

Mathematics for Political and Social Research (i.e., Extended Math Camp)

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<https://www.daveasiegel.com/teaching/math-course/>

<https://www.youtube.com/c/DavidSiegelmath/playlists>

Course Description

This online, free course is designed to introduce and/or review the mathematical tools underlying most work in quantitative social science, particularly political science. This includes both statistical and formal modeling techniques. Topics to be covered include: (i) basic building blocks, (ii) calculus, (iii) probability, (iv) linear algebra, and (v) multivariate calculus and optimization. The course is intended to prepare students for later methods classes, provide students with a basic understanding of a variety of mathematical skills, and explore the importance of these skills in the context of social science research. Upon successful completion of the course students will be able to:

- apply basic mathematical operations,
- understand when and why certain mathematical operations are required by social scientists,
- recognize and hopefully avoid the various pitfalls of faulty application of mathematics, and
- attain basic mathematical literacy with respect to research published in the discipline.

Course Format

The course material has been broken up into individual video lectures and again into modules that closely mirror the course's textbook. All videos are posted on YouTube and free to access; a link to the channel is provided at the top of the page. You may want to subscribe in case additional content is added later. More resources can be found on the webpage linked to on the previous line.

The topics of each module in each lecture are provided below, along with the corresponding reading from the textbook. It is recommended that you do the listed reading before watching the lecture, just as you would do the reading before coming to class. Comfort with math, and later expertise, is all about repetition.

The course is meant to be modular, and you can skip around to a certain degree. For example, you can easily understand most of the three lectures on linear algebra right after Lecture 2. Further, the topics do vary a bit in difficulty. Lectures 12, 14, 15, and 16 cover more advanced topics than are often taught in first year graduate courses and can therefore often be put off for later. Finally, one of the advantages of this format is the ability to pause; if at any time you begin feeling overwhelmed or frustrated, simply pause and come back to it later. And do leave time to work through the course; trying to watch all 30+ hours of video in a week is definitely not recommended!

As there is no way to learn mathematics without actually practicing mathematics, the course offers multiple ways to practice. The textbook has examples you can work through in each chapter and exercises at the end of each chapter. Selected answers to these exercises, with some worked problems, can be found on the website listed at the top of this document. There are additional problem sets, one for each lecture, also posted there, all in one document. Solutions to these problems there as well in a different document. Finally, at the end of the each lecture is a problem session module in which I work through some example problems relevant to that lecture. Though you are of course free to do as much or as little of these problems as you would like, the more you do, the better you will understand the material and be able to use it in your studies. As importantly, the more you do, the less not understanding the material will prove a detriment to your studies.

Readings and topics follow. I hope you enjoy the course!

Readings

The textbook the course is:

Moore, Will H. and David A. Siegel. 2013. *A Mathematics Course for Political and Social Research*. Princeton, NJ: Princeton University Press.

Other potentially helpful sources include:

- Simon, Carl and Lawrence Blume. (SB) 1994. *Mathematics for Economists*. New York: Norton.
- Hagel, Timothy. 1995. *Basic Math for Social Scientists: Concepts*. Volume 108.
- Ross, Sheldon M. *Introduction to Probability Models*. San Diego: Academic Press.
- ——. 1996. *Basic Math for Social Scientists: Problems and Solutions*. Volume 109.
- Gill, Jeff. 2006. *Essential Mathematics for Political and Social Research*. Cambridge Press.
- Fox, John. 2009. *A Mathematical Primer for Social Statistics*.

- Chiang, Alpha and Kevin Wainwright. 2005. Fundamental Methods of Mathematical Economics. Boston: McGraw Hill.
- Namboodiri, Krishnan. 1984. Matrix Algebra: An Introduction. Sage Publications.
- Thompson, Silvanus P. and Martin Gardner. Calculus Made Easy.
- <http://Math.com>
- <http://mathworld.wolfram.com/>

Video Topics (with Links and Readings)

Introduction: <http://youtu.be/NQyQTctveDQ>

Part I: Building Blocks

Lecture 1 Reading: Chapters 1 and 2 of Moore and Siegel

Lecture 1, Module 1 (Numbers): <http://youtu.be/VrblmS0VOMo>

Lecture 1, Module 2 (Sets and Basic Set Notation): <http://youtu.be/uflzHj51FNA>

Lecture 1, Module 3 (Constants, Variables, and Level of Measurement):
<http://youtu.be/lqUvsvYlfEc>

Lecture 1, Module 4 (Operators and Some Rules for Operators): <http://youtu.be/vNCz4cQUDSU>

Lecture 1, Module 5 (More on Sets, Set Operators): <http://youtu.be/xb-9JKxXLn8>

Lecture 1, Module 6 (Properties of Reals and Other Sets and Spaces):
<http://youtu.be/efYNpBzegzc>

Lecture 1, Module 7 (Algebra Review 1, Expressions): <http://youtu.be/3oxB9Hy7UNE>

Lecture 1, Module 8 (Algebra Review 2, Solving Equalities): <http://youtu.be/QMFxkCvqHFg>

Lecture 1, Module 9 (Algebra Review 3, Simplifying Expressions): <http://youtu.be/fKwTx9mY5bE>

Lecture 1, Module 10 (Introduction to Proofs): <http://youtu.be/9lrZDt5-wuk>

Lecture 1, Module 10b (More on Necessity and Sufficiency): <http://youtu.be/2imCMooxOqc>

Lecture 1, Problem Session: <http://youtu.be/wBdrMi0SOzQ>

Lecture 2 Reading: Chapters 3 and 4 of Moore and Siegel

Lecture 2, Module 1 (Functions 1—Definitions): <http://youtu.be/SrkXssCc6Jo>

Lecture 2, Module 2 (Functions 2—Some Properties): <http://youtu.be/9V - 3rcvf0>

Lecture 2, Module 3 (Functions 3—Examples of Functions): <http://youtu.be/z3rvMOG9 sY>

Lecture 2, Module 4 (Functions 4—Additional Properties): <http://youtu.be/0ptOg6d-F2g>

Lecture 2, Module 5 (Relations 1—Use, Definitions, Examples): <http://youtu.be/V7QVhXGpDNk>

Lecture 2, Module 6 (Relations 2—Characteristics and Utility Representation):
<http://youtu.be/DYUYOYWcLfg>

Lecture 2, Module 7 (Sequences and Series): <http://youtu.be/hts7LMsG290>

Lecture 2, Module 8 (Limits of Sequences and Series): <http://youtu.be/2ohar8xQBQO>

Lecture 2, Module 9 (More on Sets—Open, Closed, Compact, Convex):
<http://youtu.be/uzB83sQsRdK>

Lecture 2, Module 10 (Limits of Functions and Continuity): <http://youtu.be/kFnC6rRNbrs>

Lecture 2, Problem Session: <http://youtu.be/mAXNQmxON E>

Part II: Calculus in One Dimension

Lecture 3 Reading: Chapter 5 of Moore and Siegel

Lecture 3, Module 1 (Why Calculus?): <http://youtu.be/iRP Q6SSBrA>

Lecture 3, Module 2 (Instantaneous and Discrete Change): <http://youtu.be/YVA6WG3Yg4Y>

Lecture 3, Module 3 (Notation): <http://youtu.be/y220JnAr6g>

Lecture 3, Module 4 (Limits and Derivatives and the Definition of a Derivative):
<http://youtu.be/Rmz47ipBRz4>

Lecture 3, Module 5 (Examples of Derivatives Computed from the Definition):
<http://youtu.be/uX5le2V2wXA>

Lecture 3, Module 6 (A Brief Introduction to Partial Derivatives and Multivariate Functions):
<http://youtu.be/s0K9DrCi158>

Lecture 3, Problem Session: <http://youtu.be/g64f--KV0d0>

Lecture 4 Reading: Chapter 6 of Moore and Siegel

Lecture 4, Module 1 (Computing Derivatives): <http://youtu.be/s2ytDQGwvul>

Lecture 4, Module 2 (The Derivative is a Linear Operator): <http://youtu.be/CDEISbolTfo>

Lecture 4, Module 3 (Polynomials and Powers): <http://youtu.be/OE6MGxluVHc>

Lecture 4, Module 4 (Chain Rule): <http://youtu.be/N559AWzpvf4>

Lecture 4, Module 5 (Derivatives of Exponentials): <http://youtu.be/ZBQTFEYyOhM>

Lecture 4, Module 6 (Derivatives of Logarithms): http://youtu.be/U_VIWrwH8dk

Lecture 4, Module 7 (Product (and Quotient) Rule): http://youtu.be/LX_st-fHH6A

Lecture 4, Module 8 (Putting It All Together): <http://youtu.be/KbqppU1zPCI>

Lecture 4, Problem Session: <http://youtu.be/4By5oI7kQ10>

Lecture 5 Reading: Chapter 7 of Moore and Siegel

Lecture 5, Module 1 (What are Integrals? and How are They Used?):

<http://youtu.be/GkBahLMD3ck>

Lecture 5, Module 2 (Definite Integral as a Limit of a Sum): <http://youtu.be/0y1o-7AKqS8>

Lecture 5, Module 3 (Indefinite Integral and the Antiderivative): <http://youtu.be/73bbUMB7rco>

Lecture 5, Module 4 (Fundamental Theorem of Calculus): <http://youtu.be/jZfN3MP1IA>

Lecture 5, Module 5 (Integrals of Functions): <http://youtu.be/XapcygYvRoo>

Lecture 5, Module 6 (The Integral is a Linear Operator): <http://youtu.be/OMLU6QDAdJQ>

Lecture 5, Module 7 (Integration by Substitution): http://youtu.be/QHNdlXt__mc

Lecture 5, Module 8 (Integration by Parts): <http://youtu.be/I4ydnFXbNt8>

Lecture 5, Module 9 (Putting It All Together): <http://youtu.be/nYL0sRWYvsw>

Lecture 5, Problem Session: <http://youtu.be/9YRdobtUcbM>

Lecture 6 Reading: Chapter 8 of Moore and Siegel

Lecture 6, Module 1 (Extrema in One Dimension): <http://youtu.be/K9IXm1F4cvk>

Lecture 6, Module 2 (Higher-order Derivatives): http://youtu.be/T8Ief_ck8qg

Lecture 6, Module 3 (Concavity and Convexity): <http://youtu.be/zSUTLw6r9js>

Lecture 6, Module 4 (Critical and Inflection Points): <http://youtu.be/Y5Sz2YME5m8>

Lecture 6, Module 5 (First Derivative Test): <http://youtu.be/Gatr-Ui-Sxs>

Lecture 6, Module 6 (Second Derivative Test): <http://youtu.be/4bwJwykVm08>

Lecture 6, Module 7 (Summary of Method of Finding Extrema): http://youtu.be/p1zMh1zNV_0

Lecture 6, Module 8 (Examples: Maximizing Utility and Linear Models and OLS Regression):

<http://youtu.be/S4o3zREA5hc>

Lecture 6, Problem Session: <http://youtu.be/vsxdk3VWsa0>

Part III: Probability

Lecture 7 Reading: Chapter 9 of Moore and Siegel

Lecture 7, Module 1 (Introduction to Probability): <http://youtu.be/1HclKOWgiOU>

Lecture 7, Module 2 (Classical Probability): http://youtu.be/Fop6_GGws0

Lecture 7, Module 3 (Notation, Rules and Properties): <http://youtu.be/rNlyLOQoVus>

Lecture 7, Module 4 (Computing Probabilities): http://youtu.be/782cvyO_UnE

Lecture 7, Module 5 (Bayes' Rule): <http://youtu.be/GkBB6qEuLDU>

Lecture 7, Module 6 (Odds and Relative Risk Ratios): <http://youtu.be/TotsUW2xnIk>

Lecture 7, Problem Session: <http://youtu.be/9mHULv6Hv5M>

Lecture 8 Reading: Chapter 10 of Moore and Siegel

Lecture 8, Module 1 (Random Variables and Distributions): <http://youtu.be/CQUnbYG0LBA>

Lecture 8, Module 2 (Empirical Distributions): <http://youtu.be/Hiv6H3gEgD0>

Lecture 8, Module 3 (The Probability Mass Function): <http://youtu.be/Vd5C2ZYbxw>

Lecture 8, Module 4 (The Cumulative Distribution Function): <http://youtu.be/tLTC8FryUgc>

Lecture 8, Module 5 (Bernoulli and Binomial Distributions): http://youtu.be/DSg-4_nv5yQ

Lecture 8, Module 6 (Event Count Distributions): <http://youtu.be/h6N0zn1w5go>

Lecture 8, Module 7 (Expectations and Moments of Discrete Random Variables):
<http://youtu.be/LoDpAFYID9g>

Lecture 8, Module 8 (Expected Utility): <http://youtu.be/lmCyCzsfuto>

Lecture 8, Problem Session: <http://youtu.be/63zpNMc0Oao>

Lecture 9 Reading: Chapter 11 of Moore and Siegel

Lecture 9, Module 1 (Introduction to Continuous Probability Distributions): <http://youtu.be/wB-FioDqiD8>

Lecture 9, Module 2 (PDF, CDF, and Parameters): <http://youtu.be/sivT85zz3PU>

Lecture 9, Module 3 (Joint Distributions): <http://youtu.be/n09-IJXilsc>

Lecture 9, Module 4 (Expectations and the Uniform Distribution): <http://youtu.be/igvBmZfVLnY>

Lecture 9, Module 5 (Moments of Continuous Distributions): <http://youtu.be/puvKTJYAVmA>

Lecture 9, Module 6 (Stochastic Dominance): http://youtu.be/6iE_5y4r2FI

Lecture 9, Module 7 (Normal (Gaussian) Distribution): http://youtu.be/ln1t_KLRqkM

Lecture 9, Module 8 (Logistic Distribution): http://youtu.be/0F_D-aFfXEw

Lecture 9, Module 9 (Duration and other Frequently Used Distributions):
<http://youtu.be/lzGkgo45too>

Lecture 9, Problem Session: <http://youtu.be/djqlhmD51o>

Part IV: Linear Algebra

Lecture 10 Reading: Chapter 12 of Moore and Siegel

Lecture 10, Module 1 (Of Scalars and Vectors): <http://youtu.be/iQ8-asNn9EU>

Lecture 10, Module 2 (Vector Algebra): <http://youtu.be/-1860RpSlnk>

Lecture 10 Module 3 (Vector Multiplication): <http://youtu.be/xYRI5PXSSxg>

Lecture 10, Module 4 (Matrices): <http://youtu.be/eW5XoZhQAE8>

Lecture 10, Module 5 (Matrix Algebra): <http://youtu.be/ncGdSW50dUc>

Lecture 10, Module 6 (Matrix Operators): <http://youtu.be/HbLxS8aL1n0>

Lecture 10, Module 7 (Matrix Inverse): <http://youtu.be/aaxpunlIP4A>

Lecture 10, Module 8 (Properties of Vectors and Matrices): <http://youtu.be/9ScMH2BEe10>

Lecture 10, Module 9 (Matrices and OLS): <http://youtu.be/1mUIWW9wXNk>

Lecture 10, Problem Session: <http://youtu.be/xvaUD0dajU4>

Lecture 11 Reading: Chapter 13 of Moore and Siegel

Lecture 11, Module 1 (Linear Algebra and Vector Spaces): <http://youtu.be/vOX2iROHL6U>

Lecture 11, Module 2 (Linear Independence and Spanning Vectors):
<http://youtu.be/ntuITl6XQSQ>

Lecture 11, Module 3 (Solving Systems of Equations): <http://youtu.be/ek7TaYeRZMk>

Lecture 11, Module 4 (Substitution): <http://youtu.be/hk4JmJkB-oA>

Lecture 11, Module 5 (Elimination): <http://youtu.be/Sez4R7du73U>

Lecture 11, Module 6 (Matrix Inversion): http://youtu.be/Md_R7EmGADY

Lecture 11, Module 7 (Cramer's Rule): <http://youtu.be/kiykoATSIAs>

Lecture 11, Module 8 (Applications in Statistics and Game Theory):
<http://youtu.be/Wt0V3umilHQ>

Lecture 11, Problem Session: <http://youtu.be/Ar50peCqMIY>

Lecture 12 Reading: Chapter 14 of Moore and Siegel

Lecture 12, Module 1 (Eigenvalues): <http://youtu.be/MmOwpcUF2CE>

Lecture 12, Module 2 (Computing Eigenvalues): <http://youtu.be/s-9rNuuhIOg>

Lecture 12, Module 3 (Eigenvectors): <http://youtu.be/pnyHMrxQJNA>

Lecture 12, Module 4 (Matrix Decomposition and Eigenvector Centrality): <http://youtu.be/-dLBjPDMhfo>

Lecture 12, Module 5 (Markov Chains and Stochastic Processes): <http://youtu.be/FKJpIRZdmaU>

Lecture 12, Module 6 (Ergodicity and Limiting Distributions): <http://youtu.be/4z4jMa6pves>

Lecture 12, Module 7 (Computing Steady States): <http://youtu.be/h3Po7xqliQ>

Lecture 12, Problem Session: <http://youtu.be/rnwzkPqh7AA>

Part V: Multivariate Calculus

Lecture 13 Reading: Chapter 15 of Moore and Siegel through 15.2.2, 15.2.5

Lecture 13, Module 1 (Functions of Several Variables): <http://youtu.be/8iTJbWY2J5w>

Lecture 13, Module 2 (Partial Derivatives): http://youtu.be/W9vPDK_Zkxk

Lecture 13, Module 3 (Multidimensional Integrals): <http://youtu.be/X99j8C2ly1k>

Lecture 13, Problem Session: <http://youtu.be/U8XgBODp3vA>

Lecture 14 Reading: Chapter 15 of Moore and Siegel from 15.2.2 through end, other than 15.2.5

Lecture 14, Module 1 (Gradients): <http://youtu.be/kSMxUEYEIwk>

Lecture 14, Module 2 (Total Derivatives): <http://youtu.be/Eahl4dG4a3s>

Lecture 14, Module 3 (The Jacobian): <http://youtu.be/EzBkAd9tuFQ>

Lecture 14, Module 4 (The Hessian): <http://youtu.be/msb5ffgZEQk>

Lecture 14, Module 5 (Concavity and Convexity): <http://youtu.be/0VpB4kJ2FsE>

Lecture 14, Problem Session: <http://youtu.be/1q0h06CrPeM>

Lecture 15 Reading: Chapter 16 of Moore and Siegel

Lecture 15, Module 1 (Unconstrained Optimization I: Method): http://youtu.be/YCT5IbFP_Tw

Lecture 15, Module 2 (Unconstrained Optimization II: Examples): <http://youtu.be/yiOSKnh2QiE>

Lecture 15, Module 3 (Equality Constraints I: Method): <http://youtu.be/oDtwQh79Nh4>

Lecture 15, Module 4 (Equality Constraints II: Examples): <http://youtu.be/RvC8er7sY0M>

Lecture 15, Module 5 (Inequality Constraints I: Method): <http://youtu.be/hTsNcDGkpGg>

Lecture 15, Module 6 (Inequality Constraints II: Examples): <http://youtu.be/fgsM2vQnvNQ>

Lecture 15, Problem Session: <http://youtu.be/-lOefcSSVH0>

Lecture 16 Reading: Chapter 17 of Moore and Siegel

Lecture 16, Module 1 (Optima and Comparative Statics): <http://youtu.be/KN9tdiv2X30>

Lecture 16, Module 2 (Implicit Differentiation in One Dimension): <http://youtu.be/z-ID3lpOu8M>

Lecture 16, Module 3 (Implicit Differentiation in Multiple Dimensions):
<http://youtu.be/O6w8f9RVRu8>

Lecture 16, Problem Session: <http://youtu.be/6TbzEXHw9ns>