Abstract:

Past research in the field of voting behavior has suggested that both voter turnout and voter choice should be influenced by the ‘closeness’ of electoral contests. This paper argues that operationalizing closeness presents fundamentally different challenges in two- as compared to three-candidate elections. We first demonstrate that closeness indices used in past research on three-candidate elections have violated at least one of three axiomatic criteria that any such measure should satisfy. We then propose a new measurement, grounded in probability ratios, and prove formally that ratio-indices satisfy all required theoretical criteria. Empirical analyses using this new measurement strategy provide strong and consistent evidence of strategic voting across two distinct electoral contexts, suggesting that the Calculus of Voting may transcend time and place in ways not uncovered in past research. The paper’s measurement strategy should be generally applicable to research on voting in popular elections, parliaments, and organizations.
1. Introduction

The competitiveness, or ‘closeness’, of electoral contests has figured prominently in past studies of voter behavior. This has been particularly true of research grounded in the *Calculus of Voting*, an expected utility model which has suggested that both voter turnout and voter choice should be influenced by the election’s competitiveness.\(^1\) The argument is intuitive: when an election is competitive voters will understand that their votes are more ‘meaningful’. In turn they should be more likely to turnout,\(^2\) and potentially to cast strategic votes for their second-most preferred candidate.\(^3\) Empirical tests of the Calculus of Voting not only help to verify or falsify the model, but also contribute to our larger understanding of the ‘health’ of contemporary democracy. To the extent that voters respond to their strategic environment in sophisticated ways, we must reconsider commonly held wisdoms that most voters are apathetic, ill-informed, and/or unresponsive ideologues.

A key operational step in testing such hypotheses is accurately measuring the closeness of electoral contests. In two-candidate elections, one simple approach involves using *raw margin* between the two competitors’ expected probabilities of winning. For parties A and B and expected probabilities of winning \(p_A\) and \(p_B\), the closeness of the

\(^1\) The Calculus of Voting was originally developed by Riker and Ordeshook (1968) and McKelvey and Ordeshook (1972).

\(^2\) Although by no means an exhaustive list, for additional work on turnout and competitiveness see Hinich and Ordeshook (1969), Rosenthal and Sen (1973), Grofman et al. (1998), Endersby et al. (2002), and Adams et al. (2006).

\(^3\) A large amount of past research on strategic voting is reviewed in the process of developing Sections 2 through 5 below, with a particular eye to its relationship to the current paper. For a schematic review of more recent contributions see Cox (1997); Fey (1997); Alvarez and Nagler (2000); Blais (2000); Niou (2001); Fisher (2004); Fieldhouse et al. (2007); Merolla and Stephenson (2007); Myatt (2007); Herrmann and Pappi (2008); Kselman and Niou (2010); Meffert and Gschwend (2011); Kiewiet (2013); Artabe and Gardeazabal (2014); Abramson et al. (2016).
election of the election would thus be captured by \(1 - |p_A - p_B|\).\(^4\) If both parties have an equal chance of winning (\(p_A = p_B = 50\%\)) then the measure would equal ‘1’, while if one party has a 60% chance of winning and the other a 40% chance it would equal ‘.8’, and so on. The challenge of measuring electoral competitiveness becomes more complicated in contests with more than two parties. In an important contribution, Endersby et al. (2002) provide a number of distinct strategies for measuring district-level electoral competitiveness in elections with more than two candidates.\(^5\) However, such district-level indices are not suitable for a direct, individual-level operationalization of the Calculus of Voting. As demonstrated in Section 2, individual-level tests must rather operationalize a voter’s perceived *probability of being pivotal between two candidates*. A variety of approaches have been proposed to measure these pivotal probabilities in elections with more than two candidates (Rosenthal and Sen 1973; Cain 1978; Black 1978; Ordeshook and Zeng 1997). Section 3 below demonstrates that all such approaches have violated at least one of three axiomatic principles which any such closeness measure should satisfy. In turn, individual-level evidence from past studies may need to be revisited.

Having identified the challenge, the current paper then makes two positive contributions. First, our core theoretical result (Proposition 1) establishes that a measurement strategy grounded in *probability ratios* satisfies all necessary axiomatic criteria: they exhibit the proper comparative statics and the proper absolute size with

\(^4\) Cox (1988) argues that, at least for studies of voter turnout in two-candidate contests, this raw vote margin is a better measure than the ‘two-party margin’, which divides the raw margin by the denominator (\(p_A + p_B\)).

\(^5\) With these measures in hand the authors identify an aggregate relationship between district-level competitiveness and district-level voter turnout in the 1993 and 1997 Canadian general elections.
respect to one another. Second, Section 4 deploys this new measure in empirical analysis of strategic voting in three-candidate elections, which occurs when a voter chooses her second-most-preferred candidate rather than her most-preferred candidate because: her second-most-preferred candidate has a better chance defeating her least-preferred candidate, and this second-preference is strongly preferred to this last-preference. Formal theory predicts that strategic choice should occur when a voter’s 2nd most-preferred and least-preferred candidates are in a close race for first place (Black 1978; Hoffman 1982; Fisher 2004; Kselman and Niou 2010). While findings in past studies have varied, the results in Section 4 suggest that strategic behavior looks remarkably similar in two distinct settings (Canada 1988 and the United Kingdom 2010). Indeed, rather than undermining past empirical results, our findings suggest that they are likely understating the extent to which voter behavior is consistent with the strategic choice hypotheses emerging from the Calculus of Voting; and that the latter model may transcend time and place in ways not previously uncovered.6

While the current application is to strategic voting in three-candidate plurality rule contests, our methodology for measuring closeness can be extended to any plurality rule election with \( N > 3 \) candidates, as well as to the emerging literature on strategic voting in proportional representation systems with coalition governments (e.g. Indriadson 2011; Kedar 2011; Hermann 2014). Our operational strategy should also be used in future individual-level tests of the relationship between electoral closeness and voter turnout and abstention. Beyond these traditional lines of research, Bowler and Lanoue (1992),

---

6 As an additional advance, we demonstrate that operationalizing strategic voting implies an interactive regression framework, in which the marginal effect of expectations on the likelihood of strategic choice depends on a voter’s relative preferences.
Kang (2004), and Kselman and Niou (2011) have studied the phenomenon of protest voting: choosing a party or candidate in order to send a ‘signal’ of dissatisfaction with some aspect of the political status quo. All three papers have argued that such signaling behavior should be impacted by the competitiveness of electoral contests, and in particular may occur in non-competitive contests. The Concluding Section 5 addresses these and other extensions to voting processes in organizations and parliaments.

2. The Calculus of Voting (COV)

The calculus of voting is an expected utility model of voting behavior developed by Riker and Ordeshook (1968) and McKelvey and Ordeshook (1972) that applies to winner-take-all elections. The authors’ original objective was to formalize a voter’s calculus for voter turnout, but the framework can also be used to analyze the choice between voting for one’s first- and second-most-preferred candidates in a 3-candidate contest. Kselman and Niou (2010) present a proof the condition for strategic voting in 3-candidate elections. Define $E_i^j$ as the expected utility of voter $i$ for choosing candidate $j$, where $j \in \{1,2,3\}$ represent $i$’s first, second, and third-most preferred candidates respectively. Henceforth, we will refer to a voter’s first-preference as ‘1’, her second-preference as ‘2’, and her third-preference as ‘3’. According to the COV voter $i$ will choose 2 if and only if $E_i^2 - E_i^1 > 0$. For a more derivation we refer the interested reader to the aforementioned paper; here we simply reproduce the authors’ final result:

---

7 Though most known for its results on candidate convergence, Downs’ seminal work (1957) contains the first informal specification of voter turnout in expected utility terms.  
8 Black’s important paper (1978) is the first effort at doing so, and generates a result similar to that presented below. Cain’s seminal paper on strategic voting (1978) also conducted a partial expected utility comparison.
\[ E_i^2 - E_i^1 > 0 \text{ iff} \]
\[
\left( p_{23}^1 + p_{23}^2 \right) (U_2 - U_3) > \\
\left( 2p_{12}^0 + p_{12}^1 + p_{12}^2 \right) (U_1 - U_2) + \left( p_{13}^1 + p_{13}^2 \right) (U_1 - U_3),
\]

where \( U_j \) is \( i \)'s utility for their \( j^{th} \) preference, and \( p_{jk}^* \) the perceived probability of a first-place tie between the voter’s \( j^{th} \) and \( k^{th} \) preferences, given the vote choice denoted in the superscript. So, for example, \( p_{23}^1 \) is the probability candidates 2 and 3 tie given that \( i \) votes for 1; and \( p_{12}^0 \) is the probability candidates 1 and 2 tie given that \( i \) abstains from voting. These pivotal probabilities are what both we in this paper and past research conceptualize as the ‘closeness’ of two candidates in a three-candidate election. When the left-hand side of (1) is larger than the right-hand side, voters will strategically choose their second preference.\(^9\)

To highlight the result’s qualitative implications we now implement the following standard utility normalization: \( U_1 = 1, \ U_3 = 0, \) and \( U_2 = \frac{U_2 - U_3}{U_1 - U_3} = \sigma. \) As well we employ the following simplifying notation:

\[
4p_{12} = 2p_{12}^0 + p_{12}^1 + p_{12}^2, \\
2p_{13} = p_{13}^1 + p_{13}^2, \\
2p_{23} = p_{23}^1 + p_{23}^2.
\]

We can then rearrange (1) above as follows:

---

\(^9\) This is the same strategic voting condition derived in Hoffman (1982). Ferejohn and Fiorina (1974) also develop an expected-utility model of voter choice, built upon a similar calculus as that found in McKelvey and Ordeshook (1972). Inequality (3) from their paper (p. 530) contains an identical qualitative implication to inequality (1) above.
$$E_i^2 - E_i^1 > 0 \text{ iff }$$

$$(p_{23} + 2p_{12}) \cdot \sigma > 2p_{12} + p_{13}. \quad (3)$$

This reduced form inequality helps to clarify the COV’s basic predictions about the likelihood of choosing 2. Firstly, it implies that the most prominent probabilities in a voter’s calculus will be $p_{23}$ and $p_{13}$, with increases in the former (latter) having a positive (negative) impact on the likelihood of choosing 2. Increases in the parameter $\sigma$, which captures $i$’s relative preference for 2, should also increase the likelihood of choosing 2. However, importantly, the impact of the terms $p_{23}$ and $\sigma$ should be interactive; for example, if $\sigma \geq 0$ then increases in $p_{23}$ should have little effect on the likelihood of choosing 2, and vice versa. This implies the need for an interactive regression framework. In using purely linear regression models, past papers on strategic voting thus provide imprecise estimates of the effects of $p_{23}$ and $\sigma$, and in turn miss an important opportunity to provide more fine-grained evidence in favor of the COV as a model of strategic choice.

Finally note that the term $p_{12}$ has competing effects, as it enters into both sides of the inequality; though the effect should be slightly stronger on the right-hand side (where it is not discounted by $\sigma$ …). To see why, note that when 1 and 2 are in a close race for first place, voters have competing incentives. On the one hand, the chance of being pivotal between these two candidates impels them to choose 1, especially if 1 is significantly more preferred than 2. On the other hand, in these situations 2 may also be competing well against 3, thus potentially impelling voters to choose 2, especially if 1 is not significantly more preferred than 2. For these reasons, it is ultimately the two
candidates’ relative standing with respect to 3, as captured by $p_{13}$ and $p_{23}$, that should dictate a voter’s utility maximizing choice. As discussed below, much past research (e.g. Cain 1978; Abramson et al. 1992) has emphasized $p_{12}$ in their empirical analyses, and interpreted as support for the COV evidence that choosing 2 became more likely as the ‘closeness’ of the race between 1 and 2 decreased. Based on the derivation and analysis of (3) above, the predicted relationship appears to be much more ambiguous. We replicate and discuss these results further in Section 4 and Appendix B.

3. Operationalizing the Calculus of Voting

While some research has studied strategic voting using aggregate-level data, the most promising tests of an individual-level theory like the COV are grounded in individual-level surveys. In particular, to test the above inequality we need survey data from a 3-candidate plurality-rule election which allows us to operationalize: a.) a respondent’s utility for a candidate ($U_j$); b.) the ‘closeness’ of a contest between two candidates ($p_{jk}$); and c.) a respondent’s vote choice. Beginning with the latter, to

---

10 For example, Cain (1978) finds that 3rd party support in Britain tends to decrease in competitive districts, i.e. districts where the Labour and Conservative parties are in a tight race for 1st place. Galbraith and Rae (1989) and Johnston and Pattie (1991) demonstrate that district-level vote swings between the 1983 and 1987 British General Elections are consistent with strategic behavior (e.g. swings from the Alliance to Labour were more likely where the latter had a better chance of defeating the Conservatives). As well, Cox (1997) and Bawn (1999) find that major party candidates are favored over minor party candidates in electoral districts where the race for first place is close. Aggregate level evidence is suggestive but suffers from problems of ecological inference (Freedman et al. 1991; Achen and Shivley 1995; King 1997).

11 For the sake of reference, Appendix D provides links to our data sources and reproduces verbatim the survey items used in our empirical operationalization.

12 This paper uses individual-level survey data to implement a direct operationalization of inequality (3). A distinct set of papers uses respondents’ self-reported motivations (Heath
operationalize strategic choice we create a dummy variable $V_2$ which assumes the value ‘0’ if the respondent chooses their first preference and ‘1’ if they choose their second preference. So, for example, a British voter who prefers Labor-to-Liberal-to-Conservative and chooses Liberal would have a value of ‘1’, while if they chose Labor they would have a value of ‘0’. On other hand if they chose Conservative they would have no value (and would be eliminated from the analysis). A tiny number of survey respondents actually do express the intention to vote for their least-preferred party. Discarding these respondents is a necessary operational step: the ‘0’ category would be polluted were it to contain not only the choice for 1 but also the tiny subset of respondents which chooses 3.

Moving to the measurement of preferences, the most common mechanism for measuring utility for a particular candidate/party has been the *Feeling Thermometer*, which asks survey respondents to score candidates/parties on a scale of 1-100 capturing their general ‘attraction’ to said candidate/party (Cain 1978; Black 1978; Abramson et al. 1991; Niemi et al. 1992; Franklin et al. 1994; Fisher 2004), identifying as strategic anyone who reports having been motivated by considerations of ‘viability’ and/or the desire to avoid ‘wasted-votes’. Alvarez and Nagler (2000) suggest that the higher overall rates of strategic behavior often uncovered by such studies may result from *response bias* in post-election surveys. Artabe and Gardeazabla (2014) provide a methodology for eliminating response bias in self-reported motivations, finding a much lower overall level of strategic behavior (2.19%) in the 2008 Spanish General Election.

In the body of the paper we will use *expected* vote choices from pre-election surveys rather than *reported* vote choices in post-election surveys, given post-hoc bias problems associated with the latter (Alvarez and Nagler 2000). We have re-run all analyses with data on reported vote choice in post-election surveys. The results are qualitatively identical and available upon request.

In the British data, only 15 out 6,830 respondents included in our analysis choose their least-preferred candidate, while on the Canadian data the rate is a bit higher: 30 out of 1,610 respondents. All are dropped in subsequent empirical analyses.
1992; Herrmann and Pappi 2008).\textsuperscript{15} Blais and Nadeau (1996), Blais et al. (2001) and Blais et al. (2005) employ a composite feeling thermometer which simultaneously incorporates preferences for candidates, party leaders, and parties. Alvarez and Nagler (2000) and Alvarez et al. (2006) estimate a Multinomial Probit (MP) model whose right-hand side includes demographic, programmatic, and regional inputs to raw voter preferences. This approach allows for a more complete model of voter preferences, which likely captures elements missed by the raw Feeling Thermometers. Furthermore, the use of Multinomial Probit allows the authors to avoid the ‘Independence of Irrelevant Alternatives’ assumption implicit in simple Probit analysis. Nonetheless, in this paper we will adopt the simpler Feeling Thermometer approach, which facilitates testing of the reduced form condition above; and which we believe implies only minimal cost in the consideration of Irrelevant Alternatives.\textsuperscript{16}

3a. Measuring ‘Closeness’ in 3-Candidate Contests: an Axiomatic Approach

To measure closeness, i.e. the probability of a first-place tie between candidates \( j \) and \( k \) (\( p_{jk} \)), we follow other studies by first measuring the probability that a voter gives some party \( k \) of winning the electoral contest, denoted \( p_k \). In any single-member-district election, the relevant parameter \( p_k \) is the probability that a voter’s \( k \)th preference wins in

\textsuperscript{15} Naturally, these scores are recoded to replace preferences over party labels with preferences over ordinal utility-rankings. Consider a British voter who has the Conservatives as the most-preferred party, the Liberal Democrats as the 2nd most-preferred party, and Labor as their least preferred party. For this voter \( U_1 = U_{con} \), \( U_2 = U_{lib} \) and \( U_3 = U_{lab} \).

\textsuperscript{16} Regarding Irrelevant Alternatives, the 3-candidate Calculus of Voting suggests that the choice for 3 is never optimal. Furthermore, the central predictive parameters \( p_{13}, p_{32}, \) and \( \sigma \) imply an explicit and simultaneous comparison of candidates 1 and 2 with an eye to their independent relation to 3.
a voter’s particular electoral district. In the past, the most commonly used proxy for these expectations have been district-level vote-shares from the current (e.g. Cain 1978) or previous (e.g. Alvarez and Nagler 2000) electoral cycles. In this paper we confine our analysis to the 1988 Canadian National Election Study and the 2010 British National Election Study, where respondents were explicitly asked to assign parties a district-level probability of winning. In these cases we will measure $p_k$ using respondents’ answers, normalizing such that $p_1 = \frac{p_1}{p_1 + p_2 + p_3}$, $p_2 = \frac{p_2}{p_1 + p_2 + p_3}$, and $p_3 = \frac{p_3}{p_1 + p_2 + p_3}$.

Implicit in the discussion of how to measure $p_k$ is a larger question: how do voters develop expectations as to a party’s district-level probability of winning? Do they do so based on parties’ past success in the district? Or based on the current candidate’s qualities? Even more specifically, in three-candidate contests how do voters use expected vote-shares to determine probabilities of winning? For example, consider a voter who believes parties A, B, and C are expected to receive district vote shares of 50%, 40%, and

---

17 In parliamentary systems with SMD elections the COV is a district-level theory: voters may choose their second-most-preferred party because their most-preferred party has no chance of winning in their particular district. Abramson et al. (2016) introduce aggregate-level strategic considerations into their argument. For example, a voter may choose because has no chance of winning the general election.

18 Importantly, we do not mean to suggest that the measurement strategy developed below is only valid for cases in which this survey item exists. Regardless of the manner in which researchers measure the parameter $p_k$, a strategy must be employed to use these raw data as ‘inputs’ into the measure of $p_{jk}$. We choose to study two cases where the ideal survey item exists, as we feel that this is the best way to test an individual-level theory. Other measures, grounded for example in district level vote-shares, eliminate the possibility for district-level variance in voter expectations. However, regardless of the empirical strategy used to measure $p_k$, the methodology developed below is relevant to the use of these ‘inputs’ in operationalizing the closeness parameters of $p_{jk}$.

19 As with the utility measures, for a British voter who most prefers the Conservatives, has the Liberal Democrats as the 2nd most-preferred party, and Labor as their least preferred party, \{p_{cos}, p_{lab}, p_{lab}\} will be replaced by \{p_1, p_2, p_3\} respectively.
10% respectively. Will these expected vote shares translate directly into probabilities of winning? Or might voters assign party C a 0% chance of winning, given that she is so far behind the two leading parties? While beyond the scope of this paper in the Conclusion we report on a series of preliminary findings regarding this crucial issue.

Once we have the raw $p_k$ measures we then need to generate indices for the probabilities of 1st place ties $p_{jk}$. Past studies of closeness in 3-party contests have employed a wide variety of approaches to measuring $p_{jk}$, including a raw margin approach (e.g. Cain 1978; Abramson et al. 1992; Blais and Nadeau 1996), a conditional difference approach (Rosenthal and Sen 1973), a multiplicative approach (Ordeshook and Zeng 1997), and Euclidean geometry (Black 1978; Hermann and Pappi 2008). This paper takes a step back, developing an axiomatic approach: we begin by setting forth a series of criteria which any measure $p_{jk}$ should satisfy, and then develop an operational strategy to satisfy these criteria.

Note at the outset that measuring the probability of a tie between $j$ and $k$ is a 3-dimensional problem, which will require considering a voter’s expectations as to the eventual placement of all three candidates in the election. We now present a set of properties that any such 3-dimensional measure should satisfy, beginning with its comparative statics:

- **Property 1**: If either $j$ or $k$ is the expected plurality winner, the closeness of the race between $j$ and $k$ should increase as the expected probability difference separating these two candidates decreases.

---

20 We ourselves have assigned the italicized concepts; this is not the terminology used by the cited authors to describe their methodologies.
• **Property 2:** Holding their *expected probability separation constant*, the closeness of the race between candidates \(j\) and \(k\) must *increase* with their shared likelihood of winning the election against the third candidate.

To demonstrate the importance of Property 1, consider two voters who believe their least-preferred candidate, 3, will place first, their second-most-preferred candidate, 2, will place second, and their most-preferred candidate, 1, will place last in the election. The first voter has the following expectations: 1 wins with probability 10\%, 2 with probability 30\%, and 3 with probability 60\%. The second voter expects 1 to win with probability 10\%, 2 with probability 40\%, and 3 with probability 50\%. Both voters expect 1 to be the loser and receive 10\% of the vote. However the probability of a 1st place tie between 2 and 3 (\(p_{23}\)) should be higher for the second voter, for whom the *expected probability margin separating 2 and 3 is smaller*.

To demonstrate the importance of Property 2, again consider two voters whose first candidate is expected to place last. The first has the following expectations: \(p_1 = 0\%\), \(p_2 = 45\%\), and \(p_3 = 55\%\); while the second believes \(p_1 = 20\%\), \(p_2 = 35\%\), and \(p_3 = 45\%\). Both voters thus perceive the two leading candidates 2 and 3 to be separated by 10 percentage points, but \(p_{23}\) should clearly be higher for the first voter, for whom 2 and 3 have a greater shared lead over 1. Conversely, \(p_{23}\) should clearly be lower for the second voter, for whom 2 and 3’s shared probability of defeating 1 is lower.

Creating a measure for \(p_{jk}\) which displays the proper comparative statics is a necessary but not sufficient condition for operationalizing the strategic voting conditions stated in (3). In addition, any such measure must generate values that satisfy the following ‘reasonability’ criterion:
• **Property 3**: The probability of a 1st place tie between two leading candidates must be higher than the probability of any 1st place tie which includes the trailing candidate.

For example, among voters who perceive 3 to be winning, 2 to be in 2nd place, and 1 to be losing, Property 3 requires that $p_{23} > p_{12}$, $p_{13}$: the probability of a 1st place tie between the two leading candidates 2 and 3 ($p_{23}$) must be higher than the probability of any first place tie involving the expected plurality loser 1.

Before developing our measurement strategy we demonstrate that measures used to date in studies of strategic voting do not simultaneously satisfy Properties 1-3. The papers by Cain (1978) and Abramson et al. (1992) use a *raw margin* approach grounded in the margin separating the two candidates’ respective probability of winning: $|p_j - p_k|$. As noted in the Introduction, the most straightforward way of transforming this measure of margin into a measure of closeness is by simply subtracting it from ‘1’: $p_{jk} = 1 - |p_j - p_k|$. Cain (1978), on the other hand, uses the reciprocal of the absolute margin $p_{jk} = 1/|p_j - p_k|$ to rescale this measure as one of closeness rather than distance. Regardless, Properties 1-3 above apply, and it is straightforward to see that these approaches *satisfy Property 1*: whether using the subtracted margin or the reciprocal, the difference-based measures increase as the race between the two candidates gets closer.

However consider two voters, one with expectations $p_1 = 0\%$, $p_2 = 45\%$, and $p_3 = 55\%$; and the second with expectations $p_1 = 20\%$, $p_2 = 35\%$, and $p_3 = 45\%$. By the raw margin measure $p_{jk} = 1 - |p_j - p_k|$ the value of $p_{23} = .9$ will be identical for both voters. However, by Property 2 the probability of a 1st place tie between 2 and 3 should be higher
for the first voter, since 2 and 3’s shared lead over 1 is much larger. As well, consider a 
voter with expectations \( p_1 = 10\% \), \( p_2 = 30\% \), and \( p_3 = 60\% \). By the raw margin measure
\[
p_{jk} = 1 - \left| p_j - p_k \right|
\]
the value of \( p_{12} = .8 > p_{23} = .7 \), which violates Property 3, by which the 
probability of a 1\textsuperscript{st} place tie must be higher for the second set of candidates.

In their piece on turnout and abstention in French elections, Rosenthal and Sen (1973) present a \textit{conditional difference} measure which is consistent with Properties 1 and
3, but which does not rectify the above problem regarding Property 2. Their measure 
conditions the difference in probability of winning between candidates \( j \) and \( k \) on their 
shared closeness to the expected plurality winner, denoted by \( \hat{p} \). In particular, their 
function for the probability a first place tie is:
\[
p_{jk} = (1 - \left| p_j - p_k \right|) \cdot (1 - \hat{p} + \max[p_j, p_k]).
\]
(4)
To see that this measure does not solve the above problem regarding Property 2, note that 
when either \( j \) or \( k \) is the candidate with the highest probability of winning (4) reduces to
\[
p_{jk} = 1 - \left| p_j - p_k \right|,
\]
the same difference measures as above. Again, consider two voters, the 
first of which has \( p_1 = 0\% \), \( p_2 = 45\% \), and \( p_3 = 55\% \), while the second has \( p_1 = 20\% \), \( p_2 = 
35\% \), and \( p_3 = 45\% \). By Property 2, the probability of a first-place tie between the two 
leading candidates \( p_{23} \) should clearly be higher for the first voter, as the trailing candidate 
is further behind. However, by the Rosenthal and Sen measure this probability will be 
identical for both ( \( p_{23} = .9 \)).

Black (1978) adopted a creative approach to addressing the 3-dimensional 
challenge implied by the COV. This strategy employs the geometry of a \textit{simplex}, a
geometric tool used to plot interdependent 3-dimensional observations in a 2-dimensional space. To measure the closeness of a race between candidates $j$ and $k$, Black calculates the Euclidean distance between a voter’s actual position in the simplex and the line which would make these two candidates equal, *holding the third candidate’s vote share constant.* More particularly, define a probability vector as \{ $p_1$, $p_j$, $p_k$ \}, and consider the Euclidean distance between this probability vector and some other vector \{ $p_1$, $\hat{p}_j$, $\hat{p}_k$ \} where the latter represents the vector in which $j$ and $k$ would be tied for the 1st place, holding $p_1$ constant. For example, for the starting vector \{.2,.5,.3\} the associated vector would be \{.2,.4,.4\}. The Euclidian distance between these two probability vectors can be written as:

$$p_{jk} = \frac{1}{2} \left[ (p_j - \hat{p}_j)^2 + (p_k - \hat{p}_k)^2 + (p_1 - p_1)^2 \right].$$

(5)

Since $\hat{p}_j = \hat{p}_k = \frac{p_j + p_k}{2}$, this can in turn be simplified to $p_{jk} = \frac{p_j - p_k}{\sqrt{2}}$, which like the difference measures above does not satisfy Property 2.

Ordeshook and Zeng (1997) propose a ‘multiplicative’ measure $p_{jk} = p_j \times p_k$ which always satisfies Properties 2 and 3 but sometimes fails to satisfy Property 1. To see this, consider the following comparison of voters. For the first $p_1 = 30\%$, $p_2 = 34\%$, and $p_3 = 36\%$, while for the second $p_1 = 10\%$, $p_2 = 34\%$, and $p_3 = 56\%$. By Ordeshook and

---

21 The equations therein contain a minor error, which was corrected by Hermann and Pappi (2008)  
22 While Hermann and Papi (2008) adopt Black’s Euclidean indicator to measure closeness between the election’s two leading candidates, they employ a slightly distinct Euclidean formula to calculate the closeness of the race between the election’s *two trailing candidates* (pg. 243).
Zeng’s measure the probability of a 1st place tie between 2 and 3 \((p_{23})\) will be higher for the second voter than for the first voter: \((.56 \times .34) > (.36 \times .34)\). In fact, by Property 1 it is the reverse that should obtain: the second voter perceives the front-runner 3 to be much further ahead of 2 than the first voter (22% ahead vs. 2% ahead).

### 3b. Probability Ratios as an Axiomatically Viable Measure

We now present a distinct measure which satisfies all three core Properties, and which has the benefit of simplicity and transparency. It is grounded in *probability ratios* of the form \(p_{jk} = \frac{p_j}{p_k}\), where \(p_j \leq p_k\) such that \(p_{jk} \leq 1\). It is straight-forward to show that probability ratios always satisfy Properties 1 and 2. Regarding Property 1 compare two voters, the first of whom believes \(p_1 = 30\%\), \(p_2 = 34\%\), and \(p_3 = 36\%\), and the second of whom believes \(p_1 = 10\%\), \(p_2 = 34\%\), and \(p_3 = 56\%\). Unlike the multiplicative measure discussed above, for which \(p_{23}\) is higher for the second voter, here \(p_{23}\) will clearly be higher for the first voter: \((.34/.36) > (.34/.36)\). These ratio measures also satisfy Property 2. Again compare two voters, the first of whom believes \(p_1 = 30\%\), \(p_2 = 34\%\), and \(p_3 = 36\%\), and the second of whom believes \(p_1 = 10\%\), \(p_2 = 44\%\), and \(p_3 = 46\%\). Although the margin separating 2 and 3 is 2% for both voters, \(q_{23}\) will be higher for the second voter as required by Property 2: \((.44/.46) > (.34/.36)\).

Given this discussion, it is tempting to use these simple ratios to measure the closeness of a race between any two candidates. However, it is also straightforward to see that simple ratios may, under certain circumstances, violate Property 3. For example, consider a voter who believes 3 will win with probability 50%, 2 with probability 26%,
and 1 with probability 24%. If we use simple ratios to measure $p_{jk}$, then $p_{12} = (0.24/0.26) > p_{23} = (0.26/0.5)$, violating Property 3. To avoid this violation, we need to develop a slightly modified approach to measuring the closeness of the race between the election’s two trailing candidates.

Suppose that among candidates $j$, $k$, and $l$, candidate $l$ is the expected plurality winner. Consider the following measurement strategy:

$$\left\{ p_{jl} = \frac{p_j}{p_l}, p_{kl} = \frac{p_k}{p_l}, p_{jk} = \frac{p_j p_k}{p^2_l} \right\}. \quad (6)$$

So, for example, the specific values for a respondent whose most-preferred candidate, 1, is expected to place last, whose second-most preferred candidate, 2, is expected to place second, and whose least-preferred candidate 3, is expected to place first would be:

$$\left\{ p_{23} = \frac{p_2}{p_3}, p_{13} = \frac{p_1}{p_3}, p_{12} = \frac{p_1 p_2}{p^2_3} \right\}. \quad (7)$$

Appendix A proves the following Proposition, which confirms the viability of the ratio measurement strategy codified in (6):

* **Proposition 1:** The measurement strategy captured in (6) always satisfies Properties 1-3.

It is important to note that the strategy for measuring ‘closeness’ captured in (6) is in fact incomplete, as it applies only to voters who express *strict preference orderings* and *strict expectation rankings*. However, a non-negligible number of respondents in both the Canadian and British surveys assign two or more candidates the same utility and/or the
same probability of winning. Appendix A extends our ratio measurement strategy to include respondents who assign two or more candidates equal probabilities of winning. On the other hand, this paper focuses only on respondents who have strict preference-rankings, i.e. who do not express indifference between 2 or more candidates.\footnote{Analyzing respondents who assign two or more candidates the same utility implies a series of operational challenges which Black (1978) attempts to deal with by setting certain values of $p_j$ to ‘0’. Respondents who assign two or more candidates the same utility can be classified as either ‘Top-Indifference’, ‘Bottom-Indifference’, or ‘Pure-Indifference’ voters. Top-Indifferent (Bottom-Indifferent) respondents are indifferent between their two most-preferred (least-preferred) candidates. Pure-Indifference respondents are indifferent between all three. Preliminary data analysis suggests that ‘Top-Indifference’ profiles may engage in what might be labeled pseudo-strategic behavior: they tend on average to choose the candidate from among their two most-preferred parties which has the highest likelihood of winning. Though beyond this paper’s scope, it will be interesting in future papers to investigate whether this choice obeys an expected utility logic. For example, does pseudo-strategic voting become more likely when one’s least-preferred party is viable?}

The following figures present a series of descriptive comparisons of our ratio measure to the measures employed in past studies. The data we will use comes from the 1988 general election in Canada and the 2010 general election in Great Britain. As noted above, these cases were chosen because both of the associated national election surveys asked respondents to assign parties district-level probabilities of winning, which obviates the need to use proxies such as district-level vote-shares.\footnote{Furthermore, both elections were overwhelmingly 3-party contests, and both use identical formal institutions (parliamentary executive, single-member districts), facilitating cross-case comparisons. In the 1988 Canadian national election study respondents were asked to assign parties a probability of winning in their district. In the 2010 British national election study, respondents were asked to assign parties a likelihood of winning on a 1-10 scale.} For all respondents with strict preference-orderings in the (N=6,815 in the British data, N=1,580 in the Canadian data), Figure 1 plots ratio scores from (6) on the x-axis and the raw margin measure $p_{jk} = |p_j - p_k|$ on the y-axis.

\begin{equation}
\end{equation}
Given that the raw margin is technically a measure of ‘distance’ rather than ‘closeness’, the two measures would be perfect substitutes if they were perfectly negatively correlated, moving down the diagonal from the Euclidean point (0,1) to the point (1,0). As cursory inspection demonstrates, this is far from the case. In particular, pure difference measures often overestimate the probability of a first place tie. For example, the absolute difference would be identical for a voter who believed candidates \( j \) and \( k \) would each

---

25 The bivariate correlation for the \( p_{ij} \) measures is \( r=-.614 \), for the \( p_{i} \) measures is \( r=-.700 \), and for the \( p_{ij} \) measures is \( r=-.418 \). Similar plots for the Canadian data are qualitatively identical, though the bivariate correlations are slightly higher at \( r=-.783 \), \( r=-.751 \), and \( r=-.475 \) respectively.

26 Due to the fact that the raw margin in fact captures ‘distance’ rather than ‘closeness’, what we see in Figure 1 is rather an underestimate of distance.
receive 10% of the vote as for a voter who believed candidates $j$ and $k$ would each receive 40% of the vote. This follows directly from the discussion above: the absolute difference captures only the distance between $j$ and $k$, and not their shared viability with respect to the election’s third candidate. On the other hand, the ratio measure captured in (6) and analyzed in Appendix A would assign a much higher value of $p_{jk}$ to the second voter than to the first, as dictated by Property 2 above.

Figure 2 presents a similar correlational analysis, now replacing the absolute difference measures with Ordeshook and Zeng’s multiplicative measure $p_{jk} = p_j \times p_k$:

**Figure 2: Ratio vs. Multiplicative Measures**

**Figure 2a: BES2010**

**Figure 2b: CES1988**
In this case, both the ratio and multiplicative measures capture ‘closeness’, and would be perfect substitutes if they were perfectly positively correlated, moving up the diagonal from the Euclidean point (0,0) to the point (1,.25).\(^\text{27}\) As is clear from the scatter plots, the two measures are highly correlated, especially at the lower end of the spectrum.\(^\text{28}\) At higher values of \(j_k\), the two sometimes diverge. For example, for a voter who believes \(p_1 = 30\%\), \(p_2 = 34\%\), and \(p_3 = 36\%\), our ratio measure yields a higher estimate of \(p_{23}\) than the multiplicative measure. The opposite is true of a voter who believes \(p_1 = 10\%\), \(p_2 = 34\%\), and \(p_3 = 56\%\), and for whom the multiplicative index is higher. In the next section, we assess the performance of all three measures in statistical analyses.

4. Statistical Tests of Strategic Voting

As derived in Section 2, direct tests of strategic voting should be grounded in the reduced form of inequality (3) from above, restated here for expository purposes:

\[
(p_{23} + 2p_{12}) \cdot \sigma > 2p_{12} + p_{13}.
\]

When the left-hand side is greater than the right-hand side, the voter’s optimal choice should be to choose their second-most-preferred candidate 2 rather than their most-preferred candidate 1. In turn we now implement the following Logistic regression on the

\(^{27}\) The multiplicative measure reaches its maximum value of \(j_k = .5 \times .5 = .25\) when candidates \(j\) and \(k\) each have a 50\% probability of winning the election’s third candidates has a ‘0’ probability of winning.

\(^{28}\) The bivariate correlation for the \(p_{12}\) measures is \(r= .837\), for the \(p_{13}\) measures is \(r= .850\), and for the \(p_{23}\) measures is \(r= .869\). The bivariate correlations for the corresponding Canadian data are \(r= .798\), \(r= .830\), and \(r= .884\) respectively.
samples of voters with strict ordinal preferences rankings (we conduct separate analyses for the UK and Canada):

\[
prob_i(V_2 = 1) = \beta_0 + \beta_1 \cdot p_{12} + \beta_2 \cdot p_{13} + \beta_3 \cdot p_{23} + \beta_4 \cdot \sigma + \beta_5 \cdot (p_{23} \cdot \sigma) + \epsilon_i . 
\] (8)

Per the qualitative expectations implied by inequality (3), we would expect the closeness of the race between 1 and 3 (coefficient \( \beta_2 \)) to have a negative effect on the likelihood of strategic voting. On the other hand, we would expect the closeness of the race between 2 and 3 and the voter’s utility differential for 2 over 3 (coefficients \( \beta_3 \) and \( \beta_4 \)) to have a positive effect on the likelihood of strategic voting. However, consistent with the inequality (3) above, this effect should be interactive. Since the empirical model is a Logistic regression, it is difficult \textit{a priori} to state explicit expectations as to the sign on the interactive coefficient \( \beta_5 \), but we can state clear expectations as to the marginal effects: namely, at very high (low) levels of \( \sigma \) the effect of \( p_{23} \) should be positive and large (small); and similarly, at very high (low) levels of \( p_{23} \) the effect of \( \sigma \) should be positive and large (small). Finally, the size and significance of \( \beta_1 \) should be marginal, though to the extent that it has any effect it should be negative, since its ‘net’ effect should be stronger on the right-hand side of (3). The regression tables below will denote the \( p_{jk} \) indicators with the label \textit{Closeness J-K}, and the \( \sigma \) indicator as \textit{Utility Differential}.

One last methodological note is important before running the regression analyses. While the comparative static implications implied by (3) are intuitive, less immediately clear is the implication that \textit{not all voters can behave strategically}. Consider a voter who
believes her most-preferred candidate \(1\) will place 1\(^{st}\) in the election, and her second and third preferences \(2\) and \(3\) will place 2\(^{nd}\) and 3\(^{rd}\) in the election respectively. As well, recall that strategic voting is defined as abandoning \(1\) when \(2\) has a better chance of winning than \(1\), so as to prevent \(3\) from winning. Thus, by definition this voter should not cast a strategic vote, since \(1\) is a perfectly viable candidate and \(3\) is expected to place last.

Recent contributions begin to address this issue. For example, Ordeshook and Zeng (1997) and Alvarez et al. (2006) argue that strategic voting should be restricted to voters whose most-preferred candidate \(1\) is expected to place last in the election. On the other hand, Blais and Nadeau (1996) and Fisher (2004) argue that strategic voting may occur as long as a voter’s most-preferred candidate is expected to place lower than their second-most preferred candidate. To resolve this issue, in the aforementioned Kselman and Niou (2010) prove the following Proposition to identify the pool of potential strategic voters:

* Proposition 2: A necessary condition for strategic voting (i.e. for satisfying inequality (3) above) is that a voter’s second-most preferred candidate has a better chance of winning than her most-preferred candidate.

A cursory look at the data used in this paper provides suggestive preliminary evidence in favor of the COV: the likelihood that a voter chooses \(2\) is significantly higher among voters for whom \(1\) is expected to place lower than \(2\). For example, in the British data, among these potential strategic voters, the total percentage of those who choose \(2\) is 21\% (329/1,585), while among the remaining respondents it is only 2\% (115/5,138). In the Canadian data, once again, the choice for \(2\) is on average much more likely among potential strategic voters (68/289=23.5\%) than among those who believe \(1\) has a better chance than \(2\) of winning the election (56/1,250=4.5\%).
Despite the trend in both data sets, the fact remains that a small number of respondents choose 2 despite the fact that their most-preferred candidate has a better chance of winning. In the British data roughly 26% (115/444) of respondents who choose 2 fit this description, while the percentage is even higher in the Canadian data: 45% (56/124). Past analyses have tested the comparative static hypotheses discussed above on the entire sample of voters. However, including voters in analyses may corrupt the empirical results, generating coefficients of ‘strategic’ behavior which are impacted by non-strategic decisions.\(^{29}\) In turn, after running the Logit model (8) on the entire sample of voters, we also verify the robustness of our results in separate analyses which are restricted to the pool of potential strategic.

Table 1 presents empirical estimates of strategic voting for both the British and Canadian samples, on both the full sample of voters and the restricted sample of potential strategic voters. We begin by using the probability ratios analyzed in Proposition 1 as our measure of closeness.

\(^{29}\) Kselman and Niou (2011) develops a formal amendment to the Calculus of Voting which captures the signaling impact of a vote, i.e. the extent to which one’s vote not only affects short-term electoral outcomes but also political parties’ downstream behavior. The authors demonstrate that: a.) such signaling incentives may explain the choice for 2 even though 1 is expected to place higher than 2, and b.) these incentives do not in any way distort the Calculus of Voting’s predictions about strategic voting among voters for whom 2 is expected to place higher than 1.
As is evident at first glance, the results are on the whole strong and consistent with theoretical expectations, though they weaken somewhat in the smaller, restricted Canadian sample (column 4). Beginning with closeness of the race between 1 and 3 (row 2 in Table 1; coefficient $\beta_2$ in the above regression model), we see that it is negative, significant, and of roughly similar substantive size in all analyses. Figure 3 presents the substantive effects, generated from the larger N analyses in columns 1 and 3.
The first plot in Figure 3 is associated with column 1 from Table 1. Holding all other variables at their means, moving from a value of ‘0’ to a value of ‘1’ on Closeness 1-3 leads to a drop of roughly 6.5% in the likelihood of choosing 2. Furthermore, respondents
who have a value of ‘1’ on this variable essentially never vote strategically. As can be seen in the second plot from Figure 3, this effect is more or less replicated in the Canadian sample. The difference is that in the Canadian sample the effect seems to be stronger: moving from a value of ‘0’ to a value of ‘1’ on $p_{13}$ leads to a drop of roughly 15% in the likelihood of choosing 2. While as in the British data respondents who have a value of ‘1’ on this variable almost never vote strategically, the drop in the likelihood of choosing 2 is more consistent across the entire range of $p_{13}$ in the Canadian sample.

Also consistent with our predictions, the effect of the closeness of the race between 1 and 2 (coefficient $\beta_1$) is negative but small in both, and only registers as significant in the larger British sample. Figure 4 presents the substantive effects.

---

30 In fact the predicted probability curve is convex, and the drop in the likelihood of choosing 2 is most pronounced between respondents with values of $p_{13}$ between ‘0’ and ‘.4’; at values above ‘.5’ the predicted probability is close to ‘0’ and only slopes mildly downward.
Looking at both plots in Figure 4 we see that, holding other variables at their mean, moving from a value of ‘0’ to a value of ‘1’ on Closeness 1-2 leads to a drop of roughly 1% in the likelihood of choosing 2 among the British sample, and a drop of roughly 2% in the Canadian sample. The effect is fairly linear across the range of Closeness 1-2 and much weaker than that of Closeness 1-3.

Rows 3 and 4 report coefficient estimate on the individual variables Closeness 2-3 and Utility Differential (coefficients $\beta_3$ and $\beta_4$). In the first three columns the coefficient values are positive and significant, which is once again consistent with theoretical expectations. However, as already noted these parameters’ substantive impact should be interactive, as captured by the interaction coefficient $\beta_5$. The first thing to note is that the sign $\beta_5$ is negative in column 1 and positive in column 3. The second thing to notice is that this term only emerges as statistically significant in the British analyses, and that overall the strength of the aggregate interaction complex weakens in important ways on
As demonstrated by Brambor et al. (2006), the interpretation of interactive analyses requires moving beyond these raw coefficients to *marginal effects plots*, which allow for a better sense of the coefficients’ meaning and statistical power. Figure presents these marginal effects, generated once again from the larger N analyses reported in columns 1 and 3 from Table 1.

**Figure 5: Closeness of the Race between 2 and 3**

In the latter, neither the interaction term nor the coefficient on *Closeness 2-3* reaches standard significance levels. We return to this issue below.

---

31 In the latter, neither the interaction term nor the coefficient on *Closeness 2-3* reaches standard significance levels. We return to this issue below.
The plots in Figure 5 present the conditional effect of \textit{Closeness 2-3} on the likelihood of choosing 2 at the highest (‘1’) and lowest (‘0’) possible values of \textit{Utility Differential}. The results are once again strongly consistent with our predictions. When $\sigma = 0$, such that voters are essentially indifferent between candidates 2 and 3, increasing $p_{23}$ has almost no consequence, and the likelihood of choosing 2 is close to ‘0’ across the board. On the other hand when $\sigma = 1$, such that voters are essentially indifferent between candidates 1 and 2, increases in $p_{23}$ have a strong and consistent effect, leading to an increase of between 60-70\% in the likelihood of choosing 2 in both samples. In both samples, respondents who are more or less indifferent between their top two candidates, and who see 2 as being much more likely to defeat 3 in the election, vote strategically roughly 80\% of the time. As well, it becomes evident from these plots why the coefficient on $\beta_3$ falls just below statistical significance in the larger Canadian sample: at low levels of $p_{23}$, the slope when $\sigma = 1$ as opposed to $\sigma = 0$ is distinct but not as
differentiated as in the British data; as $p_{23}$ increases, so does the distinction between its
effect when $\sigma = 1$ as opposed to $\sigma = 0$.

In the British data, these results are robust to analyzing only the restricted set of
potential strategic voters (column 2 from Table 1). However, as already mentioned, the
interactive results weaken significantly in the restricted Canadian sample, in which
neither the coefficient on Closeness 2-3 nor that on the interactive term reaches standard
significance levels (the coefficient on Utility Differential is marginally significant). On
the one hand, this may not be surprising given that the degrees of freedom in column 4’s
analysis are greatly reduced as compared to the sample sizes in columns 1-to-3. This
interpretation of is reinforced by the fact that, although weaker, the results contain the
same substantive implications as those from the larger N analyses. However, they also
suggest the need for caution in future research on strategic voting, in order to ensure that
supposed evidence of strategic behavior is not being biased by the choice for 2 for non-
strategic reasons.

Overall, Table 1 and the associated substantive effects plots provide strong and
consistent evidence in favor of the COV’s basic logic, and suggest that this logic is robust
to distinct electoral contexts. This is reinforced by the high percentage of correct
predictions the model makes: in columns 1 to 4 respectively, the model correctly predicts
94%, 82%, 92%, and 77% of the respondents’ vote choices, a strong finding given that
we include nothing more than the basic instrumental parameters captured in the Calculus
of Voting. To what extent do these findings result from the use our new measure of
‘closeness’? To answer this question, we have conducted a series of comparative analyses
in which we replace our ratio-measure with the closeness indices used in past research.
As noted above, nearly all past papers on strategic voting have used the raw margin or margin-related measures (e.g. the Euclidean measure) to capture the ‘closeness’ of a race between two candidates in a three-candidate contest. As such, in Appendix B we present the results of an analysis that substitutes the raw margin measure \( p_{jk} = |p_j - p_k| \) into the regression model in equation (8).

The results in Table B1 of Appendix B are notably different from those reported in Table 1 above. Firstly, and importantly, they change significantly from columns 1 to 2 and 3 to 4, i.e. when they are generated using only the pool of potential strategic voters as opposed to the entire sample of respondents. The analyses conducted on the entire sample of voters yield results which are either ambiguous, or which in fact contradictory to the hypotheses of the COV. When analyzing only the pool of potential strategic voters the results strengthen somewhat, but remain weaker than those in Table 1 and are only partially consistent with the expectations generated by inequality (3) above (see the discussion in Appendix B).

As discussed in the Introduction, past research using these raw margin measures did not implement the fully-specified regression model detailed in equation (8), rather focusing on one or two parameters such as \( p_{12} \) and the utility differential \( (U_1 - U_2) \). In turn, these contradictory tendencies went unnoticed. By embedding these measures in a fully-specified analysis, we now see that these papers were likely uncovering some evidence of strategic choice as predicted by the COV; and furthermore that this evidence was being driven strongly by potential strategic voters. Among such voters, and

---

32 For example, when analyzing the entire sample of voters, the choice for 2 becomes more (less) likely when 1 and 3 (2 and 3) are in a close race, findings which are in direct contradiction to the expectations from (3) above.
especially among voters whose most-preferred candidate 1 is expected to be the plurality loser, difference-based measures do capture the basic idea that as 1 falls further behind, as voters become more indifferent between 1 and 2, and as the race between 2 and 3 tightens strategic voting should become more likely. However, in not satisfying Property 2 above, and in not implementing the fully-specified and interactive analysis captured in (8), difference-based measures captured these dynamics imprecisely, and axiomatically they remain unsuitable as general measures of closeness in multi-candidate contests.

While most past studies used such difference-based measures in their analyses, Ordeshook and Zeng (1997) use a multiplicative measure which, recalling Figure 2 above, exhibits a fairly strong empirical correlation to our ratio measure. Perhaps not surprisingly, when introducing their measure into the regression model detailed in equation (8), the results largely replicate those from (Appendix C, Table C1). This is true not only of the discrete coefficient estimates, but also of the sign and size of substantive results. The variable \( p_{12} \) has little to no effect on the likelihood of choosing 2, the variable \( p_{13} \) has a strong negative effect on the likelihood of choosing 2, and the interaction terms behave exactly as predicted and exactly as displayed in Figure 5 (see Appendix C). In the Conclusion we return to this similarity as part of a more general discussion of extensions to this paper’s axiomatic measurement approach.

5. Concluding Discussion

This paper’s contributions are simultaneously theoretical, methodological, and substantive. At the theoretical level, we develop an axiomatic approach to measuring the closeness of a race between two candidates in a three-candidate plurality-rule election.
More particularly, we identify three core properties that any reasonable closeness measure should satisfy, and demonstrate that past indices used in empirical research have violated at least one of these three criteria. Methodologically, we then propose a new way to measure closeness, and demonstrate that this new measurement scheme does indeed satisfy the three-dimensional requirements of Properties 1-3. Substantively, we then use this index to conduct an exhaustive test of strategic voting hypotheses emerging from the Calculus of Voting, demonstrating strikingly consistent and supportive results in two distinct plurality-rule contests (UK 2010 and Canada 1988). These results provide strong and novel evidence that voters do indeed behave according to the dictates of expected-utility maximization once in the voting booth.

While the paper here addressed three-candidate elections, the methodology can be extended to winner-take-all election with \( N > 3 \) candidates. Consider for argument’s a four-candidate contest. In this contest, voters may not only at times choose their second-most-preferred candidate, but also their third-most-preferred candidate. This would occur when this 3rd preference is in a close race for first place with the voter’s least-preferred candidate or party, and when this third preference is greatly preferred to this least-preferred candidate. Using the COV it would be straight-forward to derive similar voting conditions as that presented in (3) from the text. In these conditions the parameters \( p_{12}, p_{13}, \) and \( p_{23} \) would once again appear, but now they would be joined by the parameters \( p_{14}, p_{24}, \) and \( p_{34} \). Measuring these closeness parameters in a four-candidate contest presents additional challenges, as for any pair of candidates one would need to take into account their relative position as compared to the remaining two candidates. While this presents additional
challenges, the core importance of Properties 1 and 2 above persists: the measurement strategy will need to ensure that the parameters increase when the first place contest between two candidates tightens, and increase when the candidates’ relative standing as compared to the remaining two candidates improves. We are now considering the ideal electoral setting in which to extend the analysis to elections with \( N>3 \) candidates, and look forward to addressing these theoretical and operational issues in future research.

COALITION GOVERNMENT

The methodology developed in this paper will also be essential in future studies of what motivates citizens to turn out and vote in the first place. The question of whether or not close elections stimulate turnout has long been of interest to scholars of voter behavior. Standard difference-based measures of closeness have been sufficient to study this issue in the American political context, where both Presidential and Congressional elections tend to be two-party affairs. The question becomes both theoretically and empirically more complicated in elections with more than two options. In these contexts turnout will likely be dampened when one candidate is expected to win in a landslide. However, as we move from such landslide situations to more competitive races, the decision to turnout will be affected by how individual closeness measures interact with a voter’s utility for the respective candidates. For example, if a voter is largely indifferent between three candidates in an election, increasing the closeness of the race between said candidates should have little effect on the likelihood of turnout. In contrast, if voters have strong preference differentials, then increasing closeness among the respective candidates should

\[33\] In particular, it would require another series of *reasonability criteria*, stipulating for example that the probability of a tie between the two trailing candidates must be the smallest of all six probability parameters; that the probability of a first-place tie between the leading candidate and the candidate expected to place third must be greater than the probability of a first-place tie between the candidate expected to place second and the trailing candidate; etc.
indeed increase the likelihood of turnout. This paper’s measurement strategy will be essential for testing such fine-grained interactive hypotheses in future empirical work.

Studies of protest voting also often invoke the competitiveness of electoral outcomes as a motivating behavioral condition. Protest voting can be broadly defined as casting a vote for a less-preferred party, whose primary motivation is to send a signal of disaffection to some element of the political status quo, in the hope that this signal is received and acted upon by elected officials. Thus, it represents an act of downstream instrumental rationality. That said it may carry immediate instrumental risks, insofar as it may contribute to the election of a party which, in the short-term, voters find less than appetizing. The closeness measures developed in this paper thus become a crucial element of potential protest voters’ calculus. Distinct papers cited in the paper’s introduction (Kang 2004; Kselman and Niou 2011) suggest that, among some voters, the decision to cast a protest vote in 3-candidate plurality may be optimal when the race is non-competitive, for example when their most-preferred candidate has a comfortable 1st place lead. However, among distinct subsets of voters protest voting may be attractive when elections are competitive, as it is precisely in these elections that signals of disaffection are likely to be most ‘impactful’. Once again, testing these fine-grained hypotheses will be made possible by the methodology for measuring closeness developed above.

Beyond these additional empirical applications, this paper should be seen as only a first step in developing an axiomatic approach to measuring electoral competitiveness. Note for example the interesting empirical parallels uncovered in Sections 3 and 4 between this paper’s ratio measure and the Ordeshook and Zeng multiplicative measure. The key difference between the two lies in our emphasis on Property 1, namely that a closeness
measure should always increase as the distance between the election’s expected plurality winner and one of the trailing candidates decreases. As noted above, the Ordeshook and Zeng measure sometimes fails to capture this dynamic: consider two voters, such that for the first \( p_1 = 30\% \), \( p_2 = 34\% \), and \( p_3 = 36\% \), while for the second \( p_1 = 10\% \), \( p_2 = 34\% \), and \( p_3 = 56\% \). By Ordeshook and Zeng’s measure the probability of a 1st place tie between 2 and 3 \((p_{23})\) will be higher for the second voter than for the first voter: \((.56 \times .34) > (.36 \times .34)\), which violates Property 1. However, note also that, for the second voter, although the closeness of the race between the two plurality winners has decreased, their shared lead over the plurality loser has increased. In other words, although not consistent with Property 1 above, the Ordeshook and Zeng measure is not without logic of its own: it emphasizes the two leading candidates’ shared lead over the expected loser rather than the margin separating their own expected outcomes. Might there be contexts in which voters focus more on whether or not one candidate is truly out of the race, rather than margin separating the two leading candidates? We strongly believe that Property 1 is an essential criterion, but the similarity of results using multiplicative and ratio measures suggest the possibility of future debate and analysis in this area.

Perhaps even more basic is the question of how voters form their estimates of candidates’ probability of winning \( p_j \), which are the core inputs to measuring the closeness between two candidates. One possibility is that these estimates are grounded in candidates’ respective expected vote shares, as gleaned from poll numbers and/or past election results. Indeed, this is the explicit assumption in past papers which use such vote shares as a proxy for voters’ expectations. However, consider a voter who believes their first-preference 1 will receive 10% of the vote, while their second and third-preferences 2
and 3 will receive 40% and 50% respectively. Will this translate in a one-to-one fashion in probabilities of winning, such that for said voter $p_1 = 10\%$, $p_2 = 40\%$, and $p_3 = 50\%$? Perhaps this voter will rather assign their first-preference a probability $p_1 = 0$, knowing that they are completely out of the race. And what about a voter believes their first-preference 1 will receive 30\% of the vote, while their second and third-preferences 2 and 3 will receive 30\% and 40\% respectively? Perhaps in this case the values $p_1$, $p_2$, and $p_3$ will more directly reflect expected vote shares. In preliminary experimental analyses conducted by the authors, it appears that crucial to this translation process is the relative position of the expected 2nd place candidate with respect to the expected plurality winner and loser. Although at a very early stage, this represents a promising avenue for future research into the core inputs of our measures of electoral closeness.
Appendix A: Proof of Proposition 1

Suppose among candidates $j, k, l$ is the expected plurality winner and $j$ the plurality loser. The following measurement strategy satisfies Property 1 trivially because as either the denominator increases or numerator decreases in value, the ratios $p_{jl}$ and $p_{kl}$ decrease, and vice versa, as required by Property 1.

\[
\left\{ p_{jl} = \frac{p_j}{p_l}, p_{kl} = \frac{p_k}{p_l}, p_{jk} = \frac{p_j p_k}{p_l^2} \right\}
\]

To prove that this measure satisfies Property 2, we need to prove that holding two candidates’ expected probability separation constant and increasing their viability vis-a-vis the third candidate, the ratios increase. In other words, we need to show that when transferring expected probability margins from the third candidate to the two relevant candidates, while keeping their separation constant, the ratios increase. For any positive constant $a$ and any $p_j < p_i$ it is a mathematical fact that $\frac{p_j}{p_l} < \frac{p_j + a}{p_l + a}$, which implies that the first two ratios in the above measurement strategy $p_{jl}$ and $p_{kl}$ satisfy Property 2.

Similarly, in the case of $p_{jk}$ a transfer in equal parts of expected probability margins away from candidate $l$ implies the move from $\frac{p_j p_k}{p_l^2}$ to $\frac{(p_j + a)(p_k + a)}{p_l^2 - 2a}$. In turn, the ratio increases because the denominator decreases and numerator increases in value.

To establish Property 3, we need to establish that $p_{kl} > p_{jl}, p_{jk}$. It is trivial to see that $p_{kl} > p_{jl}$ since $p_k > p_j$. As well, rearranging demonstrates that $p_{kl} > p_{jk}$ as long as $p_k < p_j$, which again is satisfied trivially.  \textbf{QED}

Move now to voters with ‘weak’ expectation orderings, and begin with voters who believe that one candidate is in the lead, and that the other two candidates have an equal probability of winning which is lower than the leader’s probability of winning. For such voters, the following measures satisfy Properties 1-3:
Move now to voters who believe that two front-runner candidates have equal chances of winning, and that the third candidate has a lower probability of winning than these two front-runners. For such voters, the following measures satisfy Properties 1-3:

\[
\begin{cases}
p_{jl} = p_{kl} = \frac{p_j = p_k}{p_l}, \quad p_{jk} = \frac{p_j p_k}{p_l^2}
\end{cases}
\]

Finally, for any voter in C13 who assigns all parties an equal chance of winning, all of the closeness measures can be set to ‘1’.

QED
Appendix B: Statistical Results, Raw Margin Index

Recall that the measure $p_{jk} = |p_j - p_k|$ is in fact one of ‘distance’ rather than ‘closeness’, and thus that our expectation as to the substantive effect of the independent variables will be reversed. Table B1 presents results of the analysis.

### Table B1: Logistic Regression Results, Raw Margin Measure

<table>
<thead>
<tr>
<th></th>
<th>Model1 UK Full</th>
<th>Model2 UK Reduced</th>
<th>Model3 Canada Full</th>
<th>Model4 Canada Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Closeness 1-2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Raw Margin)</td>
<td>0.433**</td>
<td>3.224***</td>
<td>0.181</td>
<td>2.400***</td>
</tr>
<tr>
<td></td>
<td>(0.174)</td>
<td>(0.338)</td>
<td>(0.357)</td>
<td>(0.720)</td>
</tr>
<tr>
<td><strong>Closeness 1-3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Raw Margin)</td>
<td>-0.795***</td>
<td>1.169***</td>
<td>-0.706*</td>
<td>0.615</td>
</tr>
<tr>
<td></td>
<td>(0.167)</td>
<td>(0.339)</td>
<td>(0.368)</td>
<td>(0.853)</td>
</tr>
<tr>
<td><strong>Closeness 2-3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Raw Margin)</td>
<td>1.674**</td>
<td>-2.747***</td>
<td>1.786</td>
<td>-0.256</td>
</tr>
<tr>
<td></td>
<td>(0.796)</td>
<td>(0.959)</td>
<td>(1.128)</td>
<td>(1.452)</td>
</tr>
<tr>
<td><strong>Utility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differential</td>
<td>6.409***</td>
<td>4.867***</td>
<td>3.231***</td>
<td>3.949***</td>
</tr>
<tr>
<td></td>
<td>(0.570)</td>
<td>(0.679)</td>
<td>(0.763)</td>
<td>(1.225)</td>
</tr>
<tr>
<td><strong>Closeness 2-3 X</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differential</td>
<td>-0.557</td>
<td>1.706</td>
<td>-1.413</td>
<td>-2.465</td>
</tr>
<tr>
<td></td>
<td>(1.133)</td>
<td>(1.402)</td>
<td>(1.868)</td>
<td>(2.327)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-6.895***</td>
<td>-5.201***</td>
<td>-4.143***</td>
<td>-3.588***</td>
</tr>
<tr>
<td></td>
<td>(0.415)</td>
<td>(0.486)</td>
<td>(0.487)</td>
<td>(0.765)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>6,815</td>
<td>1,580</td>
<td>1,418</td>
<td>281</td>
</tr>
<tr>
<td><strong>Log Likelihood</strong></td>
<td>-1,368.798</td>
<td>-635.792</td>
<td>-393.821</td>
<td>-141.873</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

The first thing to note is that, unlike the results in Table 1 above, the results in Table B1 change significantly from columns 1 to 2 and 3 to 4, i.e. when they are generated using only the pool of potential strategic voters. Beginning with $p_{12}$, while in columns 1 and 3 this variable registers as marginally positive (consistent with the results from Table 1), moving to columns 2 and 4 we see that it becomes the strongest predictor of strategic choice among the closeness measures. This replicates a primary finding from past research, which has emphasized that choosing 2 becomes more likely as the distance $p_{12}$ increases.
As demonstrated in the following marginal effects plots, the latter effect is substantively important. The first two plots present the substantive effects figures which emerge from columns 1 and 3 of Table B1, namely those associated with analyses on the entire British and Canadian samples respectively. Consistent with regression coefficients, the marginal effect in these first two plots is minimal. The third and final plot presents substantive effects which emerge from column 2 of Table B1, namely those associated with analysis on the sample of potential strategic voters in the British data.\textsuperscript{34} In this final plot moving from a value of ‘0’ to ‘1’ on $p_{12}$ leads to an increase of roughly 50% in the likelihood of choosing one’s second-most-preferred candidate 2.

\textbf{Figure B1: Closeness of the Race between 1 and 2}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figB1}
\caption{Closeness of the Race between 1 and 2}
\end{figure}

\textsuperscript{34} The substantive effects plots associated with column 3 of Table 2 are omitted for reasons of redundancy (available on request).
While past research has often interpreted the evidence in this third plot as consistent with strategic behavior in the COV, our derivation and analysis of (3) suggests that the effect of $p_{12}$ should in fact be quite muted. What then should we make of this finding from past studies, replicated here in columns 2 and 4? A simple and blunt answer would be to dismiss the result entirely, given that these raw-margin measures do not satisfy Property 2 from the texts, and thus do not actually capture ‘closeness’ in a 3-candidate contest. However, this answer would not be entirely fair.

Consider, for example, a voter who sees her most-preferred candidate, 1, in last place, her second preference, 2, in 2nd place, and her least-preferred candidate, 3, as the expected plurality winner. In this case, it does make sense for the voter in question to become increasingly likely to choose 2 as this candidate’s distance from 1 increases, since this implies increasing competitiveness (un-competitiveness) of 2 (1) with the voter’s least-preferred candidate 3.

Similar remarks pertain to the remaining statistical coefficients in Table B1. Note that the remaining closeness coefficients in columns 1 and 3 are inconsistent with the predictions from (3) above. Firstly, the choice for 2 becomes less likely as the distance between 1 and 3 increases, while the opposite should be true: it is precisely when 1 and 3 are not in a competitive race that choosing 2 should be more appetizing. However, the result becomes consistent with the predictions of inequality (3) when we move to the restricted pool of potential strategic voters. This emerges clearly in the following marginal effect plots, the first two plots present the substantive effects figures which emerge from columns 1 and 3 of Table B1, and the third of which presents substantive effects which emerge from column 2 of Table B1.
Figure B2: Closeness of the Race between 1 and 3
Regarding the final plot, consider a voter who sees her most-preferred candidate, 1, in last place, her second preference, 2, in 2nd place, and her least-preferred candidate, 3, as the expected plurality winner. Columns 2 and 4 from Table 2 suggest that said voter will become more likely to choose 2 as the distance $p_{13}$ increases, which again makes perfect sense in the context of expected-utility maximization. Of course, the substantive effect is much smaller than that of the effect of increases in $p_{12}$, which is contrary to the theoretical expectations derived in inequality (3). Nonetheless, the sign now moves in the correct direction.

A similar set of results emerge with regards to the interaction between $\sigma$ and $p_{23}$. In columns 1 and 3 of Table B1, the results run counter to expectations: for example, when $\sigma = 1$, i.e. when a voter is more or less indifferent between 1 and 2, the likelihood of choosing 2 decreases as the margin separating 2 and 3 decreases. The opposite should be true: when a voter is indifferent between her two most-preferred candidates, she should become more likely to choose 2 when 2 and 3 are in a close race for first place. However, moving to column 2 of Table B1, the results now move in the right direction, though the effect is much weaker than predicted, and much weaker than the effect uncovered in Table 1 and Figure 5. These results are summarized in the following marginal effects plots.

**Figure B3: Closeness of the Race between 2 and 3**
Conditioning effect of p23_absdiff by Sigma (CES1988)

Conditioning effect of p23_absdiff by Sigma (BES2010, Reduced)
## Appendix C: Substantive Results, Multiplicative Index

### Table 3: Logistic Regression Results, Multiplicative Measure

<table>
<thead>
<tr>
<th></th>
<th>Model1 UK Full</th>
<th>Model2 UK Reduced</th>
<th>Model3 Canada Full</th>
<th>Model4 Canada Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closeness 1-2 (Multiplicative)</td>
<td>-1.300* (0.747)</td>
<td>-4.325*** (1.221)</td>
<td>-0.888 (1.359)</td>
<td>0.054 (2.423)</td>
</tr>
<tr>
<td>Closeness 1-3 (Multiplicative)</td>
<td>-10.069*** (0.976)</td>
<td>-9.382*** (2.173)</td>
<td>-9.140*** (1.781)</td>
<td>-10.636** (4.223)</td>
</tr>
<tr>
<td>Closeness 2-3 (Multiplicative)</td>
<td>18.528*** (2.747)</td>
<td>9.114** (3.542)</td>
<td>10.109** (4.269)</td>
<td>3.051 (5.951)</td>
</tr>
<tr>
<td>Utility Differential</td>
<td>7.308*** (0.550)</td>
<td>6.861*** (0.828)</td>
<td>2.893*** (0.709)</td>
<td>2.498** (1.206)</td>
</tr>
<tr>
<td>Closeness 2-3 X Differential</td>
<td>-12.130*** (3.952)</td>
<td>-10.401** (5.087)</td>
<td>-0.656 (7.435)</td>
<td>4.567 (9.724)</td>
</tr>
<tr>
<td>Constant</td>
<td>-7.258*** (0.405)</td>
<td>-5.234*** (0.593)</td>
<td>-3.690*** (0.477)</td>
<td>-2.594*** (0.868)</td>
</tr>
<tr>
<td>Observations</td>
<td>6,815</td>
<td>1,580</td>
<td>1,418</td>
<td>281</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-1,176.751</td>
<td>-643.547</td>
<td>-366.146</td>
<td>-142.198</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01
Figure C1: Closeness of the Race between 1 and 3
Figure C2: Closeness of the Race between 1 and 2

*Predicted effect of p12_multi on Strategic Voting (BES2010)*

*Predicted effect of p12_multi on Strategic Voting (CES1988)*
Figure C3: Closeness of the Race between 2 and 3

*Conditioning effect of p23* multi by *Sigma* (BES2010)

*Conditioning effect of p23* multi by *Sigma* (CES1988)
Appendix D: Data Sources and Survey Items


BES Campaign Internet Panel Survey - All Waves and Constituency-Level Political Variables bes2010_campaignperiodquestonnaire.pdf

**Survey Items on Preferences for the three major parties**

q63- On a scale that runs from 0 to 10, where 0 means strongly dislike and 10 means strongly like, how do you feel about the Labour Party?
q64- On a scale that runs from 0 to 10, where 0 means strongly dislike and 10 means strongly like, how do you feel about the Conservative Party?
q65- On a scale that runs from 0 to 10, where 0 means strongly dislike and 10 means strongly like, how do you feel about the Liberal Democrats?

**Survey Items on District-Level Probabilities of Winning for the three major parties**

q43- On a scale that runs from 0 to 10, where 0 means very unlikely and 10 means very likely, how likely is it that the Labour Party will win the election in your local constituency?
q44- On a scale that runs from 0 to 10, where 0 means very unlikely and 10 means very likely, how likely is it that the Conservative Party will win the election in your local constituency?
q45- On a scale that runs from 0 to 10, where 0 means very unlikely and 10 means very likely, how likely is it that the Liberal Democrats will win the election in your local constituency?

**Expected Vote Choice**

Q35 – If you do vote in the general election, have you decided which party you will vote for, or haven't you decided yet? == Yes, decided. Which party is that?

**Canadian Election Survey 1988:**
[http://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/9386](http://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/9386)

1988 CANADIAN NATIONAL ELECTION SURVEY CAMPAIGN ELECTION SURVEY: MODIFIED FORMAT

**Survey Items on Preferences for the three major parties**

d2d - How would you rate THE CONSERVATIVE PARTY? The thermometer runs from 0 to 100 degrees, where 0 represents a very negative feeling and 100 a very positive feeling?
d2e - How would you rate THE LIBERAL PARTY? The thermometer runs from 0 to 100 degrees, where 0 represents a very negative feeling and 100 a very positive feeling?
d2f - How would you rate THE NEW DEMOCRATIC PARTY? The thermometer runs from 0 to 100 degrees, where 0 represents a very negative feeling and 100 a very positive feeling?

Survey Items on District-Level Probabilities of Winning for the three major parties

f1a - (Using the 0 to 100 scale), what do you think the CONSERVATIVE party's chances are of winning IN YOUR RIDING?
f1b - (Using the 0 to 100 scale), what do you think the LIBERAL party's chances are of winning IN YOUR RIDING?
f1c - (Using the 0 to 100 scale), what do you think the NEW DEMOCRATIC party's chances are of winning IN YOUR RIDING?

Expected Vote Choice

b2 - Which party do you think you will vote for: the Conservative Party, the Liberal Party, the New Democratic Party, or another party?
REFERENCES


