Extending the Challenge in Mathematics

Parts III & IV

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Exploring Problems in Depth

- Organization and Representation
  1. How might I represent, simulate, model, or visualize these ideas in various ways?
  2. How might I sort, organize, and present this information?
  3. What patterns do I see in this data?

Exploring Problems in Depth

- Rules and Procedures
  1. What steps might I follow to solve that? Are they reversible? Is there an easier way?
  2. Do I have enough information? Too much information? Conflicting information?

Exploring Problems in Depth

- Optimization and Measurement
  1. How big is it?
  2. What is the largest possible answer? The smallest?
  3. How many solutions are possible? Which is the best?
  4. What are the chances? What is the best chance?

Exploring Problems in Depth

- Reasoning and Verification
  1. Why does that work? If it does not work, why not?
  2. Will that always work? Will that ever work?
  3. Is that reasonable? Can you prove that? Are you sure?

Question the Answers, Don't Just Answer the Questions!

- Generalizations
  1. What other patterns do I notice?
  2. Can I generalize these patterns?
  3. Are there exceptions to my rules? Under what conditions does this work or not work?

Question the Answers, Don't Just Answer the Questions!

- Comparisons and Relationships
  1. How is this like other mathematical problems or patterns that I have seen?
  2. How does it differ? What other questions does this raise?
  3. How does this relate to real-life situations or models?
  4. How are two factors or variables related? What new relationships can I find?
  5. What if I change one or more parts of the problem? How does that affect the outcomes?

Question the Answers, Don't Just Answer the Questions!

- With questions such as the ones in the previous slides, the solution to the original problem is used as a springboard to deeper, more original mathematical thinking.

(adapted from *Extending the Challenge in Mathematics*, pages 8-10)

How Should We Assess Success?

- Listen to our students
- Change our traditional methods of assessment
  - Deep mathematical thinking and creativity cannot be evaluated using a multiple choice or fill-in-the-blank exam.
- Have students develop Portfolios of their work over a semester or year that indicate the progress they make during that period.
- Let students know that we are looking for depth of reasoning and creativity.
- Have students help in the development of scoring rubrics and in analyzing and evaluating each other's work.

Possible Criteria for Assessment

(See page 11 of *Extending the Challenge* for an example scoring rubric for evaluating students' major projects.)

1. Depth of understanding
   - The extent to which core concepts are explored and developed
2. Fluency
   - The number of different correct answers, methods of solution, or new questions formulated.
3. Flexibility
   - The number of different categories of answers, methods, or questions, such as numeric, algebraic, geometric, or graphical.

Possible Criteria for Assessment

4. Originality
   - Solutions, methods, or questions that are unique and show insight.
5. Elaboration or Elegance
   - Quality of expression of thinking, including charts, graphs, drawings, models, and words.
6. Generalizations
   - Patterns that are noted, hypothesized, and verified for larger categories.
7. Extensions
   - Related questions that are asked and explored, especially those involving why and what if.

Organization of Activities

- Differentiated for students with different mathematical experiences
  - Level A for students who can reason additively
  - Level B for students who can reason multiplicatively
  - Level C for students who can reason proportionally and symbolically

Plan for Part III, Monday

- Number and Operations
  - Investigation 1: How Many Ways
    - Levels B & C, Pages 34-35
  - Fractions with JavaBars
  - Investigation 3: All in a Row
    - Pages 39-42
- Algebra
  - Investigation 1: Where am I?
    - pages 59-61

Plan for Part IV, Tuesday

- Geometry and Measurement
  - Investigation 1: Framed!
    - Pages 96-98
  - Investigation 3: Connect the Dots, Level C
    - Page 104
- Data Analysis and Probability
  - Investigation 1: Who's got the button?
    - Pages 123-125
  - Extensions with Attribute Blocks
    - Logic games and difference charts