Bright IDEA -

and

Math Left Behind?

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Why Are We Here?

- Head count (3rd graders)
  - Before BI: Virtually no G&T nomination
  - After BI: Control - 72 (10%); BI - 88 (24%)
  - Statistical Significance: “Computer goes banana”
- Math problem-based questionnaire
    - Miguel and Tara counted their marble collections. Miguel has 23 bags of ten marbles and 13 left over. Tara has 17 bags of ten marbles and 8 left over. If they put their marbles together how many marbles will there be?
- Only 17% correct, with 7% sound reason (BI vs. control - no difference)

What Should We Accomplish?

- Overarching Goal
  - Foster teachers' motivation/capacity to nurture K-2 students' mathematical talents
- Objectives
  - Promote relevant mathematical understandings
  - Nurture an image of powerful pedagogy
  - Equip you with work plan to accomplish the goal
- Process
  - Note taking; Questions (you, us); Struggles

Self-Examination

- Take your time (individually, quietly)
- Answer each and every question on the student Math Problem-Based Questionnaire
- Norm: Solution = Answer + Justification
- Done?
- In a different color pen/pencil, add next to each question what will YOU consider a 'good enough' justification from a student

Math Can Be ***M&M*** Sweet

- Demo pedagogical approach (different than used to)
- Form groups of 5 with folks from different schools
- Read hand-out (M&M Activity Part 1)
- Norms:
  - Solution = answer + justification
  - "I don't know" or "I am not sure" are 100% acceptable
  - There are NO stupid questions (suggest: avoid "what's the time?")
  - Instructors' questions do not indicate correctness of solutions
  - Sharing evolving ('unfinished') ideas/questions makes perfect …
- Questions?
- Let's play

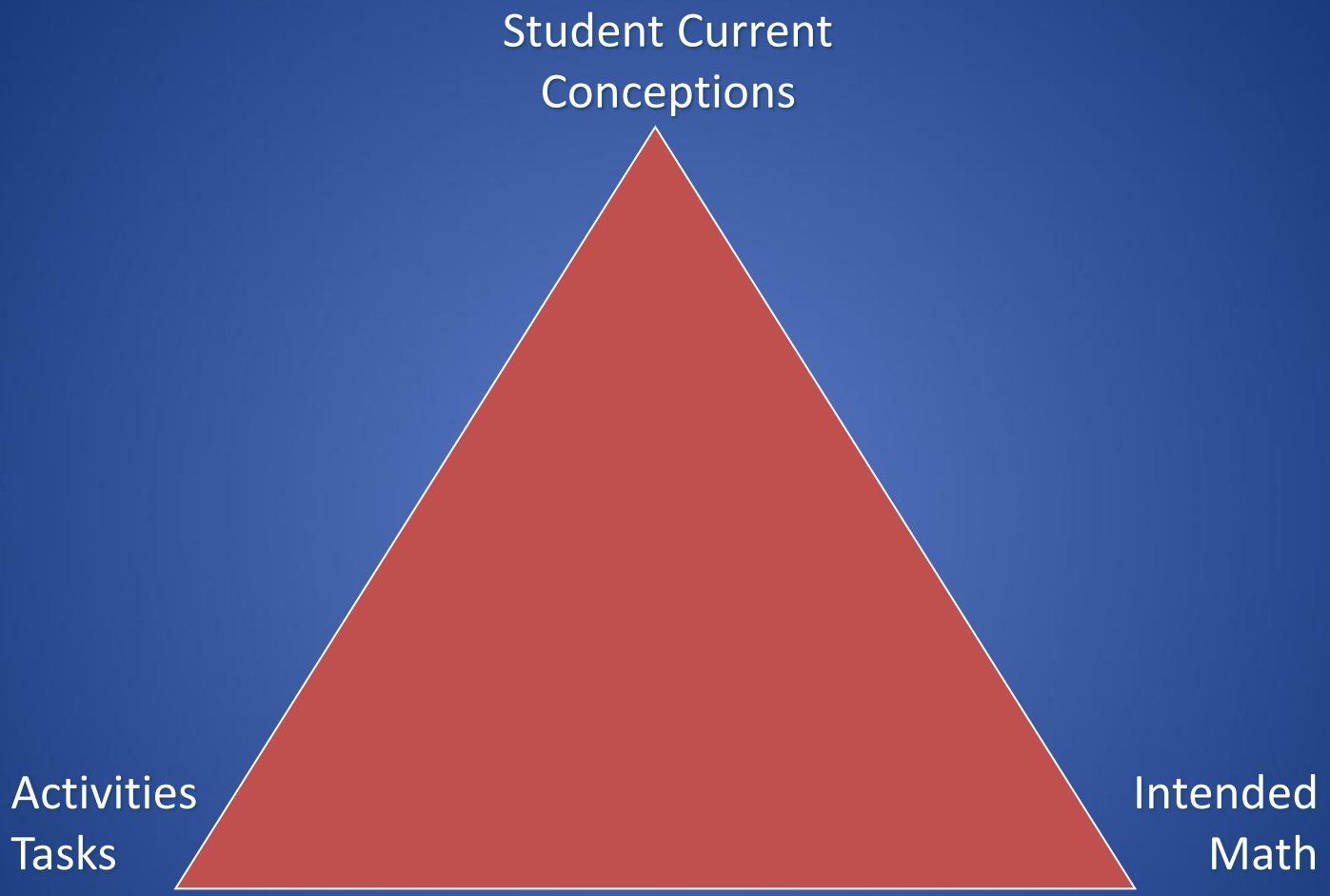
Math Can Be ***M&M*** Diverse/Exciting

- Read in group
  - M&M Activity (Part 2)
- Volunteer
  - Explain what happens next
  - Others - pitch in
- Let's learn different solutions

Sweeten Our Base-Ten Place-Value

- Munch on:
  - Some M&Ms
  - Is our group's method a base-ten, place value (PV-B_{10}) system? Why or why not? (See hand-out Part 3)
  - Each member should be able to present group's conclusions
- Group presentations
  - Ron/Matt "picks" presenter
  - Rest: critique/question
- Our statement: Which method is PV-B_{10}?

Teaching Triad



Delving Into PV-B_{10}: "Base"

- "Truncated" version of m&multiplication:
  - $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = ?$ $13 \cdot 13 \cdot 13 \cdot 13 \cdot 13 \cdot 13 = ?$
  - 5^6 13^6
  - What do we call the small number (e.g., ^{6}) on the top-right?
  - What do we call the larger number (e.g., 5 or 13)?
- In General: $b^n = (b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$ …. n times), where 'b' is called "base" and '^{n}' is called "exponent"
- **Questions?**

Delving Into PV-B10: "Place Value"

- Position (place) determines value of unit (magnitude) counted
- Direction matters: Right digit is for units of 1, defined as $\underline{base}^0$ (i.e., $\underline{b}^0 = 1$ by definition)
  - moving left increases unit exponentially

    $< \ldots \underline{b}^5 \underline{b}^4 \underline{b}^3 \underline{b}^2 \underline{b}^1 \underline{b}^0$
- Invariant Ratio: The base is also the constant ratio between any two consecutive units (e.g., 10)
- **Questions?**

Delving Into PV-B_{10}: "Base Ten"

- Exponential base is 10: Every position symbolizes a unique unit (magnitude, consecutive powers of 10)
- **Units:** Conceived of as results of recursively grouping-by-10
- **Digits:** Exactly ten, mutually exclusive digits (0; 1 through 9) to symbolize the 'count' of each different-magnitude unit
- **Unit 'Cap':** Exponential base / digit limit imply upper 'cap' of actual units per position (up to 9; exchange if ten or more)
- **Questions?**

System: Place Value AND Base Ten

- **'Truncated' form** symbolizes addition of each different-magnitude unit multiplied by its 'count'
  - Example: 3,904 means

    $3 \cdot 10^3 + 9 \cdot 10^2 + 0 \cdot 10^1 + 4 \cdot 10^0$ **OR**

    $3 \cdot 1{,}000 + 9 \cdot 100 + 0 \cdot 10 + 4 \cdot 1$
- **Commas** (every 3 digits; from the right) serve to help immediacy of position identification
  - 671955403
  - 671,955,403
- **Challenge Problem**: What happens when "adding" a 0 to the right of a 4-digit number? Why?

Roles of Zero (0)

- **Symbol** for 'no units in this position' (the famous notion of 'place holder')
- **Additive invariant:** x + 0 = x for every (x)
  - Example: the 'Tens' digit of 584 + 203 MUST be 8 (why???)
- **Origin for measuring** (e.g., number line, temperatures, etc.)
- **Diet** (as in zero M&Ms)
- 0-my-gosh (as in "let's take a break")

Learning PV-B_4

Rigorous Math for Elementary Teachers

- Threefold Purpose:
  - Promote rigorous understanding of PV, B_{10} system
  - Develop understanding of any PV system (e.g B_5)
  - Experience/appreciate children's difficulties (via a similar learning sequence)

1) Identify B_4 Digits

- Individuals
  - List all digits used for PV-B_4
- Pairs
  - Compare lists; explain why list is complete
- Entire Cohort
  - Share/finalize list & reasons

2) Rote Counting in B_4

- Entire Group
  - Sit in a circle
  - Ron begins counting (1) and the process continues to the right, each person saying one number word
  - When someone makes a mistake, s/he begins from 1
  - Our goal: To reach 1000_4 (64_{10}) without a mistake
- Two Groups - Competition
  - Correctly complete the count to 1000_4 before the other group

3) Writing Numbers in B_4

- Entire Cohort
  - Ron/Matt show how we write numbers in B_4 (while adding Unifix cubes one at a time on overhead)
  - Ron/Matt presents the 1000_4 chart; teachers suggest patterns they notice

1000_4 Chart

1_4	2_4	3_4	10_4
11_4	12_4	13_4	20_4
21_4	22_4	23_4	30_4
31_4	32_4	33_4	100_4
101_4	102_4	103_4	110_4
111_4	112_4	113_4	120_4
121_4	122_4	123_4	130_4
131_4	132_4	133_4	200_4
201_4	202_4	203_4	210_4
211_4	212_4	213_4	220_4
221_4	222_4	223_4	230_4
231_4	232_4	233_4	300_4
301_4	302_4	303_4	310_4
311_4	312_4	313_4	320_4
321_4	322_4	323_4	330_4
331_4	332_4	333_4	1000_4

1000_4 Chart

1_4	2_4	3_4	10_4
11_4	12_4	13_4	20_4
21_4	22_4	23_4	30_4
31_4	32_4	33_4	100_4
101_4	102_4	103_4	110_4
111_4	112_4	113_4	120_4
121_4	122_4	123_4	130_4
131_4	132_4	133_4	200_4
201_4	202_4	203_4	210_4
211_4	212_4	213_4	220_4
221_4	222_4	223_4	230_4
231_4	232_4	233_4	300_4
301_4	302_4	303_4	310_4
311_4	312_4	313_4	320_4
321_4	322_4	323_4	330_4
331_4	332_4	333_4	1000_4

4) Three-Way 1-to-1 Correspondence

(Number words, objects, numerals)

- Triplets
  - One person piles up several M&M candies
  - The next person counts them in B_4
  - The third person follows the counting with the 1000_4 chart and corrects/prompts as needed
  - If objects were counted correctly, switch roles and repeat the process
  - Apply to other groups of objects (fingers, tapping motions, rope jumps, chairs, teachers, etc.)

5) Writing Numbers in B_4

- Pairs

  Pose/Solve problems that require to:

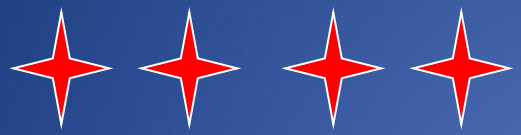
  a. write number$_4$ for a given group of objects

  b. put out a group of objects that corresponds to a given number$_4$

- Pairs, then Cohort
  - Fill and justify the missing cells

- Quiz: Five tasks of writing number$_4$ of objects

6) “Quiz”: Writing Numbers$_4$

1. 
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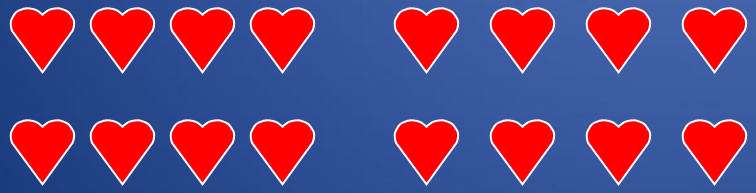
5. 

6) Guided Reflection (a)

- Individuals
  - Jot down at least 4 difficulties you faced during the learning of PV-B_4 number system that children are likely to face when learning PV-B_{10} number system
- Mid-Size Groups (6-8 teachers)
  - Each person shares one difficulty from her/his list

7) Adding Without Writing (B_4)

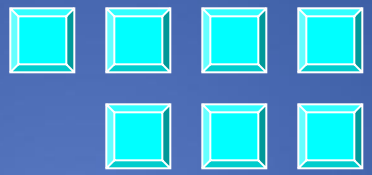
- Individuals - Entire Cohort
  - Ron will present two groups of objects, one after another
  - Each should find (mental math) how many objects are in both groups
  - After each addition, we will share several (different) solutions

7) Adding Without Writing

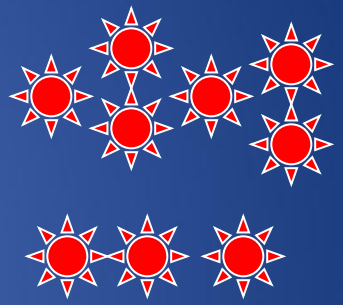
- Solutions

3_4

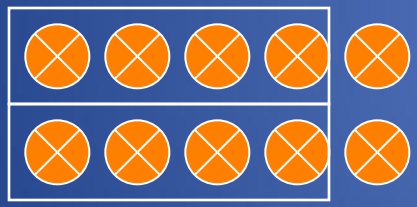
11_4



13_4



21_4

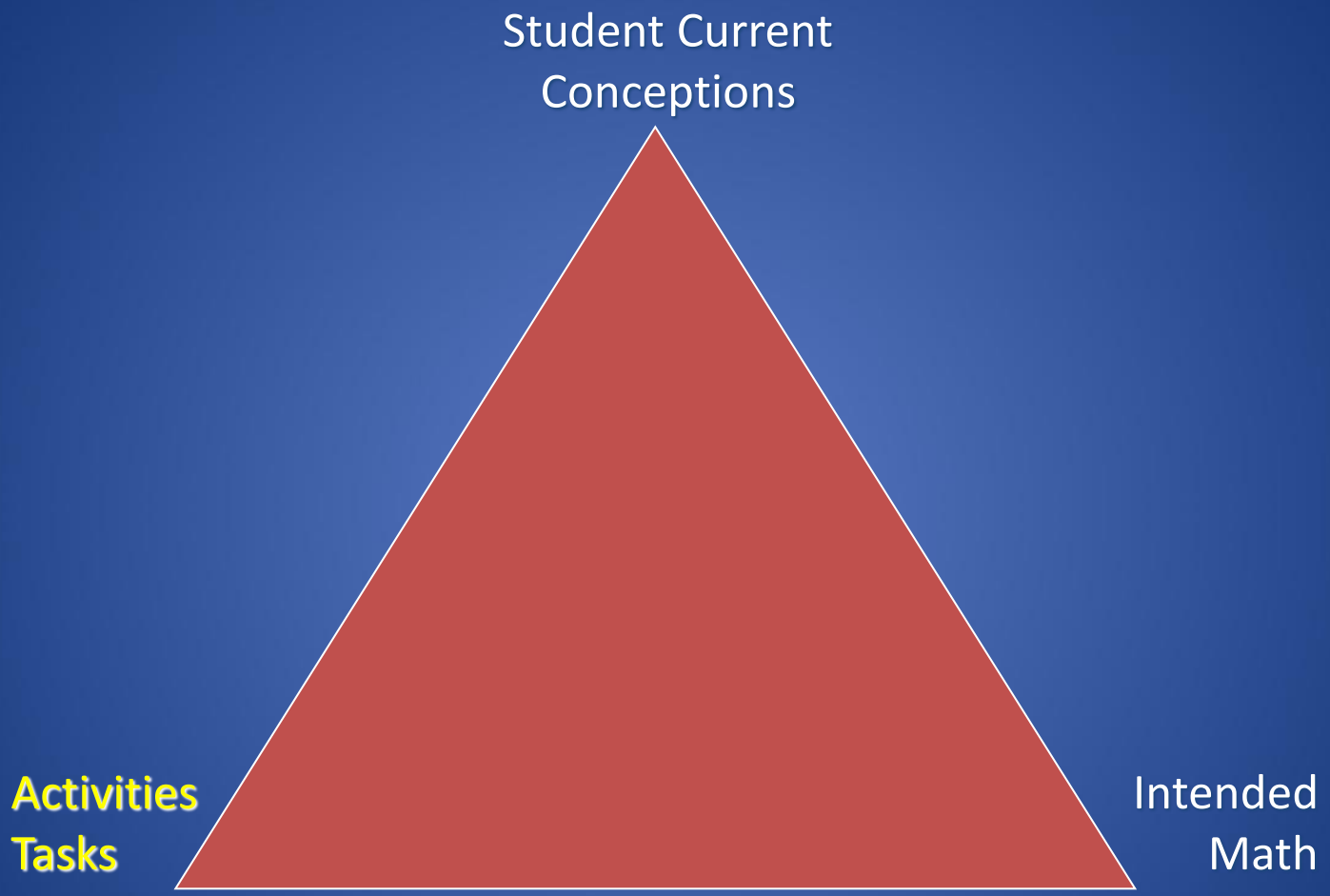


22_4

31_4

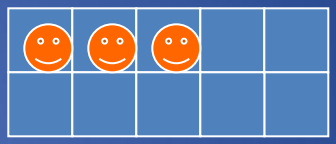
- Homework: PV-B_4 worksheet

Teaching Triad



Pedagogy: Task Selection

- Choose a task to **begin** teaching PV-B_{10}
  - 1) Ten frame 2) Base ten blocks 3)M&M
  - 4) Popsicle Sticks 5) Unifix Cubes



- Individuals
  - Decide, with explicit reasons, which of the five should be used to INITIATE students' learning of PV-B_{10} number system
- Six Mixed-Choice Groups
  - Each teacher presents her choice/reasons to the group
  - Group--school curriculum committee--tries to reach consensus
- Entire Cohort
  - Groups share their decisions; All attempt to reach a consensus

Our Inference into Your Choice of Activity to Initiate PV-B_{10}

- Entire Cohort
  - Choice seemed to be guided by:
    - Intuitive focus on the intended math (**outcome**)
    - Fix on manipulative/activity that resembles it
  - **Danger: Learning Paradox**
  - Compared to Ron/Matt's choice (e.g., M&M):
    - Focus on activity/need available to the learner **at start** (via current conceptions) **AND** likelihood of activity to undergo the intended transformation

Mathematics of Children

- Observe and take notes of children's conception of number in each tape, then discuss each segment
  - Christina, 99/10/07
  - Lori, 96/04/19
  - Christina, 99/10/14
  - Ron, 99/06/15
  - Nick and Margaret, 97/03/07
- Seven Groups: Children on developmental path
  - Articulate child's conception of number (number sense)
  - Explain why it is more/less advanced than adjacent ones

Developmental Milestones: From Rote Counting to PV-B_{10}

- Key distinction: Pre-Number vs. Number
  - Items counted; operations (mental activity) used
- Pre-Number
  - Rote Counting; 1-1 Corr; Subitizing; Perceptual (concrete) Counters; Manyness; Figurative Counters (substitute objects); Counting-All;
- Number
  - **COUNTING ON !!!** (composite unit); Double Counting; 'Through-Ten' (Purposeful Decomposition); Missing Addend
- Number System (e.g., PV-B_{10})
  - Grouping to Organize Large Quantities; Grouping by 'Tens'; Rote Counting by Ten; Counting/Adding Tens & Ones;

Counting-On: The “Culprit”

- 30-Year math-ed puzzlement:
  - Why do children regress from count-on to count-all?
- NMRSD Findings: Left behind = no number (CO)
- Observe Hannah (Cup, Pile, and Number Line tasks)
- Your turn: How do you explain the ‘regress’?

Statistical Analysis

	Cup n=37	Pile n=37	Line n=37	Items avail. pile & line n=74
count-on	30 (81%)	17 (46%)	5 (14%)	22 (30%)
count-all	7 (19%)	20 (54%)	32 (86%)	52 (70%)

- RED: Pile & Line vs. cup (Item Availability): $p < .0001$
- Blue: Pile vs. Line: $p < .005$

"If you want to get ahead, get a theory"
(Karmiloff-Smith & Inhelder, 1974-5)

"There is nothing as practical as a good theory"
(Lewin, cited in Shaffer, 1993, p. 42)

Constructivist Framework

- Assimilation: Available conceptions afford/constrain actions
- Conception: invariant activity-effect relationship (AER compound)
- Stages:
  - Participatory: Provisional AER, can be called upon only if prompted
  - Anticipatory: Spontaneously accessible AER (recognized in situation, transfer supporting)

Number

- Four primary lines of research about addition/subtraction: Carpenter et al., Fuson et al., Baroody, Steffe et al.
- Counting-on: Two invariant *AERs*
  - Start counting singletons from *n* (*SCSF-n*)
  - Keep track via double counting (*KTDC*)
- *SCSF-n & KTDC* combined into Counting-On Numerically (*CON)*: Attributing abstract concept of number

Participatory stage of *SCSF-n invariant AER*

- Andy
  - cup (8+8): 1, 2, 3, 4, oops; [8; 9-10-11-12-13-14-15-16] (clearly explains advantage of starting at 8)
  - pile (8+5): [8; 9-10-11-12-13] (regenerated partic. *SCSF-n*)
  - line (7+6): count-all; (8+7) – begins count-all, Matt interrupts – 'is there another way'?; Andy– brings forth SCSF-n, can't *KTDC*
- Explanation
  - Cup & Pile: items separated – prompt where to start
  - Pile vs. Line: items available – not separated on line
  - Typical case of regress due to participatory *SCSF-n*
- Hannah: Typical case of regress due to participatory *KTDC*

Tasks for Teaching Counting-On

- Matt: Insert examples here

?????