CS230 Recitation/Assignment 01 - Logic (Updated)

Recitation 01/22, Assignment Due 01/28

1 Recitation

1. (Ungraded activity) Icebreaking and getting to know each other in recitations.

2. (Continued from class meeting) Let $C(x)$ represent "x can have cake", and $I(x)$ represent "x can have ice cream". Let $DS(x)$ represent "x drives sober" and $PO(x)$ represent "x gets pulled over".

   (a) Translate the following sentence from English to a proposition *literally* (i.e., without rephrasing or simplifications): "Shao-Heng can have cake or ice cream, but not both."

   (b) What does the sentence "Drive sober, or get pulled over." most likely *actually* mean? Translate that to a *quantified predicate.*

3. (Ungraded whole-section activity) Let P be the proposition *"I have Passed the assignment of module 1"*, Q be the proposition *"I have got 80% of the questions in the preparation Quiz of module 1 in Canvas"*, and R be the proposition *"I have attended Recitation for module 1"*. Derive the truth table for S, the proposition *"I have Satisfactorily completed module 1"*, according to the information in Canvas. You will need this truth table again in later parts of the assignment.

P	Q	R	S

4. (Ungraded activity) Finding assignment partner.

5. (De Morgan's law for three variables) We have already seen De Morgan's law for two variables:

$$\neg(x \wedge y) = \neg x \vee \neg y; \quad \neg(x \vee y) = \neg x \wedge \neg y.$$

   We will derive the case for three variables:

$$\neg(x \wedge y \wedge z) = \neg x \vee \neg y \vee \neg z; \tag{1}$$

$$\neg(x \vee y \vee z) = \neg x \wedge \neg y \wedge \neg z. \tag{2}$$

(a) Show (1) using a truth table.

x	y	z	$x \wedge y$	$x \wedge y \wedge z$	$\neg(x \wedge y \wedge z)$	$\neg x$	$\neg y$	$\neg z$	$\neg x \vee \neg y$	$\neg x \vee \neg y \vee \neg z$

(b) Show (2) using a chain of equivalences. You may (should) use De Morgan's law for two variables.

Similarly, the general case of De Morgan's law for k variables is:

$$\neg(x_1 \wedge x_2 \wedge \ldots \wedge x_k) = \neg x_1 \vee \neg x_2 \vee \ldots \vee \neg x_k; \quad (3)$$

$$\neg(x_1 \vee x_2 \vee \ldots \vee x_k) = \neg x_1 \wedge \neg x_2 \wedge \ldots \wedge \neg x_k. \quad (4)$$

You may find these useful later in the assignment. We will not prove the general case right now; we will revisit it when we get to the topic of *inductions*.

6. Here is the definition of the limit in calculus (don't worry, this question does not actually use calculus):

Definition 1. *Let $f(x)$ be a function defined on an interval that contains $x = a$ (except possibly at $x = a$). Then we say "the limit of $f(x)$ as x approaches a is L", written as*

$$\lim_{x \to a} f(x) = L,$$

if for every positive number ε there is some positive number δ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

For the parts below, you can assume $f(x)$ is defined for all real numbers.

(a) Write a predicate that means "the limit of $f(x)$ as x approaches a exists" without using any English words.

(b) Now write a predicate that means "the limit of $f(x)$ exists everywhere" without using any English words. How many layers of quantifiers is this?

2 Gradescope Assignment (Complete outside recitations)

1. (Predicate design) For each subproblem, design a predicate $P(x, y)$ such that the statements are satisfied. Specify the domain of each variable. *An example has been provided for each subproblem to help you think through them.* ***For each subproblem, your answer should not use the same predicate as the one in the example provided.***

   (a) $\Big[\exists x \forall y [P(x, y)]\Big] = T, \quad \Big[\forall x \exists y [P(x, y)]\Big] = F.$
   Example. Let $x, y \in \mathbb{Z}$. Let $P(x, y)$ be the predicate $\frac{y}{x} \in \mathbb{Q}$.

   (b) $\forall x \forall y \big[P(x, y) = P(y, x)\big]$.
   Example. Let $x, y \in \mathbb{Z}$. Let $P(x, y)$ be the predicate $x = y$.

   (c) $\Big[\exists x \exists y [P(x, y)]\Big] = F, \quad \Big[\forall x \forall y [P(y, x)]\Big] = T.$
   Example. Let $x \in \mathbb{Z}^-$. Let $y \in \mathbb{Z}^+$. Let $P(x, y)$ be the predicate $x > y$.

2. (Inference rules in predicate logic) In class, we have seen two valid syllogisms, *Modus Ponens* and *Modus Tollens*:

$$\begin{array}{rl} & p \\ & \underline{p \to q} \\ \therefore & q \end{array} \qquad \begin{array}{rl} & \neg q \\ & \underline{p \to q} \\ \therefore & \neg p \end{array}$$

for two propositions p and q.[1] Do they generalize to predicates? Below are 16 syllogisms about two predicates $P(x)$ and $Q(x)$. The first 8 resemble *Modus Ponens* while the last 8 resemble *Modus Tollens*.

$$(1)\ \begin{array}{rl} & \forall x P(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \forall x Q(x) \end{array} \quad (2)\ \begin{array}{rl} & \forall x P(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \exists x Q(x) \end{array} \quad (3)\ \begin{array}{rl} & \forall x P(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \forall x Q(x) \end{array} \quad (4)\ \begin{array}{rl} & \forall x P(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \exists x Q(x) \end{array}$$

$$(5)\ \begin{array}{rl} & \exists x P(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \forall x Q(x) \end{array} \quad (6)\ \begin{array}{rl} & \exists x P(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \exists x Q(x) \end{array} \quad (7)\ \begin{array}{rl} & \exists x P(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \forall x Q(x) \end{array} \quad (8)\ \begin{array}{rl} & \exists x P(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \exists x Q(x) \end{array}$$

$$(9)\ \begin{array}{rl} & \forall x \neg Q(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \forall x \neg P(x) \end{array} \quad (10)\ \begin{array}{rl} & \forall x \neg Q(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \exists x \neg P(x) \end{array} \quad (11)\ \begin{array}{rl} & \forall x \neg Q(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \forall x \neg P(x) \end{array} \quad (12)\ \begin{array}{rl} & \forall x \neg Q(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \exists x \neg P(x) \end{array}$$

$$(13)\ \begin{array}{rl} & \exists x \neg Q(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \forall x \neg P(x) \end{array} \quad (14)\ \begin{array}{rl} & \exists x \neg Q(x) \\ & \underline{\forall x[P(x) \to Q(x)]} \\ \therefore & \exists x \neg P(x) \end{array} \quad (15)\ \begin{array}{rl} & \exists x \neg Q(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \forall x \neg P(x) \end{array} \quad (16)\ \begin{array}{rl} & \exists x \neg Q(x) \\ & \underline{\exists x[P(x) \to Q(x)]} \\ \therefore & \exists x \neg P(x) \end{array}$$

   (a) Which ones are valid? ***This question will not be graded****, but thinking about it is useful for this assignment as well as the next assignment.*

   (b) Choose one among **(1) to (6)** that is *invalid*. Design your own predicates $P(x)$ and $Q(x)$, along with the domain of x, and show a counterexample in which both premises for the chosen syllogism is true, while the conclusion is not true.

   (c) Choose another among **(9) to (14)** that is *invalid*. Design your own predicates $P(x)$ and $Q(x)$, along with the domain of x, and show a counterexample in which both premises for the chosen syllogism is true, while the conclusion is not true. You may reuse the same predicates and domain.

3. (Canonical CNF and DNF) Recall the truth table of propositions P, Q, R, and S in Recitation activity 3. First, add a column for $\neg S$ in your truth table. We now express $\neg S$ in disjunctive normal form (DNF) and conjunctive normal form (CNF) in P, Q, and R. ***You don't need to submit the truth table for this question.***

   (a) Express $\neg S$ in DNF by first creating a conjunctive clause for each row with a T in the $\neg S$ column in the truth table, then combining all the clauses by disjunctions. *See AIDMA Example 2.94 for a demonstration for two variables.*

   (b) Express $\neg S$ in CNF following the below procedure:

      i. Obtain the DNF of S using the same procedure as above. You should already have it in the table from the recitation work.

      ii. Now negate the entire DNF. This expression is $\neg S$.

      iii. Use De Morgan's law to "push the outer negation into the clauses" until the expression becomes in CNF.

[1] These are proved valid by showing both $\big(p \wedge (p \to q)\big) \to q$ and $\big(\neg q \wedge (p \to q)\big) \to \neg p$ are tautologies.