

# Vertical Market with Bargaining

Allan Collard-Wexler  
Duke

October 26, 2016

# Outline

**Markets with Bargaining**

**Nash Bargaining and Rubinstein Bargaining Model**

**Nash Bargaining and Rubinstein Bargaining Model**

**Buyer Size and Negotiated Contracts**

# Markets with Bargaining

- ▶ Many of the market power issues we are looking into involve *bilateral oligopoly*.
- ▶ These are markets where input suppliers and final goods producers are both concentrated: there is monopsony and monopoly power.
- ▶ In these markets, we think that prices are negotiated.
- ▶ The idea of “countervailing market power” is an old idea. Galbraith (1954) coined it to discuss the effects of consequences of say GM bargaining with UAW on wages. The idea being that concentration in the demand side for labor (GM), is countered by concentration in the supply of labor (the UAW union).

# Motivation: Surplus Division in Bilateral Oligopoly

- ▶ Many current antitrust cases hinge on understanding concentrated upstream and downstream markets.
  - ▶ Hachette and Amazon E-Book pricing case.
  - ▶ Accountable Care Organization (ACO), and Hospital Mergers.
  - ▶ Cable TV Mergers.
  - ▶ Net Neutrality debate: Verizon and Netflix.
  - ▶ Rise of large chain stores (Walmart or Tesco).
- ▶ Comcast and Time Warner announced that they are merging: over 30% of U.S. Cable TV consumers.
- ▶ How should we think of these mergers?
  - ▶ These firms do not compete over the same customers: non-overlapping local cable monopolies?
  - ▶ However, providers of content; e.g. ESPN, Big Ten Network, are more worried about the merger.
  - ▶ Should we approve a merger in a context of bilateral oligopoly?

# Overview of Health Care

- ▶ Surge of work in economics on healthcare. Most interestingly at the intersection of industrial organization and health.
- ▶ Much of this is spurred by new healthcare programs, that were designed (for good or bad) to be served with a regulated private market:
  - ▶ Medicare Part D: Drug Insurance for old people (above 65).
  - ▶ Affordable Care Act: Health Insurance for people with bad jobs, but not too poor.
- ▶ Some of this interest is due to the large amount of merger activity among hospitals: about a third of the FTC's caseload.
- ▶ Health Care is 18% of the economy. A very large share of this is public money. This makes the U.S. healthcare sector the 6th largest economy in the world: larger than Canada (and the U.K.).
- ▶ Hospital Services are 6% of the economy.
- ▶ Data on health care, in some places, is amazing: all admissions, reimbursements from medicare.

# Arrow on Healthcare

Ken Arrow “Uncertainty and the Welfare Economics of Healthcare” (AER 1963).

- ▶ Consumers don't know what they are buying.
- ▶ Consumers don't pay the price for what they consume.
- ▶ Physicians might not have the same incentives as patients: there is an agency problem.
- ▶ We do not feel it “fair” to exclude people from life saving medical care.
- ▶ Health insurers do not know how much an insurer will use health care.
- ▶ Government is the largest player in this market, and regulates the market extensively.

# Healthcare in the United States: a short survey (aside)

- ▶ Medicare Part D: Drug Insurance for the Old. Privately provided.
- ▶ Medicare Part A and B: Medical Insurance for the old: Privately provided, but regulated prices.
- ▶ Medicare Advantage: Medicare, but provided by private health insurance companies.
- ▶ Medicaid: Health Care for the poor. Provided by the states, but federally funded.
- ▶ Affordable Care Act (ACA) – Obamacare: Health Insurance provided on competitive exchanges.

# Nash Bargaining

Nash (1950), not Nash (1951) which introduces Nash-Equilibrium as a solution concept, presents an axiomatic solution for bargaining problems. Consider the bargaining problem over  $x$ , where you can think of there being one dollar on the table, and  $x$  denotes the share that goes to player 1, and thus,  $1 - x$  goes to player 2. More generally, think of players as receiving utilities  $u_i(x)$ . So we have:

$$\mathcal{U} = \{(v_1, v_2) \mid v_1 = u_1(x), v_2 = u_2(x)\}$$

Other examples include:

- ▶ Split the dollar:  $u_1 = x$ ,  $u_2 = 1 - x$ .
- ▶ Manufacturer-Retailer (from a couple of lectures ago), where  $t$  is the wholesale price and demand is  $P = a - Q$ :

$$u_1 = \pi^M(t) = (t - c) \frac{a - t}{2}$$

$$u_2 = \pi^R(t) = \frac{(a - t)^2}{4}$$



# Nash Bargaining

The Nash Bargaining Solution is the  $x$  that maximizes the Nash Product:

$$\mathcal{N} = \max_x (u_1(x) - d_1)(u_2(x) - d_2)$$

where  $d_1$  and  $d_2$  denote the reservation value, the utility that 1 and 2 get if they don't agree. This is to make sure that payoffs are split relative to what the parties would get absent agreement, and, in particular, insures that bargaining is better than walking away from the table.

# Generalized Nash Bargaining

The Generalized Nash Bargaining Solution is the  $x$  that maximizes the Nash Product:

$$\mathcal{GN} = \max_x (u_1(x) - d_1)^\alpha (u_2(x) - d_2)^{1-\alpha}$$

where  $\alpha$  denotes the weights on the utilities of agent 1 and agent 2.

# Solving Nash Bargaining

Suppose we have a split the dollar game: The Nash Bargaining Solution is the  $x$  that maximizes the Nash Product:

$$\mathcal{N} = \max_x x(1 - x)$$

taking first-order conditions with respect to  $x$ :

$$\frac{\partial \mathcal{N}}{\partial x} = 0$$

$$(1 - x) - x = 0$$

$$x = \frac{1}{2}$$

# Solving Generalized Nash Bargaining

Suppose we have a split the dollar game: The Generalized Nash Bargaining Solution is the  $x$  that maximizes the Nash Product:

$$\mathcal{GN} = \max_x x^\alpha (1-x)^{1-\alpha}$$

taking first-order conditions with respect to  $x$ :

$$\begin{aligned}\frac{\partial \mathcal{GN}}{\partial x} &= 0 \\ \alpha x^{\alpha-1} (1-x)^{1-\alpha} - (1-\alpha) x^\alpha (1-x)^{-\alpha} &= 0 \\ x &= \alpha\end{aligned}$$

So the generalized solution allows for differences in surplus split

## Manufacturer Retailer Example

Recall the profit functions are:

$$u_1 = \pi^M(t) = (t - c) \frac{a - t}{2}$$

$$u_2 = \pi^R(t) = \frac{(a - t)^2}{4}$$

Which gives:

$$\mathcal{N} = \max_t \left( (t - c) \frac{a - t}{2} \right) \left( \frac{(a - t)^2}{4} \right)$$

taking first-order conditions with respect to  $t$ ,  $\frac{\partial \mathcal{N}}{\partial t} = 0$ :

$$\frac{a - t}{2} \frac{(a - t)^2}{4} + \frac{a - t}{2} \frac{(a - t)^2}{4} - 2 \frac{a - t}{4} (t - c) \frac{a - t}{2} = 0$$

$$a - t + c - t - 2t + 2c = 0$$

$$t = \frac{a + 3c}{4}$$

## Manufacturer Retailer Example Ctd.

- ▶ Recall that the solution for the game where the manufacturer sets the wholesale price is  $t = \frac{a+c}{2}$ .
- ▶ What would the wholesale price be if, instead, it was the retailer who set it (and then the manufacturer produces)?

$t = c$  right?

- ▶ So how should we look at the bargaining solution of  $t = \frac{a+3c}{4}$ ?

## Outside Options

Suppose we have a split the dollar game, but firm 1's outside option is 0.8, but firm 2's outside option is 0.1.

The Nash Bargaining Solution is the  $x$  that maximizes the Nash Product:

$$\mathcal{N} = \max_x (x - 0.8)(1 - x - 0.1)$$

taking first-order conditions with respect to  $x$ :

$$\begin{aligned}\frac{\partial \mathcal{N}}{\partial x} &= 0 \\ (0.8 - x) - (x - 0.9) &= 0 \\ x &= 0.85\end{aligned}$$

Basically, what is left is 0.1 units of surplus  $1 - (0.8 + 0.1) = 0.1$ , which is split 50-50.

# Axioms for Nash Bargaining

Nash (1950) shows that the Nash Bargaining solution is the only one that satisfies the following axioms:

- ▶ Pareto optimality
- ▶ Invariant to affine transformations or Invariant to equivalent utility representations.
- ▶ Symmetry: if the payoffs are the same, then the split of surplus should not depend on the identity of the firm.
- ▶ Independence of irrelevant alternatives: adding an option that won't get chosen should not change the split of surplus.



# Rubinstein Bargaining Model

Setup of Rubinstein (1982):

- ▶ Consider  $U$  and  $D$ , who bargain over pie of size 1 (which  $D$  gets)
- ▶ In odd periods,  $D$  makes offer of  $p \in [0, 1]$ , where  $D$  pays  $p$  to  $U$  (and keeps  $1 - p$ ).  $U$  can accept or reject. If  $U$  rejects, in even periods,  $U$  makes counteroffer
- ▶ Discount factors are given by  $\delta_{i,U} = \exp^{-r_i \Lambda}$ , where  $\Lambda$  is time between periods.
- ▶ Thus the pie is melting:  $1, \delta, \delta^2, \delta^3, \dots, 0$ .

# Review / Fundamentals

## Nash Bargaining and Rubinstein Bargaining Model

Proof of Proposition (Sutton) Uniqueness:

- ▶ Let  $\underline{v}_D$  and  $\bar{v}_D$  denote the lowest and highest payoffs  $D$  can get in any SPE starting in an odd period.
- ▶ Consider any even period. Note  $D$  will accept anything with payoffs greater than  $\delta_D \bar{v}_D$  and reject anything less than  $\delta_D \underline{v}_D$ . So  $U$  can get at least  $(1 - \delta_D \bar{v}_D)$  and at most  $(1 - \delta_D \underline{v}_D)$ .
- ▶ Consider odd period.  $D$  needs to offer at least  $\delta_U(1 - \delta_D \bar{v}_D)$  to get  $U$  to accept.  $U$  will also definitely accept if  $D$  offers  $\delta_U(1 - \delta_D \underline{v}_D)$ . Thus:

$$\bar{v}_D \leq \delta_U(1 - \delta_D \bar{v}_D)$$

$$\underline{v}_D \geq \delta_U(1 - \delta_D \underline{v}_D)$$

- ▶ This implies:

$$\underline{v}_D \geq \frac{1 - \delta_U}{1 - \delta_D \delta_U} \geq \bar{v}_D$$

But since  $\underline{v}_D \leq \bar{v}_D$ ,  $D$  must receive  $(1 - \delta_U)/(1 - \delta_D \delta_U)$  in any SPE beginning in odd period.

# Review / Fundamentals

## Nash Bargaining and Rubinstein Bargaining Model

### Proposition (Rubinstein 1982)

$$\text{Let } p_R^N = \frac{1 - \delta_D}{1 - \delta_D \delta_U}$$

There is a unique SPE in the infinite-horizon sequential bargaining game.

### Relationship between Nash bargaining and Rubinstein solution

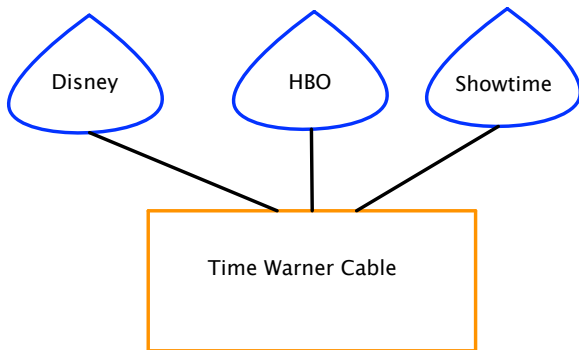
Nash bargaining solution:

$$p_{NB}^N = \arg \max_p [1 - p]^{b_D} \times [p]^{b_U} = \frac{b_U}{b_U + b_D}$$

Binmore Rubinstein Wolinsky (1986) note:

$$p_{NB}^N = \lim_{\Lambda \rightarrow 0} p_R^N$$
$$\frac{b_U}{b_U + b_D} = \lim_{\Lambda \rightarrow 0} \frac{1 - \delta_D}{1 - \delta_D \delta_U}$$

# Example of a Bilateral Market



$$\pi^{TW}(D, HBO, S) = 1$$

$$\pi^{TW}(D, S) = 0.9$$

$$\pi^{TW}(D, HBO) = 0.8$$

$$\pi^{TW}(HBO, S) = 0.8$$

$$\pi^{TW}(D) = 0.5$$

$$\pi^{TW}(HBO) = 0.5$$

$$\pi^{TW}(S) = 0.5$$

## Existing Literature: Who should make take-it or leave-it offers?

Suppose we want to evaluate the effects of a merger between all three upstream firms.

- ▶ Suppose that Upstream Firms (HBO, Showtime, Disney) Make take-it or leave-it offers.
- ▶ Suppose that Downstream Firms (Time Warner Cable) Make take-it or leave-it offers.

# Existing Literature: Who should make take-it or leave-it offers?

Suppose we want to evaluate the effects of a merger between all three upstream firms.

- ▶ Suppose that Downstream Firms (Time Warner Cable) Make take-it or leave-it offers.
  - ▶ Prices Before Merger  
 $p^{HBO} = 0, p^S = 0, p^D = 0.$
  - ▶ Prices After Merger  
 $p^{HBO} = 0, p^S = 0, p^D = 0.$
- ▶ Suppose that Upstream Firms Make take-it or leave-it offers.
  - ▶ Prices Before Merger  
 $p^{HBO} = 0.1, p^S = 0.2, p^D = 0.2$  (Nash Equilibrium)
  - ▶ Prices After Merger  
 $\sum p^{HBO} + p^S + p^D = 1$  (Price for the bundle of all channels), instead of  
 $\sum p^{HBO} + p^S + p^D = 0.5$  bundle price before the merger.

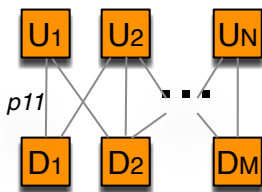
# Existing Literature: Who should make take-it or leave-it offers?

Suppose we want to evaluate the effects of a merger between all three upstream firms.

- ▶ Suppose that Downstream Firms (Time Warner Cable) Make take-it or leave-it offers.
  - ▶ Prices Before Merger  
 $p^{HBO} = 0, p^S = 0, p^D = 0.$
  - ▶ Prices After Merger  
 $p^{HBO} = 0, p^S = 0, p^D = 0.$
- ▶ Suppose that Upstream Firms Make take-it or leave-it offers.
  - ▶ Prices Before Merger  
 $p^{HBO} = 0.1, p^S = 0.2, p^D = 0.2$  (Nash Equilibrium)
  - ▶ Prices After Merger  
 $\sum p^{HBO} + p^S + p^D = 1$  (Price for the bundle of all channels), instead of  
 $\sum p^{HBO} + p^S + p^D = 0.5$  bundle price before the merger.

## “Work Horse” Model:

- ▶  $N$  Upstream firms  $U_1, \dots, U_N$
- ▶  $M$  Downstream firms  $D_1, \dots, D_M$ .
- ▶  $\mathcal{G}$  is set of all agreements.
- ▶ Primitives:  $\pi_i^U(\mathcal{A})$  and  $\pi_j^D(\mathcal{A})$ 
  - ▶ for all  $\mathcal{A} \subseteq \mathcal{G}$
  - ▶ Allows for externalities
- ▶  $U_i$  and  $D_j$  bargain over  $p_{ij}$



### Horn Wolinsky (1988) (Generalized)

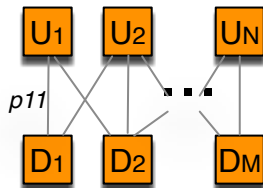
$$p_{ij}^N = \arg \max_p [\pi_j^D(\mathcal{G}) - \pi_j^D(\mathcal{G} \setminus ij) - p]^{b_{i,D}} \times [\pi_i^U(\mathcal{G}) - \pi_i^U(\mathcal{G} \setminus ij) + p]^{b_{i,U}}$$
$$= \frac{b_{i,U} \Delta \pi_j^D(\mathcal{G}, ij) - b_{j,D} \Delta \pi_i^U(\mathcal{G}, ij)}{b_{i,U} + b_{j,D}}, \forall i = 1, \dots, N, j = 1, \dots, M.$$

Each price maximizes Nash product given other prices: hence  
“Nash-in-Nash bargains”



## “Work Horse” Model:

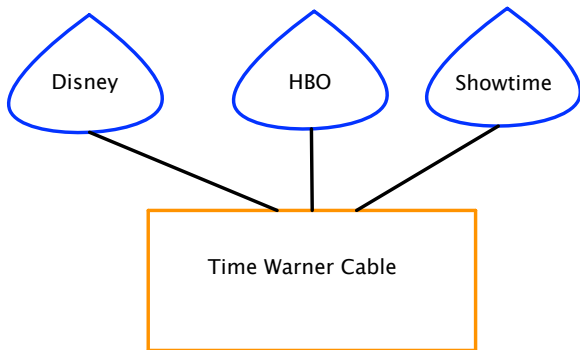
- ▶  $N$  Upstream firms  $U_1, \dots, U_N$
- ▶  $M$  Downstream firms  $D_1, \dots, D_M$ .
- ▶  $\mathcal{G}$  is set of all agreements.
- ▶ Primitives:  $\pi_i^U(\mathcal{A})$  and  $\pi_j^D(\mathcal{A})$ 
  - ▶ for all  $\mathcal{A} \subseteq \mathcal{G}$
  - ▶ Allows for externalities
- ▶  $U_i$  and  $D_j$  bargain over  $p_{ij}$



### Horn Wolinsky (1988) (Generalized)

$$\begin{aligned} p_{ij}^N &= \arg \max_p [\pi_j^D(\mathcal{G}) - \pi_j^D(\mathcal{G} \setminus ij) - p]^{b_{j,D}} \times [\pi_i^U(\mathcal{G}) - \pi_i^U(\mathcal{G} \setminus ij) + p]^{b_{i,U}} \\ &= \frac{b_{i,U} \Delta \pi_j^D(\mathcal{G}, ij) - b_{j,D} \Delta \pi_i^U(\mathcal{G}, ij)}{b_{i,U} + b_{j,D}}, \forall i = 1, \dots, N, j = 1, \dots, M. \end{aligned}$$

Each price maximizes Nash product given other prices: hence  
“Nash-in-Nash bargains”



$$\pi^{TW}(D, HBO, S) = 1 \qquad \pi^{TW}(HBO, S) = 0.8$$

$$\pi^{TW}(D, S) = 0.9 \qquad \pi^{TW}(HBO) = 0.5$$

$$\pi^{TW}(D, HBO) = 0.8 \qquad \pi^{TW}(S) = 0.5$$

$$\pi^{TW}(D) = 0.5$$

# Effects of a Merger

- ▶ Say HBO and Disney merge. What is the effect on the prices that they receive?

$$\underbrace{MC}_{\text{marginal contribution}} = \pi^{TW}(D, HBO, S) - \pi^{TW}(S) \\ = 1 - 0.5 = 0.5$$

$$\underbrace{P^{\{D, HBO\}}}_{\text{price}} = \underbrace{\frac{1}{2}}_{\text{even surplus split}} \times 0.5 = 0.25$$

- ▶ Price without a merger:

$$p^D + p^{HBO} = 0.05 + 0.10 = 0.15$$

# Some Nomenclature

## Horn Wolinsky (1988) (Generalized)

$$p_{ij}^N = \arg \max_p [\pi_j^D(\mathcal{G}) - \pi_j^D(\mathcal{G} \setminus ij) - p]^{b_{j,D}} \times [\pi_i^U(\mathcal{G}) - \pi_i^U(\mathcal{G} \setminus ij) + p]^{b_{i,U}}$$

- ▶ A high  $\pi_j^D(\mathcal{G} \setminus ij)$  means that the firm has high outside options: it can extract a higher price.
- ▶ A high  $b_{j,D}$  means that the firm can extract most of the surplus split.
- ▶ Notice that there is never disagreement in this model, but the threat of disagreement disciplines the players in the game.

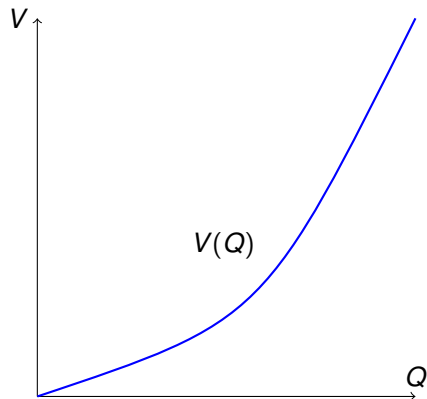
## Chipty and Snyder, Stole and Zwiebel

- ▶ Do mergers, or unions, raise or lower prices.
- ▶ Suppose the buyer has a valuation of  $V(Q)$ , where  $Q$  is quantity.
- ▶ Each Supplier can produce 1 unit, unless they merge, and have size  $X$ . They have marginal costs of  $c$ .
- ▶ Will larger suppliers get better prices?
- ▶ Prices are determined by:

$$p = \frac{b_S}{b_S + b_B} ([V(Q) - V(Q - 1)] - c)$$

# Chitty and Snyder, Stole and Zwiebel Ctd.

Convex Value Function



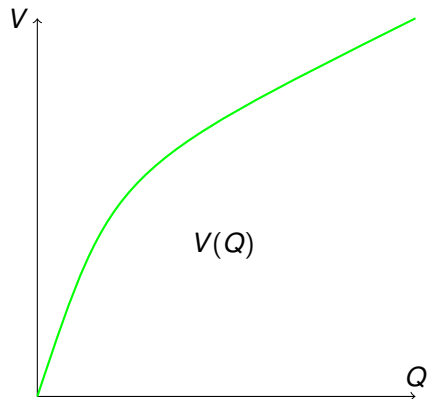
We will obtain:

$$\frac{V(\bar{Q}) - V(\bar{Q} - A)}{A} \leq \frac{V(\bar{Q}) - V(\bar{Q} - B)}{B}$$

for any  $A \leq B$ . This means that larger firms pay more,

# Chitty and Snyder, Stole and Zwiebel Ctd.

Concave Value Function



We will obtain:

$$\frac{V(\bar{Q}) - V(\bar{Q} - A)}{A} \geq \frac{V(\bar{Q}) - V(\bar{Q} - B)}{B}$$

for any  $A \geq B$ . This means that larger firms pay more,

# Evidence on the shape of $V(Q)$ from advertising

FIGURE 4.—SERIES ESTIMATION OF ADVERTISING REVENUE FUNCTION

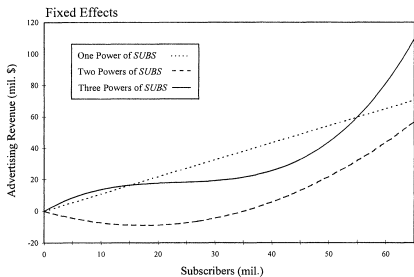


FIGURE 5.—CUBIC ADVERTISING REVENUE FUNCTION WITH CONFIDENCE SLEEVE

