CS230 SP24

LDoC recap/review

04/24/2024

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  - Therefore, do not fixate on the numerical part of the answers.

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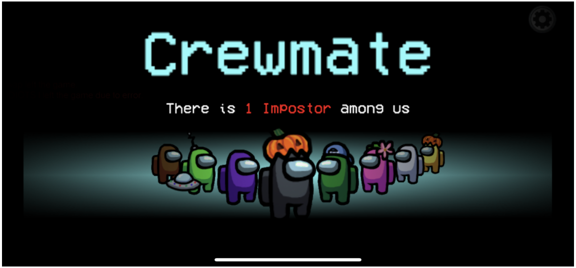

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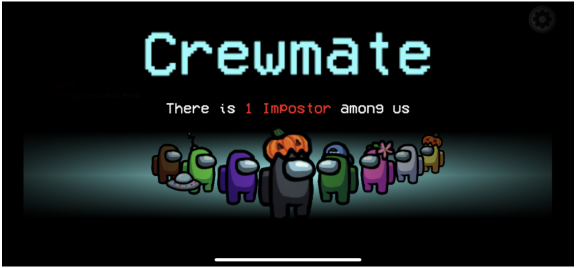

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- Disclaimer: I wrote this based on the **Among us** fandom wiki[1] and simplified as necessary. Please do not nitpick if you are an **Among us** expert.

[1] https://among-us.fandom.com/wiki/

Prelude

- In the most basic setting, players in a game are secretly divided into two teams: Crewmates and Impostors.
  - Impostors' goal is to blend in and kill/eliminate all crewmates.
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    Select two players from ten, $\binom{10}{2} = 45$.

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  Two. Impostors and Crewmates.

Selecting the impostor team

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- Note that all the above are known without specifying the underlying probability distribution.

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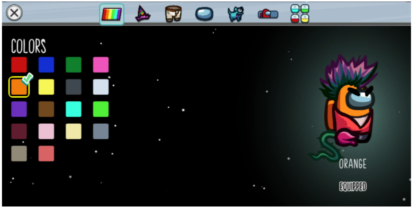
  - For every two consecutive games, we can define an indicator variable with expectation $\frac{1}{45}$. Using the **linearity of expectation**, the expectation of the sum of these variables is $\frac{44}{45}$. Did you say 1?

Dressing up



- Players in the lobby must select one of the 18 available colors: Red, Blue, Green, Pink, Orange, Yellow, Black, White, Purple, Brown, Cyan, Lime, Maroon, Rose, Banana, Gray, Tan, and Coral. Every player has to have a different color.

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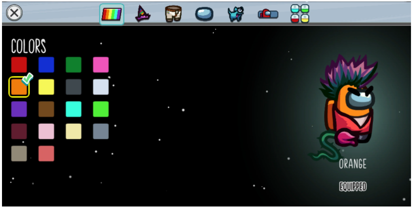

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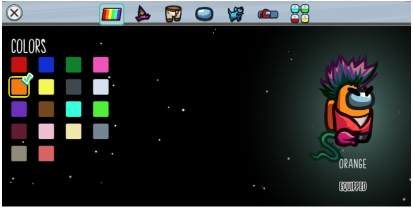

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  - $18 \times 17 \times 16 \times \ldots \times 9 = \frac{18!}{8!}$.

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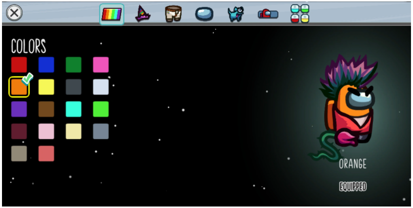

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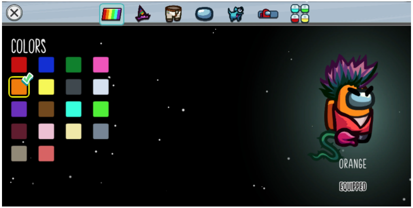

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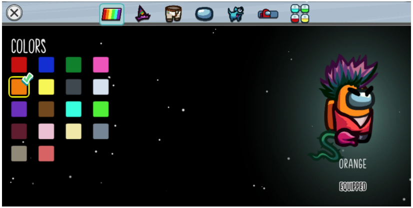

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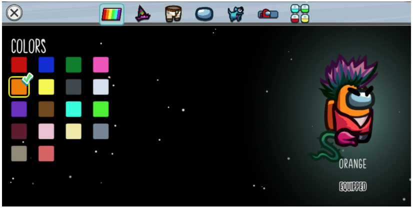

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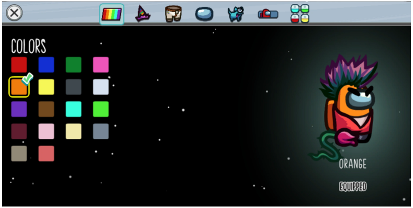

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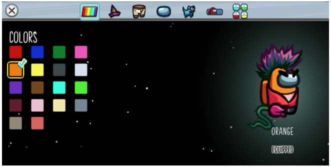
  - Therefore, all k players have the same color.
- **What is wrong with this induction?**
  The last step in the inductive step does not hold when $k = 1$ and $(k + 1) = 2$. In this case, the two sets P_1 and P_2 are disjoint.

Dressing up more



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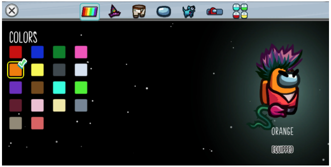

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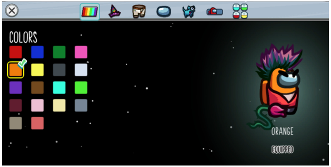

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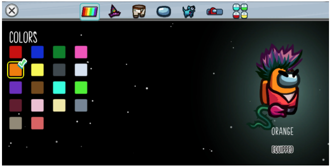

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- In how many different ways can one player customize their look, **disregarding color** (because that part depends on other players)?
  - Product rule: $(99+1)(12+1)(4+1)(2+1) = 19500$.
    Note the $+1$s in each category account for choosing nothing.

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  - In the early days of **Among us** the votes are shown directly with color coding (see figure), so it is clear who voted whom.

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  - If one player gets a majority of votes, the player gets ejected from the spaceship and eliminated from the game.
  - In the early days of **Among us** the votes are shown directly with color coding (see figure), so it is clear who voted whom.
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  - In the early days of **Among us** the votes are shown directly with color coding (see figure), so it is clear who voted whom.
- Assume this is an emergency meeting where all **10** players are alive (and thus can vote). How many different voting results can there be?
  - Each player has 11 actions (including skipping), so $11^{10} = 25937424601$.

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- Assume now **9** players are alive and thus can vote. How many different voting results can be displayed now?
  - Stars and bars. We need to distribute 9 votes across **10** candidates (treating all skipped votes as one "candidate"). So 9 stars and **9** bars. This is $\binom{9+9}{9} = \binom{18}{9} = 48620$.

Not voting out someone

- **Anonymous** voting with **8** players alive. **Nobody skipped.** How many different ways can we not vote out anyone?
  (Remember the player who got **the** highest number of votes is voted out, but no one is voted out if it is a tie.)

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  1. If there is a two-way tie of $3 \leq x \leq 4$ votes, the remaining $(8 - 2x)$ votes distribute across all other 6 players (so 5 bars).[4]

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  3. Finally... there may be a 8-way tie with 1 vote.

  $$1$$

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Communication is key

- According to **Among us**'s official Twitter account[5] in 2020, Impostors won roughly 60% of the time while Crewmates only won roughly 40% of the time.[6]

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  - We can denote I, C, and W as being an Impostor, being a Crewmate, and winning the game. Assume all the hidden assumptions hold, we have

    $$\Pr[W] = \Pr[W|I] \cdot \Pr[I] + \Pr[W|C] \cdot \Pr[C] = 0.6 \times 0.2 + 0.4 \times 0.8 = 44\%.$$

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  - We can apply Bayes's Theorem:

$$\Pr[I|W] = \frac{\Pr[W|I] \cdot \Pr[I]}{\Pr[W]} = \frac{0.6 \times 0.2}{0.44} = 27.3\%.$$

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Reactor meltdown

- Besides killing everyone, another way Impostors may win is through enacting a Reactor Meltdown. Crewmates then have to rush to the scene and resolve the issue in a certain time limit (e.g., 30 seconds). If Crewmates fail to do so, Impostors win the game.

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  - What is the **minimum** probability that some Crewmate resolves the reactor? It is just $1/5$. In this multiverse,
    either all Crewmates arrive in time (with probability $1/5$)
    or they all don't make it (with probability $4/5$).

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  - This evaluates to about 26.3%.

Death time

- Right before the countdown goes to 0, someone reported a body, which triggered a meeting to save the game.
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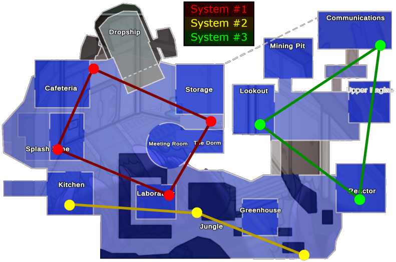
Venting



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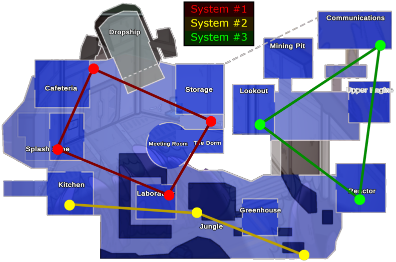

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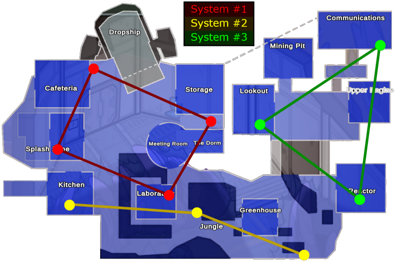

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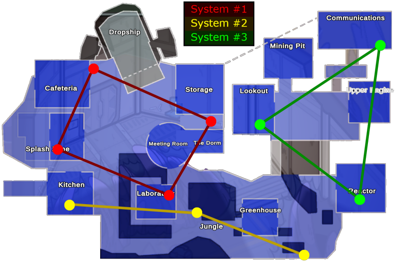

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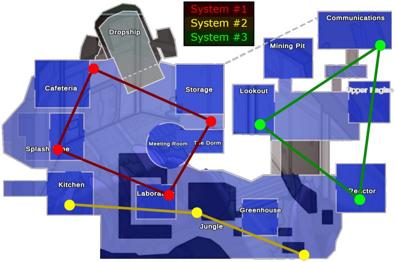

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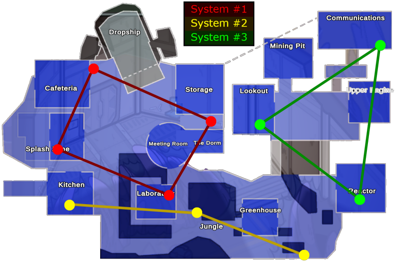

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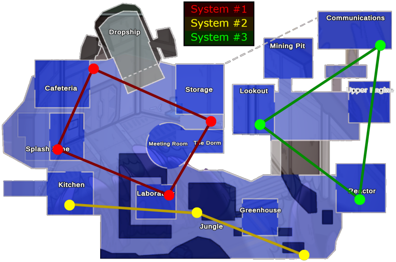
  3, but **not these colors on the graph**. We need only two colors for the red system (an even cycle) or the yellow system (a path), but the green system is a triangle that needs three colors.
- How many **red venting walks** of length **at most** 8 are there?
  - This includes entering a vent and staying there (length-0 walks).

Venting



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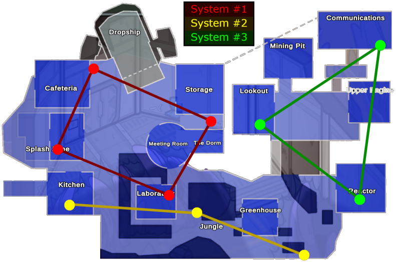

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- For each venting walk with length k, there are two walks with length $(k + 1)$ because there are two connecting red vents for the next step

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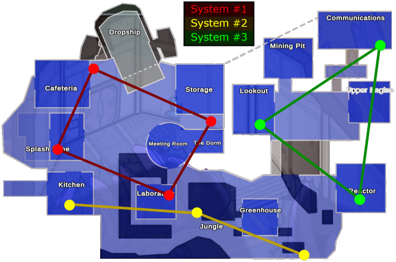

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- For each venting walk with length k, there are two walks with length $(k+1)$ because there are two connecting red vents for the next step
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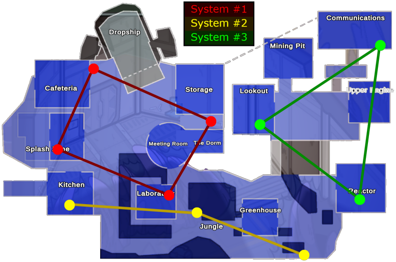
Venting



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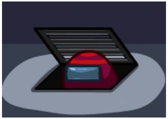
Don't get caught



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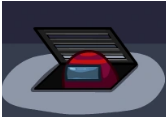

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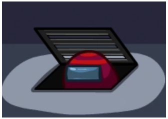

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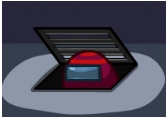

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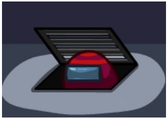

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- What is the variance of this number?
  - The variance of a geometric r.v. is $\frac{1-p}{p^2}$. We are interested in the r.v. that is this r.v. **minus 1**, but that doesn't affect the variance. Plugging in $p = 0.1$, this evaluates to $\frac{0.9}{0.01} = 90$.

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- We don't know. But using Markov's Bound,

$$\Pr[\texttt{READING} > 360] < \frac{\mathbf{E}[\texttt{READING}]}{360} = \frac{5}{6}.$$

It is cold in here

- For the temperature inside the laboratory, `READING` will show a random integer (it now can be negative). The probability distribution of `READING` is still unknown, but its **expected value** is -30 and its **variance** is 36. What is $\Pr[\texttt{READING} > 0]$?

It is cold in here

- For the temperature inside the laboratory, READING will show a random integer (it now can be negative). The probability distribution of READING is still unknown, but its **expected value** is -30 and its **variance** is 36. What is $\Pr[\mathtt{READING} > 0]$?
- We don't know. But note that the standard deviation is $\sigma_{\mathtt{READING}} = \sqrt{36} = 6$. Using Chebyshev's Bound,

$$\begin{aligned}\Pr[\mathtt{READING} > 0] &\leq \Pr[|\mathtt{READING} - \mathbf{E}[\mathtt{READING}]| > 30] \\ &= \Pr[|\mathtt{READING} - \mathbf{E}[\mathtt{READING}]| > 5 \cdot \sigma_{\mathtt{READING}}] \\ &< \frac{1}{5^2} = \frac{1}{25} = 4\%.\end{aligned}$$

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WHO IS THE IMPOSTOR??

Postlude

- **Impostor syndrome** can happen when people *"doubt their skills, talents, or accomplishments and have a persistent fear of being exposed as frauds"*.[8]
- **NO ONE IS IMPOSTOR HERE. WE ALL BELONG.**

[8] Langford, J., & Clance, P. R. (1993). The imposter phenomenon: Recent research findings regarding dynamics, personality and family patterns and their implications for treatment. Psychotherapy: theory, research, practice, training, 30(3), 495.