# Extending the Challenge in Mathematics

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# Characteristics of a Mathematically Promising Student

- Mathematical Frame of Mind
  1. Loves exploring patterns and puzzles
  2. Sees mathematics and structure in a variety of situations
  3. Recognizes, creates, and extends patterns
  4. Organizes and categorizes information
  5. Has a deep understanding of simple mathematical concepts, including a strong number sense

# Characteristics of a Mathematically Promising Student

- Mathematical Formalization and Generalization
  1. Generalizes the structure of a problem, often from only a few examples
  2. Uses Proportional Reasoning
  3. Thinks logically and symbolically with quantitative and spatial relations
  4. Develops proofs and other convincing arguments

# Characteristics of a Mathematically Promising Student

- Mathematical Creativity

  1. Processes information flexibly--switches from computation to visual to symbolic to graphic representations as appropriate in solving problems

  2. Reverses processes--can switch from a direct to a reverse train of thought (reversible reasoning)

  3. Solves problems in unique ways, tries unusual methods

  4. Strives for mathematical elegance and clarity in explaining reasoning

# Characteristics of a Mathematically Promising Student

- Mathematical Curiosity and Perseverance

  1. Is curious about mathematical connections and relationships--asks "why" and "what if"

  2. Has energy and persistence in solving difficult problems

  3. Digs beyond the surface of a problem, continues to explore after the initial problem has been solved

# An Open Approach to Problem Solving

Start at any point on the diagram and proceed in any order

# An Open Approach to Problem Solving

- <u>Relate</u> the problem to other problems you have solved.

- <u>Investigate</u> the problem. Think deeply and ask questions.

- <u>Evaluate</u> your findings. Did you answer the question? Does the answer make sense?

- <u>Communicate</u> your results. How can you best let others know what you have discovered?

- <u>Create</u> new questions to explore. What else would you like to find out about this topic? Start a new investigation.

# Questions! Questions! Questions!

- Think Deeply About Simple Things
  - How might I model or organize my thoughts
  - Why did that work?
  - Why didn't that work?
  - How is this like any other problem I have solved?
  - How is this different from other problems?
  - Is that always true?
  - Will that ever work?
  - What patterns do I notice?
  - What is the largest possible answer? The smallest?
  - How many solutions are possible?
  - How might I best convince others of my results?

# Questions! Questions! Questions!

- Question the Answers: Don't Just Answer the Questions
  - What other questions came up as I solved the original problem?
  - What if part of the problem were not there or a new part were added?
  - Can I do that another way? How many ways might I…?
  - What other patterns do I notice?
  - What generalizations might I make? Are they always true?
  - What other problems might I solve in a similar way?

# Organization of Activities

- Differentiated for students with different mathematical experiences
  - Level A for students who can reason additively
  - Level B for students who can reason multiplicatively
  - Level C for students who can reason proportionally and symbolically

# Plan for Part 1, Thursday

- Number and Operations
  - Investigation 2: Same Sums
    - Pages 36-38
- Algebra
  - Investigation 2: Heads or Tails?
    - Pages 62-65
  - Investigation 3: Field of Dreams
    - Additive combinations only, pages 66-73

# Plan for Part 2, Friday

- Data Analysis and Probability
  - Investigation 2: On Your Mark
    - Pages 126-128
  - Investigation 3: Bull's Eye
    - Pages 129-134
- Geometry and Measurement
  - Investigation 2: Shape Up!
    - Pages 99-101
  - Investigation 3: Connect the Dots
    - Pages 102-103