# Problem Set 2: Static Entry Game and Moment Inequalities

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You can download the data for this problem set from my website. I have data on the location of automobile repair shops in Isolated Towns in the United States, where the isolated towns are constructed in the same manner as Bresnahan and Reiss.

Table 1: Summary statistics for automobile repair data

| Variable | Mean | Std. Dev. | N |
|---|---|---|---|
| Placeid | 338747 | 161192 | 447 |
| Houseunits | 5079.3 | 5432 | 447 |
| Population | 12017.6 | 14282 | 447 |
| Autoroute | 0.28 | 0.45 | 447 |
| Est811111 | 24.6 | 23 | 447 |
| Est811112 | 1.6 | 2.6 | 447 |
| Est811113 | 1.9 | 3.5 | 447 |

Note that:

- Est811111 are automobile repair shops.

- Est811112 are automobile transmission shops.

- Est811113 are automobile exhaust shops.

For this problem, I've also attached a Simulated Annealing Minimizer in Matlab that you might find useful to minimize certain functions.

1. Produce Summary Statistics for data:

   (a) Produce Population, Housing and Autoroute summary statistics.
   (b) Produce the distribution of the number of firms per market for transmission and exhaust sectors (811112 and 811113).
   (c) Regress both the number of establishments in transmission on population and the number of establishments in exhaust on population. What is the correlation of the residual in these two regressions. What does this indicate?

2. Estimate a Bresnahan-Reiss Model for general transmission repair shops (i.e. 811112) using the following functional form for profits:

   $$\pi(X, N) = \underbrace{X_m}_{\text{Log Population}} \beta + \alpha + \theta(N_m \geq 2, N_m \geq 3, \max(0, N - 3)) + \epsilon_m$$

   where $\epsilon_m$ is distributed as $\mathcal{N}(0, 1)$.

   You can estimate this model by maximum likelihood, and the notes should have enough detail for you to figure out how to do this.

   (a) Write down the likelihood for this model.
   (b) Estimate the model.

3. Mazzeo model with Moment Inequalities

   I want you to look at the competitive interactions between transmission and exhaust repair shops. To do this, we need to estimate a Mazzeo model. The Mazzeo model has the following *necessary* conditions:

   $$\pi_h(X_m, N_l, N_h) + \epsilon_h \geq 0$$
   $$\pi_l(X_m, N_l, N_h) + \epsilon_l \geq 0$$

   $$\pi_h(X_m, N_l, N_h + 1) + \epsilon_h < 0$$
   $$\pi_l(X_m, N_l + 1, N_h) + \epsilon_l < 0$$

   $$\pi_h(X_m, N_l, N_h) + \epsilon_h > \pi_l(X_m, N_l + 1, N_h - 1) + \epsilon_l$$
   $$\pi_l(X_m, N_l, N_h) + \epsilon_l > \pi_h(X_m, N_l - 1, N_h + 1) + \epsilon_h$$

which are everyone who is in makes positive profits, if another person enters they will make negative profits and firms choose not to switch their type.

So construct the following moment inequalities using the expectations of these things:

$$
\begin{aligned}
\mathbf{E}\pi_h(X_m, N_l, N_m, N_h) + \epsilon_h = \pi_h(X_m, N_l, N_h) &\geq 0 \\
\mathbf{E}\pi_l(X_m, N_l, N_m, N_h) + \epsilon_l = \pi_l(X_m, N_l, N_h) &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{E}\pi_h(X_m, N_l, N_h + 1) + \epsilon_h = \pi_h(X_m, N_l, N_h + 1) &< 0 \\
\mathbf{E}\pi_l(X_m, N_l + 1, N_h) + \epsilon_l = \pi_l(X_m, N_l + 1, N_h) &< 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{E}\pi_h(X_m, N_l, N_h) - \pi_l(X_m, N_l + 1, N_h - 1) - \epsilon_l + \epsilon_h &< 0 \\
\mathbf{E}\pi_l(X_m, N_l, N_h) + \pi_h(X_m, N_l - 1, N_h + 1) - \epsilon_h + \epsilon_l &< 0
\end{aligned}
$$

We can form sample analogues to these expections:

$$
\begin{aligned}
\nu_1 &= \left(-\sum_m \pi_h(X_m, N_l, N_h) Z_m\right)^+ \\
\nu_2 &= \left(-\sum_m \pi_h(X_m, N_l, N_h) Z_m\right)^+ \\
\nu_3 &= \left(\sum_m \pi_h(X_m, N_l, N_h + 1) Z_m\right)^+ \\
\nu_4 &= \left(\sum_m \pi_l(X_m, N_l + 1, N_h) Z_m\right)^+ \\
\nu_5 &= \left(\sum_m -(\pi_h(X_m, N_l, N_h) - \pi_l(X_m, N_l + 1, N_h - 1)) Z_m\right)^+ \\
\nu_6 &= \left(\sum_m -(\pi_l(X_m, N_l, N_h) + \pi_h(X_m, N_l - 1, N_h + 1)) Z_m\right)^+
\end{aligned}
$$

Use $Z = [1 \; X]$ as instruments.

Define the criterion function as:

3

$$Q(\theta) = \sum_{j=1}^{6} \nu_j^2$$

Use the following profit function:

$$\pi_h(X_m, N_l, N_h) = X\beta_h + \alpha_h + \theta_{hh}\ln(N_h + 1) + \theta_{hl}\ln(N_l + 1)$$
$$\pi_l(X_m, N_l, N_h) = X\beta_l + \alpha_l + \theta_{lh}\ln(N_h + 1) + \theta_{ll}\ln(N_l + 1)$$

As well, normalize $\beta_h = 1$ and $\beta_l = 1$ since we need to put some normalization for this model to be identified (and we don't have any $\epsilon$'s to do it for us).

(a) Minimize the Criterion function. Is this model partially identifed (in which case $Q(\theta) = 0$) or not identified? Interpret the coefficients of this model.

(b) If the model is partially identified, then find the identified set, i.e. the upper and lower bounds on $\theta$. Also assume that the coefficients on competition are negative. Interpret the coefficients.

Hint: To answer this question you will need to use FMINCON in Matlab. You need to to find the lowest and highest parameter for each component of $\theta$ which is still in the identified set $Q(\theta) = 0$. Also assume that all the $\theta$'s are between -100 and 100, since this will help shrink the identified set a bit and that the effect of competition is always negative, i.e. less than 0.