Class starts after this song

Bad Bunny, Jhay Cortez – Dakiti (2020)

requested by Prince Ahmed (Recitation 08D)

One fun fact about me is that I was raised in Rome so I speak Italian fluently. Feel free to reach out if you'd like to practice Italian or maybe even have me explain the content in Italian for some extra flair!

Duke

CS230 Spring 2024
EM C: Voting and Social Choice

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Poll (Not a PI) - seasons

- In general, think about a poll with n voters and m candidates.
- Treating voters as distinct,
  how many possible "raw votes" are there if:
  - Each vote is for one candidate? m^n
  - Each vote is a total order/full ranking? $(m!)^n$
- Why total order?

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Finding “common ground”

Vote 1: *Spring* $\succ$ *Summer* $\succ$ *Fall* $\succ$ *Winter*

Vote 2: *Fall* $\succ$ *Summer* $\succ$ *Winter* $\succ$ *Spring*

Vote 3: *Winter* $\succ$ *Summer* $\succ$ *Spring* $\succ$ *Fall*

Rank aggregation

- A voting rule (rank aggregation mechanism) is a function:
  - Domain = *set of possible raw votes* ($(m!)^n$ *of them)*
  - Codomain = *set of candidates*

$$f\left(\begin{array}{l}\text{Vote 1: } \textit{Spring} \succ \textit{Summer} \succ \textit{Fall} \succ \textit{Winter} \\ \text{Vote 2: } \textit{Fall} \succ \textit{Summer} \succ \textit{Winter} \succ \textit{Spring} \\ \text{Vote 3: } \textit{Winter} \succ \textit{Summer} \succ \textit{Spring} \succ \textit{Fall}\end{array}\right) = \textit{Summer}$$

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Think, pair, share:
What makes a good rule?

aka

What properties do we want our function to satisfy?

Non-dictatorship

- Note that the n voters are distinguishable, so our function f can treat different voters differently.

- However, we do NOT want there to be a voter whose vote solely dictates the result of f

Majority

"If a is ranked first by more than half of the voters, a should win"

Unanimity

“If a is ranked first by **all** voters, a should win”

Weak unanimity

"If everyone ranks a above b, then b should not win"

Pareto Efficiency

“If everyone ranks a above b,
the result should prefer a over b (so b will not win)”

Voter-equality

We should ensure complete fairness among the voters:
"any permutation of individual votes should get the same result"

$$f\begin{pmatrix} \textit{Spring} \succ \textit{Summer} \succ \textit{Fall} \succ \textit{Winter} \\ \textit{Fall} \succ \textit{Summer} \succ \textit{Winter} \succ \textit{Spring} \\ \textit{Winter} \succ \textit{Summer} \succ \textit{Spring} \succ \textit{Fall} \end{pmatrix} = f\begin{pmatrix} \textit{Winter} \succ \textit{Summer} \succ \textit{Spring} \succ \textit{Fall} \\ \textit{Spring} \succ \textit{Summer} \succ \textit{Fall} \succ \textit{Winter} \\ \textit{Fall} \succ \textit{Summer} \succ \textit{Winter} \succ \textit{Spring} \end{pmatrix}$$

Monotonicity

- If a voter moves a higher in their individual ranking, this should "never harm a" *(vague)*
  - *(**weak monotonicity**) if a wins given the current votes,*
    then it should still win if a voter moves a even higher in their individual ranking
    (while keeping other parts of the ranking intact)
  - *(**strong monotonicity**) if a wins given the current votes,*
    then it should still win if a voter moves a even higher in their individual ranking
    (as long as no other candidate "jumps ahead" a)

Independence of Irrelevant Alternatives

- If the current rule ranks a above b given the current votes, it should still prefer a over b:
  - *If we add a new candidate c to the poll (so every voter inserts c into their ranking)*
  - *If we remove an existing candidate c from the poll (voters delete c from rankings)*
  - *Note that c might become the new winner or a might, but b cannot become the winner*

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Arrow's impossibility theorem (1951)

- There is no voting rule that is simultaneously:
  - Non-dictatorial (no voter solely dictates the result of f)
  - Pareto efficient (if everyone ranks a above b, the result ranks a over b)
  - IIA (If the rule ranks a above b given the current votes,
    it should still prefer a over b when we add/remove an irrelevant c)

  as long as there are 3 candidates.

Muller-Satterthwaite's impossibility theorem (1977)

- There is no voting rule that is simultaneously:
  - Non-dictatorial (no voter solely dictates the result of f)
  - Weak unanimous (if everyone ranks a above b, b will not win)
  - Strong monotonic (if a wins given the current votes,
    then it should still win if a voter moves a even higher in their individual ranking, as long as no other candidate jumps ahead of a)

as long as there are 3 candidates.

All rules are imperfect.
But some are useful.

- Scoring rules
- Runoffs
- Pairwise elections-based rules

Scoring rules

$$f\left(\begin{matrix}\text{Spring} \succ \text{Summer} \succ \text{Fall} \succ \text{Winter} \\ \text{Fall} \succ \text{Summer} \succ \text{Winter} \succ \text{Spring} \\ \text{Winter} \succ \text{Summer} \succ \text{Spring} \succ \text{Fall}\end{matrix}\right) = argmax\left\{\begin{matrix}\text{score(Spring)}, \\ \text{score(Summer)}, \\ \text{score(Fall)}, \\ \text{score(Winter)}\end{matrix}\right\}$$

4 | 3 | 2 | 1

- Plurality: $(1,0,0,\ldots,0)$
- Veto (anti-plurality): $(1,1,1,\ldots,1,0)$ Sp: 2 Su: 3 Fa: 2 Wi: 2
- Borda: $(m,(m-1),(m-2),\ldots,1)$ or $((m-1),(m-2),\ldots,1,0)$

Wi = Fa = Sp: 7 Su: 9

Runoffs

- Use whatever (plurality, anti-plurality, Borda, etc.) to determine the final two candidates a, b
- Then whichever is ranked higher by more voters wins

Single Transferable Vote (STV)

- Use plurality, drop only one candidate each round
- Repeat for $m - 1$ rounds; the single candidate left wins

Spring $\succ$ ~~*Summer*~~ $\succ$ *Fall* $\succ$ ~~*Winter*~~
Spring $\succ$ ~~*Summer*~~ $\succ$ *Fall* $\succ$ ~~*Winter*~~
Fall $\succ$ ~~*Summer*~~ $\succ$ ~~*Winter*~~ $\succ$ *Spring*
Fall $\succ$ ~~*Summer*~~ $\succ$ ~~*Winter*~~ $\succ$ *Spring*
~~*Winter*~~ $\succ$ ~~*Summer*~~ $\succ$ *Spring* $\succ$ *Fall*

r1: summer gets 0 pts and lost

r2: winter gets 1 pt and lost

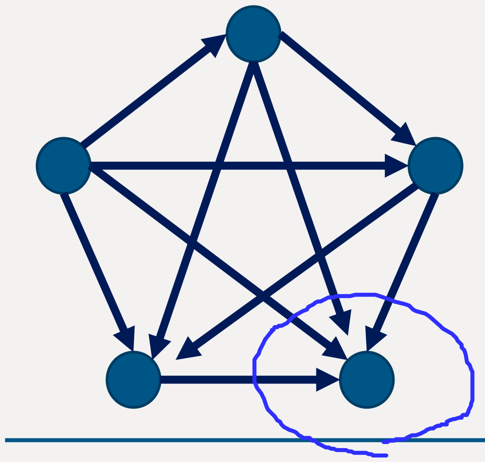
r3: spring: 3 pt, fall: 2pt

Pairwise Elections

- Let's consider all pairs of candidates (how many?) $\binom{m}{2}$
- Prefer a over b if more voters ranked a higher than b

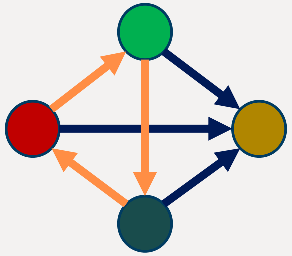
Pairwise Election Graphs

- Directed graph with m vertices (each representing a candidate)
- Edge from (pairwise) loser to winner (assume n odd so no ties)



- Who wins if the graph is acyclic?
- Is the graph always acyclic?

Spring $\succ$ *Summer* $\succ$ *Fall* $\succ$ *Winter*
Fall $\succ$ *Summer* $\succ$ *Winter* $\succ$ *Spring*
Winter $\succ$ *Summer* $\succ$ *Spring* $\succ$ *Fall*



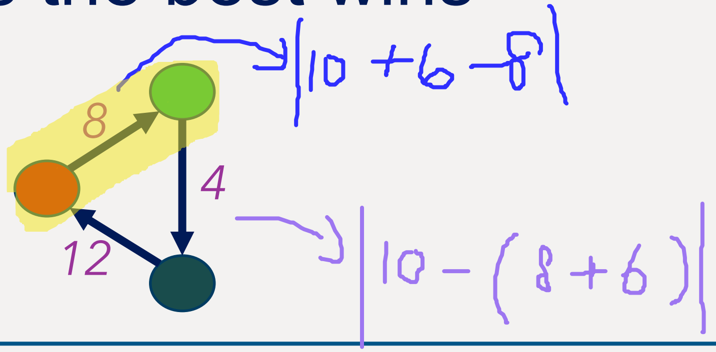
copeland: every candidate gets 2pt

simpson: spring's worst loss is by 4, which is the best so spring wins

Pairwise Elections-Based Rules

- Copeland: every candidate gets 2 points for each election won (if n even and there are ties, award 1 point to each candidate)
- Simpson: weight each edge by gap/margin; the candidate whose "worst loss" is the best wins

(*Spring* ≻ *Fall* ≻ *Winter*) x 10
(*Fall* ≻ *Winter* ≻ *Spring*) x 8
(*Winter* ≻ *Spring* ≻ *Fall*) x 6



- Slater: create an acyclic version of the graph by reversing as few edges as possible (NP-hard)
- Kemeny: create an acyclic version of the graph by reversing as little edge weight as possible (NP-hard)

Manipulating

- One severe limitation of everything discussed today is we assumed people would vote truthfully (i.e., according to their individual preference). This is often not true.

- Need game theory to rigorously study manipulation (take CS323/535)