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Coordination with flexible information acquisition

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Abstract

We study flexible information acquisition in a coordination game. "Flexible" acquisition means that players choose not only how much but also what kind of information to acquire. Information acquisition has a cost proportional to reduction of entropy. Hence, players will collect the information most relevant to their welfare but can be rationally inattentive to other aspects of the fundamental. When information is cheap, this flexibility enables players to acquire information that makes efficient coordination possible, which also leads to multiple equilibria. This result contrasts with the global game literature, where information structure is less flexible and cheap information leads to a unique equilibrium with inefficient coordination.

JEL classification: C72; D82

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1. Introduction

This paper studies a coordination game in which players can acquire information flexibly. Coordination requires that the payoffs are almost known to every player. In practice, payoffs may not be perfectly observable, and it maybe difficult to achieve such common knowledge approximately. The global game literature models this by endowing players with additive private signals on payoffs. Accordingly, researchers obtain a well-known limit unique equilibrium with inefficient coordination (e.g., Carlsson and van Damme [2], Frankel, Morris and Pauzner [4]). We re-examine this result in an environment where players can flexibly acquire information about the payoffs. Rather than fixing an additive information structure and letting players choose the precision of signals, we allow them to choose any information structures. An information structure exclusively specifies both the amount (measured by reduction of Shannon's entropy) and the substance of the information. Hence, information acquisition is flexible in the sense that the players choose not only how much but also what kind of information to acquire. This paper thus addresses the following questions: What information will be acquired under this flexibility? And how does this flexibility affect the efficiency of coordination? These questions are interesting because a player's incentive to acquire information is shaped by his payoff structure, which further depends on his opponent's information acquisition. This strategic concern together with the flexibility of information acquisition leads to nontrivial implications.

We examine our concept of flexible information acquisition in the following game. Two players coordinate investment in a risky project whose future cash flow is driven by a randomly fluctuating fundamental. Each player decides whether to invest, but the payoff depends not only on the realized fundamental but also on the other player's action. Given the other player's choice, a player's payoff from investing increases as the fundamental improves. And for any given realization of the fundamental, a player's gain from "invest" over "not invest" is strictly greater when the other player also invests. That is, players' actions are strategic complements. Before making a decision, each player can independently purchase private information about the fundamental in the form of an information structure, i.e., the conditional distribution of the signal given the fundamental. The player then takes action according to the realized signal.

The players' information acquisition strategy is determined by two factors. The first is the effect of the fundamental: intuitively, given the other player's action, a player wants to collect information that induces investment with high (low) probability in the high (low) states of the fundamental. The second, and more interesting, factor is the player's incentive to match the other's informational choice, so as to minimize the probability of miscoordination. As a result, the strategic complementarity between actions creates a motive for the coordination of information as well. Indeed, coordination in information acquisition arises thanks to the fact that information acquisition is flexible. Especially, when the cost is low, players can coordinate to achieve approximate common knowledge of any "cutoff event" with the cutoff taking intermediate values.[2] Consequently, lowering the information cost makes possible efficient coordination in investing through efficient coordination in acquiring information; it also gives rise to multiple equilibria, insofar as there is a multiplicity of ways of coordinating information acquisition.

[2] By approximate common knowledge, we mean the common-p belief (see Monderer and Samet [9]) with p close to 1. By "cutoff event", we mean the event that the fundamental is above/below some cutoff.

This paper contributes to our understanding of the impacts of information acquisition on coordination. Especially, a key feature, flexibility is highlighted. To see the point, consider an extended global game model where the players can purchase more accurate signals but cannot change the additive information structure. Intuitively, cheaper information induces the players to acquire more precise private signals, which lead to the inefficient, unique equilibrium commonly seen in standard global game models. The reason why efficient coordination is no longer sustainable and multiplicity disappears is precisely the rigidity of information acquisition, embodied in the constraint that players must pay equal attention to all possible realizations of the fundamental, as the additive information structure forces the observational error to be invariant in the fundamental. As a result, players coordinate only in choosing overall precision but cannot coordinate to achieve approximate common knowledge for any "cutoff event". This rigidity contrasts sharply with the mechanism of flexible information acquisition described above.

We proceed as follows. Section 2 sets up the model. Section 3 characterizes the equilibria and gains some initial insight through comparative statics. Section 4 reports our main results, first comparing our approach with the extended global game model and exploring the origins of the difference and then comparing the welfare implications of flexible information acquisition in strategic and non-strategic settings. Section 5 concludes with a discussion of several extensions of the model. All the proofs are given in Appendix A.

Relation to the literature. The flexible information acquisition is modeled in the rational inattention framework of Sims [16]. In applied work, rational inattention is studied chiefly in two cases: linear-quadratic (e.g., Mackowiak and Wiederholt [8]) and binary-action. A prime instance of the latter is Woodford [20]. Our model too adopts the binary-action setup. Our work differs from most other rational inattention models in that we study information acquisition in a strategic environment rather than in decision problems.

Our model is closely related to the global games literature in that both study coordination under incomplete information. The global game models are characterized by additive information structures (e.g., Morris and Shin [10,12], and Goldstein and Pauzner [5]), which are less flexible than that in our setup. This difference in flexibility differentiates our model from the global games literature.

The equilibria of games with incomplete information are known to be highly sensitive to the belief environment (Rubinstein [15], Kajii and Morris [7], and Weinstein and Yildiz [18], etc.). Here, the focus shifts from the belief environment to information acquisition, which determines the beliefs in equilibrium. So rather than analyze the effects of exogenously assumed beliefs, our model studies how the equilibria depend on the properties of information acquisition.

Another related strand of work makes information acquisition endogenous but rigid, where players can purchase Gaussian additive signals before selecting actions (e.g., Hellwig and Veldkamp [6], Myatt and Wallace [13], Szkup and Trevino [17], Colombo, Femminis and Pavan [3], and Pavan [14]). Both Hellwig and Veldkamp [6] and our model show that if players' actions are strategic complements, then so are their information choices. However, our research provides a different insight into the creation of approximate common knowledge and multiplicity. Whereas in Hellwig and Veldkamp [6], public noise is assumed in the signals and players have the option to access it to create approximate common knowledge, in our approach approximate common knowledge stems from the flexibility of information acquisition.

Table 1
Payoff matrix conditional on fundamental.

	Invest	Not invest
Invest	θ, θ	$\theta - r, 0$
Not invest	$0, \theta - r$	$0, 0$

2. The model

We define our game as follows. Two players[3] play a coordination game with payoffs shown by Table 1.

Here θ is "the fundamental state" distributed according to a prior distribution P with support $\Theta \subset \mathbb{R}$. We assume that P is absolutely continuous with respect to the Lebesgue measure over $\mathbb{R}$. The action set of player $i \in \{1, 2\}$ is $A_i = \{0, 1\}$, where 1 stands for "invest" and 0 stands for "not invest". If both players invest, each gets a payoff θ. If only one player invests, he gets $\theta - r$. The payoff for not investing is normalized to zero, regardless of the other player's action. Parameter $r > 0$ is the cost of miscoordination. It measures the degree of strategic complementarity.

The players start with common prior P. Before selecting an action, they can simultaneously and privately acquire information about θ. Information acquisition is modeled in the rational-inattention framework of Sims [16]. It is flexible in the sense that any information structure is feasible. In general, player i's strategy is characterized by a triplet (S_i, q_i, σ_i), where $S_i \subset \mathbb{R}$ is the set of realizations of player i's signal, $q_i(s_i|\theta)$ is the probability measure of that signal conditional on θ, and σ_i is a mapping from S_i to $[0, 1]$, with $\sigma_i(s_i)$ denoting the probability of choosing action 1 upon observing $s_i \in S_i$. As a standard result in the rational inattention literature (e.g., Woodford [20]), player i's information structure chosen in equilibrium contains at most two signal realizations.[4] Hence, without loss of generality, his strategy can be represented by a function

$$m_i(\theta) = \Pr(a_i = 1 | \text{ fundamental} = \theta).$$

That is, when the fundamental is θ, player i receives signal 1 (signal 0) with probability $m_i(\theta)$ $(1 - m_i(\theta))$ and then takes action 1 (action 0) as instructed.

Given player j's strategy m_j, player i's expected payoff from playing m_i is

$$U_i(m_i, m_j) = \int m_i(\theta) \cdot \big[\theta - r \cdot \big(1 - m_j(\theta)\big)\big] \cdot dP(\theta).$$

This expression is derived from Table 1. As a standard setup in the rational inattention literature (e.g., Woodford [20]), the information cost associated with a strategy m is given by $\mu \cdot I(m)$, where $I(m)$ is the amount of information conveyed by m, and $\mu > 0$ is a scaling parameter that controls the difficulty of information acquisition. Specifically,

$$\begin{aligned} I(m) = &\int \big[m(\theta) \ln m(\theta) + \big(1 - m(\theta)\big) \ln\big(1 - m(\theta)\big)\big] dP(\theta) \\ &- p_I \ln p_I - (1 - p_I) \ln(1 - p_I), \end{aligned} \tag{1}$$

[3] The "two-player" setup is not as restrictive as it seems. All our results remain valid when there is a continuum of players if we redefine the payoff for "invest" as $\theta - r \cdot (1 - m)$, where m is the fraction of players that invest.

[4] Essentially, acquiring more elaborate signals (e.g., a third signal) is costly but provides no extra benefit to the player, since the player must in any case take either action 1 or action 0.

where

$$p_I = \Pr(a = 1) = \int m(\theta) dP(\theta) \tag{2}$$

is the unconditional probability of investing. Mutual information $I(m)$ measures function m's variability, which reflects the informativeness of actions about the fundamental. For example, when $m(\theta)$ is constant, the actions convey no information about θ and the corresponding mutual information is nil. This is because the integrand in the first term of (1) is strictly convex, so $I(m)$ is zero if and only if $m(\theta)$ is constant. A nice property of our technology of information acquisition, therefore, is that information acquisition exists if and only if $m(\theta)$ varies over θ, if and only if information cost is strictly positive. Also note that the functional form ("shape") of m determines not only the quantity but also the qualitative nature of the information. For instance, a player can concentrate his attention on some event by making $m(\theta)$ highly sensitive to θ if the event occurs. In this sense, our technology of information acquisition is flexible, inasmuch players can decide both the quantity and the qualitative nature of their information by unrestricted choice of m.

Taking information cost into account, player i's overall expected payoff (in terms of m_i and his opponent's strategy m_j) is

$$V_i(m_i, m_j) = U_i(m_i, m_j) - \mu \cdot I(m_i). \tag{3}$$

For simplicity, in the rest of the paper we abstract from the story of information acquisition and treat the problem as a two-player game with preference (3) and strategy profile (m_1, m_2). We assume that the players choose strategies from $L^1(\Theta, P)$, i.e., the space of all P-integrable functions on Θ equipped with the distance

$$\rho(m_1, m_2) = \int_{\Theta} \left| m_1(\theta) - m_2(\theta) \right| dP(\theta).$$

Since m is the probability of taking action 1, we can further restrict the players' strategies to

$$\Omega \triangleq \left\{ m \in L^1(\Theta, P) : \forall \theta \in \Theta, m(\theta) \in [0, 1] \right\}.$$

We write $G(r, \mu)$ for the game with strategic complementarity r and information cost μ.

3. The equilibria

A Nash equilibrium of game $G(r, \mu)$ is a strategy profile (m_1, m_2) that solves the following problem:

$$m_i \in \arg\max_{\widetilde{m}_i \in \Omega} V_i(\widetilde{m}_i, m_j) = \arg\max_{\widetilde{m}_i \in \Omega} U_i(\widetilde{m}_i, m_j) - \mu \cdot I(\widetilde{m}_i),$$

where $i, j \in \{1, 2\}$ and $i \neq j$.

In order to analyze the best response of this game, we look first at the general binary-action decision problem with flexible information acquisition. Consider a decision maker who has to choose an action $a \in \{0, 1\}$ and will receive a payoff $u(a, \theta)$, where $\theta \in \Theta$ and P are defined as above. Before deciding, the decision maker can acquire information in a manner specified in Section 2. Let

$$\Delta u(\theta) = u(1, \theta) - u(0, \theta)$$

be the payoff gain from action 1 over action 0. Then the decision maker's problem is

$$\max_{\widetilde{m}\in\Omega} \int_{\Theta} \widetilde{m}(\theta)\cdot \Delta u(\theta) dP(\theta) - \mu \cdot I(\widetilde{m}).$$

The payoff gain $\Delta u(\theta)$ determines the incentive to acquire information by shaping strategy m, as demonstrated by Lemma 2 of Woodford [19]. We adapt this lemma to our context in the following proposition.

Proposition 1. *Let* $\Pr(\Delta u(\theta) \neq 0) > 0$ *exclude the trivial case of the decision maker always being indifferent between the two actions. Let* $m \in \Omega$ *be an optimal strategy and*

$$p_1 = \int_{\Theta} m(\theta) dP(\theta)$$

be the corresponding unconditional probability of taking action 1. Then,

i) *the optimal strategy is unique;*
ii) *there are three possibilities for the optimal strategy:*
   a) $p_1 = 1$ *(i.e.,* $m(\theta) = 1$ *for all* $\theta \in \Theta$*) if and only if*

$$\int_{\Theta} \exp\bigl(-\mu^{-1}\Delta u(\theta)\bigr) dP(\theta) \leq 1;$$

   b) $p_1 = 0$ *(i.e.,* $m(\theta) = 0$ *for all* $\theta \in \Theta$*) if and only if*

$$\int_{\Theta} \exp\bigl(\mu^{-1}\Delta u(\theta)\bigr) dP(\theta) \leq 1;$$

   c) $p_1 \in (0, 1)$ *if and only if*

$$\int_{\Theta} \exp\bigl(\mu^{-1}\Delta u(\theta)\bigr) dP(\theta) > 1 \quad \textit{and} \quad \int_{\Theta} \exp\bigl(-\mu^{-1}\Delta u(\theta)\bigr) dP(\theta) > 1;$$

   in this case, the optimal strategy m *is characterized by*

$$\Delta u(\theta) = \mu \cdot \left[\ln\left(\frac{m(\theta)}{1 - m(\theta)}\right) - \ln\left(\frac{p_1}{1 - p_1}\right)\right]$$

   for all $\theta \in \Theta$.

Given player j's strategy m_j, player i's payoff gain from investing rather than not investing is

$$\Delta u_i(\theta) = \theta - r \cdot \bigl[1 - m_j(\theta)\bigr].$$

As is shown in this proposition, the payoff gain function $\Delta u_i(\theta)$ determines player i's incentive to acquire information. In our coordination game, there are two motives for acquiring information: to reduce uncertainty about the fundamental and to coordinate the investment decision with the other player's by coordinating information acquisition. Given this second motive, player i should pay attention to the events to which player j pays attention. But if player j never acquires

information (e.g., always invests), this second motive for player i is lacking. Moreover, if the fundamental is very likely to be positive ex ante, the first motive does not hold either. Then player i may find it optimal always to invest and never acquire information. This confirms player j's no information-acquisition strategy and so constitutes an equilibrium. Because such equilibria are trivial, we exclude them by the following assumption.

Assumption 1.

$$\int_{\Theta} \exp\left(-\mu^{-1}\theta\right) dP(\theta) > 1 \quad \text{and} \quad \int_{\Theta} \exp\left(\mu^{-1}\theta\right) dP(\theta) > e^{\mu^{-1}r}.$$

The basic intuition here is that common prior P should not be concentrated within the interval $[0, r]$, insofar as if it is, the players are both confident of event $\theta \in [0, r]$. Once a player always invests (or does not invest), the other's payoff gain is very likely to be positive (negative) ex ante, eliminating the incentive to acquire information.[5,6]

Proposition 2. *Strategy profile (m_1, m_2) is an equilibrium of game $G(r, \mu)$ if and only if there exists an $m \in \Omega$, such that $m_1(\theta) = m_2(\theta) = m(\theta)$ and*

$$\theta - r \cdot \left[1 - m(\theta)\right] = \mu \cdot \left[\ln\left(\frac{m(\theta)}{1 - m(\theta)}\right) - \ln\left(\frac{p_I}{1 - p_I}\right)\right] \tag{4}$$

for all $\theta \in \Theta$, where p_I is given by (2).

The proposition states that all equilibria are symmetric. This is because the strategic complementarity between the players' actions creates the motive for coordination in the acquisition of information. Given the private nature of their information acquisition, the players achieve coordination by choosing the same information structure (i.e., the same $m(\cdot)$). Moreover, the equilibrium strategy m is characterized by (4), which is a direct implication of Proposition 1. Suppose player j is playing m. The left-hand side of (4) is player i's marginal benefit from increasing the conditional probability of "invest". Since $\mu > 0$ is the marginal cost of acquiring an extra bit of information and

$$\left[\ln\left(\frac{m(\theta)}{1 - m(\theta)}\right) - \ln\left(\frac{p_I}{1 - p_I}\right)\right]$$

is the "derivative" of the amount of information with respect to $m(\theta)$, the right-hand side of (4) is player i's marginal cost of increasing $m(\theta)$. Hence (4) states that the marginal cost must equal the marginal benefit. Also note that

$$\ln\left(\frac{p_I}{1 - p_I}\right)$$

[5] For example, if the common prior is $N(t, \sigma^2)$, this assumption is equivalent to

$$\sigma^2 > r \cdot \mu \quad \text{and} \quad t \in \left(r - \mu^{-1}\sigma^2/2, \mu^{-1}\sigma^2/2\right);$$

or if the common prior is a uniform distribution over interval $[-A, r + A]$, the assumption holds when A is large enough.

[6] We show in Yang [21], a working-paper version of this paper, that non-information acquisition equilibria exist once this assumption is not satisfied.

is the average odds of investing (ratio of "invest" to "not invest"), while

$$\ln\left(\frac{m(\theta)}{1-m(\theta)}\right)$$

is the odds conditional on θ. Therefore (4) indicates that your opponent's strategy m shapes your marginal benefit

$$\theta - r\cdot\big[1-m(\theta)\big],$$

which in turn determines the deviation from your average odds of investing. In fact, Eq. (4) summarizes all the previous derivations and is sufficient, as well as necessary, for all the equilibria. The equilibrium analysis of the next two sections is conducted through this equation.

It is easy to verify that the graph

$$\left\{(\theta,m):\theta - r\cdot(1-m) = \mu\cdot\left[\ln\left(\frac{m}{1-m}\right) - \ln\left(\frac{p_I}{1-p_I}\right)\right]\right\}$$

is central-symmetric[7] in the $\theta\sim m$ plane about the point $(\theta_0, 1/2)$, where

$$\theta_0 = r/2 - \mu\cdot\ln\left(\frac{p_I}{1-p_I}\right). \tag{5}$$

Combining (4) and (5) gives

$$\theta-\theta_0 = \mu\cdot\ln\left(\frac{m(\theta)}{1-m(\theta)}\right) + r\cdot\left(\frac{1}{2}-m(\theta)\right).$$

Hence any solution to (4) has an expression $m(\theta-\theta_0)$ and can be indexed by θ_0, i.e.,

$$\theta-\theta_0 = \mu\cdot\ln\left(\frac{m(\theta-\theta_0)}{1-m(\theta-\theta_0)}\right) + r\cdot\left(\frac{1}{2}-m(\theta-\theta_0)\right). \tag{6}$$

In other words, any solution is a translation of function $m(\theta)$, which is implicitly defined by

$$\theta = \mu\cdot\ln\left(\frac{m(\theta)}{1-m(\theta)}\right) + r\cdot\left(\frac{1}{2}-m(\theta)\right).$$

As a result, a solution to (4) is jointly determined by its position θ_0 and its "shape" $m(\theta)$. Note, however, that not every $\theta_0\in\mathbb{R}$ is sufficient to make $m(\theta-\theta_0)$ a solution. The position θ_0 is endogenously determined in equilibrium.

We first analyze the "shape" of the equilibrium. The "shape" $m(\theta)$ is determined by $R \triangleq \frac{r}{4\cdot\mu}$, the ratio of strategic complementarity r to the marginal cost of information acquisition μ. Fig. 1 shows how $m(\theta)$ varies as R increases.

What information is acquired in equilibrium? Since the probability of investing is highly sensitive to θ where the slope $|\frac{dm(\theta)}{d\theta}|$ is steep, $|\frac{dm(\theta)}{d\theta}|$ reflects player i's attentiveness around θ.[8] Under this interpretation, Fig. 1 reveals that players actively acquire information for intermediate values of the fundamental but are rationally inattentive to tail values. This coincides with our

[7] Literally, the graph is central-symmetric about $(\theta_0, 1/2)$ provided that m is defined over the whole real line, i.e., when $\mathrm{supp}(P)=\mathbb{R}$. However, our derivation of the equilibria requires neither $\mathrm{supp}(P)=\mathbb{R}$ nor $\theta_0\in\mathrm{supp}(P)$. The position parameter θ_0 is purely used to index the equilibria.

[8] $|\frac{dm(\theta)}{d\theta}| \triangleq \infty$ when $m(\theta)$ is discontinuous at θ.

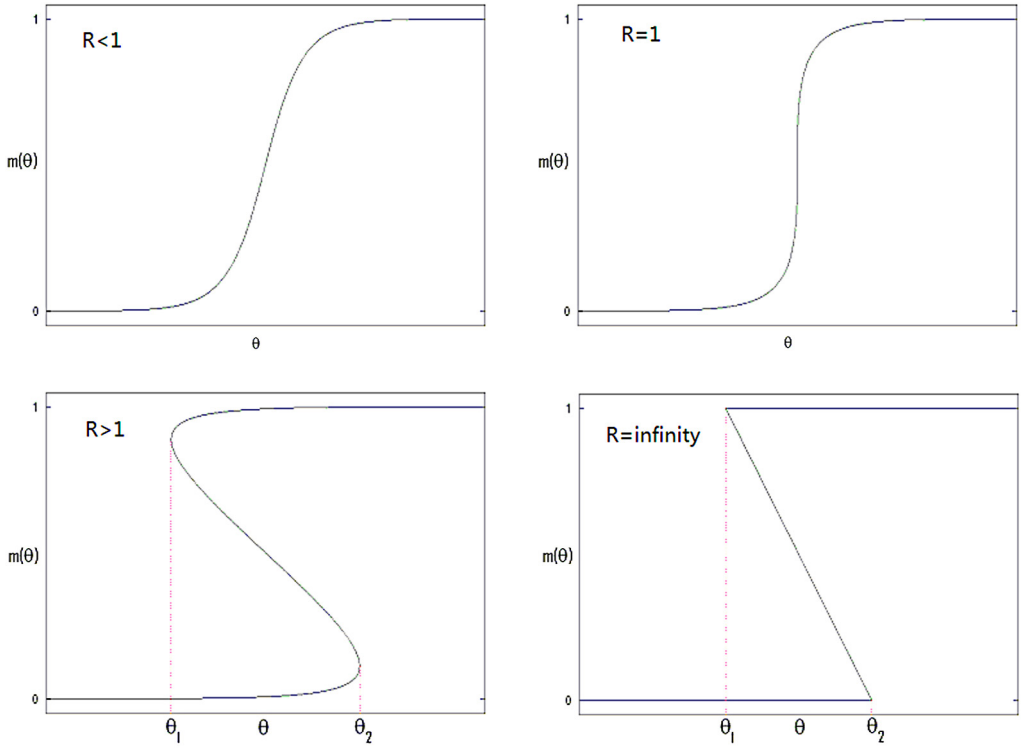

Fig. 1. Shapes of equilibria.

intuition. When θ is too high (low), the players should invest (not invest) anyway. Hence the information about θ at tail values is largely irrelevant to their payoffs. When θ takes intermediate values, each player's gain from investing depends crucially on the value of θ and its implication concerning the other player's action. Therefore, the information about θ in the intermediate region is payoff-relevant and attracts most of their attention.

How does information acquisition affect coordination? First, the equilibrium strategy curve becomes flatter as μ increases. Higher cost of information directly weakens the players' ability to acquire it. Hence players' responses will display more idiosyncratic errors. Moreover, expecting their counterparts to react in a noisier fashion, the players now have less incentive to coordinate. Thus the equilibrium strategy becomes even less decisive.

Second, multiple equilibria might emerge[9] as $R = \frac{r}{4 \cdot \mu}$ exceeds unity. As the lower-left subgraph of Fig. 1 shows, there exist $\theta_1 < \theta_2$ such that multiple values of $m(\theta)$ satisfy (6) for all θ within $[\theta_1, \theta_2]$. Note that while strategic complementarity r measures players' motive for coordination, the cost of coordination is given by μ, as acquiring information is a prerequisite to coordinating investment decisions. Hence, the condition $R = \frac{r}{4 \cdot \mu} > 1$ means that when coordination motive dominates coordination cost, the players have multiple ways of coordinating information acquisition, which leads to multiplicity.

[9] We prove it later.

Third, the Monotonic Likelihood Ratio Property (MLRP), a frequent assumption in applied models with incomplete information, could be violated by our players when $R = \frac{r}{4\cdot\mu} > 1$.[10] When coordination motive exceeds coordination cost, a player has both the incentive and the ability to coordinate with the other player's non-MLRP strategy.

Finally, as the lower-right subgraph of Fig. 1 shows, when the information cost goes to zero this equilibrium approximates the switching strategy. This result coincides with the equilibria of coordination games with complete information.

When $R = \frac{r}{4\cdot\mu} \leq 1$, there is a unique shape of $m(\theta)$ that can satisfy (6). But when R exceeds unity there are infinitely many shapes that satisfy (6).

Define the set of possible equilibrium shapes as

$$\mathbf{M}(r,\mu) \triangleq \left\{ m \in \Omega : \theta = \mu \cdot \ln\left(\frac{m(\theta)}{1-m(\theta)}\right) + r \cdot \left(\frac{1}{2} - m(\theta)\right) \right\}.$$

Note that

$$\#\mathbf{M}(r,\mu) = \begin{cases} 1 & \text{if } R = \frac{r}{4\cdot\mu} \leq 1 \\ \infty & \text{if } R = \frac{r}{4\cdot\mu} > 1 \end{cases}.$$

Given r and μ, an equilibrium $m(\theta - \theta_0)$ is determined by its shape $m \in \mathbf{M}(r,\mu)$ and by its position θ_0. According to (5), the equilibrium condition for θ_0 is

$$\theta_0 = r/2 - \mu \cdot \ln\left(\frac{\int m(\theta-\theta_0)\cdot dP(\theta)}{1-\int m(\theta-\theta_0)\cdot dP(\theta)}\right).$$

Hence, searching for an equilibrium with any given shape $m \in \mathbf{M}(r,\mu)$ is equivalent to seeking a fixed point θ_0 in the following mapping[11]:

$$g(\theta_0, m) \triangleq r/2 - \mu \cdot \ln\left(\frac{\int m(\theta-\theta_0)\cdot dP(\theta)}{1-\int m(\theta-\theta_0)\cdot dP(\theta)}\right). \tag{7}$$

As $R = \frac{r}{4\cdot\mu} > 1$ allows multiple shapes, a natural question, explored in Section 4, is whether this leads to multiple equilibria.

4. Information acquisition: rigidity versus flexibility

This section sets out our main results. Multiple equilibria emerge when strategic complementarity dominates information cost. We contrast this result to the result of an extended global game model, showing why the roles of rigid and flexible information acquisition differ so greatly. We also show how and why flexible information acquisition allows more efficient coordination.

Proposition 3. *For any possible shape $m \in \mathbf{M}(r,\mu)$, there exists $\theta_0 \in \mathbb{R}$ such that $m(\theta-\theta_0)$ is an equilibrium. Moreover, if $R = \frac{r}{4\cdot\mu} > 1$, then game $G(r,\mu)$ has infinitely many equilibria.*

[10] We say a player's strategy satisfies MLRP if the conditional probability of investing increases in the fundamental; that is, if the information structure is more likely to suggest investing when the fundamental is higher.

[11] It is worth noticing that common prior P does not affect $\mathbf{M}(r,\mu)$, the set of possible equilibrium shapes. It only changes θ_0, the position of the equilibrium, as shown by (7).

This proposition demonstrates the existence of the equilibrium and also provides a sufficient condition for multiple equilibria. Since strategic complementarity exceeds information cost, multiple modes of information acquisition are sustainable in equilibrium. Flexibility plus relatively low information cost enables players to achieve approximate common knowledge of various "cutoff events" (i.e., $\{\theta \geq t\}$ with various t's), which leads to multiplicity. In contrast, the players in global game models never achieve approximate common knowledge of any "cutoff event" (see Morris and Shin [11]), due to the "infection" effect. This leads to the limit uniqueness result. In our game, although multi-signal information structures are feasible, the players automatically acquire binary signals, as a result of the flexibility in information acquisition. The "infection" argument thus does not apply here as it relies on multiple overlapping information sets.[12,13]

To illustrate the indispensable mechanism of flexibility, it is instructive to contrast the role of information acquisition in our model from that in an extended global game model, where the players can purchase more accurate signals but cannot change any other aspect of the information structure. Specifically, player i observes a private signal $x_i = \theta + \beta_i^{-1/2} \cdot \varepsilon_i$, where ε_i is idiosyncratic noise and β_i represents the precision of his private information. Player i can increase precision β_i by incurring some information cost. Here the information acquisition is rigid in that the additive nature of the signal generating process is not adjustable. Intuitively, lowering informational cost induces the players to acquire more accurate private signals. As a well known result in the global game literature, unique equilibrium is guaranteed if private information is sufficiently accurate relative to public information (e.g., Morris and Shin [12]). Hence we can retrieve the standard limit uniqueness result in the extended global game model when information becomes cheap enough.[14] But by Proposition 3, our model with flexible information acquisition generates the opposite prediction: lowering the cost of information enhances approximate common knowledge and facilitates multiplicity.

How should we understand this radical discrepancy? Strategic complementarity of actions produces the motive for coordination in acquiring information. This motive evolves into actual coordination in our model with flexibility, especially when the cost of information is lower. Hence, we recreate approximate common knowledge of the payoffs, resulting in multiplicity.

In the global games approach, where private noise is additive to the fundamental, the players are constrained to pay equal attention to all possible values of θ, in the sense that the distribution of the observational error is invariant with respect to θ. As a result, they coordinate only in choosing overall precision, while the potential motive for coordinating attention allocation for different levels of θ cannot materialize. This mechanism of rigidity contrasts diametrically with the mechanism at work when information acquisition is flexible.

Rigid and flexible information acquisition also entail different welfare implications. In the extended global game model, the limit unique equilibrium is a switching strategy

[12] For games with larger action space, the equilibrium strategies involve multiple signals that may generate contagious information sets. Then the "infection" argument may still be applicable in games with flexible information acquisition. We thank a referee for pointing out this possibility. Since global games with multiple actions are difficult to handle, we leave this possibility for future research.

[13] Whilst the binary-action assumption causes the binary-signal information structure and thus the failure of the "infection" argument, it does not necessarily imply the failure of limit uniqueness. In an ongoing project with Stephen Morris, we find limit uniqueness in a binary-action coordination game where information acquisition is less flexible than it is here but still prescribes a binary-signal information structure in equilibrium. Therefore, the flexibility, rather than the binary-action assumption, seems the driving force for our results.

[14] We refer the interested readers to Yang [21], the working-paper version of this paper, for a formal derivation of the extended global game model with information acquisition.

$$a(x) = \begin{cases} 0 & \text{if } x \leq r/2 \\ 1 & \text{if } x > r/2 \end{cases},$$

where x is a signal of θ with small additive noise and each player takes action $a(x)$ when observing x. This equilibrium is inefficient. Both players would have had higher payoffs if they had committed to the most efficient strategy

$$\widetilde{a}(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}.$$

However, this strategy is not sustainable in equilibrium. A player with a signal just above zero will rationally assign a large probability to the other player's signal being negative, and thus be reluctant to invest for fear of miscoordination. This problem could be overcome if the players could commit to ignoring the exact values of their signals and considering only the signs. For example, if there were a third party who observed the signals and told the players (privately) only the signs of their own signals, each player would find it optimal to invest if and only if the signal were positive. That is, players can achieve efficient coordination by discarding information through a commitment device. In a trivial sense, insofar as they map continuous signals to binary actions players always discard some information. But the point is that while, some information is not reflected in players' actions, it is still used in making inferences about others' beliefs and consequent actions. This is the way in which information matters, which sets it apart from all other economic resources. Therefore, "discarding information" means committing to forget it, i.e., refraining from making inferences based on it.

By contrast, in our model with flexible information acquisition, $\forall \widehat{\theta} \in [0, r]$,

$$m(\theta) = \begin{cases} 0 & \text{if } \theta \leq \widehat{\theta} \\ 1 & \text{if } \theta > \widehat{\theta} \end{cases}$$

is an equilibrium when informational cost μ vanishes. Hence the most efficient strategy with cutoff $\widehat{\theta} = 0$ can be supported in equilibrium. Here, flexible and costly information acquisition helps players to acquire only information that is valuable for efficient coordination and to ignore information that is detrimental to it even if its cost goes to zero; they could have a certain welfare gain over the case of rigid information acquisition. Unlike the extended global game model, where players can only discard harmful information by an explicit commitment device, our model with flexibility helps them choose the quantity and the qualitative nature of their information, whereby they act as if they had committed to discarding the information harmful to coordination. In other words, we can interpret this as an implicit commitment device inhabiting the flexibility of information acquisition. It is worth underscoring that this contrast was not discernible in one-person decision problems with information acquisition, and not only because by definition they do not entail coordination but also because where cost is not a consideration more information is always more desirable, no matter whether acquisition is flexible or rigid.

Whilst introducing flexibility in information acquisition makes the efficient outcome possible, it also creates other less efficient equilibria. So we do not view this flexibility as a solution to the inefficiency problem in the global game approach. Instead, we highlight the characteristics of information acquisition that can lead to dramatically different results. We also leave the equilibrium selection problem for the future research.

5. Discussion

Beyond the model analyzed in the main text, our results are also robust to other specifications of the information cost and payoff structures. A sufficient condition for flexibility and thus

the limit multiplicity result is that the information cost strictly respects Blackwell's ordering[15] and satisfies Lipschitz continuity. An information cost respects Blackwell's ordering if it assigns lower cost to less informative structures. As a result, both players find it optimal to consider only binary information structures, precluding all structures with more than two signal realizations, which therefore contain redundant and even harmful information. In addition, the Lipschitz continuity prevents the information cost from varying wildly and hence guarantees the availability of all potentially valuable information structures. It is worth noticing that the standard and extended global game models violate both Blackwell's ordering and Lipschitz continuity. On the other hand, Lipschitz continuity is not necessary, as it is violated by the entropic information cost in our main model. We call for future research to provide a necessary and sufficient condition that clearly distinguishes limit multiplicity from limit uniqueness.

We can also study the impact of public information through this model. Public information is common knowledge since it is directly observable and cost-free. It affects players' decisions by changing the common prior about the fundamental. Thus the impact of public information can be studied through comparative static analysis with respect to common prior P. It turns out to be that multiple equilibria emerge under highly precise public information.[16]

In our model, the payoff gain from investing over not investing is continuous with respect both to the fundamental and to the opponent's probability of investing. In many important applications of global game theory, however, the payoff gain is discontinuous (e.g., Morris and Shin [10]). This discontinuity implies infinite strategic complementarity. As a consequence the coordination motive always dominates information cost, so that an infinite number of ways of coordinating information acquisition can be supported in equilibrium. Therefore, the insight developed in previous sections suggests multiple equilibria no matter how large μ is.[17]

Finally, our model can be naturally extended to accommodate multiple players and costless but constrained information acquisition.

Appendix A

Proof of Proposition 2. Given player j's strategy m_j, player i's payoff gain is

$$\Delta u_i(\theta) = \theta - r \cdot \big[1 - m_j(\theta)\big].$$

Assumption 1 implies that

$$\int \exp\big(\pm\mu^{-1}\Delta u_i(\theta)\big) dP(\theta) > 1.$$

According to Proposition 1, this inequality further implies that a strategy profile (m_1, m_2) constitutes an equilibrium if and only if for all $\theta \in \Theta$,

$$\theta - r \cdot \big(1 - m_1(\theta)\big) = \mu \cdot \left[\ln\left(\frac{m_2(\theta)}{1 - m_2(\theta)}\right) - \ln\left(\frac{p_{I2}}{1 - p_{I2}}\right)\right], \tag{8}$$

$$\theta - r \cdot \big(1 - m_2(\theta)\big) = \mu \cdot \left[\ln\left(\frac{m_1(\theta)}{1 - m_1(\theta)}\right) - \ln\left(\frac{p_{I1}}{1 - p_{I1}}\right)\right], \tag{9}$$

[15] See Blackwell [1].

[16] We refer the interested readers to Yang [21], the working-paper version of this paper, for a formal analysis of the impact of public information.

[17] We refer the interested readers to Yang [21], the working-paper version of this paper, for an example of games with discontinuous payoff gain.

where

$$p_{Ii} = \int m_i(\theta) dP(\theta) \in (0, 1) \tag{10}$$

is player i's unconditional probability of investing.

To prove the proposition, it then suffices to show that all equilibria are symmetric.

Eqs. (8) and (9) imply

$$\begin{aligned}&\left[\ln\left(\frac{p_{I2}}{1-p_{I2}}\right) - \ln\left(\frac{p_{I1}}{1-p_{I1}}\right)\right]\\ &\quad = \left[\ln\left(\frac{m_2(\theta)}{1-m_2(\theta)}\right) - \ln\left(\frac{m_1(\theta)}{1-m_1(\theta)}\right)\right] + \frac{r}{\mu}\big(m_2(\theta) - m_1(\theta)\big).\end{aligned} \tag{11}$$

If $p_{I2} = p_{I1}$, (11) becomes

$$0 = \left[\ln\left(\frac{m_2(\theta)}{1-m_2(\theta)}\right) - \ln\left(\frac{m_1(\theta)}{1-m_1(\theta)}\right)\right] + \frac{r}{\mu}\big(m_2(\theta) - m_1(\theta)\big),$$

and we must have $m_2(\theta) = m_1(\theta)$ a.s. Now suppose $p_{I2} \neq p_{I1}$. Without loss of generality, let $p_{I2} > p_{I1}$. Denote $z = \ln(\frac{p_{I2}}{1-p_{I2}}) - \ln(\frac{p_{I1}}{1-p_{I1}}) > 0$. Let $\ln(\frac{m_2(\theta)}{1-m_2(\theta)}) = x(\theta)$ and $\ln(\frac{m_1(\theta)}{1-m_1(\theta)}) = y(\theta)$. Then (11) implies $m_2(\theta) > m_1(\theta)$ and thus

$$x(\theta) < y(\theta) + z$$

for all $\theta \in \Theta$. Note that $p_{Ii} = \int m_i(\theta) \cdot dP(\theta) = \mathbf{E}m_i(\theta)$, $i \in \{1, 2\}$, $m_2(\theta) = \frac{\exp(x(\theta))}{1+\exp(x(\theta))}$ and $m_1(\theta) = \frac{\exp(y(\theta))}{1+\exp(y(\theta))}$, thus

$$\begin{aligned}e^z &= \frac{\mathbf{E}[\frac{\exp(x(\theta))}{1+\exp(x(\theta))}]}{\mathbf{E}[\frac{1}{1+\exp(x(\theta))}]} \div \frac{\mathbf{E}[\frac{\exp(y(\theta))}{1+\exp(y(\theta))}]}{\mathbf{E}[\frac{1}{1+\exp(y(\theta))}]}\\ &< \frac{\mathbf{E}[\frac{\exp(y(\theta)+z)}{1+\exp(y(\theta)+z)}] \cdot \mathbf{E}[\frac{1}{1+\exp(y(\theta))}]}{\mathbf{E}[\frac{1}{1+\exp(y(\theta)+z)}] \cdot \mathbf{E}[\frac{\exp(y(\theta))}{1+\exp(y(\theta))}]},\end{aligned}$$

i.e.,

$$\begin{aligned}&\int \frac{\exp(y(\theta_1))dP(\theta_1)}{1+\exp(y(\theta_1)+z)} \cdot \int \frac{dP(\theta_2)}{1+\exp(y(\theta_2))} + \int \frac{\exp(y(\theta_2))dP(\theta_2)}{1+\exp(y(\theta_2)+z)}\\ &\quad \cdot \int \frac{dP(\theta_1)}{1+\exp(y(\theta_1))}\\ &> \int \frac{dP(\theta_1)}{1+\exp(y(\theta_1)+z)} \cdot \int \frac{\exp(y(\theta_2))dP(\theta_2)}{1+\exp(y(\theta_2))} + \int \frac{dP(\theta_2)}{1+\exp(y(\theta_2)+z)}\\ &\quad \cdot \int \frac{\exp(y(\theta_1))dP(\theta_1)}{1+\exp(y(\theta_1))},\end{aligned}$$

i.e.,

$$\begin{aligned}&\int \frac{A+B-C-D}{[1+\exp(y(\theta_1)+z)][1+\exp(y(\theta_2))][1+\exp(y(\theta_2)+z)][1+\exp(y(\theta_1))]}\\ &\quad \times dP(\theta_1)dP(\theta_2) > 0,\end{aligned} \tag{12}$$

where

$$A = \exp\big(y(\theta_1)\big)\big[1 + \exp\big(y(\theta_2) + z\big)\big]\big[1 + \exp\big(y(\theta_1)\big)\big],$$
$$B = \exp\big(y(\theta_2)\big)\big[1 + \exp\big(y(\theta_1) + z\big)\big]\big[1 + \exp\big(y(\theta_2)\big)\big],$$
$$C = \exp\big(y(\theta_2)\big)\big[1 + \exp\big(y(\theta_2) + z\big)\big]\big[1 + \exp\big(y(\theta_1)\big)\big]$$

and

$$D = \exp\big(y(\theta_1)\big)\big[1 + \exp\big(y(\theta_1) + z\big)\big]\big[1 + \exp\big(y(\theta_2)\big)\big].$$

Let $y(\theta_1) = u$ and $y(\theta_2) = v$, then the numerator in the integral becomes

$$A + B - C - D = \big[e^u - e^v\big]^2\big[1 - e^z\big] < 0,$$

where the last inequality follows the fact that $z > 0$. Therefore, the left hand side of (12) is strictly negative, which is a contradiction. Therefore, $m_2(\theta) = m_1(\theta)$ a.s. □

Proof of Proposition 3. Let $m \in \mathbf{M}(r, \mu)$ be an arbitrary shape. Let $\theta_0(p_I)$ be defined by (5) and

$$m(\theta, p_I) = m\big(\theta - \theta_0(p_I)\big).$$

By definition, $m(\theta, p_I)$ satisfies

$$\theta - r \cdot \big(1 - m(\theta, p_I)\big) = \mu \cdot \left[\ln\left(\frac{m(\theta, p_I)}{1 - m(\theta, p_I)}\right) - \ln\left(\frac{p_I}{1 - p_I}\right)\right] \quad \text{for all } \theta \in \Theta. \tag{13}$$

Here $p_I \in (0, 1)$ is treated as an index and $m(\theta, p_I)$ is an equilibrium if and only if

$$p_I = \int_{\Theta} m(\theta, p_I)\, dP(\theta). \tag{14}$$

Therefore, our objective is to show the existence of $p_I \in (0, 1)$ satisfying (14).

We show

$$\int_{\Theta} m(\theta, p_I)\, dP(\theta) < p_I$$

for p_I sufficiently close to 1.

By (13),

$$m(\theta, p_I) < \frac{\frac{p_I}{1-p_I}}{e^{-\mu^{-1}\theta} + \frac{p_I}{1-p_I}} \quad \text{for all } \theta \in \Theta.$$

Hence it suffices to show

$$\int_{\Theta} \frac{\frac{p_I}{1-p_I}}{e^{-\mu^{-1}\theta} + \frac{p_I}{1-p_I}}\, dP(\theta) \le p_I.$$

Let

$$w = \frac{1}{1 - p_I}$$

and

$$v(\theta) = e^{-\mu^{-1}\theta} - 1,$$

then it suffices to show

$$\int_{\Theta} \frac{1}{1 + v(\theta)/w} dP(\theta) \leq 1. \tag{15}$$

By Assumption 1,

$$\int_{\Theta} v(\theta) dP(\theta) > 0. \tag{16}$$

Hence there exists $N > 0$ s.t.

$$\int_{\Theta \cap [-N, +\infty)} v(\theta) dP(\theta) > 0.$$

Let

$$B = \max\left(e^{\mu^{-1}N} - 1, 1\right),$$

then

$$\left|v(\theta)\right| \leq B$$

for all $\theta \in [-N, +\infty)$. Choose $\overline{w} > 0$ s.t.

$$\frac{1}{1 + v(\theta)/w} < 1 - \frac{v(\theta)}{w} + \frac{2B^2}{w^2}$$

for all $\theta \in [-N, +\infty)$ and $w > \overline{w}$. Choose

$$w > \max\left(\overline{w}, \frac{2B^2}{\int_{\Theta \cap [-N, +\infty)} v(\theta) dP(\theta)}\right), \tag{17}$$

then

$$\begin{aligned}
&\int_{\Theta \cap [-N, +\infty)} \frac{1}{1 + v(\theta)/w} dP(\theta) \\
&\quad < \int_{\Theta \cap [-N, +\infty)} \left[1 - \frac{v(\theta)}{w} + \frac{2B^2}{w^2}\right] dP(\theta) \\
&\quad \leq \Pr(\theta \geq -N) + \frac{2B^2}{w^2} - w^{-1} \int_{\Theta \cap [-N, +\infty)} v(\theta) dP(\theta) \\
&\quad < \Pr(\theta \geq -N),
\end{aligned} \tag{18}$$

where the last inequality follows (17). Hence,

$$\begin{aligned}
&\int_{\Theta} \frac{1}{1 + v(\theta)/w} dP(\theta) \\
&\quad \leq \int_{\Theta \cap [-N, +\infty)} \frac{1}{1 + v(\theta)/w} dP(\theta) + \int_{\Theta \cap (-\infty, -N)} 1 \cdot dP(\theta) \\
&\quad < 1.
\end{aligned}$$

Therefore, (15) holds and if we let

$$\overline{p}_I = \frac{w-1}{w},$$

we have

$$\int_{\Theta} m(\theta, \overline{p}_I) dP(\theta) < \overline{p}_I.$$

Similar argument also leads to

$$\int_{\Theta} m(\theta, p_I) dP(\theta) > p_I$$

for p_I sufficiently close to 0.

The absolute continuity of common prior P implies that

$$\int_{\Theta} m(\theta, p_I) dP(\theta) - p_I$$

is a continuous function of $p_I \in (0, 1)$. Hence there exists a $p_I^* \in (0, 1)$ s.t.

$$\int_{\Theta} m\big(\theta, p_I^*\big) dP(\theta) = p_I^*.$$

According to (5), let

$$\theta_0^* = r/2 - \mu \cdot \ln\left(\frac{p_I^*}{1-p_I^*}\right),$$

then $m(\theta - \theta_0^*)$ is an equilibrium with shape m. Moreover, the game has infinitely many equilibria when $r/4\mu > 1$, since $\#M = \infty$. $\square$

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