CS230 SP24

LDoC recap/review

04/24/2024

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  - Therefore, do not fixate on the numerical part of the answers.

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Among us is an online multiple social deduction game made by Innersloth and inspired by the real-world game *Mafia*. It was a pretty big thing during the pandemic days.

- Disclaimer: I wrote this based on the *Among us* fandom wiki[1] and simplified as necessary. Please do not nitpick if you are an *Among us* expert.

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[1] https://among-us.fandom.com/wiki/

Prelude

- In the most basic setting, players in a game are secretly divided into two teams: Crewmates and Impostors.
  - Impostors' goal is to blend in and kill/eliminate all crewmates.
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    Select two players from ten, $\binom{10}{2} = 45$.

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  Two. Impostors and Crewmates.

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- Note that all the above are known without specifying the underlying probability distribution.

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  - For every two consecutive games, we can define an indicator variable with expectation $\frac{1}{45}$. Using the linearity of expectation, the expectation of the sum of these variables is $\frac{44}{45}$. Did you say 1?

Dressing up

- Players in the lobby must select one of the 18 available colors: Red, Blue, Green, Pink, Orange, Yellow, Black, White, Purple, Brown, Cyan, Lime, Maroon, Rose, Banana, Gray, Tan, and Coral. Every player has to have a di erent color.

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  - $18 \quad 17 \quad 16 \quad \ldots \quad 9 = \frac{18!}{8!}$.

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  - Consider the set P_1 of all the players except a. By inductive hypothesis, these k players all have the same color.
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  The last step in the inductive step does not hold when $k = 1$ and $(k + 1) = 2$. In this case, the two sets P_1 and P_2 are disjoint.

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  - Players can wear the same things as others. They are already distinct in colors, after all.
- In how many different ways can one player customize their look, disregarding color (because that part depends on other players)?
  - Product rule: $(99 + 1)(12 + 1)(4 + 1)(2 + 1) = 19500$. Note the +1s in each category account for choosing nothing.

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  - In the early days of Among us the votes are shown directly with color coding (see figure), so it is clear who voted whom.
- Assume this is an emergency meeting where all 10 players are alive (and thus can vote). How many different voting results can there be?
  - Each player has 11 actions (including skipping), so $11^{10} = 25937424601$.

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- Assume now 9 players are alive and thus can vote. How many di erent voting results can be displayed now?
  - Stars and bars. We need to distribute 9 votes across 10 candidates (treating all skipped votes as one \candidate"). So 9 stars and 9 bars. This is $\binom{9+9}{9} = \binom{18}{9} = 48620$.

Not voting out someone

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$$\sum_{x=3}^{4} \binom{8}{2} \binom{8 - 2x + 5}{5}$$

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3. Finally... there may be a 8-way tie with 1 vote.

$$1$$

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Communication is key

- According to Among us's o cial Twitter account [5] in 2020, Impostors won roughly 60% of the time while Crewmates only won roughly 40% of the time[6].

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[6] This may have included all settings, not just 10-player 2-impostor games. Moreover, most public lobbies were played without a side voice channel for players to verbally communicate. In late 2020, when a group of streamers played with a Discord voice channel to aid discussion before voting, Impostors only won roughly 30% of the time whereas Crewmates won roughly 70% of the time.

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  - We can denote I, C, and W as being an Impostor, being a Crewmate, and winning the game. Assume all the hidden assumptions hold, we have

$$\Pr[W] = \Pr[W \mid I] \cdot \Pr[I] + \Pr[W \mid C] \cdot \Pr[C] = 0.6 \cdot 0.2 + 0.4 \cdot 0.8 = 44\%.$$

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Communication is key

- According to Among us's official Twitter account[5] in 2020, Impostors won roughly 60% of the time while Crewmates only won roughly 40% of the time[6].
- Recall that every player has a 20% chance to be an Impostor in a 10-player, 2-impostor game. Given all this information, what is the expected winning chance for an "average player" in each game?
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$$\Pr[I \mid W] = \frac{\Pr[W \mid I] \cdot \Pr[I]}{\Pr[W]} = \frac{0.6 \cdot 0.2}{0.44} = 27.3\%.$$

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Reactor meltdown

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  - What is the minimum probability that some Crewmate resolves the reactor? It is just $1/5$. In this multiverse, either all Crewmates arrive in time (with probability $1/5$) or they all don't make it (with probability $4/5$).

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  - This evaluates to about 26.3%.

Death time

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  Partial order.
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Venting

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  3, but not these colors on the graph. We need only two colors for the red system (an even cycle) or the yellow system (a path), but the green system is a triangle that needs three colors.
- How many red venting walks of length at most 8 are there?
  - This includes entering a vent and staying there (length-0 walks).

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- For each venting walk with length k, there are two walks with length $(k+1)$ because there are two connecting red vents for the next step
- $T(k+1) = 2 \times T(k), \; \forall k \geq 0$
- $$\sum_{k=0}^{8} T(k) = 4 + 4 \times 2^1 + \cdots + 4 \times 2^8 = 4 \times \frac{1-2^9}{1-2} = 2044$$

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  - Geometric random variable with \success probability" 0.1, so expected number of trials to rst \success" is $\frac{1}{0.1} = 10$. Note that this means \caught on the 10-th try", so the number in question is actually $10 - 1 = 9$.

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- What is the variance of this number?

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- What is the variance of this number?
  - The variance of a geometric r.v. is $\frac{1-p}{p^2}$. We are interested in the r.v. that is this r.v. minus 1, but that doesn't a ect the variance. Plugging in $p = 0.1$, this evaluates to $\frac{0.9}{0.01} = 90$.

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- We don't know. But using Markov's Bound,

$$\Pr[\text{READING} \geq 360] \leq \frac{E[\text{READING}]}{360} = \frac{5}{6}.$$

It is cold in here

- For the temperature inside the laboratory, READING will show a random integer (it now can be negative). The probability distribution of READING is still unknown, but its expected value is 30 and its variance is 36. What is Pr[READING > 0]?

It is cold in here

- For the temperature inside the laboratory, READING will show a random integer (it now can be negative). The probability distribution of READING is still unknown, but its expected value is 30 and its variance is 36. What is $\Pr[\text{READING} > 0]$?
- We don't know. But note that the standard deviation is $\sigma_{\text{READING}} = \sqrt{36} = 6$. Using Chebyshev's Bound,

$$
\begin{aligned}
\Pr[\text{READING} > 0] &\le \Pr[|\text{READING} - E[\text{READING}]| > 30] \\
&= \Pr[|\text{READING} - E[\text{READING}]| > 5\,\sigma_{\text{READING}}] \\
&< \frac{1}{5^2} = \frac{1}{25} = 4\%.
\end{aligned}
$$

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WHO IS THE IMPOSTOR??

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WHO IS THE IMPOSTOR??

Postlude

- **Impostor syndrome** can happen when people \doubt their skills, *talents, or accomplishments and have a persistent fear of being exposed as frauds".*[8]
- **NO ONE IS IMPOSTOR HERE. WE ALL BELONG.**

[8] Langford, J., & Clance, P. R. (1993). The imposter phenomenon: Recent research findings regarding dynamics, personality and family patterns and their implications for treatment. Psychotherapy: theory, research, practice, training, 30(3), 495.