent the lottery, as s could have chosen not to increase competition at all and simply have seen its revenue decrease due to the resident's movement. This implies that the decrease in $b_s^A(1)$ resulting from the population shift must be less than that in $b_s^{\sim A}(1)$.

If the person is moved to s's side of cutline, then the incentive to shift the cutline is r's. If the cutline is in s, then the same argument may be made for r's choice of a competitive level: r would never choose a level of competition that would cause a decrease in revenues more than would be gained from the addition of one lottery player. Since r's gain is a loss to s, and since we have assumed it is cheaper for s to compete for an individual in s than it is for r to compete for that individual, we again have that the decrease in $b_s^A(1)$ resulting from the individual's being moved must be less than the decrease in $b_s^{\sim A}(1)$ resulting from the same shift of the individual. Finally, if the cutline is in r, then moving one interested individual in s can have no marginal effect on r's desire to compete in equilibrium, since r has already deemed it not to be beneficial to compete for individuals in s in this case. Thus, we have that $\delta_s(1)$—s's revenue gain from a lottery adoption under competition—is increasing in the proportion of s's population in its own border region. This leads to our first spatial expectation:

> <u>Defensive Competition Expectation</u>: If r has a lottery, the probability that s will adopt one is positively related to the proportion of its adult population within D miles of r (and thus with access to r's lottery).

[4] The cutline—which separates individuals that play the lottery in s from those that play in r—is defined on pp. 15-16 of the paper.

Now consider the impact of an increase in the population in *r*'s border region relative to the population of *s*. If *s* has no lottery, this increase has no effect on *s*'s revenue, regardless of whether *r* has a lottery; i.e., the increase has no effect on $b_s^{\sim A}(0)$ or $b_s^{\sim A}(1)$. In contrast, an increase in the population in *r*'s border region relative to the population of *s* clearly increases $b_s^{A}(0)$, as more residents of *r* would play *s*'s monopoly lottery. However, an increase in the population in *r*'s border region relative to the population of *s* has an uncertain effect on $b_s^{A}(1)$. On the one hand, there are more potential players of *s*'s lottery; on the other hand, there are more individuals over whom *s* and *r* may compete, lessening overall revenue due to competitive losses. Assume that the decrease in $b_s^{A}(1)$ due to competition is greater than the increase in $b_s^{A}(1)$ due to additional players, perhaps because it is cheaper for *r* to compete for individuals living in the two states' border regions than it is for *s*. (Our conclusions hold more strongly if this assumption is not true, so the assumption is unimportant.) The only time there is an incentive for either state to increase competition in this case is when the cutline is in *r*. But by the same argument introduced earlier regarding the proportion of *s*'s population in its own border region, the decrease in $b_s^{A}(1)$ can never be more than the increase in $b_s^{A}(0)$, regardless of where the extra individual is located. Since $\delta_s(0) = b_s^{A}(0) - b_s^{\sim A}(0)$ and $\delta_s(1) = b_s^{A}(1) - b_s^{\sim A}(1)$, and since an increase in the population in *r*'s border region relative to the population of *s* has no effect on $b_s^{\sim A}(0)$ or $b_s^{\sim A}(1)$, the increase in $\delta_s(0)$ can never be less than the decrease in $\delta_s(1)$. For fixed populations of *s* and *r*, however, increasing the population in *r*'s border region relative to the population of *s* also increases the proportion of *r*'s total population in its own border region. Thus, we cannot fully understand the effect on *s* of the population in *r*'s border region relative to the population of *s* until we understand the effect of the size of this population relative to *r*'s total population on *r*.

Earlier we saw that increasing the proportion of *s*'s population in its own border region decreases both $b_s^{A}(1)$ and $b_s^{\sim A}(1)$ and has no impact on both $b_s^{A}(0)$ and $b_s^{\sim A}(0)$, leading to an increase in $\delta_s(1)$ and no change in $\delta_s(0)$. However, for fixed populations of *s* and *r*, increasing the proportion of *s*'s population in its own border region also increases the population in *s*'s border region relative to the population of *r*.

To untangle effects, we consider the net change to the parameters due to a change in the proportion of individuals in each region. First consider *r*'s border population. As we have seen, increasing this population relative to the population in *s* increases $\delta_s(0)$, decreases $\delta_s(1)$ but to a lesser degree, and increases $\delta_r(1)$. One can use the comparative statics derived in Part I of this on-line appendix to determine the net effect of marginal changes in these parameters arising from an increase in the population in *r*'s border region relative to the population of *s*. Given the relative rarity of lottery adoptions absent other states' prior adoptions—implying low probabilities that *r* and *s* will adopt when neither state has a lottery [i.e., low values of $\pi_s(0)$ and $\pi_r(0)$]—we believe that assumptions (B1) and (B2) are likely to hold. Accordingly, these comparative statics imply that the net effect of increasing the population in *r*'s border region relative to the population of *s* will be to increase the likelihood that *s* will adopt. This leads to a second spatial expectation:

<u>Offensive Competition Expectation</u>: If *r* does not have a lottery, the probability that s will adopt one is positively related to the number of adults in *r* who are within *D* miles of *s* relative to the adult population of *s*.

Now reconsider *s*'s border region. Increasing the proportion of *s*'s population in this region increases $\delta_s(1)$ and $\delta_r(0)$, and decreases $\delta_r(1)$. Again we can compute the net effect of a marginal increase in the proportion of *s*'s population in its own border region on the probability that it will adopt using our comparative statics. This time our assumptions imply a decrease in the probability of adoption, leading to a third spatial expectation:

<u>Anticipatory Competition Expectation</u>: If *r* does not have a lottery, the probability that *s* will adopt one is negatively related to the proportion of its adult population within *D* miles of *r* (i.e., the proportion of *s*'s population that would have access to a lottery in *r* if *r* were to adopt one).

Part III

Figure 4 from Berry and Baybeck (2005)

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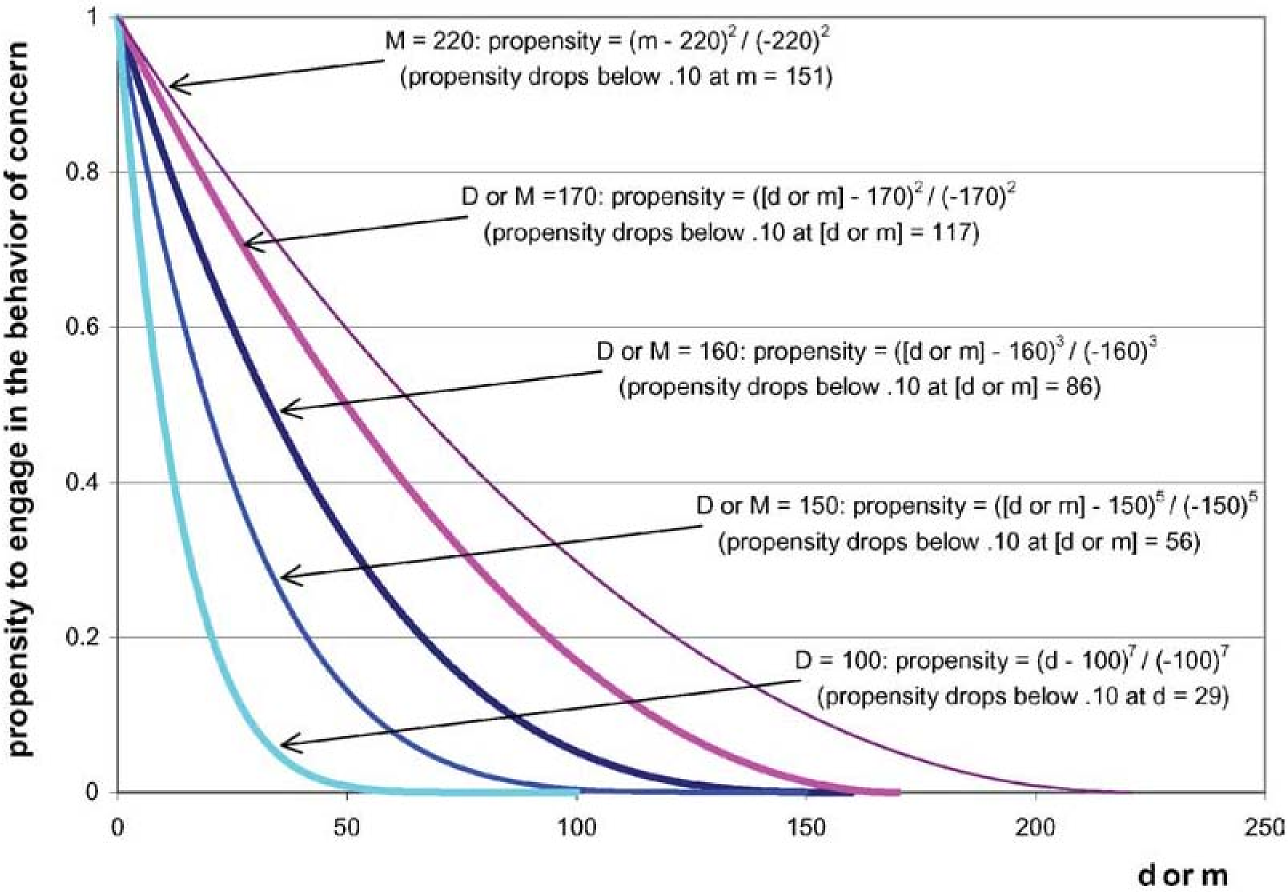
FIGURE 4. Specific Functions Mapping an Individual's Geographic Location into Propensity to Engage in the Behavior of Concern



<u>Reference</u>

Berry, William D., and Brady Baybeck. 2005. "Using Geographic Information Systems to Study Interstate Competition." *American Political Science Review* 99(4):505-19.

Part IV

Probit MLEs for Learning and Competition Variables in Models Specifying Ideologically-Based Learning

Assumed value of D:	100	150	160	170	100	150	160	170
Competition Variables								
Defensive Competition	1.14**	0.54	-0.04	-0.32	1.02*	0.66	0.19	-0.31
	(1.95)	(1.00)	(-0.08)	(-0.59)	(1.30)	(0.78)	(0.24)	(-0.43)
Offensive Competition	0.70	0.35**	0.43***	0.36**	0.68	0.36**	0.44***	0.36**
	(1.05)	(1.78)	(2.42)	(2.28)	(1.01)	(1.73)	(2.41)	(2.22)
Anticipatory Competition	-1.79	-1.97**	-2.67***	-2.42***	-1.70	-2.04**	-2.79***	-2.43***
	(-0.94)	(-1.78)	(-2.64)	(-2.52)	(-0.85)	(-1.70)	(-2.62)	(-2.47)
Learning Variables								
Ideological Distance	-0.05***	-0.05***	-0.05***	-0.05***	-0.05***	-0.05***	-0.05***	-0.05***
(Ideologically-Based Learning)	(-4.94)	(-4.93)	(-4.80)	(-4.78)	(-4.65)	(-4.69)	(-4.66)	(-4.66)
Previously Adopting Neighbors					0.03	-0.04	-0.08	-0.00
(Regional Learning)					(0.21)	(-0.19)	(-0.40)	(-0.02)

Note: All columns report coefficients from models in which one or both of our "learning variables" are added to the models estimated in Table 1 of the *JOP* article. *Z*-statistics based on robust standard errors (obtained using the probit procedure in Stata 9) are in parentheses below coefficients.

*$p < .10$, **$p < .05$, ***$p < .01$ (one tailed)