Differentiated Products and Evidence on Horizontal Mergers

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Differentiated Products

- We drop the assumption that firms offer homogeneous products
- Differentiated product models are among the most realistic and useful of all models in Industrial Organization.
- If you understand the basic elements of product differentiation theories, then you should have an awareness of the economics underlying:
  1. product placement.
  2. niche markets.
  3. product design to target certain types of consumers.
  4. brand proliferation, etc.

What is a Differentiated Products

- Differentiated products are “similar", but not identical (they are close, but not perfect, substitutes).
  1. different cars.
  2. different breakfast cereals.
- There are two major models of differentiation:
  1. **Horizontal differentiation**: if all products were the same price, consumers disagree on which product is most preferred.
     Eg. films, cars, clothes, books, cereals, ice-cream flavors, Starbucks (by geographic location), ...
  2. **Vertical differentiation**: if all products were the same price, all consumers agree on the preference ranking of products, but differ in their willingness to pay for the top ranked versus lesser ranked products
     Eg. computers, airline tickets, different quantities, car packages, ...

Price Competition Among Horizontally Differentiated Firms: AKA Bertrand with Differentiated Products

The main idea comes from Hotelling (1929), referred to as "Hotelling's Linear City"

1. Imagine a beach that is one mile long
2. Suppose that 1,000 people are uniformly distributed along the beach
3. Thus, in each quarter mile there are 250 people, for example
4. There are two ice-cream sellers, one at each end of the beach
5. People like ice-cream, but an individual may have to walk as far as half a mile to get to the nearest seller
6. Say the cost of walking one mile (because of the effort) is t
7. Ignore the fact that the individual would have to walk back to their spot on the beach after they buy an ice-cream (we can include it or not, makes no difference)

Hotelling Model Ctd.

- If a person walks half a mile to buy an ice-cream, and then pays price p for the ice-cream when she gets there, the total cost to the individual is $\frac{1}{2}t + p$
- The point is, the total price a person pays to consume a product is equal to the price of the product plus some amount reflecting how close the product is to their ideal product.
- The closer a person is to the ice-cream seller, the closer the product is to their ideal point, the lower will be the overall price the person pays to consume the ice-cream.

Hotelling Model Ctd.

- A person on the beach located at any distance x from the ice-cream seller at the left-hand-side, will be at a distance of $(1 - x)$ from the seller on her right-had-side.
- Say that the price at the left-seller is p_l and the price at the right-seller is p_r. For the person sitting at x, the effective prices are:

$$p_l + xt$$

$$p_r + (1 - x)t$$

- We generally presume there exists the option of not buying an ice-cream at all, in which case the person pays nothing but also gains nothing. Suppose all people regardless of where they are located on the beach, have willingness-to-pay of v for an ice-cream.

Hotelling Model Consumer's Decision.

- What is the consumer's problem? Maximize utility:

$$\max(v - (p_l + xt), v - (p_r + (1-x)t), 0)$$

  You don't always go to the closest guy: it depends on relative prices p_l, p_r.
- Suppose everyone buys an ice-cream. Then there is a marginal consumer sitting at x^* for whom

$$u(\text{go left}) = u(\text{go right})$$

$$v - (p_l + x^* t) = v - (p_r + (1 - x^*)t)$$

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Hotelling Model Ctd.

Rearranging, we get:

$$p_l + x^* t = p_r + (1 - x^*)t$$

solve for x^*:

$$x^* = \frac{1}{2t}(p_r - p_l + t)$$

Hotelling Model Ctd.

All people to the left of x^* will strictly prefer going to the left seller.

All people to the right of x^* will strictly prefer going to the right seller.

Since we assumed there were 1000 people uniformly distributed along the beach, this means that

$$q_l = 1000\left(\frac{1}{2t}(p_r - p_l + t)\right) = 500 - \frac{500}{t}p_l + \frac{500}{t}p_r$$
$$q_r = 1000\left(\frac{1}{2t}(p_l - p_r + t)\right) = 500 + \frac{500}{t}p_l - \frac{500}{t}p_r$$

These are demand functions for horizontally differentiated products!

Hotelling Model Ctd.

Points to note:

1. if $p_l > p_r$, then $q_l \neq 0$ (unlike homogeneous goods)
2. demand is downward sloping in firms' own-price, and upward sloping in their competitors' price
3. instead of a beach + walking cost, we could think about the location of my "tastes" and various products in "product space"
4. i.e., although people may differ in their preferences, they may still be willing to consume a product that is not closest to their ideal, as long as the price is low enough
5. we can also solve this model when the firms are located at points other than the ends. We can also have more than one dimension.

Differentiated Product Demand System

In general, the demand functions will have a form like:

$$q_1 = a_1 - b_1 p_1 + \theta p_2$$

$$q_2 = a_2 + \theta p_1 - b_2 p_2$$

where $a_1, a_2, b_1, b_2, \theta$ are all positive numbers.

Cross-price elasticities:

We can look at the percentage change in demand for firm 1 due to a one percent increase in price by firm 2:

$$\epsilon_{1,2} = \frac{\partial q_1}{\partial p_2} \frac{p_2}{q_1}$$

Since products are imperfect substitutes, the cross-price elasticities will be positive. A one percent increase in price by firm 2 will lead to an x percent increase in demand for firm 1. Own-price elasticities are negative.

Hotelling Model 3 Firm Example

Go back to the ice-cream example.

Firm 1 located at $x = 0$.
Firm 2 located at $x = \frac{1}{5}$.
Firm 3 located at $x = 1$.

(Note firms 1 and 2 are much closer to each other than firms 2 and 3.)
To illustrate the point here, let's compute the demand functions.

Hotelling Model 3 Firm Example Ctd.

1. Assuming firm 2 has positive demand, no consumer is choosing between firms 1 and 3.
2. What is the location of the consumer who's indifferent between firms 1 and 2? (As before...)

$$x^*_{1,2} = \frac{1}{10} + \frac{1}{2t}(p_2 - p_1)$$

3. What is the location of the consumer who's indifferent between firms 2 and 3?

$$x^*_{2,3} = \frac{3}{5} + \frac{1}{2t}(p_3 - p_2)$$

4. Everyone to the left of $x^*_{1,2}$ goes to firm 1.

$$q_1 = 100 - \frac{500}{t}p_1 + \frac{500}{t}p_2$$

5. Everyone between $x^*_{1,2}$ and $x^*_{2,3}$ goes to firm 2.

$$q_2 = 500 + \frac{500}{t}p_1 - \frac{1000}{t}p_2 + \frac{500}{t}p_3$$

6. All consumers located to the right of $x^*_{2,3}$ go to firm 3...

$$q_3 = 400 + \frac{500}{t}p_2 - \frac{500}{t}p_3$$

Cereal Differentiated Product

TABLE VII

MEDIAN OWN AND CROSS-PRICE ELASTICITIES[a]

#	Brand	Corn Flakes	Frosted Flakes	Rice Krispies	Froot Loops	Cheerios	Total	Lucky Charms	P Raisin Bran	CapN Crunch	Shredded Wheat
1	K Corn Flakes	−3.379	0.212	0.197	0.014	0.202	0.097	0.012	0.013	0.038	0.028
2	K Raisin Bran	0.036	0.046	0.079	0.043	0.145	0.043	0.037	0.057	0.050	0.040
3	K Frosted Flakes	0.151	−3.137	0.105	0.069	0.129	0.079	0.061	0.013	0.138	0.023
4	K Rice Krispies	0.195	0.144	−3.231	0.031	0.241	0.087	0.026	0.031	0.055	0.046
5	K Frosted Mini Wheats	0.014	0.024	0.052	0.043	0.105	0.028	0.038	0.054	0.045	0.033
6	K Froot Loops	0.019	0.131	0.042	−2.340	0.072	0.025	0.107	0.027	0.149	0.020
7	K Special K	0.114	0.124	0.105	0.021	0.153	0.151	0.019	0.021	0.035	0.035
8	K Crispix	0.077	0.086	0.114	0.034	0.181	0.085	0.030	0.037	0.048	0.043
9	K Corn Pops	0.013	0.109	0.034	0.113	0.058	0.025	0.098	0.024	0.127	0.016
10	GM Cheerios	0.127	0.111	0.152	0.034	−3.663	0.085	0.030	0.037	0.056	0.050
11	GM Honey Nut Cheerios	0.033	0.192	0.058	0.123	0.094	0.034	0.107	0.026	0.162	0.024
12	GM Wheaties	0.242	0.169	0.175	0.025	0.240	0.113	0.021	0.026	0.050	0.043
13	GM Total	0.096	0.108	0.087	0.018	0.131	−2.889	0.017	0.017	0.029	0.029
14	GM Lucky Charms	0.019	0.131	0.041	0.124	0.073	0.026	−2.536	0.027	0.147	0.020
15	GM Trix	0.012	0.103	0.031	0.109	0.056	0.026	0.096	0.024	0.123	0.016
16	GM Raisin Nut	0.013	0.025	0.042	0.035	0.089	0.040	0.031	0.046	0.036	0.027
17	GM Cinnamon Toast Crunch	0.026	0.164	0.049	0.119	0.089	0.035	0.102	0.026	0.151	0.022
18	GM Kix	0.050	0.279	0.070	0.101	0.106	0.056	0.088	0.030	0.149	0.025
19	P Raisin Bran	0.027	0.037	0.068	0.044	0.127	0.035	0.038	−2.496	0.049	0.036
20	P Grape Nuts	0.037	0.049	0.088	0.042	0.165	0.050	0.037	0.051	0.052	0.047
21	P Honey Bunches of Oats	0.100	0.098	0.104	0.022	0.172	0.109	0.020	0.024	0.038	0.033
22	Q 100% Natural	0.013	0.021	0.046	0.042	0.103	0.029	0.036	0.052	0.046	0.029
23	Q Life	0.077	0.328	0.091	0.114	0.137	0.046	0.096	0.023	0.182	0.029
24	Q CapN Crunch	0.043	0.218	0.064	0.124	0.101	0.034	0.106	0.026	−2.277	0.024
25	N Shredded Wheat	0.076	0.082	0.124	0.037	0.210	0.076	0.034	0.044	0.054	−4.252
26	Outside good	0.141	0.078	0.084	0.022	0.104	0.041	0.018	0.021	0.033	0.021

[a] Cell entries i, j, where i indexes row and j column, give the percent change in market share of brand i with a one percent change in price of j. Each entry represents the median of the elasticities from the 1124 markets. The full matrix and 95% confidence intervals for the above numbers are available from http://elsa.berkeley.edu/~nevo.