Name: Solutions

Please answer all questions in the space provided (including the sheets following each of the longer questions). You do NOT need to provide explanations for your answers unless the question specifically asks for an explanation. If it does, a brief explanation is all that is needed. Feel free to use the back of each sheet if you need to. Unless otherwise stated, assume that tastes satisfy the usual rationality, continuity, monotonicity and convexity assumptions, that all goods are essential, and that indifference curves do not have kinks or flat spots. Do not assume anything more specific about tastes unless you are specifically asked to do so. There are 100 points on the exam, and you have 1 hour and 30 minutes to complete the exam. The total number of points for each part is indicated in each question. No calculators, books, notes, or other electronic devices are permitted. Good luck!

Before starting the exam, you must sign the following statement:

I pledge to obey the Duke University Honor Code during this exam.

Signed: _____________________________________________________________
Problem 1 - [35 points total]

Transit activists in Durham have been concerned with the growing sprawl of the city and the affordability and ridership of public transportation options. Consider a typical Durham citizen, Harvey, who divides his exogenous income $I$ between public bus rides and all other consumption. The current price of each public bus ride is $p$.

(a) [4 points] In a graph with “Number of Public Bus Rides” on the horizontal axis and “Dollars of Other Consumption” on the vertical axis, illustrate Harvey’s initial budget constraint. Label the vertical and horizontal intercepts along with the slope. Choose an optimal bundle $A$ on this budget constraint. Illustrate an indifference curve that passes through bundle $A$.

(b) [4 points] A Durham transit activist named Speed wants to subsidize the price of each bus ride, lowering the price from $p$ to $p-s$. Illustrate the new budget constraint Harvey would face. Be sure to label the slope and any new intercepts.

(c) [5 points] Harvey considers a public bus trip to be a regular inferior good. Illustrate the substitution and income effects of the subsidy. Locate a possible new optimal bundle $C$ on Harvey’s new budget constraint. What will happen to the number of public bus trips that Harvey consumes as a result of this subsidy? Explain.

(d) [4 points] What will happen to Harvey’s other consumption as a result of this subsidy? Explain.

(e) [4 points] In a new picture that copies both budgets and bundles $A$ and $C$, illustrate the total amount $S$ that the government spends on the subsidy.

(f) [4 points] The mayor of Durham revises the city budget and decides the city is spending too much money on the transit subsidy. The mayor suggests that the government could save money by providing the minimum lump sum subsidy that makes Harvey no worse off than he is under the current per-unit subsidy. Illustrate this amount $L$ on your picture from part (e).

(g) [6 points] If Speed the transit activist wants to maximize the number of public bus rides, how would he feel about the lump sum subsidy compared to the per unit subsidy? How would he feel about the lump sum subsidy compared to no subsidy? Explain.

(h) [4 points] Speed offers a final proposal. He suggests they maintain the per-unit subsidy price of $p-s$ but they simultaneously impose a lump sum tax on Harvey’s income that is exactly equal to the current subsidy expenditure $S$, leaving Harvey with $I-S$ in exogenous income. If this final proposal were implemented, would the income tax revenue fully offset the final subsidy expenditure? Explain.
ECON201 - Midterm 1

(a) The initial budget constraint is the solid black line in Figure 1.1 below. The vertical intercept is \(I\), the horizontal intercept is \(I/p\), and the slope is \(-p\). The optimal bundle \(A\) is an interior optimal bundle, indicated by the tangency between the indifference curve and the black budget at bundle \(A\).

(b) The new budget constraint is the solid blue line in Figure 1.1 below. The vertical intercept is still \(I\), the horizontal intercept is \(I/(p_s)\), and the slope is \(- (p_s)\).

(c) **The number of public bus rides will increase.** The substitution effect points right. As the opportunity cost of public bus rides falls, Harvey should consume more public bus rides. This is illustrated as the movement from \(A \rightarrow B\) in Figure 1.1, where bundle \(B\) is tangent to the dashed blue compensated budget. Harvey considers a public bus ride to be a regular inferior good. Therefore, the income effect points left but is smaller in magnitude than the substitution effect. This is illustrated as the movement from \(B \rightarrow C\) in Figure 1.1. (One could also note that a regular inferior good has a downward sloping uncompensated demand curve, so lowering the price from \(p\) to \(p_s\) would increase the quantity demanded of public bus rides.)

(d) **We cannot tell what will happen to Harvey’s other consumption.** The substitution effect points down (\(A \rightarrow B\)). The income effect must point up because if public bus rides are regular inferior then all other consumption must be normal (\(B \rightarrow C\)). Because there are competing income and substitution effects, we cannot say for sure whether other consumption increases or decreases. Compare Figure 1.1, where other consumption decreases, to Figure 1.2, where other consumption increases.

(e) This is illustrated in Figure 1.3 as the purple vertical distance between the two solid budget lines at bundle \(C\).

(f) This is illustrated in Figure 1.3 as the red vertical distance between the black solid budget and the parallel dashed red budget. The dashed red budget is the budget under the lump sum subsidy that makes Harvey just as happy as he is under the current per-unit subsidy, indicated by the tangency at bundle \(D\).

(g) Speed prefers the per-unit subsidy to the lump sum subsidy. This is a pure substitution effect from \(C \rightarrow D\) in Figure 1.3, where the number of bus rides is clearly lower under the lump sum. Speed also prefers no subsidy to the lump sum subsidy. This is a pure income effect from \(A \rightarrow D\) in Figure 1.3, where the number of bus rides is lower under the lump sum subsidy because bus rides are a regular inferior good.

(h) No, the income tax revenue would not fully offset the final subsidy expenditure. If Harvey were taxed an amount \(S\) he would face the dashed blue budget, and he would further increase his public bus rides to a point such as \(X\). The government would end up spending even more on the subsidy. The net spending (i.e., the new, larger subsidy expenditure minus the tax revenue) is illustrated in Figure 1.4.
Problem 2 - [35 points total]

This problem explores the labor supply decisions of two coworkers, Tom and Rob. Tom and Rob both have homothetic tastes over leisure and consumption. Tom and Rob are both endowed with 60 leisure hours per week and face an hourly wage of $20.

(a) [6 points] In a graph with "Hours of Leisure" on the horizontal axis and "Dollars of Consumption" on the vertical axis, draw the budget constraint for the coworkers. Label the slope and the intercepts. Tom and Rob both optimize at bundle Z where they work 50 hours per week. Locate bundle Z on your budget constraint. How much consumption does each worker enjoy at his optimal bundle?

(b) [4 points] Tom discovers a commune he could join. The commune does not offer any wage labor opportunities but instead guarantees each member $800 in exogenous income without a single hour of "work." In exchange, the commune requires each member to perform 20 hours of "community service." Draw Tom’s budget constraint if he joins the commune, reflecting his reduced leisure endowment. Label any new intercepts.

(c) [7 points] After considering his options, Tom decides he is exactly indifferent between staying at his job and joining the commune. Illustrate Tom’s indifference curve through bundle Z. If Tom joins the commune, will he enjoy more or less leisure? Can you tell exactly how much more or less leisure he will have? Explain.

(d) [5 points] Tom tells his friend Rob about the commune. Rob is thrilled about the discovery and is much happier to join the commune than to stay at his job. (Rob is not indifferent.) Since Tom and Rob both have homothetic tastes and an identical optimal bundle at their old job, what can account for their difference in enthusiasm toward the commune?

(e) [4 points] After a short while, the commune dissolves. Tom returns to his old life. Just weeks later, the government passes a wage tax that lowers his wage from $20 to $15. With the new tax in place, Tom continues to work 50 hours per week. On a new picture, re-illustrate Tom’s original budget and bundle. Then illustrate the after-tax budget constraint for Tom, labeling the slope and intercepts. Locate Tom’s new optimal bundle and label it X.

(f) [5 points] In a new graph with “Hours of Labor” on the horizontal axis, illustrate the uncompensated labor supply curve for Tom. Then illustrate the compensated labor supply curve for Tom. Be sure to label any exact values that you can. Using the appropriate curve, shade the deadweight loss from the wage tax.

(g) [4 points] If Rob had returned to his old job and also faced the new wage tax, how would the deadweight loss from taxing Rob compare to the deadweight loss from taxing Tom? Explain.
(a) The initial budget constraint is the solid black line shown in Figure 2.1 below. The horizontal intercept is the 60 hour leisure endowment, and the vertical intercept is the $1200. The slope is -20. Bundle Z is located at 10 hours of leisure and contains $1000 in other consumption.

(b) The commune budget constraint is the solid blue line shown in Figure 2.1 below. The horizontal intercept is the 40 hour leisure endowment and the vertical intercept is $800.

(c) If Tom is exactly indifferent then his indifference curve through bundle Z is also tangent to the commune budget at the kink point K as shown. This is a pure substitution effect, so if Tom joins the commune he will enjoy more leisure. In fact, because he will optimize at the kink point, we know Tom will increase his weekly leisure by 30 hours (from the 10 hours at the old job to 40 hours at the commune).

(d) Rob must have more substitutable preferences over leisure and consumption. This is illustrated in Figure 2.2 below. Rob can move from the indifference curve labeled $u_Z$ to the indifference curve labeled $u_K$, so he is happier joining the commune.

(e) The after-tax budget constraint is the solid green budget in Figure 2.3. The vertical intercept is $900 and the horizontal intercept is back to the 60 hours of leisure. Bundle X is directly below bundle Z at 10 hours of leisure.

(f) The uncompensated labor curve is shown in Figure 2.4 below. Since the uncompensated labor supply curve illustrates Tom’s actual labor decision, it is perfectly vertical at 50 hours of labor. The uncompensated curve contains bundles X and Z. The relevant compensated curve that we need to show deadweight loss is the compensated curve that goes through bundle X, since this is the optimal bundle with the tax. Another bundle on this compensated curve is bundle Y, as illustrated by the tangency between the dashed compensated budget and the indifference curve in Figure 2.3. The compensated labor supply is upward sloping and contains bundles X and Y. The deadweight loss from the wage tax should be measured on the compensated curve and is shaded in red.

(g) The deadweight loss would have been larger under Rob. The deadweight loss comes from the substitution effect - the larger the substitution effect, the larger the deadweight loss. Since we determined in part (d) that Rob has more substitutable preferences, we know that the wage tax would have induced a larger substitution effect and thus would have created more deadweight loss for Rob than for Tom.
Problem 3 - [30 points total]

Grant is a lawyer who recently signed a two-year job contract. He expects to make the same income this year and next year. He is required to work a fixed number of hours each year, so we will treat his income as exogenous for this problem. Grant also spends money on consumption in each of these years. Assume throughout the problem that consumption in each year is a normal good for Grant.

(a) [4 points] Denote Grant’s fixed income in each year by \( I \). Assume Grant has no other sources of income. Grant is able to save and borrow at interest rate \( r \). On a graph with “Dollars of Consumption This Year” on the horizontal axis and “Dollars of Consumption Next Year” on the vertical axis, begin by illustrating Grant’s budget constraint. Label the endowment point, both intercepts, and the slope.

(b) [4 points] Suppose that banks actually do not have the same interest rate for saving and borrowing. Grant can continue to borrow at interest rate \( r \) but must save at the interest rate \( r' \), where \( r' < r \). On the same graph, illustrate Grant’s new budget constraint. Label any new intercepts.

(c) [4 points] When the borrowing rate is \( r \) and the savings rate is \( r' \), Grant would like to save money now to consume more than his income next year. Illustrate this in your graph. Label the optimal bundle \( A \) and include the indifference curve through \( A \).

(d) [8 points] The government is worried about the capital supply and Congress proposes a subsidy that would return the interest rate for savers to \( r \). What will happen to Grant’s consumption this year? What will happen to Grant’s consumption next year? Explain.

(e) [4 points] Suppose that the uncompensated capital supply curve is upward sloping. Does this change either of your answers from part (d)?

(f) [3 points] On a new graph with both budget constraints and bundle \( A \), illustrate a new optimal bundle \( C \) for Grant that is consistent with the information in (e). Indicate the total amount \( S \) the government spends on the interest rate subsidy.

(g) [3 points] Suppose that the uncompensated capital supply curve had been downward sloping instead. Then the total amount the government would have spent on the subsidy would have been \( S' \). How does \( S' \) compare to \( S \)? Explain.
(a) The initial budget constraint is the solid black budget in Figure 3.1 below. The endowment point is labeled E, where Grant just consumes his exogenous income I this year and his exogenous income I next year. The vertical intercept is $I + I(1+r)$ and the horizontal intercept is $I + I/(1+r)$. The slope of the budget is $-(1+r)$.

(b) Grant’s budget does not change to the right of E. However to the left of E (where Grant is a saver) the budget rotates down to the red segment. The new vertical intercept is $I + I(1+r')$.

(c) Since Grant chooses to save, bundle A must be on the red budget constraint. A is optimal as indicated by the tangent indifference curve.

(d) **Grant’s consumption next year will definitely increase, but we don’t know what will happen to his consumption this year.** For future consumption, the income and substitution effects both point in the same direction, so we know future consumption will increase. For consumption this year, the substitution effect says consume less and the income effect says consume more. Since there are competing income and substitution effects, we cannot say for sure what will happen to current consumption. Compare Figure 3.1, where the net effect is to consume less this year, to Figure 3.2, where the net effect is to consume more this year.

(e) **Now Grant’s consumption this year will definitely fall.** If the uncompensated capital supply curve is upward sloping, then as the interest rate increases, Grant will save more. Saving more means he will consume less today for sure, so this would be consistent with Figure 3.1 and not Figure 3.2.

(f) This is illustrated in Figure 3.3 below. If the uncompensated capital supply curve is upward sloping then as the interest rate increases, Grant will save more and consume less this year. Thus, bundle C must be to the left of bundle A. The total expenditure $S$ is then the blue vertical distance between the black and red budget segments at bundle C.

(g) **$S'$ must be less than $S$.** If the uncompensated capital supply curve had been downward sloping instead, then as the interest rate increased Grant would have saved less. The government would be subsidizing less savings, so the total expenditure would fall. This is illustrated in Figure 3.4, where $S'$ would correspond to the smaller green vertical distance.
Figure 3.1

- Dollars of Consumption Next Year
- Dollars of Consumption This Year

Figure 3.2

- Dollars of Consumption Next Year
- Dollars of Consumption This Year
Dollars of Consumption Next Year

Dollars of Consumption This Year

Figure 3.3

Figure 3.4