Big Ideas in the Mathematical Development of Children in Grades K-2

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An Idea Worth Considering

- Most, if not all important mathematical ideas can be taught via problem solving.

  That is-

- Tasks or problems are posed to engage children in thinking about and developing the intended mathematics

John A. Van De Walle

Elementary and Middle School Mathematics: Teaching Developmentally (fourth edition)

Allyn and Bacon, 2001

A Problem for Teachers

- No matter how lucidly and patiently teachers explain to their students, they cannot understand *for* their students.

Deborah Schifter & Cathy Fosnot

Reconstructing Mathematics Education:

Stories of Teachers Meeting the Challenge of reform

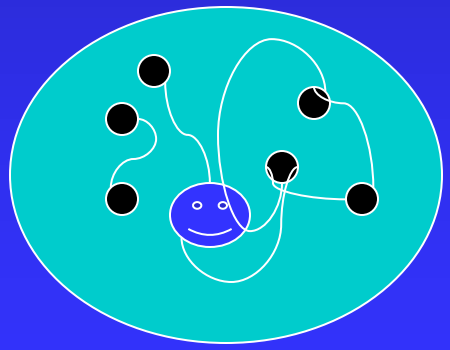
Teachers College press, 1993

Important Theoretical Perspectives

- Piaget
  - Children construct their own knowledge and understanding
  - Each child's knowledge and understanding is unique
  - The promotion of *reflective thought* is the most important ingredient of effective instruction

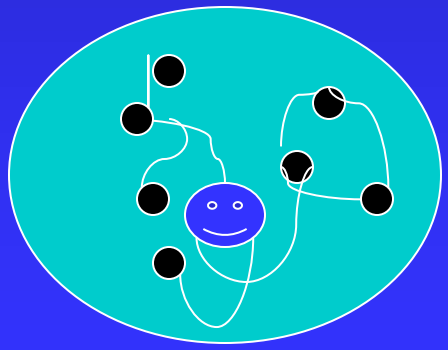
Important Theoretical Perspectives

- Skemp's Schema building and testing
  - The tools of construction are the ideas we already have
  - These ideas are connected in networks called schemas
  - The more connections in a network, the better our understanding of the ideas
- Three modes of Schema building and testing
  - Mode 1: Through practical activity
  - Mode 2: Through communication
  - Mode 3: Through creative thinking



Important Theoretical Perspectives

- Skemp's Relational vs Instrumental Understanding
  - A Continuum of Understanding (John A. Van de Valle)
    - Understanding of an idea can be thought of as a measure of the quality and number of connections that the idea has with existing ideas
    - Understanding depends on the existence of appropriate ideas and on the creation of new connections



Important Theoretical Perspectives

- Steffe's Number Sequences
  - Prenumerical
    - Can only count what is visible -- cannot "count-on"
  - Initial Number Sequence
    - Can "count-on" when adding more to an already counted set
  - Tacitly Nested Number Sequence
    - Can "double count" or count her counting acts
    - Can *make* composite units
  - Explicitly Nested Number Sequence
    - Can operate with composite units (multiplicative)
    - Partitioning operations and strategic reasoning
  - Generalized Number Sequence
    - Can operate with composite units of composite units
    - Can operate on *numerical relations* (distributive and proportional reasoning)

Important Theoretical Perspectives

- Dienes
  - Structured Mathematical Games
    - E.g. Logic Games with Attribute Blocks
  - Multi-embodiment Principle
    - E.g. Multi-Base Blocks
  - Relations and Operations
    - E.g. Input-Output machines

Important Theoretical Perspectives

- Vygotsky
  - Importance of social interaction
  - Knowledge is socially constructed
  - Zone of Potential Development
  - Role of the Teacher
- Davidov
  - Quantitative Reasoning
  - Quantitative relations before numerical computation
  - Reasoning with unknown quantities

A New Idea: Let's start with…

Quantitative Reasoning

- Comparing Quantities
  - Same
  - Different
  - More
  - Less
- Ordering Quantities
  - From least to most
  - From most to least

Quantitative Reasoning

- Measuring Quantities
  - Defining a unit quantity
  - Comparing a given quantity to the unit
  - Expressing a given quantity in terms of a unit
- Example: the Hawaii *Measure Up* Project
  - Second Grade Activities
  - Multiplicative Reasoning

Numerical Reasoning

- Initial Counting Activities
- Steffe's Number Sequences
  - Pre-numerical
  - Initial Number Sequence
  - Tacitly Nested Number Sequence
  - Explicitly Nested Number Sequence
  - Generalized Number Sequence

Number versus Quantity

- What is a "number"?
  - The answer to "How *many*?"
- How does a child determine "How many?"?
  - By counting.
- What is a "quantity"?
  - The answer to "How *much*?"
- How does a child determine "How much?" of something?
  - By comparing with some known *unit* quantity
    - I.e. by *measuring*.

Counting and Measuring

- *Counting* entails the act of *unitizing* --
  - I.e. making something into a unit item -- a singleton -- and recording that *act* with a spoken number-word
- *Measuring* entails *comparing* a quantity with some other (unit) quantity --
  - I.e. visually or physically segmenting one quantity into segments (parts) equivalent to the other quantity

Unitizing and Comparing

- However, before counting and before measuring, children engage in *unitizing* and *comparing* things and quantities
- These are the *two basic operations* underlying all of a child's mathematical development

Uniting and Partitioning

- A third operation that must also be developed is that of *uniting* (joining together) to make what Steffe calls a *composite unit* (the building block for multiplicative reasoning).
- The *inverse operation* of uniting is *partitioning* -- splitting a unit quantity into equal parts.
  - *Partitioning* is the basic operation underlying reasoning with fractional quantities.

The Part-Whole Relation

- The basic *part-whole relation* is a *comparison* of two quantities with the added complexity that one quantity is derived from the other (the *part* from the *partitioning* of the *whole* or the *whole* from the *iteration* and *uniting* of multiple *parts*)
- Thus a *partitioned whole* is a *composite unit*
  - JavaBars illustration

Multiplicative Reasoning

- *Multiplicative Reasoning* is, therefore, a necessary pre-cursor for developing an understanding of fractions.
- Multiplicative Reasoning involves the *coordination* of two levels of units: singletons and composite units.

Algebraic Reasoning

- From early years on
- Reasoning with unknown quantities
- Discerning Patterns
- Classifying and sorting
- Working with relations
- Discerning the structure of relations among quantities
- Working with functional relations and variables

Classification & Logical Reasoning

- Sorting by attributes
- Venn Diagrams
- Similarity Relations
- Difference Relations
- Simple If-Then statements
- Logical Reasoning

Spatial Reasoning

- Necessary for quantitative reasoning
- Topological before geometrical
- Three-dimensional before 2-D
- Van-Hiele levels
  - Visual: “Looks like”
  - Properties
  - Relations
  - Formal
  - Theoretical

Role of Teacher

- To construct models of students' mathematical schemes through interaction with students (Piaget).
- To cooperate with students' second-order "director systems" to improve their schema building (Skemp).
- To provide problem situations situated in the students' zone of potential development (Vygotsky) or students' zone of potential construction (Steffe).

Strategies for Effective Teaching

- Create an interesting mathematical environment
- Pose worthwhile mathematical tasks
- Use cooperative learning groups
- Use physical models and technology as *thinking tools*
- Require justification of student responses
- Encourage discourse and writing
- Listen actively
- Let the *students* do the thinking!

Adapted from John A. Van de Walle, 2001

Curriculum Examples
that emphasize *Conceptual Mathematics*

- Singapore Primary Mathematics Program
- SAIL: Structured Activities for Intelligent Learning (Skemp)
- Hawaii Measure Up Project
- EDC'c Math Workshop
- ArAl from Italy
- Dynamic Geometry for Young Learners