Class starts after this song

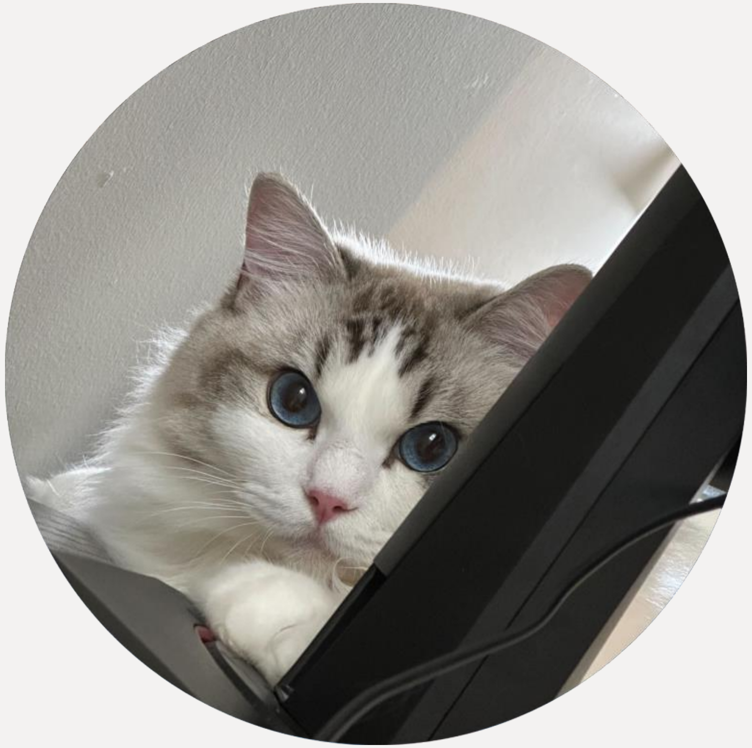
Noisestorm - Crab Rave (2018)
requested by Alex Chao (TA-of-CM1)

This is Leo (unfortunately not mine). Occasionally, I think about replacing my roommate with a cat, but unfortunately cats can't pay rent. I enjoy cooking food for others and climbing rocks.

Duke

Featured: Violet Pang

- I like surfing and skiing. In my leisure time at Durham, I enjoy going to gyms, playing switch games, and cooking. I have a ragdoll cat named Taro.



Logistic Bulletin Board

- Recitation section 08D **class change**:
  - ~~was Gross 105~~
  - now moved to **LSRC A247**

Today's class will also end a bit early
Come to the front after class if (1) you're on the waitlist
INCLUSIVE-or (2) you've emailed us about recitation sections

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Before we start

- You don’t necessarily need to complete ALL the questions in the textbook before attempting the Canvas quiz (or at all)
  - Read the concepts
  - Use Canvas quiz to test your understanding of concepts
  - Depending on what you find yourself struggle on, read again or attempt the textbook questions
- It is okay to be a bit confused between prepare and class

Before we start

- Most of you got to *about* 80% in three tries or lower
  - Including 7/9 = 77.78% for this first quiz

- Is the 1-hour cooldown period too long? (Changed to 15 mins)

CS230 Spring 2024
Module 01: Logic

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Why logic?

- You may have completed the required reading and the Canvas quiz, and likely already know the basics about logic
- But one thing the textbook does NOT say (explicitly) is **WHY should we learn logic?**

Rationale 1

- Logic is the foundation of all mathematical proofs, so if you want to learn proofs, you should learn logic (first)
- You want to learn proofs in CS230

- *Therefore, you should learn logic*

Rationale 2

- English is hard, because it is often imprecise
- *Learning logic helps us become vigilant in everyday talking and reasoning*

Peer Discussion!

- Turn to your neighbor
- Introduce yourself:
  - Name, year, major, etc.
  - What is another interesting course you're taking?
- Then discuss the question in the QR code
  - Remember everyone should have an opinion

Natural language is often imprecise

- Waiter could’ve said “you can have cake or ice cream, *but not both*”
- Waiter could’ve also said “you can have cake *exclusive-or* ice cream”…
- What about the traffic sign?

Propositions/predicates

- Propositions have a truth value *per se*
- Strictly speaking, predicates *don't* have a truth value
  … until we assign value(s) to the variable(s)
  - *But it's often vague whether we've assigned value(s)*
  - *As a result, even the textbook sometimes calls predicates propositions*
- Fully quantified predicates like
  ∃x [Logic is the x-th module in CS230]
  become propositions because they have truth values

Food for thought

- Is *CS230 is enjoyable* a proposition?

- Is *The Duke Blue Devils will win the national NCAA basketball championship in **2024*** a proposition?

- Is $x^2 \geq 0$ a tautology?

Lesson: always declare the domain of variables (unless it's REALLY REALLY clear from context)

Example 2.44. Assume that x is a real number.

PI: Implication

$(x \wedge y) \rightarrow (x \vee y)$ is a: (select all that apply)

- [x] proposition that is always true
- [ ] proposition that is sometimes true
- [ ] proposition that is never true
- [x] tautology
- [ ] contingency
- [ ] contradiction

- You all did okay on this question
- Let’s do a trickier one in peer instruction

What does $p \to q$ actually mean?

- It really means $\neg p \vee q$ and nothing else
- Even if it evaluates to True, it says *nothing* about whether there is any cause-and-effect relation between p and q
- $F \to F$ and $F \to T$ are both True (in fact $F \to q$ is always True!)
- Similarly $(x \vee y) \to (x \wedge y)$ can be True if $(x \vee y)$ evaluates to False

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Chains-of-equivalences

Example 2.52. Prove the idempotent law $p \vee p = p$ by using the other equivalences.

Proof: It is easier to prove backwards ($p = p \vee p$). We have

It's a Not Yet without the equal signs. If these are all omitted, it's not a proof, just many logical statements (that happen to be equivalent) written together.

$$
\begin{aligned}
p &= p \vee F && \text{(by identity)}\\
&= p \vee (p \wedge \neg p) && \text{(by negation)}\\
&= (p \vee p) \wedge (p \vee \neg p) && \text{(by distribution)}\\
&= (p \vee p) \wedge T && \text{(by negation)}\\
&= p \vee p && \text{(by identity)}
\end{aligned}
$$

It's a Satisfactory without these elaborations/steps. Omitting these doesn't affect technical correctness, but it makes the proof harder to understand.

Thus, $p \vee p = p$. $\square$

Modus Ponens

- Logic is the foundation of all mathematical proofs, so if you want to learn proofs, you should learn logic (first)
- You want to learn proofs in CS230
- *Therefore, you should learn logic*

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\qquad} \\ \therefore\ q \end{array}$$

Modus Ponens

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$$

Modus Ponens

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Modus Tollens

p	q	$p \to q$	$\neg q$	$\neg p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

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Modus Tollens

- If the sky is blue, then it is raining
- It is not raining

- *Therefore, the sky is not blue*

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore\ \neg p \end{array}$$

“Scientific method”?

- If my hypothesis is true, I will see this result.
- I run an experiment and get this result.

- *Therefore, my hypothesis is true.*

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

“Scientific method”?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

Late Policies

- **If:**
  - we don’t want to measure your time management skills **and**
  - deadlines for Gradescope assignments are for logistic reasons,
- **Then:**
  - no one should be punished **if** they don’t incur any logistic cost/inconvenience to the staff

Class starts after this song

I am a senior studying computer science with a concentration in AI/ML with minors in math and gender, sexuality, and feminist studies. I am a member of the rowing team at Duke. In my free time, I like to bake and crochet.

Bon Jovi - Livin' on a Prayer (1986)
requested by Paige Knudsen (TA-of-CM1, Recitation 04D)

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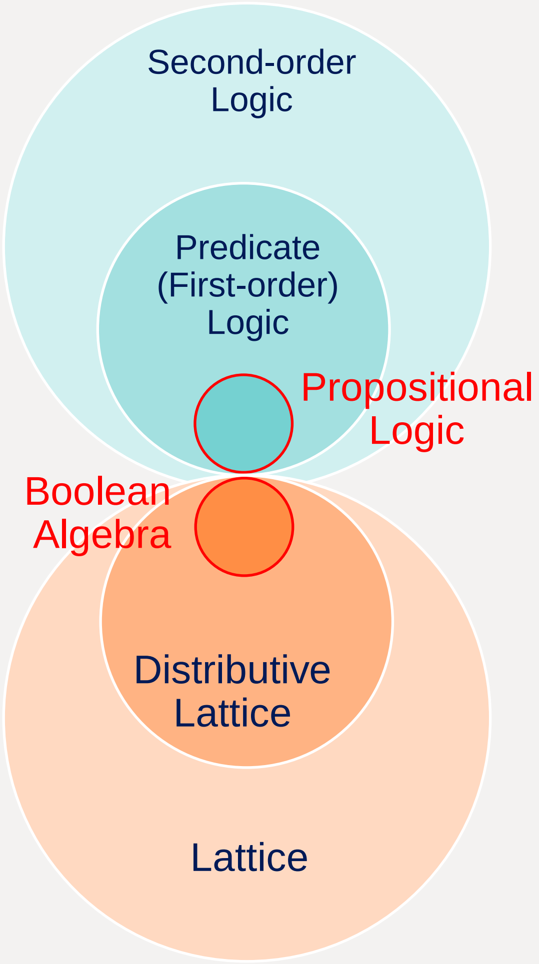
Boolean algebra/logic = Propositional logic?

- If you have taken hardware/systems courses, you may have already seen *Boolean algebra/logic*
- You may even feel Boolean algebra/logic is awfully similar to propositional logic



PI: DNF/CNF

- DNF: **disjunctions (or's)** of conjunctive clauses (and's)
  - $q \wedge \neg r$
  - $(d \wedge n \wedge f) \vee (d \wedge n \wedge \neg f) \vee (\neg d \wedge n \wedge f) \vee (\neg d \wedge \neg n \wedge f)$

- CNF: **conjunctions (and's)** of disjunctive clauses (or's)
  - $q \wedge \neg r$
  - $(c \vee n \vee f) \wedge (c \vee n \vee \neg f) \wedge (\neg c \vee n \vee f) \wedge (\neg c \vee \neg n \vee f)$ **Which one is satisfiable?**

Applications

- DNF: **disjunctions (or's)** of conjunctive clauses (and's)
  - $q \land \neg r$
  - $(d \land n \land f) \lor (d \land n \land \neg f) \lor (\neg d \land n \land f) \lor (\neg d \land \neg n \land f)$

- A lot easier to check for *satisfiability* (just check every clause)
- Similar to the idea of *proof-by-cases*

Applications

- CNF: **conjunctions (and’s)** of disjunctive clauses (or’s)
  - $q \land \neg r$
  - $(c \lor n \lor f) \land (c \lor n \lor \neg f) \land (\neg c \lor n \lor f) \land (\neg c \lor \neg n \lor f)$
- The problem of whether a CNF* is *satisfiable*
  is one of the 21 fundamental problems in theory-of-computing
  - *CNF with exactly 3 literals per clause

PI: Order of Quantifiers

☐	$\forall s\,\exists d\,[\mathsf{IsDeanOf}(d, s)]$	Every student has a dean (true)
☐	$\exists d\,\forall s\,[\mathsf{IsDeanOf}(d, s)]$	There's a dean who is every student's dean (false)

Takeaway: Order of quantifiers matters. A LOT.

Recap: common sets of numbers

```
\mathbb{Z} (may need \usepackage{amsfonts})
```

- $\mathbb{Z}$: the set of all integers
  - $\mathbb{Z}^+$: the set of all positive integers
  - $\mathbb{Z}^-$: the set of all negative integers
- $\mathbb{R}$: the set of all real numbers ($\mathbb{R}^+$ and $\mathbb{R}^-$ follow similarly)
- $\mathbb{N}$: the set of natural numbers
  - $0 \notin \mathbb{N}$ or $0 \in \mathbb{N}$? No consensus
- $\mathbb{Q}$: the set of rational numbers
  - Rational numbers can be represented in the form of ${}^{a}/_{b}$, where $a, b \in \mathbb{Z}$, $b > 0$

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Recap: division

- In the following, $a, b, c \in \mathbb{Z}$
- $a \mid b$ (a divides b): there exists $k \in \mathbb{Z}$ such that $ak = b$
  - Also said as b is divisible by a, b is a multiple of a, a is a factor of b
  - $\forall x \in \mathbb{Z}\ [1 \mid x]$
- $a \nmid b$: there exists no such $k \in \mathbb{Z}$ (a does not divide b)

  \nmid
  - $\forall x \in \mathbb{Z}^+ [0 \nmid x]$
- c is a common divisor of a, b if $c \mid a$ and $c \mid b$
- c is the greatest common divisor (GCD) of a, b if there exists no other common divisor of a, b larger than c

Recap: primes

- $p \in \mathbb{Z}^+$ is a prime number (or p is *prime*) if p is divisible by exactly two natural numbers, 1 and p
- $p \in \mathbb{Z}^+$ is a composite number (or p is *composite*) if p is divisible by more than two natural numbers, i.e., there exists $1 < q < p$ such that $q \mid p$
- p, q are *coprime* if the only natural number that divides both is 1
- 1 is *neither* a prime number *nor* a composite number

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Recap: prime factorization

- Every positive integer greater than 1 can be written as multiplications of (not necessarily distinct) prime numbers.
  - $16 = 2 \times 2 \times 2 \times 2$
  - $525 = 3 \times 5 \times 5 \times 7 = 5 \times 7 \times 3 \times 5$
- (Uniqueness) There is only one such way to do this for every positive integer such that the prime numbers are *sorted*.

Peer Discussion: Quantifiers

Let the domain of x be integers.
Let $P(x)$ represent $x \leq 0$, i.e., x is negative; let $Q(x)$ represent $x^2 \leq 0$, i.e., x^2 is negative.
Accordingly, $Q(x)$ is always false regardless of x, while $P(x)$ is a contingency (so $\forall x P(x)$ is false).

- $\forall x [P(x) \rightarrow Q(x)]$

This is false because $P(x) \rightarrow Q(x)$ is false when x is negative. We may represent $P(x) \rightarrow Q(x)$ by $R(x)$; then $R(x)$ is false when x is negative, so $\forall x R(x)$ is false.

- $\forall x P(x) \rightarrow \forall x Q(x)$

This is true simply because $\forall x P(x)$ is false. Anything is implied by a false proposition (think back to the "vacuous promise" argument. It doesn't even matter whether $\forall x Q(x)$ is true or false (it is false).

Higher order logic systems

- Propositional logic (0-order logic) is simple yet very restrictive
- Predicate logic (1-order logic) is richer and still simple enough, so it is a good “playground” for us
- There are still many things that cannot be represented by predicate logic statements, e.g.,
  “every non-empty set of positive integers contains a smallest element”

Memorization?

- This table will be part of our “official reference sheet” for exam 1
- Memorizing things in this course almost *never* helps in the long term

Name	*Equivalence*
commutativity	$p \vee q = q \vee p$ $p \wedge q = q \wedge p$
associativity	$p \vee (q \vee r) = (p \vee q) \vee r$ $p \wedge (q \wedge r) = (p \wedge q) \wedge r$
distributive	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
identity	$p \vee F = p$ $p \wedge T = p$
negation	$p \vee \neg p = T$ $p \wedge \neg p = F$
domination	$p \vee T = T$ $p \wedge F = F$
idempotent	$p \vee p = p$ $p \wedge p = p$
double negation	$\neg(\neg p) = p$
DeMorgan's	$\neg(p \vee q) = \neg p \wedge \neg q$ $\neg(p \wedge q) = \neg p \vee \neg q$
absorption	$p \vee (p \wedge q) = p$ $p \wedge (p \vee q) = p$

Table 2.3: Common Logical Equivalences