

The Elusiveness of Systematic Jumps

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2 Abstract

We test for the presence of jumps and measure the price variance of 40 major stocks and the index they form using intra-day returns. Subsequently, we find that jumps can be classified into two groups: systematic and idiosyncratic. Idiosyncratic jumps are firm specific and are usually larger than systematic jumps which affect stocks collectively. Systematic jumps are virtually non-detectable when jump test statistics are applied to individual stocks. The elusiveness of systematic jumps is a consequence of their moderate size and the higher price variance of individual stocks. We also uncover encouraging evidence for a new jump detection scheme.

3 Introduction

In this paper, we examine the relationship between jumps in individual stocks and jumps in the market by utilizing a daily statistic that tests for jumps on 40 of the largest stocks in the United States. The equally weighted index formed by these stocks is shown to represent a large portion of the US market. To the best of our knowledge, this method of relating firm specific jumps to market jumps has yet to be used in the literature.

Jumps that are found in the index are virtually undetectable when we try to locate them in individual stocks using the same detection scheme. This is very paradoxical as it is hard to intuit why an equally weighted index does not remotely resemble any of its components. We discover that this is due to the volatility of individual stocks and the relative size of jumps that affect the entire market. In the process, we find that it might be feasible to detect jumps that affect the market by measuring the covariance of returns in individual stocks.

We revert to basic finance as an introduction.

In financial markets, one would encounter the changing prices of assets such as treasury bills, government and corporate bonds, options, currencies, commodities or stocks. This is often represented by ticks of the asset's price in time. Representations of these price ticks over long periods at different sampling rates may look like the plots in the following figure.

Figure 1 shows the price of Proctor&Gamble's (PG) common stock from 2001 to 2005. The sampling rate is increased from once every 40 trading days to once every 5 trading days as we move from the top panel to the bottom panel.

We would like to draw attention to the growing level of detail in the price of PG. We can see that the space between any two tick marks in the top panel of Figure 1 is replaced with more ticks as the sampling frequency is increased. This gives us higher resolution and a clearer view of PG's price.

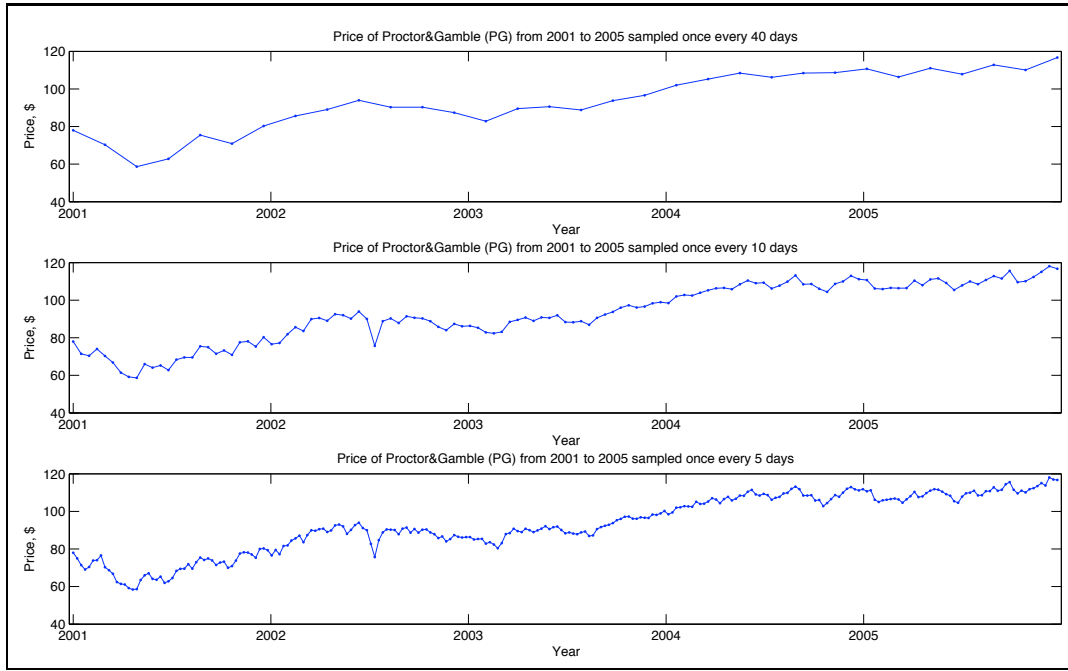


Figure 1: Price of Proctor&Gamble (PG) over 5 years sampled at different frequencies.

Let us now pass a magnifying glass over the price of PG and look at the price as it evolves within a single day. Instead of sampling the price of PG once every few days as we did in Figure 1, let us now sample the price of PG every 30 seconds. Data at such high frequency has only become recently available and is revealing new detail about price evolution.

Figure 2 gives the price of PG on August 3, 2004 sampled once every 30 seconds. Take note of the discontinuity that occurs a little after 11am where PG gains about 30 cents over the period of about 2 minutes.

The plots in Figure 1 and the plot in Figure 2 are similar in that they show the evolution of price over time. However, to someone who tries to model the behavior of price, the plots are different in a very fundamental aspect. This difference is a consequence of the extra detail revealed at finer sampling intervals.

The plots in Figure 1 can be represented by a drift, which is the gradual shift in price (in this example, from about \$80 to about \$120) over long periods, (for

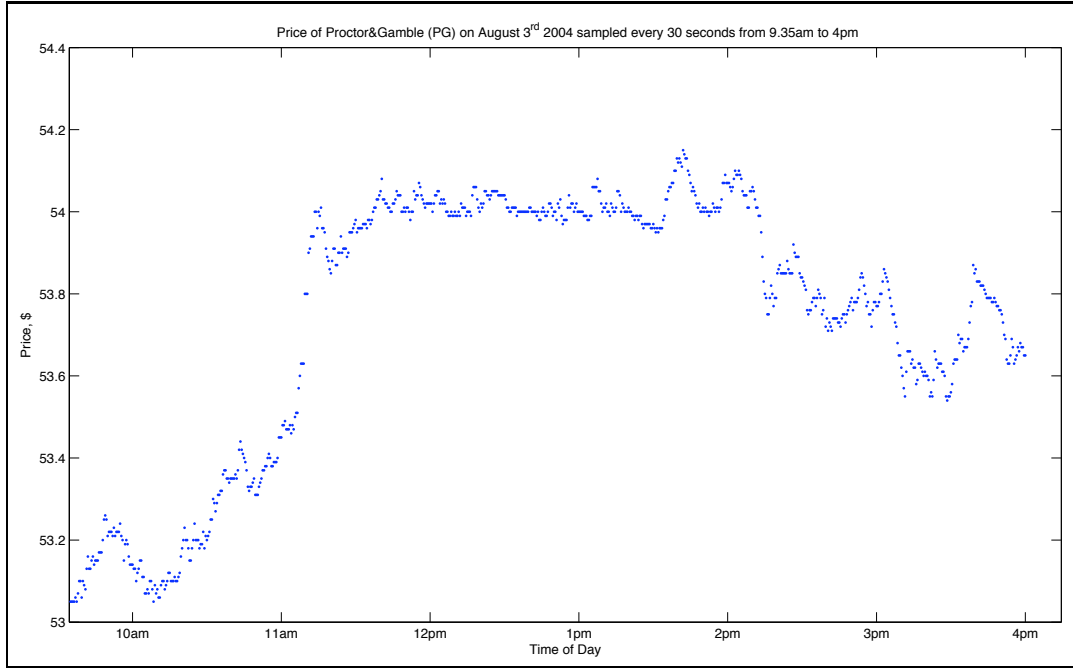


Figure 2: Price of PG sampled once every 30 seconds on August 3, 2004.

example, the entire sample period of 5 years), and by stochastic volatility, which are the fluctuations in the drift resembling a random walk.

In Figure 2, one would notice that there is a discontinuity in the price of PG a little after 11am. This discontinuity is called a jump. We can confirm this by observing that in Figure 1, the prices can be visually connected with a continuous line, but in Figure 2 the continuous evolution of price is broken when the jump occurs. This discontinuity makes it difficult for us to think about price in Figure 2 in the same light as prices in Figure 1.

One may argue that sampling even more finely would reveal ticks which connect the jump to the rest of the prices in a continuous manner, but in theory, jumps would occur even with the continuous record. The continuous record is the ideal situation where the price of the asset is known for all time. The existence of jumps even with the continuous record is supported by evidence generated from the highest sampling rates currently available.

As we shall see later on in this paper and as reported by among others, Andersen, Bollerslev, and Diebold (2004), Barndorff-Nielsen and Shephard (2006a), Huang and

Tauchen (2005) and Eraker et al. (2003) jumps in large indexes and exchange rates often coincide with unexpected macroeconomic announcements which are likely to impact the market in a drastic fashion. These announcements include interest rate changes, oil prices, legislative alterations and security concerns.

While these macroeconomic events affect individual stocks to a certain degree, it is not a stretch of imagination that individual firms can also be affected by sudden unexpected firm specific information that may force an abrupt revaluation of the firms' stock. Examples include lawsuits against a cigarette company, announcements of war for a defense company or legislation of privacy issues that affect an Internet search engine.

But why are we so concerned about jumps? Among other things, jumps contribute toward the volatility of assets. Volatility is central to asset pricing, asset allocation and risk management. These practices are usually aided by models which incorporate what we know about an asset's volatility. Until recently, these models which contain only drift and stochastic volatility elements do not incorporate jumps and hence do not accurately represent reality. Why do we say that?

Recall that at high sampling frequencies such as in Figure 2, details like jump discontinuities are revealed in price series. At lower sampling frequencies, such as in Figure 1 the jumps are present, but it is practically impossible to discern them from stochastic volatility.

In general, models which only incorporate only drift and stochastic volatility work over long periods but do not reflect reality with higher resolution price series. Models that include jump components fit high frequency data a lot better but present a host of computational and analytical difficulties. Many of these problems are becoming increasingly tractable with the advent of ultra high frequency data and advances in the literature on this subject.

The model that we are working with in this paper represents the change in the logarithm of price by the sum of a drift term, a stochastic volatility term and a jump term which better captures the dynamics we described earlier. More analysis

is presented in subsequent sections of this paper. With better specified models and high frequency data as utilized here, it is of interest to formulate methods to describe the magnitude, arrival and nature of jumps in a price series.

There are different approaches for doing this. A particularly elegant method proposed by Barndorff-Nielsen and Shephard (2004) involves the comparison of two measures of volatility. While this method works on any period of time, we introduce it with a trading day as the time period.

The first measure is the well known sum of squared intra-day returns and it asymptotically goes to the integrated variance of returns and the sum of squared jumps over the course of the day. This is known as the realized variance. The second measure is called the bi-power variation. It is the scaled sum of products between the absolute values of adjacent returns in a day. This measure asymptotically yields the integrated variance within a day, but is robust toward jumps.

From here, it naturally follows that the difference of the two is the pure jump contribution. Barndorff-Nielsen and Shephard (2006b) study the distribution of the difference and ratio of these statistics and arrive at tests for jumps in a day. Forms of these jump test statistics have been shown by Andersen et al. (2004), Barndorff-Nielsen and Shephard (2006a) and Huang and Tauchen (2005) to identify large movements in the price of assets. These price movements often coincide with macroeconomic news announcements as previously mentioned.

As mentioned earlier, we extend on this literature by utilizing the statistics on 40 individual stock price series and with the index they form assuming that the index represents the market. We also find encouraging initial evidence of a new scheme for detecting jumps via measuring the covariance of individual stock returns.

The rest of the paper proceeds as follows. Section 2 describes briefly the data that is used in this study. Section 3 reviews the statistics previously described. Section 4 puts the statistics in context with the data used. Section 5 presents the results and discusses them. Section 6 concludes. Figures, citations and in depth explanations and presentations of methodology are included in the Appendix.

4 Methods

High frequency data revolutionized how financial economists study financial markets by allowing them to view and model asset pricing on a different scale altogether. These advantages come with minor inconveniences related to high frequency data which fortunately, can be readily dealt with.

First we consider the process of selecting stocks, the observation period and the exchange observed. There are many stocks to choose from, different ranges of dates and different exchanges on which these stocks are traded. Since we are aggregating and comparing stocks to form an index representing the market, it is important that the mechanisms that generate the price series are similar from stock to stock.

The next issue is dealing with errors in the recording of price. The study started with about 50 stocks of which 40 were chosen after the data was processed for error. Even so, some errors remain in these 40 more reliable stocks. Some of these errors are undetectable; for instance, the specialist entering the price of \$1.90 as \$1.91 by mistake. These errors fortunately do not affect results appreciably. However, entering \$1.90 as \$19.00 would influence results tremendously. How do we detect such errors? More importantly, how do we do so for such a large amount of data?

There is also concern on how often to sample the prices. One would think that for such a study, it would be advantageous to sample as often as possible. On the contrary, it is very important not to over-sample as this introduces market microstructure noise which corrupts the statistics Hansen and Lunde (2006). In this paper, we use signature plots described by Andersen et al. (2001) to determine the sampling frequency.

With those main considerations in mind, the final goal of data collection is the price series of stocks. Returns are then constructed by taking the difference of log prices. Following that, statistics are computed.

Detailed explanations of how this process takes place are provided in Appendix 1.

5 Statistics

5.1 Price Evolution Model

Let us consider a scalar log-price $p(t)$ that evolves in continuous time. The differential of log price $dp(t)$ can be modeled by

$$dp(t) = \mu(t)dt + \sigma(t)dw(t) + d\mathcal{L}_J(t), \quad (1)$$

where $\mu(t)dt$ and $\sigma(t)$ are drift and instantaneous volatility, $w(t)$ is standardized Brownian motion, $d\mathcal{L}_J(t)$ is a pure jump Lévy process with increments $\mathcal{L}_J(t) - \mathcal{L}_J(s) = \sum_{s \leq \tau \leq t} \kappa(\tau)$, and $\kappa(\tau)$ is the jump size. This notation is adopted from Basawa and Brockwell (1982). This paper focuses on a special class of Lévy processes known as the **Compound Poisson Process** where the jump intensity is constant and the jump size $\kappa(t)$ is independent identically distributed.

5.2 Returns

In reality, prices are sampled $p(0), p(1), p(2) \dots p(M)$ and the j^{th} return on day t is given by

$$r_{t,j} = p\left(t - 1 + \frac{j}{M}\right) - p\left(t - 1 + \frac{j-1}{M}\right); \quad j = 1, 2, \dots, M. \quad (2)$$

where M is the sampling frequency corresponding to the number of returns for day t . Barndorff-Nielsen and Shephard (2004) studied the general measures of realized intra-day price variance and their work resulted in the use of two very convenient measures of integrated volatility to study jumps. These measures are **Realized Variance** and **Bi-Power Variation**.

Integrated volatility refers to the cumulative volatility over an arbitrary period of time. While any arbitrary period can be used, the rest of this section pegs the period as one day and it more convenient for the presentation of ideas. We also measure the

integrated volatility over single day periods for the rest of this paper.

5.3 Realized Variance and Bi-power Variation

Realized variance is given by

$$RV_{t,i} = \sum_{j=1}^M r_{t,j,i}^2, \quad (3)$$

and as noted in Andersen, Bollerslev, and Diebold (2002), $RV_{t,i}$ satisfies

$$\lim_{M \rightarrow \infty} RV_{t,i} = \int_{t-1}^t \sigma_i^2(s) ds + \sum_{j=1}^{N_t} \kappa_{t,j,i}^2, \quad (4)$$

where N_t is the number of jumps in day t and $\kappa_{t,j}$ is the size of the jump. This makes RV_t a consistent estimator of the integrated variance of day t , plus the contribution of jumps to the total variation. Note that in this paper, subscript i is used to denote individual assets. $RV_{t,i}$ is presented as an example and the rest of the equations in this section will exclude the subscript for brevity.

Bi-power variation is given by

$$BV_t = \mu_1^{-2} \left(\frac{M}{M-1} \right) \sum_{j=1}^M |r_{t,j-1}| |r_{t,j}|, \quad (5)$$

where

$$\mu_a = E(|Z|)^a, \quad Z \sim N(0, 1), \quad a > 0.^1$$

Here, the term μ_1^{-2} is absorbed into the definition of BV_t to make it directly comparable to RV_t . $M/(M-1)$ is the adjuster for degrees of freedom. The results of Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen, Graversen, Jacod, Podolskij, and Shephard (2005) imply that under reasonable assumptions about the dynamics of (1),

¹ $\mu_p \equiv 2^{\frac{p}{2}} \frac{\Gamma(\frac{1}{2}(p+1))}{\Gamma(\frac{1}{2})} = E(|Z|^p)$, See Barndorff-Nielsen and Shephard (2004) for further explanations.

$$\lim_{M \rightarrow \infty} BV_t = \int_{t-1}^t \sigma^2(s) ds. \quad (6)$$

This makes BV_t a jump robust measurement of integrated variance.

Naturally, it follows that $RV_t - BV_t$ is a consistent estimator for the jump contribution to variation and as emphasized by Barndorff-Nielsen and Shephard (2004, 2006a) can form the basis for a jump detection scheme. Using this measure, Andersen et al. (2004) suggested that there are too many large within day returns in equity, fixed income and foreign exchange prices to be consistent with standard continuous time stochastic volatility models.

5.4 Relative Jump

From RV_t and BV_t we also consider the **Relative Jump** measure,

$$RJ_t = \frac{RV_t - BV_t}{RV_t}, \quad (7)$$

as a measure of the proportion of jump contribution to the total variation. An equivalent statistic, $-RJ_t$, called the ratio statistic is proposed and studied by Barndorff-Nielsen and Shephard (2006a). As in Huang and Tauchen (2005), we prefer the term relative jump since RJ_t naturally gives us the proportion of jump contribution to total price variance, if any. In theory, $RJ_t \geq 0$, but finite sampling sometimes results in $RJ_t < 0$ in which we set RJ_t to zero as recommended by Barndorff-Nielsen and Shephard (2004).

5.5 Asymptotic Distributions and Jump Test Statistics

Given that $RV_t - BV_t$ measures the jump contribution to price variance, it is of interest to compute the jump contribution on a day-by-day basis and to flag days where the jump contribution is abnormally large. This indicates that the day would contain at least one jump. This would be similar to the exercise of pivoting a sample

about its mean and comparing the result to a Z -table where one would be able to determine within a confidence level if the the sample's deviation from the mean was significant or merely by chance.

To do this, one would have to first know the joint distribution of RV_t and BV_t . Under the null hypothesis of no jumps and some other regularity conditions, Barndorff-Nielsen and Shephard (2006a) first give the joint distribution of RV_t and BV_t conditional on the volatility path as $M \rightarrow \infty$ as

$$M^{\frac{1}{2}} \left[\int_{t-1}^t \sigma^4(s) ds \right]^{-\frac{1}{2}} \begin{pmatrix} RV_t - \int_{t-1}^t \sigma^2(s) ds \\ BV_t - \int_{t-1}^t \sigma^2(s) ds \end{pmatrix} \xrightarrow{\mathcal{D}} N \left(0, \begin{bmatrix} \nu_{qq} & \nu_{qb} \\ \nu_{qb} & \nu_{bb} \end{bmatrix} \right), \quad (8)$$

where

$$\begin{bmatrix} \nu_{qq} & \nu_{qb} \\ \nu_{qb} & \nu_{bb} \end{bmatrix} = \begin{bmatrix} \mu_4 - \mu_2^2 & 2(\mu_3\mu_1^{-1} - \mu_2) \\ 2(\mu_3\mu_1^{-1} - \mu_2) & (\mu_1^{-4} - 1) + 2(\mu_1^{-2} - 1) \end{bmatrix}.$$

From Footnote (1), we evaluate $\mu_1 = \sqrt{\frac{2}{\pi}}$, $\mu_2 = 1$, $\mu_3 = 2\sqrt{\frac{2}{\pi}}$ and $\mu_4 = 3$, giving

$$\nu_{qq} = 2,$$

$$\nu_{qb} = 2,$$

and

$$\nu_{bb} = \left(\frac{\pi}{2} \right)^2 + \pi - 3.$$

To determine the scale of $RV_t - BV_t$ in units of conditional standard deviation, one needs to estimate the integrated quarticity $\int_{t-1}^t \sigma^4(s) ds$. We shall see further on how the integrated quarticity is used to construct jump test statistics. Andersen et al. (2004) suggest using the jump robust realized **Tri-Power Quarticity** statistic which is a special case of the multi-power variations studied in Barndorff-Nielsen and Shephard (2004). The tri-power quarticity is given by

$$TP_t = M\mu_{4/3}^{-3} \left(\frac{M}{M-2} \right) \sum_{j=3}^M |r_{t,j-2}|^{\frac{4}{3}} |r_{t,j-1}|^{\frac{4}{3}} |r_{t,j}|^{\frac{4}{3}} \quad (9)$$

and asymptotically goes to

$$\lim_{M \rightarrow \infty} TP_t = \int_{t-1}^t \sigma^4(s) ds. \quad (10)$$

even in the presence of jumps. There are other estimators for integrated quarticity such as the **Quad-Power Quarticity** (QP_t) also based off Barndorff-Nielsen and Shephard (2004). While both TP_t and QP_t estimate $\int_{t-1}^t \sigma^4(s) ds$ unbiasedly, we utilize TP_t in this paper because it is suitable for this study as documented by Huang and Tauchen (2005) in their intensive Monte Carlo survey of these statistics' performance.

5.6 Jump Test Statistics

As previously mentioned, one approach to detecting jumps is to compute a measure of jump contribution to price variation like $RV_t - BV_t$ on a day-by-day basis and to flag days where the price movements are very large given a conditional distribution of the measure. Following the joint distribution of RV_t and BV_t , Barndorff-Nielsen and Shephard (2006a) find that a measure that would in theory yield a normal distribution under the null hypothesis of no jumps. This time series measured once a day from intraday returns is

$$z_{TP,t} = \frac{RV_t - BV_t}{\sqrt{(\nu_{bb} - \nu_{qq}) \frac{1}{M} TP_t}} \quad (11)$$

where for each day t evaluated, $z_{TP,t} \xrightarrow{\mathcal{D}} N(0,1)$ as $M \rightarrow \infty$ under the no jump assumption. Subsequently, $z_{TP,t}$ is used by Andersen et al. (2004) to test for the presence of jumps in the DM/\$ exchange rate, the S&P500 market index and the 30-year US Treasury bond yield.

From the results of Andersen, Bollerslev, Diebold, and Labys (2001), Andersen,

Bollerslev, Diebold, and Ebens (2001) and Barndorff-Nielsen and Shephard (2002) one might expect to be able to improve the performance of the statistics under finite sampling conditions by basing the test statistic on the logarithm differences of the variation measures instead of the absolute difference. This makes the statistic better behaved by introducing a natural scale to the jump contribution in price variance. In this case, the statistic is

$$z_{TP,l,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(\nu_{bb} - \nu_{qq}) \frac{1}{M} \frac{TP_t}{BV_t^2}}} \quad (12)$$

which is used Andersen et al. (2004).

In theory, or asymptotically, $TP_t \geq BV_t^2$. This implies that the minimum that $\frac{TP_t}{BV_t^2}$ can be is 1. Unfortunately, finite sampling conditions doesn't always guarantee this fact. Thus, another adjustment is made to make the statistics better behaved by including a maximum adjuster in the denominator of (12). This results in

$$z_{TP,lm,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(\nu_{bb} - \nu_{qq}) \frac{1}{M} \max\left(1, \frac{TP_t}{BV_t^2}\right)}}. \quad (13)$$

The relative jump measure, (7), can also be used in the numerator and the resulting statistics are,

$$z_{TP,r,t} = \frac{RJ_t}{\sqrt{(\nu_{bb} - \nu_{qq}) \frac{1}{M} \frac{TP_t}{BV_t^2}}} \quad (14)$$

and

$$z_{TP,rm,t} = \frac{RJ_t}{\sqrt{(\nu_{bb} - \nu_{qq}) \frac{1}{M} \max\left(1, \frac{TP_t}{BV_t^2}\right)}}. \quad (15)$$

One can see that the forms of the denominators in equations (12), (13), (14) and (15) are the same except for the maximum adjustment mentioned earlier. This implies that RJ_t and $\log \frac{RV_t}{BV_t}$ have identical asymptotic distributions.

As mentioned earlier, the statistic that we are utilizing in this paper to test for jumps is $z_{TP,rm,t}$ as guided by the results of Huang and Tauchen (2005). $z_{TP,rm,t} \sim N(0, 1)$ under the null of no-jumps and in this paper we use it as a jump detection tool with a 99.9% significance level. We also refer to $z_{TP,rm,t}$ as the z -statistic in this paper.

Throughout the paper, we will be also be applying RV_t , BV_t and RJ_t on the return series of 40 stocks and the index formed from the stocks. Another statistic, the U -statistic which measures covariance of returns will be mentioned in Section 5. We shall now show how these statistics are actually obtained from return series using PG again as an example.

6 Application of Statistics

This section applies the equations presented in Section 5 on the price series of individual stocks to show how the results in Section 7 are obtained.

Let us start by looking at the price and corresponding return series of Proctor and Gamble (PG) on January 4, 2001 at the 17.5 minute sampling interval where $M = 22$.

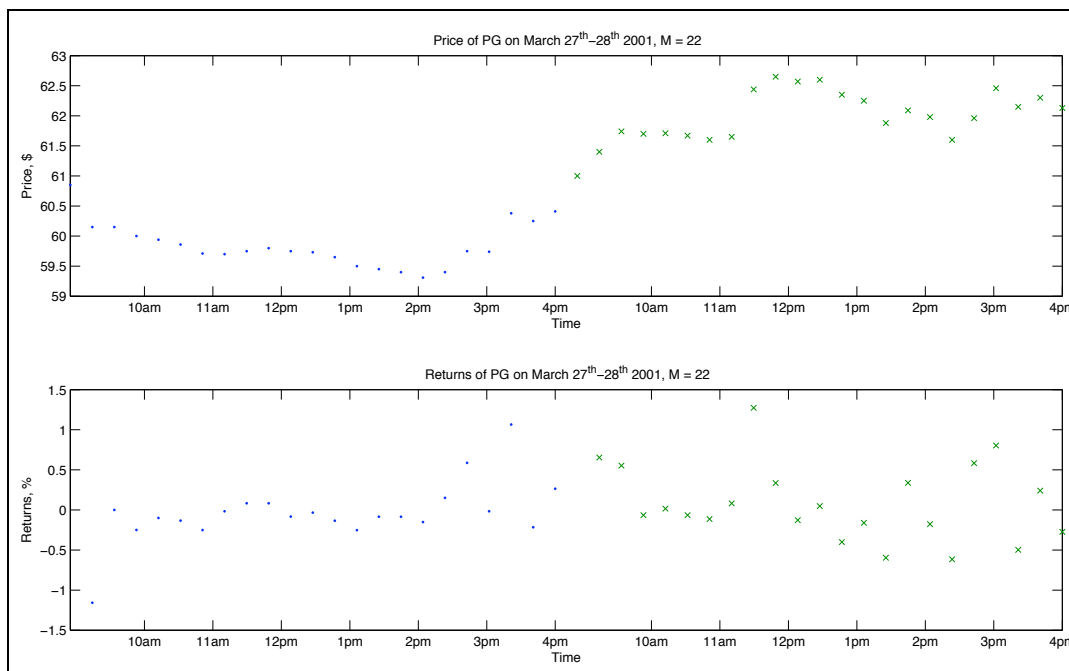


Figure 3: Price and corresponding returns of Proctor&Gamble on March 26 - 27, 2001, $M = 22$.

Figure 3 gives the price and corresponding return series of PG on March 26 and 27, 2001. Notice that the first price of each day does not correspond to any return since overnight returns are not considered in constructing any measures of variance. As previously stated, returns are calculated by taking the difference of log prices,

$$r_{t,j} = \log(p_{t,j}) - \log(p_{t,j-1})$$

Next, we calculate from the returns the values of RV_t , BV_t , RJ_t and $z_{TP,rm,t}$ using the equations given in Section 5.

Table 1: RV_t , BV_t , RJ_t and $z_{TP,rm,t}$ on March 26 and 27, 2001.

Statistic	March 26	March 27
RV_t	3.253% ²	4.945% ²
BV_t	1.037% ²	5.111% ²
RJ_t	68.14%	-0.0336%
$z_{TP,rm,t}$	3.6261	-0.2022

Table 1 gives the values of RV_t , BV_t , RJ_t and $z_{TP,rm,t}$. Take note that RJ_t and $z_{TP,rm,t}$ on March 27 is negative and is set to zero for analysis due to reasons given in Section 5. These results show that total price variation during the day which is the sum of integrated variance and squared jumps is 3.253%² and 4.945%² for each day evaluated as given by RV_t . The integrated variance portion of total variation is given by the jump robust measure of BV_t and is 1.037%² and 5.111%². The relative contribution of jumps to total price variance is 68.14% and 0%, (after the zero adjustment which is not shown in Table 1), as given by RJ_t . We also calculate $z_{TP,rm,t}$ as a test for the presence of jumps as outlined in Section 5 and obtain values of 3.6261 and 0 (with adjustment).

The z -value threshold for a jump day is the inverse normal of the standard Gaussian distribution with a p -value of 99.9 which is 3.09. Any z -value larger than or equal to $\Phi^{-1}(0.999)$ implies that the particular day is flagged as a day containing a jump return by the z -statistic. Thus, March 26 is flagged as a jump at the 99.9% significance level by the jump test used in this study.

We do the for all the 1241 days in the sample period for PG. s

Figure 4 shows the progression from a return series to the statistics that measure variance, jump contribution and presence of jumps which are used to describe PG's jump characteristics. The horizontal line in the plot of $z_{TP,rm,t}$ is the z -value threshold

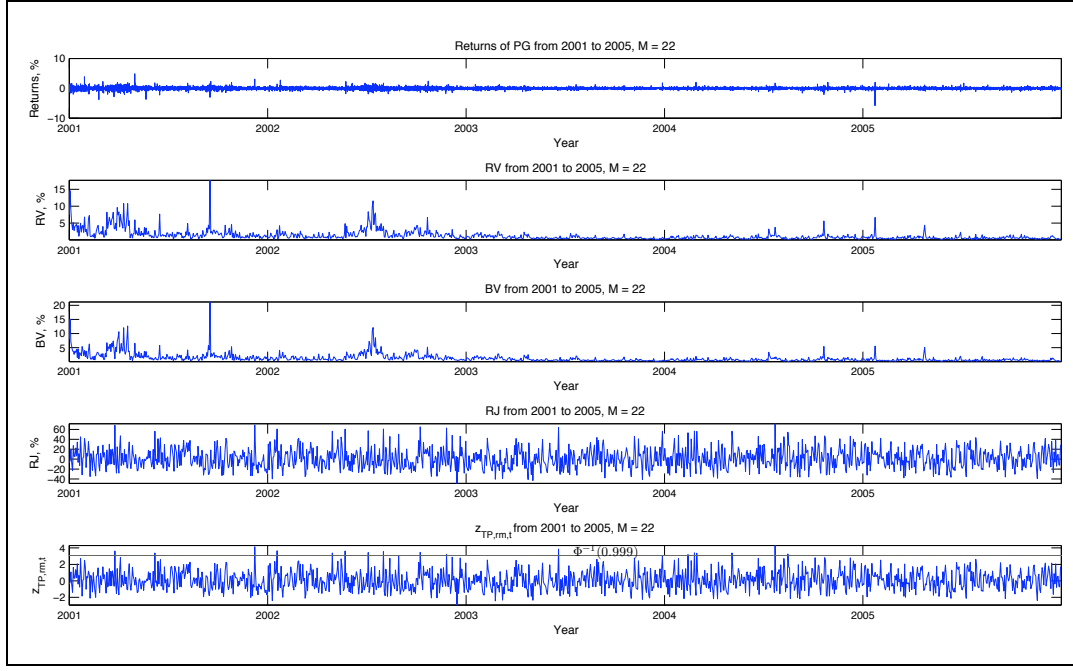


Figure 4: (Returns, Realized Variance, Bi-Power Variation, Relative Jump and $z_{TP,rm,t}$ of PG from 2001 to 2005, $M = 22$).

for the 99.9% significance level. At the 99.9% significance level, PG jumped a total of 17 times throughout the sample period.

We do the same for all 40 stocks used in this study. We also calculate the same statistics for an equally weighted portfolio of the 40 stocks. We call this portfolio AGG which stands for the AGGregate of 40 stocks. AGG's returns are formed via

$$r_{t,j,AGG} = \frac{1}{n} \sum_{i=1}^n r_{t,j,i} \quad n = 40. \quad (16)$$

7 Results and Discussion

As mentioned in Section 6, AGG is our ticker symbol for the AGGregate of 40 stocks used in this study. This is not to be confused with the Exchange Traded Fund (ETF) iShares Lehman Aggregate Bond which is traded under the ticker symbol AGG. AGG is an equally weighted portfolio constructed from 40 of the largest NYSE traded stocks as done in Eqn. (16).

At the 5 minute sampling interval, AGG's returns are 0.88 correlated with the returns of another ETF, SPY. SPY reflects the activity of the S&P500 which is widely regarded as a good representative of the entire market. Thus, we continue our analysis with AGG as our proxy for the market.

In this study, we apply the statistics previously described in Section 5 and Section 6 to the return series of 40 individual stocks and AGG which represents the market. These statistics are used to relate jumps in individual stocks to jumps in the market.

Let us initiate the discussion by asking how closely individual firms track AGG, our proxy for the market. To do this, we use the classical measure β from the Capital Asset Pricing Model (CAPM) which was originally introduced by Sharpe (1964) and Lintner (1965). Here, β is measured by regressing the returns of individual stocks and the returns of AGG at the 17.5 minutes sampling interval. We also look at $\text{corr}(z_{TP,rm,t,AGG}, z_{TP,rm,t,i})$.

The top panel of **Figure 5** shows the β of individual stocks at 17.5 minutes sampling interval over the sample period January 2001 through December 2005. The bottom panel displays the correlations of the daily z -statistics between individual stocks and AGG. The figure above is strange, but why?

Let us explain that by first establishing the requirements for AGG to jump and the implications of AGG jumping when the continuous record of price is available. The continuous record means that the price of an asset is known at all times. For AGG's

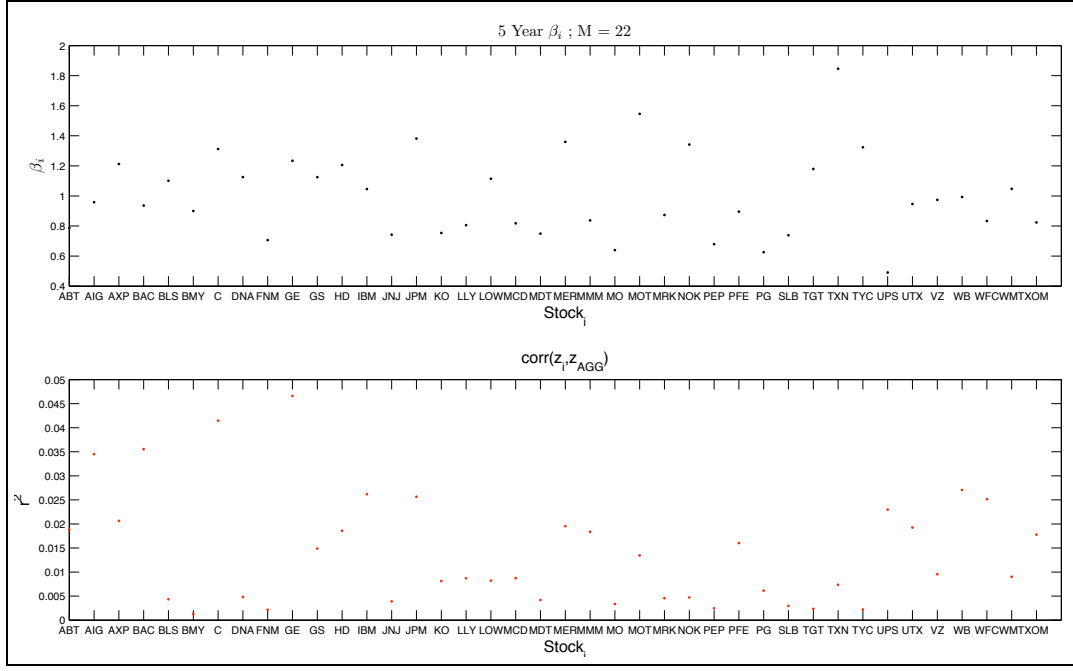


Figure 5: 5 year high frequency β and $\text{corr}(z_{TP,rm,t,AGG}, z_{TP,rm,t,i})$

price series to be completely continuous(no jumps), one of the following 2 conditions must apply to the price series of its component stocks. The price series of all the components of AGG must be continuous, or, the jumps that occur simultaneously in the components cancel each other out perfectly in magnitude. Let us rule out the possibility of the latter since its probability is effectively zero.

From the first requirement, we can deduce that when AGG jumps, at least one of its component stocks jumped. If a continuous record of price was available, we would be able to quite trivially identify the stocks that caused jumps in the index, the time when the jumps occurred and the size of the jumps. It would simply be an exercise of pointing them out. It is also true that a jump no matter how small in a single stock would result in a jump in an index of a finite number of component stocks even if the index was very large.

In reality, the continuous record does not exist; we can only sample prices at discrete time intervals. Furthermore, prices are decimalized which means we value stocks at discrete levels. The trivial exercise of pointing out jumps becomes a lot more complicated. Statistics such as the ones used in this study are required. In our

case where prices are discrete a jump in a single component stock is very unlikely to cause a jump in AGG (unless the jump in the individual stock is huge) since it only contributes to a fortieth of AGG's returns. However, jump common to many stocks would accumulate to cause a jump in AGG. Thus, for AGG to jump, we expect that its components would jump in unison in the same direction.

Let us now go back to the claim that Figure 5 was strange. The top panel tells us that the β s of the stocks are around unity which indicates that returns of these stocks closely track the returns of AGG. Naturally, we would expect that the jump characteristics of the stocks and the index would be correlated to a certain degree. However, the bottom panel in Figure 5 tells us that there is absolutely no correlation between the $z_{TP,rm,t}$ of AGG and individual stocks. This is very surprising. How exactly do the jump characteristics of stocks behave on average in relation to AGG? To do this, we relate the average $z_{TP,rm,t}$ of individual stocks on day t to $z_{TP,rm,t,AGG}$. The average z -statistic, $\overline{z_{TP,rm,t,i}}$ is given by

$$\overline{z_{TP,rm,t,i}} = \frac{1}{n} \sum_{i=1}^n z_{TP,rm,t,i}, \quad n = 40.$$

Figure 6 shows the relationship between the average z -statistics of individual stocks and the z -statistic of AGG on a given day. With $\Phi^{-1}(0.999)$ as the cutoff, AGG jumps on 7 days while the individual stocks on average do not jump. Due to scaling, it is easy to be misled by above plot that there is a relationship between the average z -statistic for individual stocks and AGG. When we inspect the scales on the axes, we can see that z_{AGG} varies a lot more than $\overline{z_{TP,rm,t,i}}$.

Figure 6 confirms in many ways the implications of Figure 5. Figure 5 tells us that the jump statistics of individual stocks and AGG are not correlated. This result contradicts with our earlier heuristic reasoning that for an index to jump, stocks on average need to jump. Figure 6 supports Figure 5 by telling us that, the average jump statistics of the components of AGG do not have anything to do with the jump

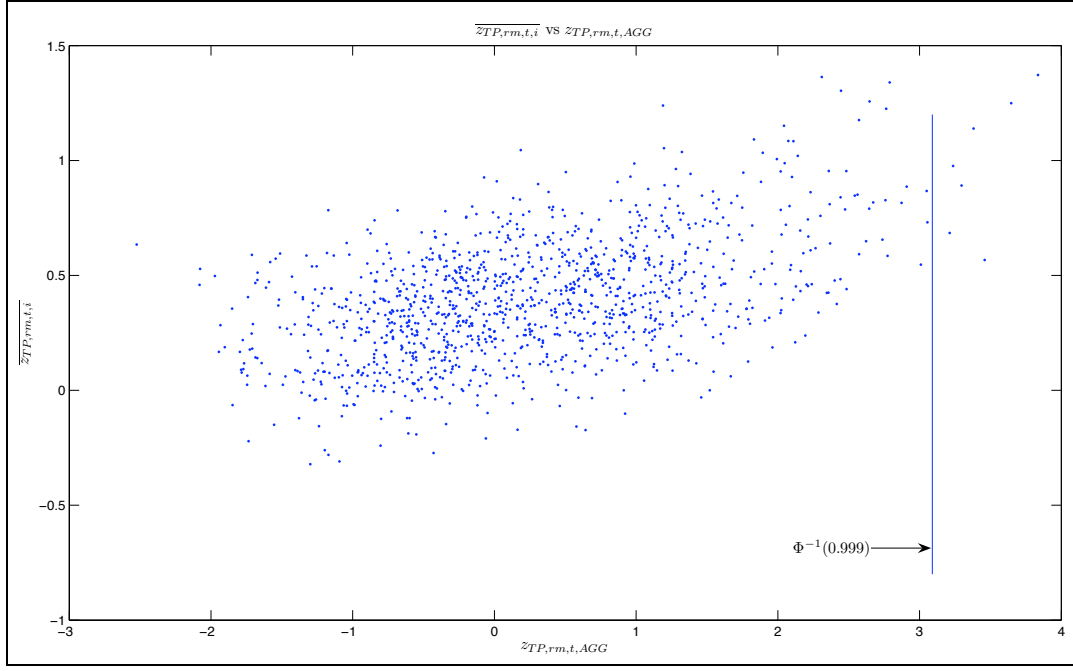


Figure 6: Scatter plot of the average z -statistic of individual stocks and the corresponding z -statistic for AGG.

statistics of AGG. Even more surprisingly, individual stocks on average never jump during the time horizon of the analysis while the index jumped 7 times at a 99.9% critical value.

This is quite a paradox! How can an index jump when its components on average don't jump? The only ways for that to happen is if the continuous record is available, but we have just reasoned that with discrete sampling, jumps from individual stocks will be too small to cause the market to jump. To resolve the paradox, let us start by taking a closer look at the 6 of the 7 days where the index jumped. We are not selectively choosing 6 days but picking the days with the largest z -statistics to avoid making figures with 7 subplots which is difficult to arrange.

In **Figure 7**, we look at the z -statistics of individual stocks for each of the 6 days where AGG is flagged as a jump day. The title of each subplot gives the date and z -statistic of AGG for the particular jump day. We observe that on each of these days, very few stocks jump.

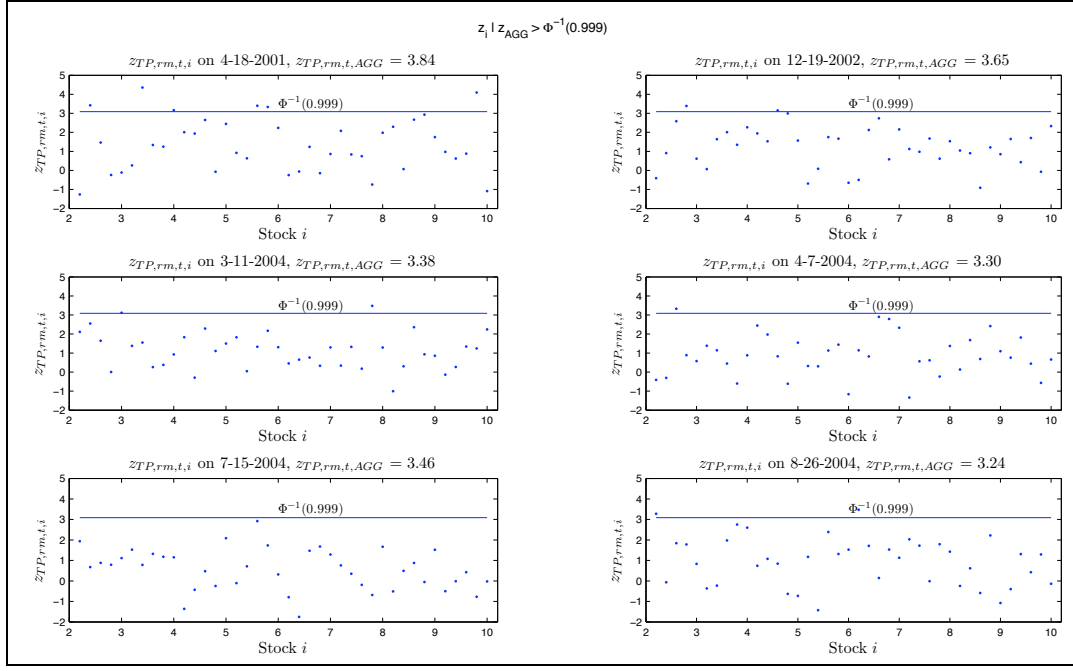


Figure 7: The z -statistics of individual stocks on the 6 days where AGG is flagged as a jump day.

Figure 7 shows that on all of the days involved, stocks on average do not jump. In fact, a few stocks jumping are sufficient to cause the index to jump. This is strange and contradicts our reasoning.

However, one may argue that the 17.5 minute sampling interval is fine enough for the record to be effectively continuous and hence a single stock jumping is enough to cause the index to jump. But then, on July 15, 2004, not a single stock jumped. Clearly, something unexplained is causing this to happen. Earlier, we intuitively reasoned that jumps in an index are likely to be caused by jumps that are common to individual stocks as opposed to a jump in a single stock. In other words, jumps in individual stocks that are highly covaried with each other cause jumps in an index. This can be more formally shown with the following calculation obtained from Tim Bollerslev based of the statistics presented earlier. He shows that jumps in an index are caused by the average cross product of jump components in individual stocks. This is summarized in Eqn. 17.

The mathematics can be shown by recalling from Eqns. (4) and (6) that,

$$RV_{t,i} \rightarrow \int_{t-1}^t \sigma_i^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j,i}^2,$$

$$BV_{t,i} \rightarrow \int_{t-1}^t \sigma_i^2(s)ds,$$

and hence,

$$RV_{t,i} - BV_{t,i} = \sum_{j=1}^{N_t} \kappa_{t,j,i}^2.$$

In an equally weighted portfolio of n stocks such as AGG where

$$r_{t,j,AGG} = \frac{1}{n} \sum_{i=1}^n r_{t,j,i},$$

$$RV_{t,AGG} = \sum_{j=1}^M \left(\frac{1}{n} \sum_{i=1}^n r_{t,j,i} \right)^2 \xrightarrow{M \rightarrow \infty}$$

$$\frac{1}{n^2} \sum_{i=1}^n \int_{t-1}^t \sigma_i^2(s)ds + \frac{1}{n^2} \sum_{i=1}^n \sum_{k=1}^n \int_{t-1}^t \sigma_i(s)\sigma_k ds \quad i \neq k$$

$$+ \frac{1}{n^2} \sum_{i=1, j=1}^{n, N_{t,i}} \kappa_{t,j,i}^2 + \frac{1}{n^2} \sum_{i=1, j=1}^{n, N_{t,i}} \sum_{k=1, q=1}^{n, N_{t,k}} \kappa_{t,j,i} \kappa_{t,q,k} \quad i \neq k$$

and

$$BV_{t,AGG} = \sum_{j=2}^M \left(\frac{1}{n} \sum_{i=1}^n r_{t,j-1,i} \cdot \frac{1}{n} \sum_{i=1}^n r_{t,j,i} \right) \xrightarrow{M \rightarrow \infty}$$

$$\frac{1}{n^2} \sum_{i=1}^n \int_{t-1}^t \sigma_i^2(s)ds + \frac{1}{n^2} \sum_{i=1}^n \sum_{k=1}^n \int_{t-1}^t \sigma_i(s)\sigma_k ds \quad i \neq k.$$

Therefore,

$$RV_{t,AGG} - BV_{t,AGG} =$$

$$\begin{aligned}
& + \frac{1}{n^2} \sum_{i=1, j=1}^{n, N_{t,i}} \kappa_{t,j,i}^2 + \frac{1}{n^2} \sum_{i=1, j=1}^{n, N_{t,i}} \sum_{k=1, q=1}^{n, N_{t,k}} \kappa_{t,j,i} \kappa_{t,q,k} \quad i \neq k \\
& = \frac{1}{n} \overline{\kappa_i^2} + \frac{n-1}{n} \overline{\kappa_i \kappa_k} \quad i \neq k.
\end{aligned}$$

When n is large, which in practice represents a large, well diversified portfolio,

$$RV_{t,AGG} - BV_{t,AGG} \rightarrow \overline{\kappa_i \kappa_k} \quad \text{as } n \rightarrow \infty \quad (17)$$

This result shows that in large, well diversified portfolios, jumps can only be caused by highly co-varied jump components of return or equivalently, common jumps. Common jumps are jumps that occur simultaneously across many stocks, or simply, jumps that pervade the market. This is consistent with our reasoning so far. Another important result is that jumps unique to individual stocks are diversified away. Since systematic jumps affect stocks in general, and idiosyncratic jumps affect individual stocks but are diversified away, we would expect to find more jumps in individual stocks than in an index such as AGG. Let us confirm that.

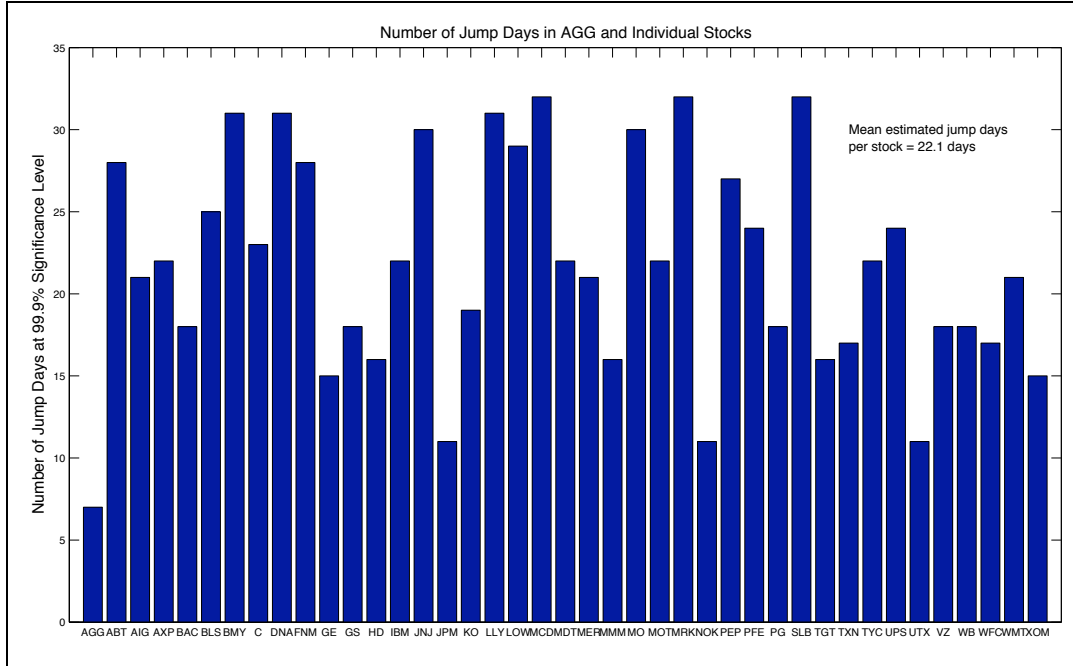


Figure 8: The number of flagged days for AGG and its components from 2001 to 2005.

Figure 8 shows the number of jump days flagged by the jump tests in AGG and its 40 component stocks between 2001 and 2005. It confirms that individual stocks are flagged more frequently than an index and our calculations show that these jumps are diversified away. We also show with the same calculations that the flagged jumps in AGG are in fact jumps that are common to the individual stocks that make up AGG. Since this property of jumps is so similar to that of risk in CAPM, we shall call jumps that are unique to individual stocks idiosyncratic jumps and jumps that pervade the market systematic jumps. As previously explained, this property of jumps sheds some light on why there are fewer flagged jumps in the index compared to individual stocks but does not explain the paradox brought up in Figures 5, 6 and 7.

Figures 5 and 6 show that jumps flagged in individual stocks and AGG are not correlated at all. The absence of correlation can be explained if systematic jumps are not detected at all in individual stocks. Figure 7 makes a very strong statement in that jumps in the index can occur even when only a few stocks jump which is contrary to our reasoning and mathematical results. In fact, on one of the 6 days we examined, none of the individual stocks were flagged as jumps.

Up to this point, we have established that for an index to jump, its components must jump in unison. Figures 5, 6 and 7 all indicate that jumps in the index are not accompanied by jumps in the individual stocks. But, we know that they have to be there since that is the only way for AGG to jump. Somehow, systematic jumps are escaping detection by the jump test statistics. Why? How?

From Eqn. (17), we can see that jumps in indexes are caused by systematic jumps which are simply jumps in individual stocks that are highly covaried with one another. To measure this covariance, we make use of a U -statistic to measure the co-movements or covariance between returns in individual stocks. After that we take a closer look at each of the 6 days. For a more detailed explanation on these statistics, see Serfling (1980). The U -statistic we use is given by

$$U_{t,j,AGG} = \frac{1}{2n(n-1)} \left[\sum_{i=1}^n \sum_{k=1}^n (r_{t,j,i} - \overline{r_{t,j}}) (r_{t,j,k} - \overline{r_{t,j}}) \right] \quad i \neq k. \quad (18)$$

We further assume that $\overline{r_{t,j}} = 0$ at $M = 22$ at 17.5mins sampling frequency. The U -statistic is a measure of how well the stocks move together. In effect, to get a large U -statistic, we need many covaried returns. More importantly, it is robust to a single large movement in one stock. Since a return in AGG is the average return of its component stocks, we expect large returns on AGG to correspond to large U -statistics. Instead of using the absolute value of the U -statistic as a gage for how well the prices of individual stocks move together, we standardize the U -statistic by pivoting about its standard deviation for a single day.

We expect large or significant standardized U -statistics to correspond to jump returns. To confirm, we plot the returns of AGG and its corresponding standardized U -statistics and the returns of individual stocks on each of the 6 days.

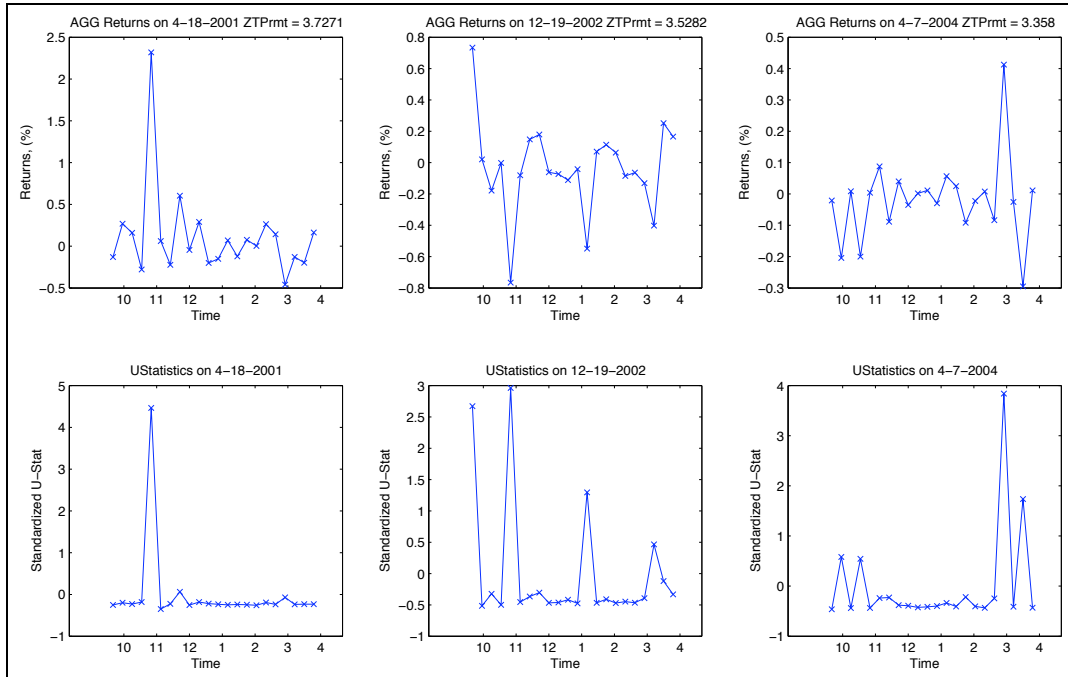


Figure 9: AGG's returns and corresponding U -statistics on flagged jump days. The top row gives the returns. The bottom row gives standardized U -statistics. Each column represents a day.

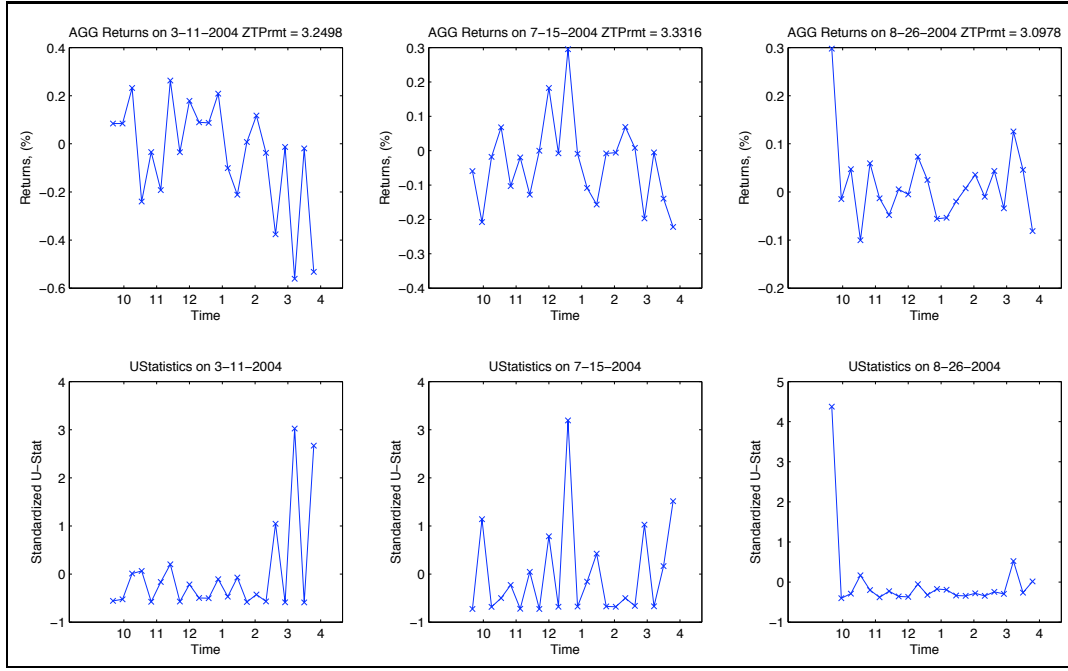


Figure 10: AGG's returns and corresponding U -statistics on flagged jump days. The top row gives the returns. The bottom row gives standardized U -statistics. Each column represents a day.

Figures 9-10 give the returns and the U -statistics of the 6 AGG jump days. Each figure carries 3 days arranged in 3 columns. The returns are given in the top row and the standardized U -statistics are given in the bottom row. The time of the day where the returns and U -statistics are measured is given on the x -axis.

The large values of the standardized U -statistics are estimators of where the co-jumps occur since they measure of the covariance of the stock returns at a particular time of the day and we know jumps in indexes are caused by highly correlated jumps in individual stocks. We use the same 99.9% threshold in our evaluation of how well returns of individual stocks move together. Each of the 6 days presented in Figures 9-10 are flagged jump days and therefore the largest return within the day is by default a jump return. With the exception of December 19, 2002, all the jump days contain standardized U -statistics that are significant at our simplistically assigned threshold which is chosen since we are using the same threshold for z -statistics. Loosely speaking, the large U -statistics are flags for jumps since we have shown that jumps in an

index are caused by covaried jump returns in individual stocks.

For now, we have confirmed that jump returns in the index are indeed caused by highly co-varying returns in the index which is also consistent with our reasoning and calculations. However, this still does not explain why these systematic jumps are not flagged by the jump test statistics when applied to the individual stocks.

To finish the explanation, let us look at the individual return series of each of the 40 stocks on the 6 days evaluated.

Figures 11-16 depict the intra-day returns for the components of AGG on the 6 days where AGG was estimated to have jumped. Time of day is on the x -axis. Returns in percent is on the y -axis. $z_{TP,rm,t}$ of the individual stock is given as the label of the y -axis.

If we look at any particular day in Figures 9-10 and were asked to point out the largest return, this would be a very easy thing to do. The largest return of the day is the jump return and more importantly, the largest return of the day stands out amongst other returns. We now look at the corresponding day in Figures 11-16 and locate the return in the individual stocks which contributed to the large return in the index. While this is arguably quite unscientific, we can convince ourselves that it is much easier to locate the jump return in the index than it is to locate the same return in the component stocks. The jump test statistic behaves in the same way in that it compares the relative contribution of jumps to the overall volatility of price.

The very same return which causes the jump test statistics to flag a day as a jump does not stand out as much in the return series of individual stocks. The individual stocks are more volatile than the index, and that hides the systematic jump and allows it to escape the test for jumps. We have just claimed visually that the individual stocks are more volatile than the index and this volatility makes the moderate sized systematic jump return in the individual stock less apparent than in the case of the index. Hence, the jump test statistics do not pick them up. They pick up the moderately sized co-jumps which aggregate in the index because the idiosyncratic component of jumps has been diversified away and the volatility of the

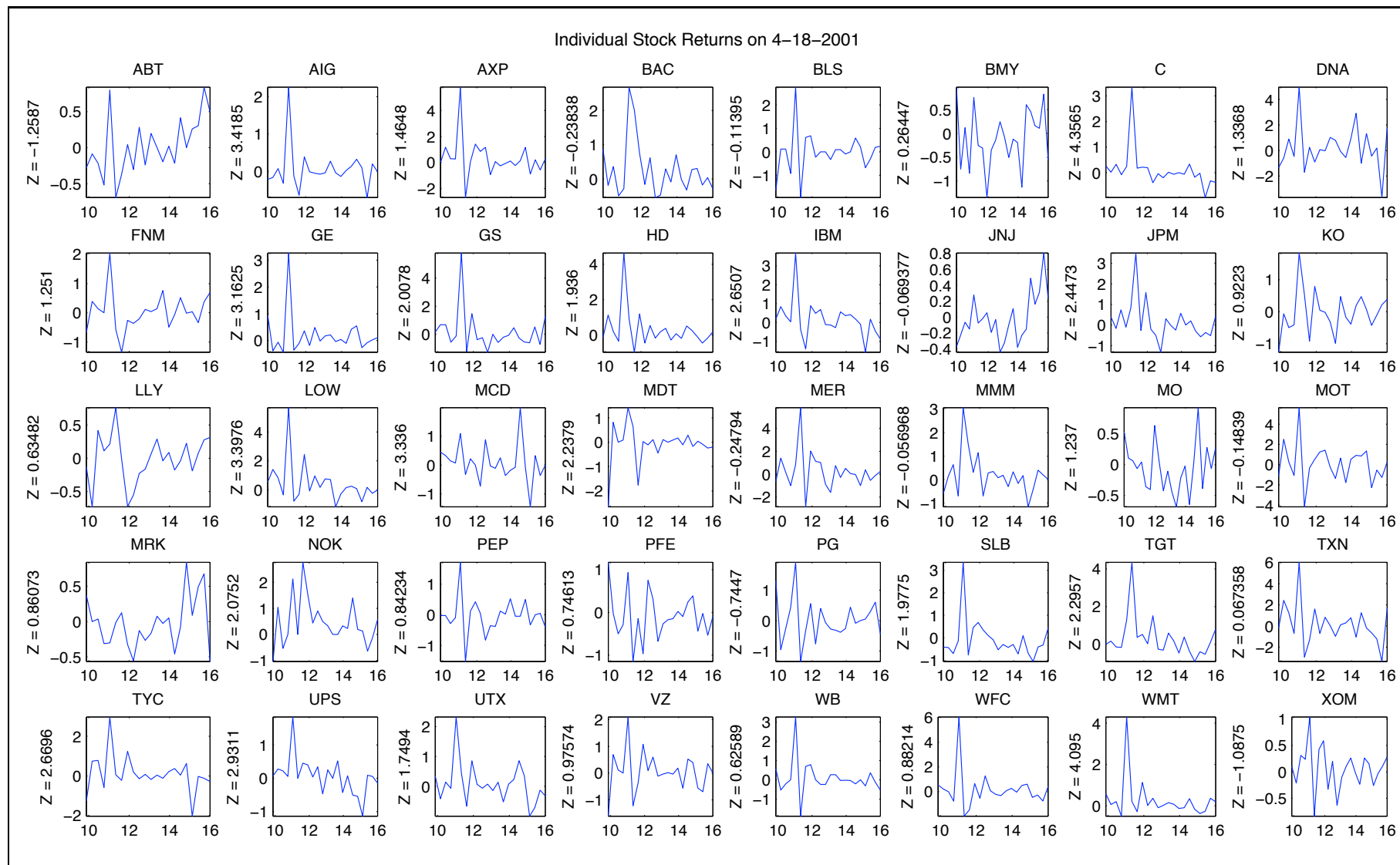


Figure 11: Intraday returns and z -statistics of individual firms on April 18, 2001

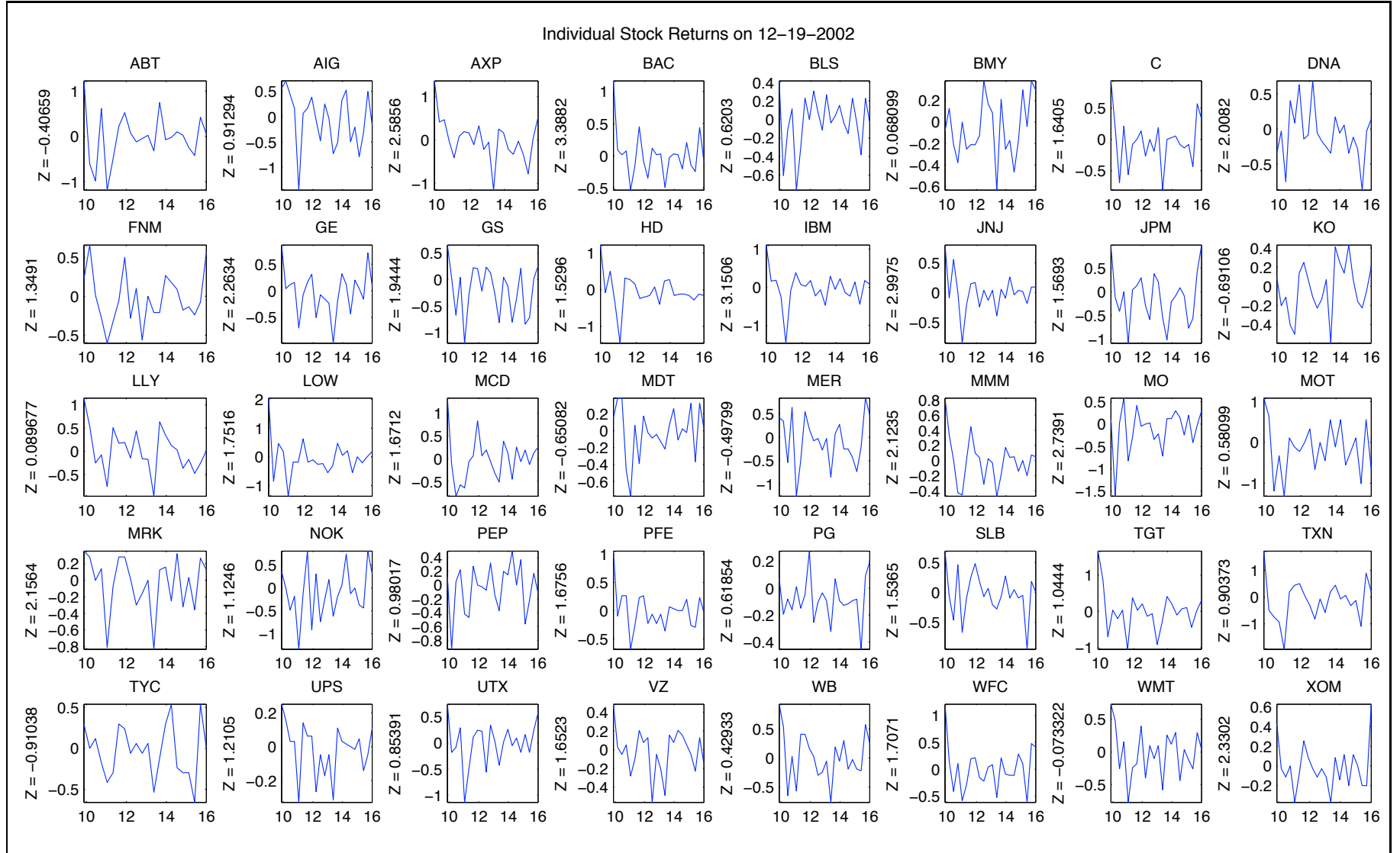


Figure 12: Intraday returns and z -statistics of individual firms on December 19, 2002

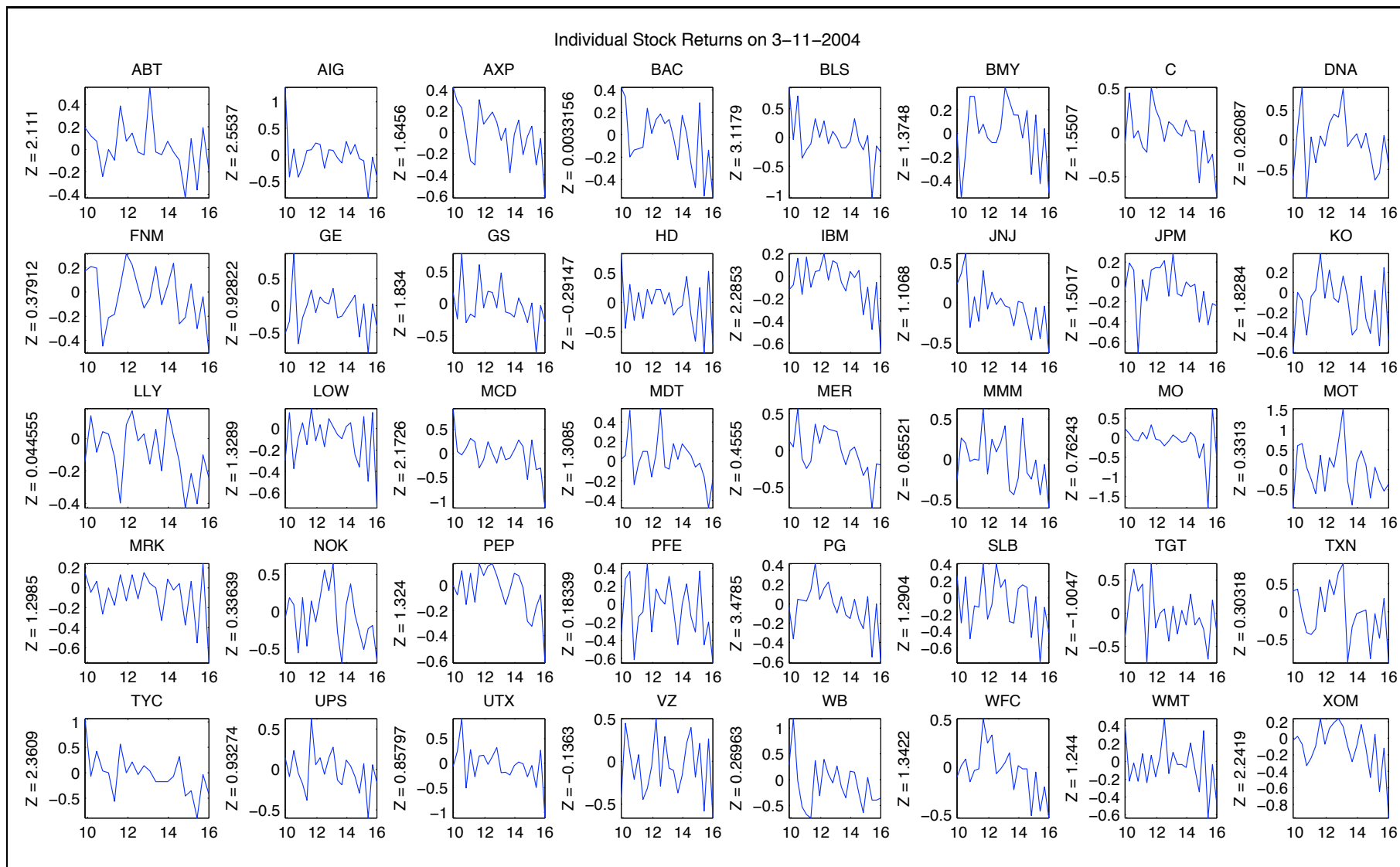


Figure 13: Intraday returns and z -statistics of individual firms on March 11, 2004

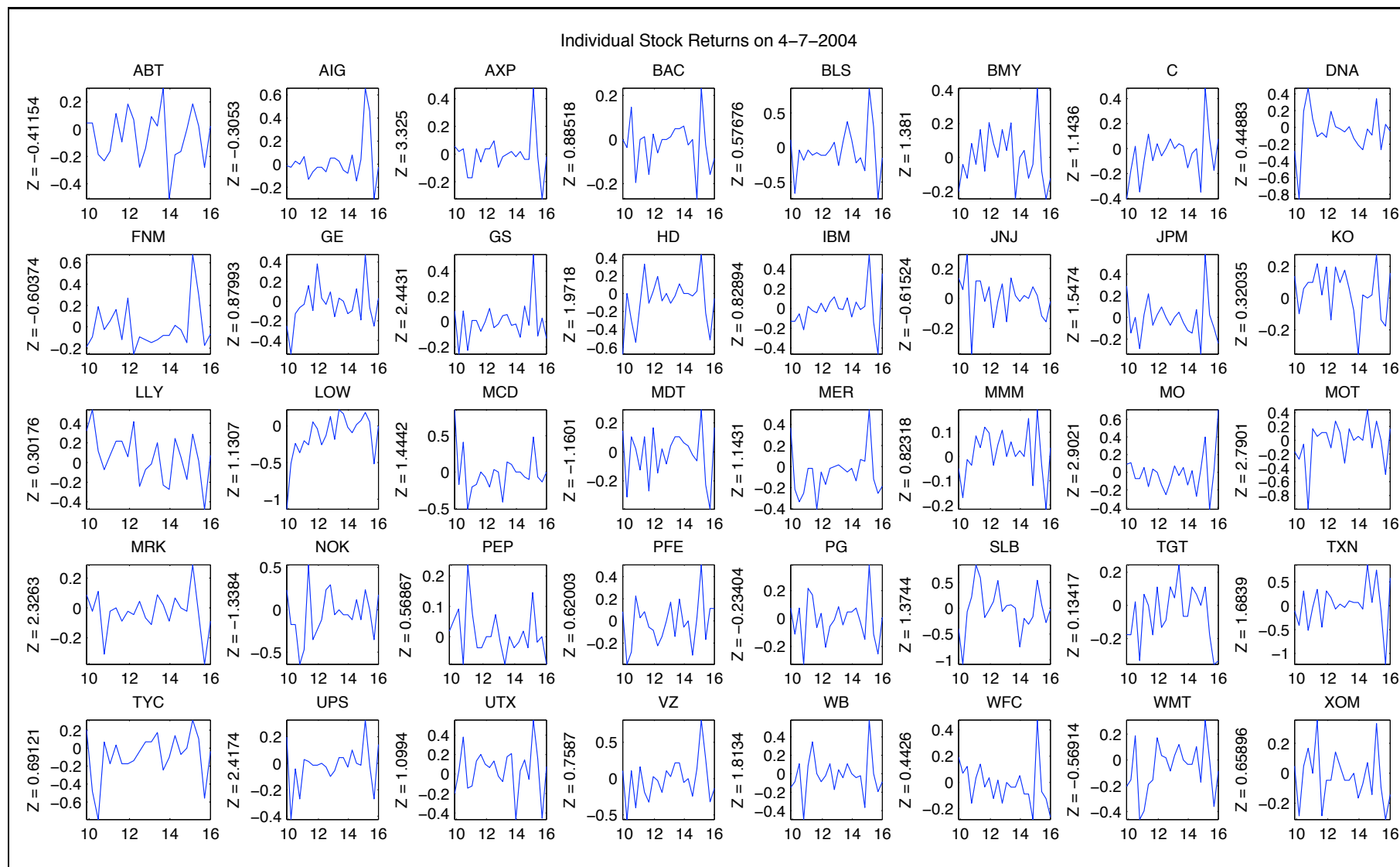


Figure 14: Intraday returns and z -statistics of individual firms on April 7, 2004

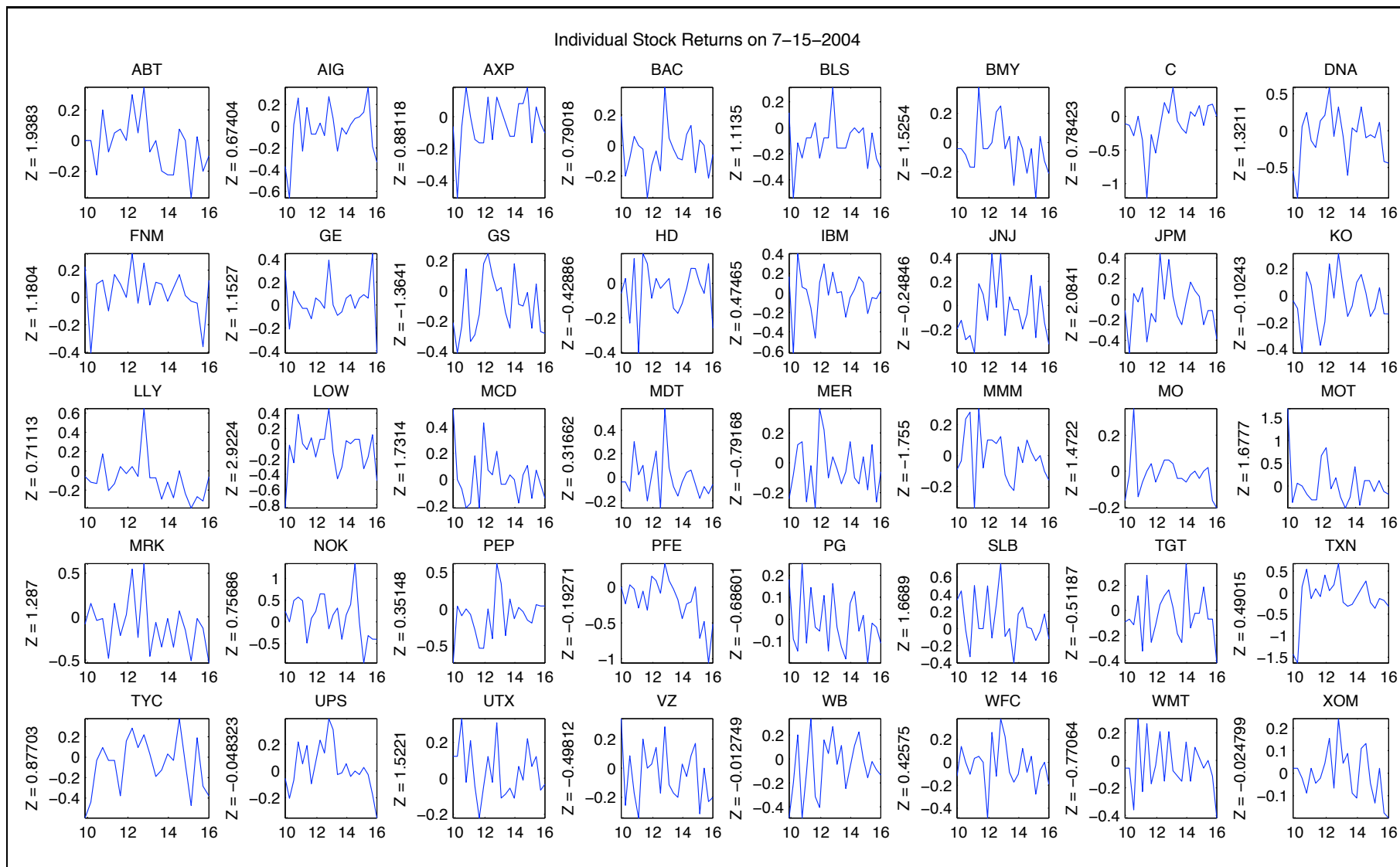


Figure 15: Intraday returns and z -statistics of individual firms on July 15, 2004

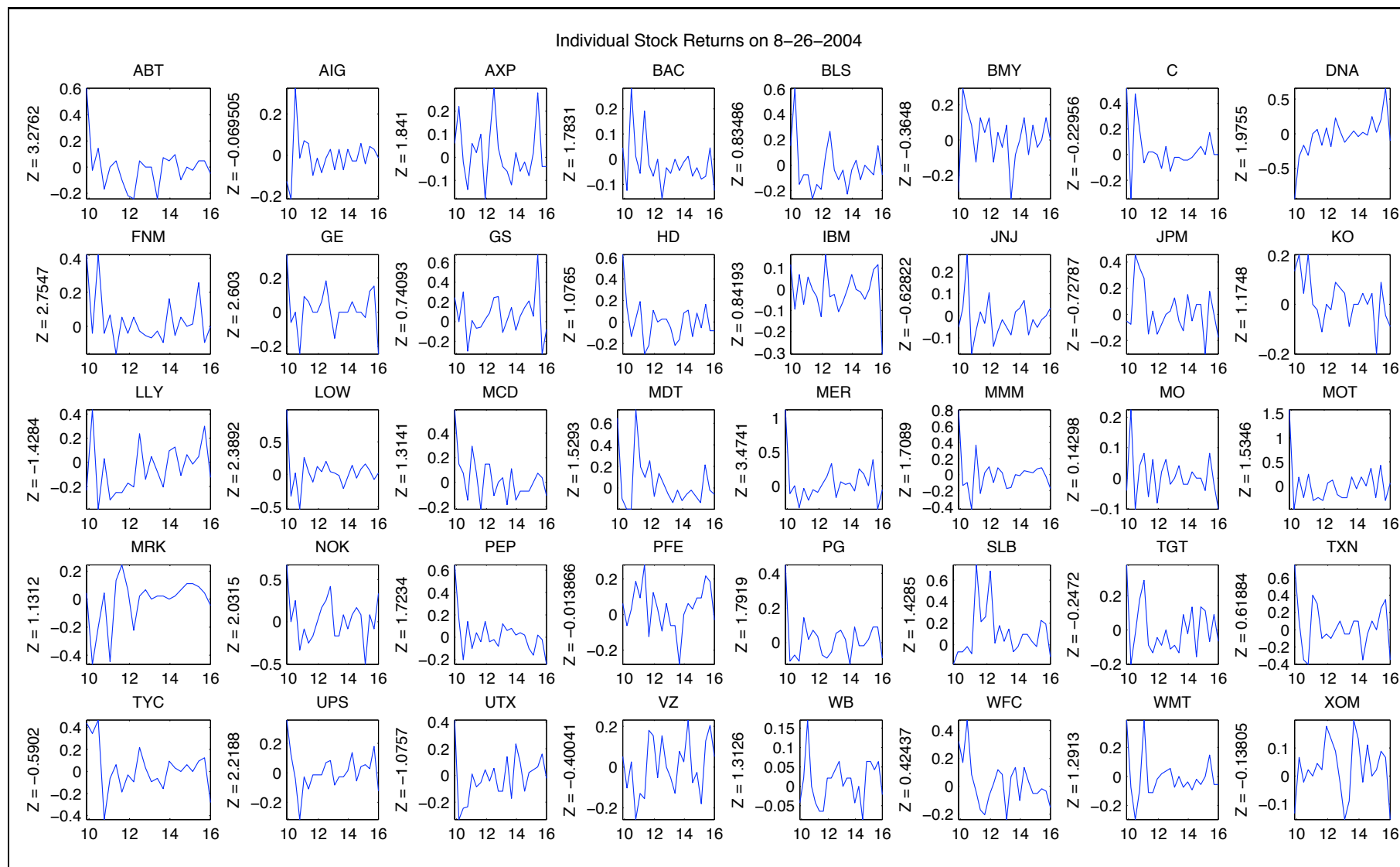


Figure 16: Intraday returns and z -statistics of individual firms on August 26, 2004

index is much lower than the volatility of the individual stocks. Let us now confirm that in a more formal manner.

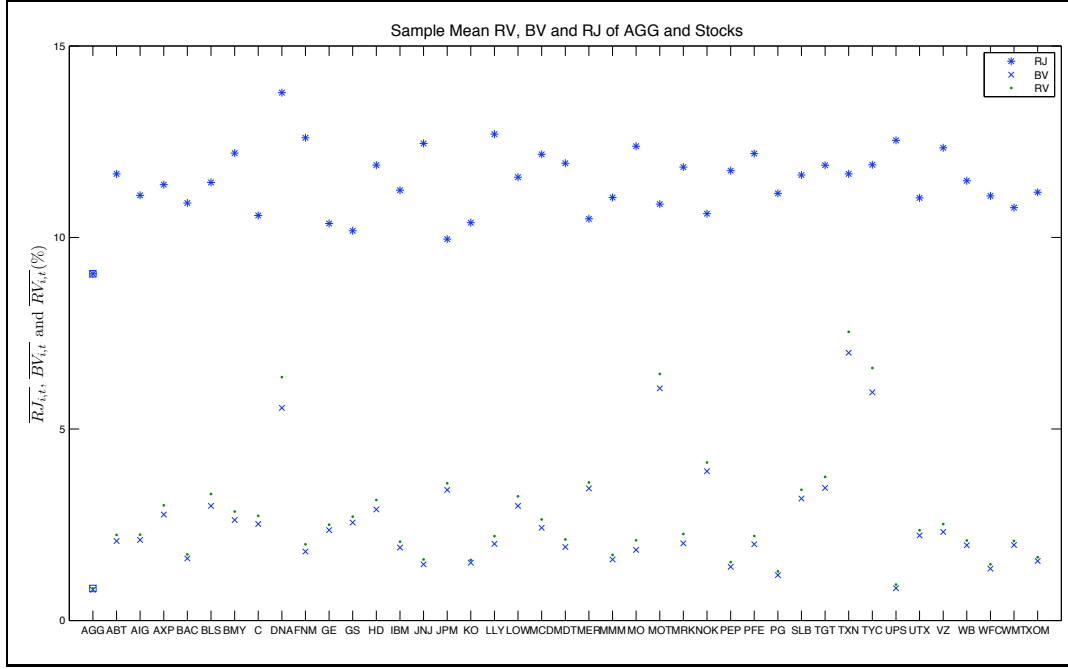


Figure 17: Sample relative jumps, realized variance and bi-power variation of AGG and individual stocks. $RJ|(RV - BV < 0)$ is set to 0.

The top panel of **Figure 17** gives the average sample relative jump of AGG and individual stocks with negative relative jump statistics set to zero. The bottom panel of Figure 17 gives the average realized variance and bi-power variation of AGG and individual stocks. The average statistic is given by the following with RJ_t as an example;

$$\overline{RJ_{t,i}} = \frac{1}{n} \sum_{t=1}^{1241} RJ_{t,i}.$$

From Figure 17 the relative contribution of jumps to the variance of AGG is about 9%. This is lower than that of the individual stocks' which is on average about 11.5%. From the bottom panel of the same figure, the price variance in AGG is also generally lower than that of individual stocks'. Because of this, the moderate sized systematic

jumps which are not detected when the BN-S statistics are applied to individual stocks but get detected when we apply the same statistics to an index.

This gives us our first important conclusion which is that systematic jump get detected only in the index because the index is less volatile than individual stocks.

We obtain our second important conclusion by recalling from Figure 8 that the jump tests flag more jump days in individual stocks than in AGG. Since individual stocks are also more volatile than AGG as evidenced by Figure 17, it would take bigger jumps to trigger the statistics and we deduce from the number of detected jumps that idiosyncratic jumps are in general larger than systematic jumps.

We can easily revisit Section 5 to see that the empirical findings are consistent with the mathematics. By inspection, the denominator of the jump test statistic, given in Eqn. (15), is robust to scaled changes in jump size and in variance. The effects of our 2 conclusions take place in the numerator of (15).

Recall from Eqn. (7) that

$$RJ_t = \frac{RV_t - BV_t}{RV_t}.$$

Increasing variance or σ^2 increases BV_t , but increases RV_t by a smaller amount since RV_t is composed of a diffusive Brownian motion term and a jump term while BV_t is jump robust. Hence, RJ_t is unambiguously smaller with the increase of variance since the numerator is decreased and the denominator is increased. This explains why jumps that are present in the index elude the jump test statistics when we try to search for them in the environment where the diffusive component is larger as in the case of individual stocks.

Increasing the jump size results in an increase in the numerator of RJ_t and an increase in the denominator. However, since the numerator, given by $RV_t - BV_t$ is the pure jump contribution and the denominator is the total variance, the effect of the increase in the numerator is always larger than the effect of the increase. Hence, RJ_t is unambiguously larger with the increase in jump size. This explains how idiosyncratic

jumps are found in individual stocks despite the higher volatility in individual stocks.

This concludes our explanation of the paradox brought up earlier. Systematic jumps are smaller than idiosyncratic jumps and are hidden in the higher volatility of stocks but get detected in an index that is less volatile while idiosyncratic jumps are diversified away.

Having explained why systematic jumps not being detected in individual price series but show up in the less volatile market index, let us now turn our attention to the number of systematic jumps detected.

This analysis detected 7 jump days out of 1241 days evaluated which represents half a percent of days evaluated being flagged as jump days under the 99.9% significance level. Huang and Tauchen (2005) report 1.6% of days as jump days between April 1997 and October 2002 using similar tests. This difference is caused by the decline in volatility post circa 2000 which might be caused by a more active market and decimalization of trading prices. See Figure 12 in Huang and Tauchen (2005) for confirmation.

What are the actual underlying events that caused jumps in these 6 days? Let us look at some selected news items which might give us an idea of the market's activities for the 6 jump days. While this does not adequately reflect the information in the market at the time of the jump, or in the case of August 26th, 2004, overnight news, it does give us a rough idea of the events on those days. We also try to select headlines which we feel would most likely cause a jump. We concede that without knowing the exact time news hits the market, we will not know for sure what the market's reaction to the news was.

4-18-2001:

Wall Street Journal (Eastern Edition), April 19th 2001, Page A1

"Yesterday the Federal Reserve Chairman unveiled the latest move in that complex strategy: a surprise half-percentage point reduction in short-term interest rates." "The Dow Jones Industrial Average ended the day up 399.10 points, or 3.9%, and the Nasdaq Composite Index rallied 156.22, or 8.1%."

12-19-2002:

Wall Street Journal (Eastern Edition), December 20th 2002, Page A1

"OPEC said it wouldn't be able to produce enough oil to meet demand should output cease from both Iraq and Venezuela." "Stocks fell and Treasury bonds jumped on worry about a war with Iraq. The industrials slipped 82.55 points to 8364.80." "Gold prices climbed to their highest level since 1997, boosted by concerns about Iraq, firm oil prices and weak stock markets" "Greenspan said it is "too soon to judge" if the Fed was right to resist puncturing the late-1990s stock bubble with higher rates"

3-11-2004:

Wall Street Journal (Eastern Edition), March 12th 2004, Page A1

"Stocks tumbled after an al Qaeda-linked group claimed responsibility for the Spanish attack. The industrials slid 168.51 to 10128.38, their biggest drop since May."

4-7-2004:

Wall Street Journal (Eastern Edition), April 8th 2004, Page A1

"Import prices rose 0.9% in March from February, their sixth-straight monthly climb, suggesting the risk of deflation is fading" "The EU predicted half the countries using the Euro will break budget-deficit rules this year and cut its growth forecast to 1.7%."

7-15-2004:

Wall Street Journal (Eastern Edition), July 9th 2004, Page A1

"OPEC endorsed an agreement to raise its daily production target by 500,000 barrels, or 2%, in an effort to curb crude-oil prices."

8-26-2004: **Wall Street Journal (Eastern Edition), August 26th 2004, Page A1**

"Global banks are rushing to set up oil-trading desks to profit from higher prices. Spending by producers on finding and extracting oil hasn't risen much". "Oil slid \$1.74 to \$43.47 on rumors reserves could be tapped. Cheney said such a move should be used only if supplies are cut."

From the news items, it seems that one cause of systematic jumps is the release of macroeconomic news. These include news pertaining interest rates, commodity

prices, exchange rates, index composition and foreign policy. Given that fact, why is it that the index only jumped on 6 days over a period of 5 years?

The Bureau of Labor Statistics and the Federal Reserve alone makes more than 6 announcements of these important macroeconomic indicators. There is no way of knowing for certain how efficient the market is at predicting macroeconomic news and how rationally it reacts, but the market is certainly susceptible to disasters that affect the health of the global economy: terrorism, illness and climate problems to name a few. Given the number of these events between 2001 and 2005, more likely than not, there are more than 6 of these deviations over a period of 5 years. Is there a way to detect the rest if they exist?

We have seen from Figures 9-10 that significantly large U -statistics coincide with the largest intra-day returns of AGG on days which were flagged as jump days by the jump test statistics developed by Barndorff-Nielsen and Shephard (2004, 2006a). This suggests that the U -statistics do pick out jumps but do so by measuring how well they co-vary. Would this allow the U -statistics to detect the jump events that we claim to exist?

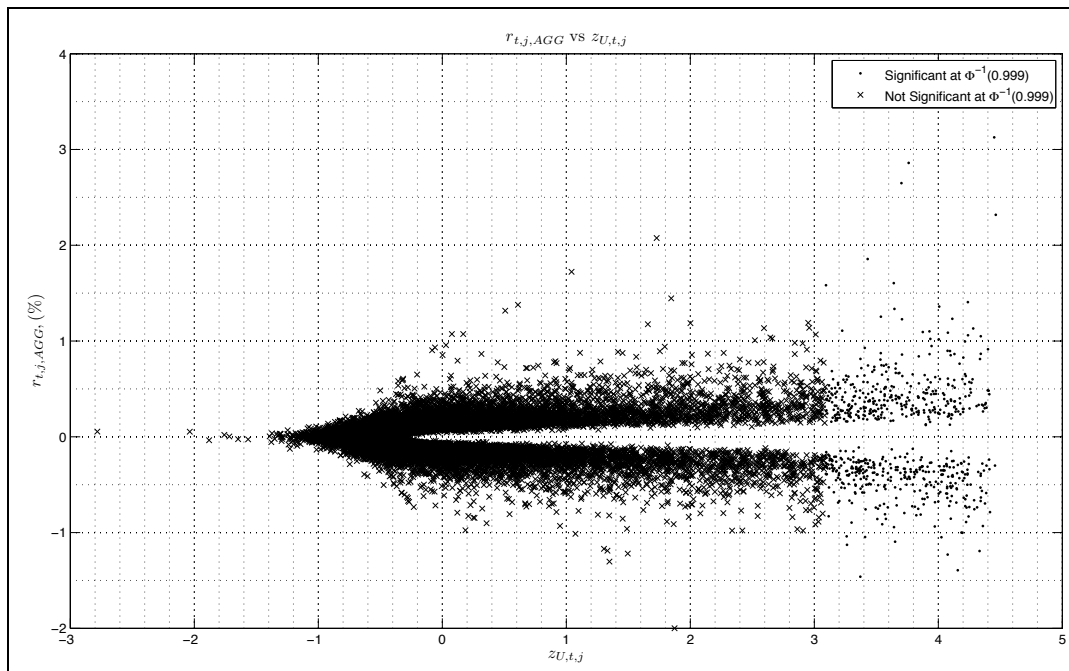


Figure 18: Standardized U -statistics vs AGG's intra-day returns.

Figure 18 scatters the returns of AGG against the pivoted U -statistics, z_U . Values of z_U s larger than the 99.9% significance level are marked with a ‘.’ while the rest are marked with ‘×’. U -Statistics are serially uncorrelated which allows us to standardize them via pivoting about the mean. If the pivoted U -Statistics were normally distributed as anticipated, we would expect that of the 27302 U -Statistics, approximately 27 would be significant at the 99.9% threshold. In this plot, there are 685 U -Statistics which meet that criterion; about 25 times more than expected.

The significant U -statistics that are detected in the 6 flagged AGG jump days make up about 1 percent of the significant U -statistics in Figure 18. How the remaining 99% of the significant U -statistics different from those in Figure 18? At this point, we do not know because variance in the returns of AGG is a combination of Brownian motion and jumps. It impossible without further work to determine how the U -statistic distinguishes between the diffusive Brownian motion and jumps. However, it is still hard to accept that 99% of the significant U -statistics in Figure 18 are caused by Brownian motion that is correlated across 40 stocks especially when the relative jump contribution in stocks is about 11.5% and in the index, about 9% (Figure 17).

We can also see that near zero returns never result in U -statistics that are significant. Furthermore, there are many returns which are large in magnitude but are not accompanied large U -statistics. For instance, the returns where the standardized U -statistics are between 1 and 2 are similar in magnitude to the returns corresponding to significant U -statistics, but themselves do not correspond to significant U -statistics.

At this stage, there is no way of pointing out which U -statistic flag jumps and which do not. We shall leave the matter realizing that there are many more U -statistics than we expect, and if a portion of them do indeed flag jumps that coincide with jump causing events, they would serve as a very useful jump detection tool.

8 Conclusion

In this paper, we elucidated on the relationships between jumps in individual stocks or idiosyncratic jumps and jumps in a well diversified index like AGG which we call systematic jumps. We found that systematic jumps escape detection when the jump test statistics developed by Barndorff-Nielsen and Shephard (2006a) are applied to individual stocks because they are more moderately sized and because individual stocks are more volatile.

Investigation was catalyzed when we wondered why $\text{corr}(z_{TP,rm,t,AGG}, z_{TP,rm,t,i})$ was so low when the stocks in general had unit β s. Unit β stocks track the market closely and hence, we would expect the jump characteristics of these stocks to be similar to that of AGG's to a certain degree. However, this was not the case and forms the paradox we try to explain.

We proceeded to find that stocks on average do not jump. However, the index jumped a total of 6 times over the period of 5 years. Next, we looked at the z -statistics of the individual stocks for the 6 days where AGG was marked as a jump day and found that on each of the days where AGG jumped, only a few individual stocks jumped. In fact, on one of the days, none of the individual stocks jumped at all. This was unusual and unanticipated since we reasoned that stocks need to jump on average to cause an index to jump.

We next showed mathematically that jumps in an index are caused by jumps that are highly covaried with each other or simply jumps that occur at the same time in many stocks. We found in the process that individual jumps are diversified away and do not contribute at all if the index is infinitely large. Since this diversification behavior was so alike that of risk in the CAPM model, we call the non-diversifiable jumps, systematic jumps and the diversifiable ones idiosyncratic jumps.

If idiosyncratic jumps were diversifiable and systematic jumps affect most stocks, then, we expect to find more jumps in individual stocks than in well diversified indexes like AGG. We confirmed that this was true.

We then used U -statistics to show that jumps in AGG were indeed accompanied by returns that were highly covaried in the individual stocks. While this was an interesting discovery, it still did not elucidate on the fact that the jump test statistics flagged jumps in AGG on days where few individual stocks are flagged as jumps. We proceeded by qualitatively looking at the returns of the individual stocks on the 6 days where AGG jumped.

It then became clear that the flagged systematic jumps that was easily located qualitatively in AGG was harder to identify in individual stocks because the individual stocks were more volatile. We then confirmed what we found qualitatively with statistics and empirical data.

We then looked up the headlines of the financial press on the 6 days to discover that the flagged jump days in AGG are usually accompanied by the release of macroeconomic news which affected the entire market. This echoes the findings of Huang and Tauchen (2005) and Andersen et al. (2004). However, we were not able to pinpoint the exact time where the information hits the market and therefore could not determine more surely if the news was the cause of the flagged jumps. Even so, the assumption that such news cause jumps is not stretched.

From there, we proceeded to investigate the possibility of jump detection via measuring the covariance of stock returns. To do so, we made use of the U -statistics again. We found that there were many more significantly large U -statistics than expected. We also noticed that near zero returns in AGG were always accompanied by near zero U -statistics but large returns in AGG were not always accompanied by significant U -statistics.

While this is exciting, we have yet to understand how the U -statistics discern between covaried diffusive returns and covaried jump returns. We also do not know what the joint distribution of individual stock returns is. We are deeply intrigued by this and will pursue this matter in future research.

To summarize our findings and contribution, systematic jumps escape detection from statistics that utilize the magnitude of stock returns and local variance in price

series of individual stocks because individual stocks are relatively more volatile and because systematic jumps are usually smaller than their idiosyncratic counterparts. This is shown empirically, mathematically, and is consistent with the statistics developed by Barndorff-Nielsen and Shephard.

Appendix

Stock Selection

Using Yahoo Finance's Stock Screener, 75 of the largest NYSE traded stocks as measured by their market cap at the beginning of June 2006 are chosen. These 75 stocks are reduced to 50 by selecting the most actively traded ones using the 10-day trading volume. It is very important that the stocks are actively traded for the statistics used to work properly and this screening process ensures this.

Removing Erroneous Time-stamps, Constraining to Regular Days and Trading Hours

Data on completed trades of these stocks are obtained from the Trade and Quote Database (TAQ) which is available via Wharton Research Data Services (WRDS). This includes trades from all the exchanges in North America and over the counter trades. Each exchange has its slight nuances and this might affect the structure of noise and the unknown variables in the data-set. To homogenize these elements in our data-set, we consider only New York Stock Exchange (NYSE) trades for our analysis. NYSE being the most active exchange is used.

We consider only trades from January 1, 2001 to December 31, 2005. Trading frequency increased significantly in the late 90s and by 2001 has reliably become active enough to yield useful data.

Also, by 2001, almost all stocks have converted from fractional to decimalized trading. Illogical data values such as time stamp errors (hour #25, minute #78, month #43 and year #3001 are examples) and negative prices are removed from the data sets. These errors represent a small number of data points compared to the data set.

Another measure to homogenize the mechanisms that generate the trades is to constrain the times where the analysis is extended to. We exclude trades that occur outside of 9.30am and 4pm which constrains our data-set to regular trading hours

only. Trades that occur during abnormal trading days are not considered. Examples of such days are Sept 11, 2001 and holidays when the NYSE is only open for partial day trading. These dates are available on the NYSE web-site.

The price series are sampled every 30 seconds using an adapted version of the previous tick method from Dacorogna, Gencay, Muller, Olsen, and Pictet (2001). Sampling starts at 9.35am, five minutes after the market opens, because the activity at the beginning of a trading day could be atypical while the market re-calibrates itself to overnight news and information. By excluding the first 5 minutes of the day, we ensure the uniformity of trading and information arrival mechanisms over the duration of the analysis.

The previous tick method simply fixes the time where prices are ideally sampled at regular intervals and selects a completed trade prior to the time should a there be no trade at that desired time. For instance, a trade completed at 9:34:58am is used in place for 9:35:00am when there is no actual trade at 9:35:00am. In this case, backtrack is 2 seconds. Backtrack is measured as time difference between the ideal sampling time and actual sampling time. The first trade of the day is used if there are not prior trades to sample. For instance, with 5 minutes sampling interval the first trade of the day which happened at 9:50:00am is used for ticks 9:35:00am, 9:40:00am and 9:45:00am.

We sample every 30 seconds from 9.35am to 4.00pm to obtain 771 price samples per day. The sample runs from January 1st 2001 through December 31st 2005 which gives 1241 normal trading days. We can conveniently reconstruct price series at longer intervals from the 30 second intervals. This is very handy for constructing the signature plots proposed by Andersen et al. (2000)

Cleaning the Data Automatically and Manually

The electronic stock market and the databases where such prices are obtained contain errors. We think of errors as a false record of the true event. But, it is very difficult to determine what the actual event is when the only information we have is price.

It is possible that the price 'error' is in fact an unusually priced trade. Fortunately for us, a large error is difficult to miss and a small error which is easily overlooked doesn't affect our analysis much.

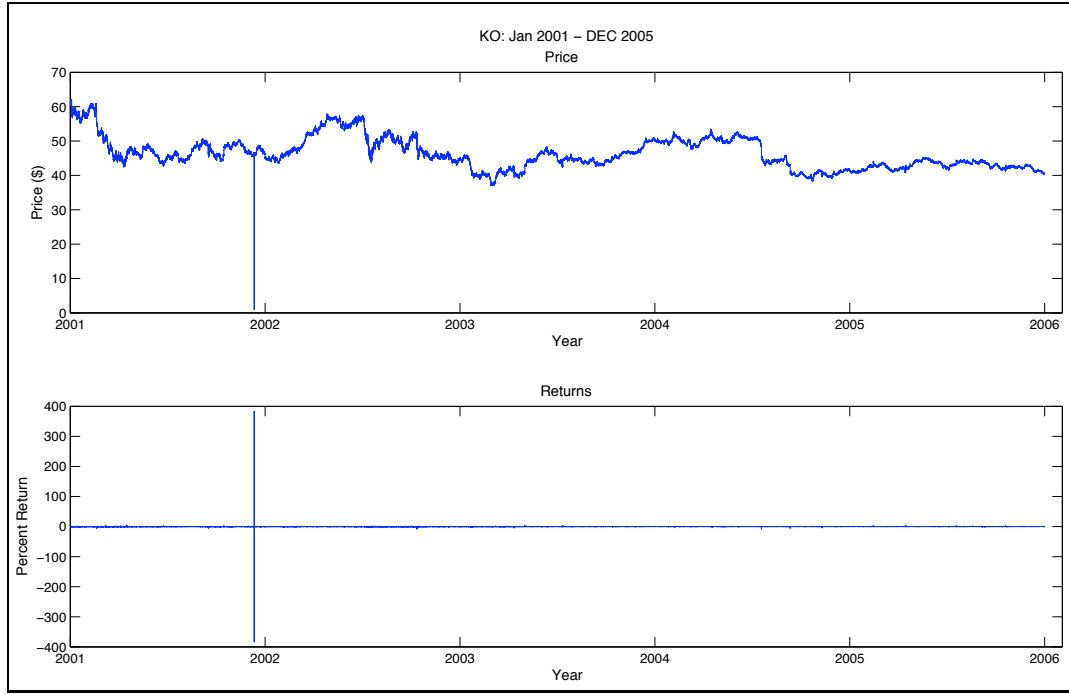


Figure 19: The price and return series of Coca Cola (KO) with an obvious error.

Figures 19 shows a very drastic error in price recording or an erroneous trade which does not reflect the value of Coca-Cola(KO) at all. The extreme price movement (losing almost all its value and regaining it in one trade) and corresponding return which goes negative and positive in large magnitudes is characteristic of such an error. In this example, the other smaller returns are too small compared to the two massive movements to be discerned. The error is probably caused by a human error in entering the trade price of KO. Since the statistics that we are concerned with are jump statistics which are extreme price movements in the applied model, these price errors influence results significantly if they are not dealt with. How do we detect and handle such errors?

In some cases, such as the one in the case of KO, it is obvious that an error is present, but in others, it is not so apparent. For instance, KO losing a couple of

dollars and regaining it in one trade. Is that an error or merely an unusual trade which actually occurred? Where do we draw the line? After trying out different values, a threshold of 1.5% in the following kernel seems to work well to remove the extreme movements at the 30 second sampling interval.

"Returns are set to zero (by changing the erroneous price, not the erroneous return) when a stock price changes by at least 1.5% in opposite directions on two consecutive ticks when sampled every 30 seconds."

This kernel removes most of the errors present in the price series. The remaining price series is then manually inspected and corrections are made if necessary. The manual inspection is necessary since a human picks up problems which the algorithm does not; for instance, two consecutive erroneous prices induced by the previous tick method. It is also convenient to change price and to recalculate the return series from there so that both series are coherent with one another.

We now have the price series at the 30 second sampling interval for 40 stocks for 1241 days each beginning Jan 1st 2001 through Dec 31st 2005. The next step is deciding which sampling frequency to use for the remaining analysis in this study.

Sampling Frequency Selection

The statistics that we use in this study are asymptotic statistics. They become more accurate as the sampling frequency is increased. In our case, we have access to high frequency data which supports the statistics proposed by Barndorff-Nielsen and Shephard (2004, 2006a). However, there is a limit to how finely we can sample as the increase in sampling frequency comes with an increase in market microstructure noise (MMN). MMN is noise introduced by a variety of sources including properties of the trading mechanism, Black (1976) and Amihud and Mendelson (1987) and discreteness of data, Harris (1990, 1991).

Market microstructure noise is unaccounted for in the estimates for realized volatility and can heavily influence results. There are various approaches to this problem and the one used in this study is to lower the sampling frequency to one where the

market microstructure noise no longer affects the data in a significant manner. The method used here is one introduced by Andersen, Bollerslev, Diebold, and Labys (2000) where signature plots of the realized variance are created and the proper sampling frequency is visually chosen. The intuition in choosing the proper sampling frequency is that market microstructure noise overwhelms realized volatility at high frequencies resulting in an increase in the estimate of RV as the sampling frequency is increased. This estimate decreases as we decrease the sampling frequency to a point where the MMN is no longer dominant. The goal is to choose the highest sampling frequency which is relatively free from MMN. While this method is simple to implement, it hinders the use of higher frequencies in sampling.

To implement, we calculate the average sample realized variance at various sampling frequencies and plot them. The sampling frequency to be used is then visually chosen by looking for a sampling frequency which is as high as possible without introducing an exponential increase in RV as microstructure noise sets in.

Figure 20 gives the signature plots of all 40 stocks. The x-axis gives sampling frequency in minutes. The y-axis gives sample realized variance in percent. Some stocks exhibit strange signature plots. The realized variance of BAC, FNM and MO increases as the sampling frequency is increased unlike the other stocks which exhibit the expected exponential decay. We are not entirely sure what its cause or significance is. To our best knowledge, this issue has not been addressed by the literature on the matter and will not be pursued further in this study.

From the signature plots of all 40 stocks, the sampling frequency for this study is pegged at 17.5 minutes which gives 22 returns per day. The final check is to ensure that at the 17.5 minute sampling frequency, the stocks used are indeed liquid enough to support the statistics.

Confirmation

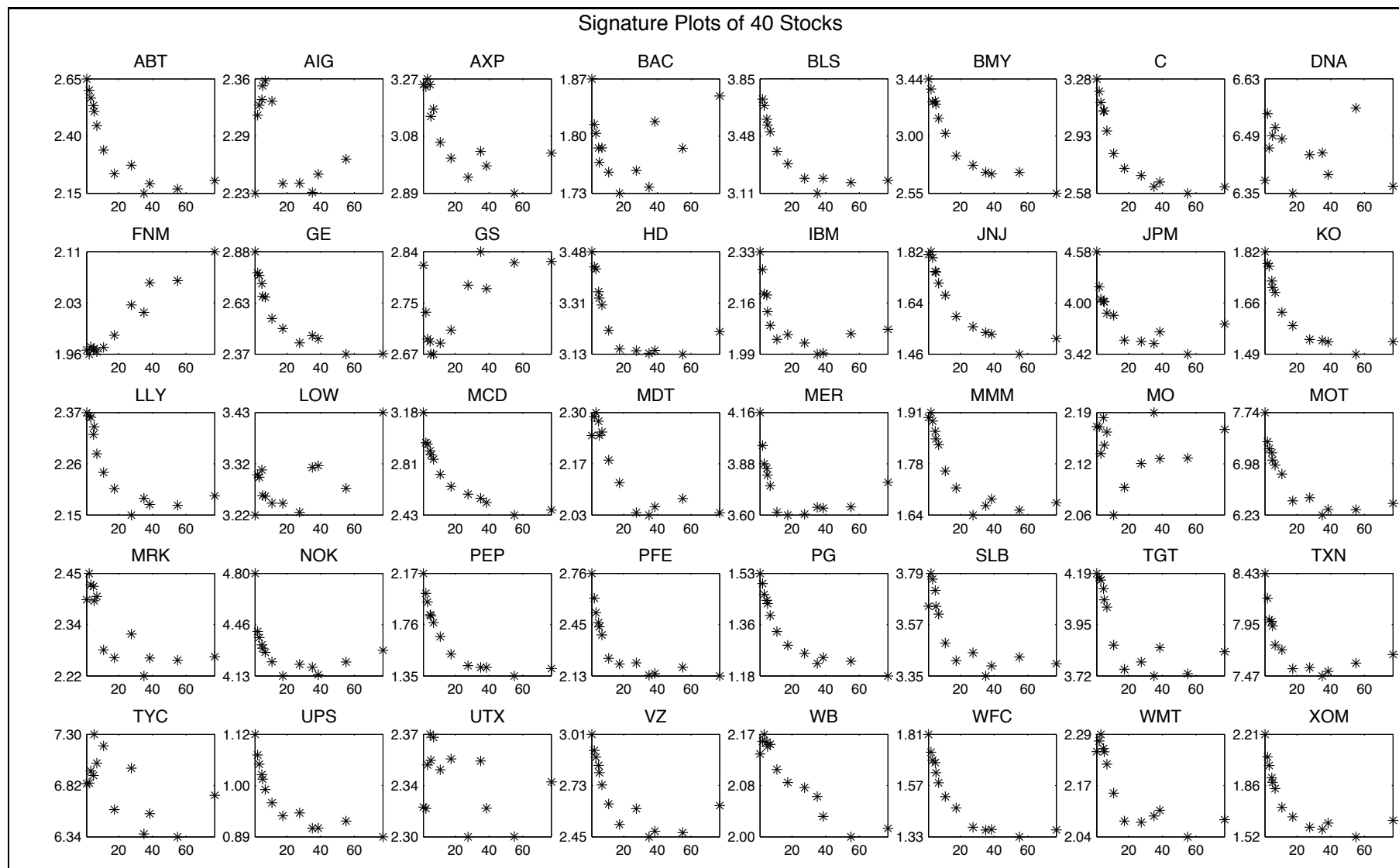


Figure 20: Signature plots of 40 stocks.

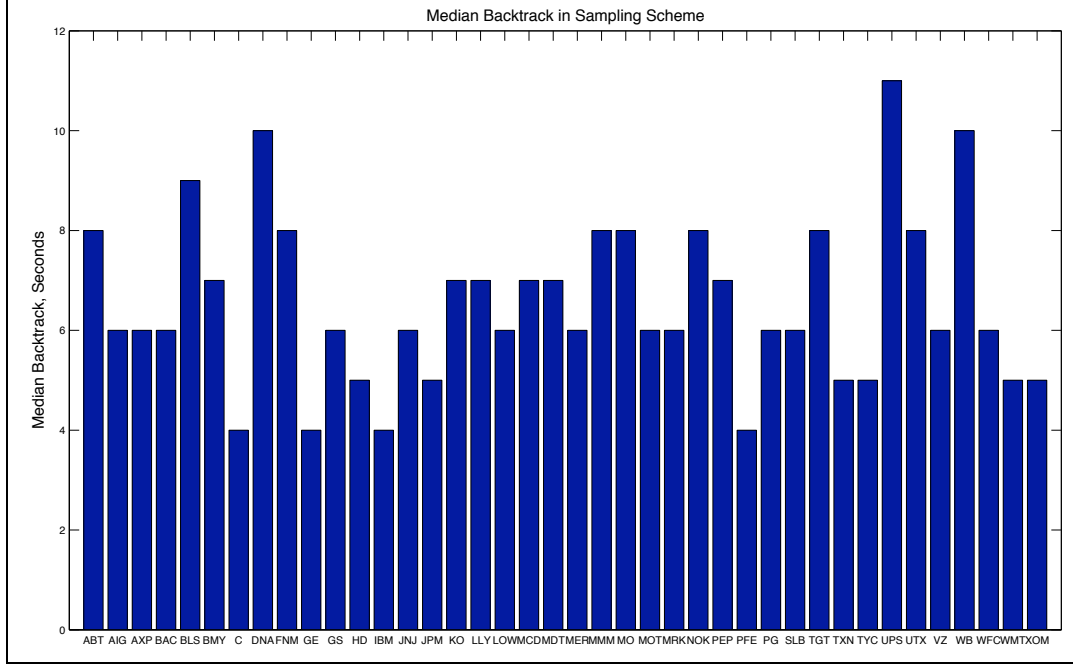


Figure 21: Median backtracking using the previous tick method.

Figure 21 gives the median backtrack from the previous tick method for each of the 40 stocks used. We can see that on average, the median backtrack with a 17.5 minute sampling interval is about 6.5 seconds. This means that on average, prices will adjust to information in 6.5 seconds. This is very fast considering we are sampling once every 17.5 minutes. We assume from here that the arrival of macroeconomic news is reflected in the price change of each individual stock on the same tick which allows us to examine multiple stocks at a given time.

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