Strategic Behavior in Online Auctions: An Analysis of Sniping

Claudia Lai

Professor Daniel Graham, Faculty Advisor

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Abstract

Sniping is a prevalent phenomenon in eBay auctions, which have a fixed end time. Such practice seems apparently inconsistent with standard auction theory – last minute bids are received with reduced probability, and should rationally be submitted earlier – yet previous literature has shown that bidders typically do not engage in early bidding, or if they do, submit low valued bids that they then raise at the last minute. Modeling of the sniping strategy in a game theoretical framework has been limited to games under abstract assumptions not applicable to the conditions under which the eBay bidding game is typically played. This paper intends to carry out analysis within a more general, more applicable framework, and finds that sniping is a collusive strategy that players engage in to prevent price inflation of an auctioned item, and weakly dominates engaging in bidding one's valuation early, so long as late bids are received with sufficiently high probability.

I. Introduction

Online auction sites have swelled in popularity since their inception in 1995. EBay, currently the world's largest online auction site, generated a revenue of $70 million dollars in August 1998 alone (Lucking-Reiley, 2000, 230). There are currently 203 million registered eBay users, and their gross merchandise volume is valued at $12.9billion (eBay, 2006). Online auction sites, in attempting to appeal to both buyers and sellers, typically strive to replicate the desirable efficiency of the classic second-price sealed bid (also Vickrey or textbook) auction (Ibid.,). In this format, players simultaneously submit their bids in sealed envelopes and the highest bidder wins the item for the price equal to the second highest bid. The structure of the Vickrey auction is such that the equilibrium strategy is for each bidder to submit his own valuation; this results in the efficient outcome where the buyer with the highest valuation wins the object. It is hard to achieve this in online auction sites, as the structure of an online auction changes payoffs entirely.

Indeed, a significant amount of literature has been devoted to studying the effect of auction structure on strategic behavior. There has also been literature on the ethics of new tactics and practices to exploit the online auction structure (Marcoux, 2003). What has garnered a significant amount of attention specifically is the way in which payoffs are affected by the structure of online auction closes (Milgrom, 2003; Roth, 2002). Online auction closes can be either 'soft closes', where the auction is automatically extended by a given amount of time after the last bid, and would be extended indefinitely until bidding stops; or 'hard closes', where the auction has a fixed deadline. Literature on auction closes has typically focused on investigating the different types of strategic behavior generated by the different types of auction closes (Barbaro and Bracht, 2006; Asker et. al, 2004; Ockenfels and Roth, 2006).

EBay itself employs a hard close. Ockenfels and Roth (2006) demonstrated that the fixed end time changes the incentive structure of the auction from that of the Vickrey auction. The hard close in particular leads to many Internet auction bidders engaging in 'sniping', the practice of placing last minute bids on auctions: eBay records over 10% of bids received in the last 10 seconds of an auction (Ockenfels, Ariely and Roth, 2003). Why sniping might be optimal is not immediately obvious; sniped bids are not registered with certainty, and since previous knowledge on Vickrey auctions dictates that there is no rationale behind bidding anything other than one's valuation of the item, it is not clear why bidders do not place these true valued bids in the early stage of an auction where such a bid would be received with certainty. Literature that has investigated the sniping phenomenon has attempted establish sniping as an optimal action in response "to the strategic structure of the auction rules in a predictable way", as opposed to "naïve time-dependent bidding" (Ockenfels & Roth, 2006, 315). Most of the work has been empirical: evidence shows that auctions are won with higher probability by sniped bids than multiple incremental early bids (Ockenfels & Roth, 2004). Asker et. al (2004) strengthened this finding in a controlled experiment, finding that experienced bidders are more likely to snipe.

There has been limited theoretical modeling of sniping. Ockenfels and Roth (2006) is the only paper that has introduced equilibria associated with sniping. This paper itself is limited in scope, and identifies a single Bayesian Nash equilibrium[1] that holds

[1] *Bayesian Nash Equilibrium* – a Nash equilibrium in a game of incomplete information, a game in which each player is not fully informed of all relevant parameters in a game, for example each player may know

only for some very specific conditions that are not necessarily immediately applicable within the general eBay environment. In fact, Ockenfels and Roth's (2006) sniping equilibrium holds only under the condition that all players in the auction have the exact same valuation for the item being auctioned, and that furthermore, everyone's valuations are common knowledge. That is to say, they assume all players value the item for the same amount and are aware of such. This abstraction weakens the argument that sniping is optimal in a dynamic eBay environment where competing players' valuations are unknown and all players' valuations fall within a given range or distributions of values. Furthermore Ockenfels and Roth (2006) were unable to identify a (strictly, or weakly) dominant strategy. Therefore, there has not been any previous work that has conclusively identified a last minute bid as a strategic best reply under conditions that mimic a realistic eBay environment.

I intend to fill this gap in the literature by proposing a strategic model adapted from Ockenfels and Roth's (2006) work to conclusively identify a last minute bid as a best reply under more realistic parameters. Furthermore I argue that there exists a weakly dominant strategy in which a player adopts a strategy that engages in both hedges against early incremental bidding by bidding early if and only if competing early bids are made, otherwise engaging only in last minute snipe bidding, and that there exists a Nash equilibrium when both players play this strategy. Lastly I argue that though this strategy is at least as good as, and in some cases better than, the strategy in which players participate in early bidding only, provided that the probability that such late bids are

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their own valuations, but not their opponents'. In Ockenfels and Roth's (2006) Bayesian game the unknown parameters are whether each bid is going to be received or not. However, each player may have expectations of these unknown parameters received is sufficiently high. That is, the benefit from sniping is conferred from the fact that it is a collusive strategy that suppresses incremental bidding and thus the final price of an item. I intend to cover only the parameters for a private-value auction[2], since most of the items on eBay apply to; common value auctions are relevant only for item resale. Thus, my thesis is that the model will show that there exists a Nash equilibrium for sniping that holds for a general game of incomplete information, when players' valuations are not identical and not common knowledge but independent and private.

This paper is organized into six main sections. Section II reviews the relevant literature; Section III gives a brief overview of the eBay bidding system; Section IV introduces a theoretical model. Section V analyzes the model and explains the results. Section VI discusses applications, further studies, and concludes the paper. The appendix provides some definitions of terms used in this paper.

[2] *Independent Private Value Auction* – An auction in which the information each player receives about the auctioned item is simply a valuation for the item independent of every other players' valuation. That is, each player's valuation for the item depends on no one else's valuation. A common value auction is one in which each player's valuation depends on other players' valuations to some degree.

II. Literature Review

Vickrey style auctions imply that there is no incentive for one to submit a maximum bid that is lower than one's true valuation, i.e. submit any bid other than your true valuation, so a lot of work done on sniping has focused on how sniping might be a strategically informed move. This is because snipers usually submit early bids as well as a higher late bid which is not necessarily received, and that furthermore this uncertainty associated with the late bid could be avoided by bidding that higher amount early. The rationale behind the Vickrey case is simple: submitting a bid lower than one's true valuation lowers the chance that the bidder wins the auction, whilst submitting a bid greater than one's true evaluation increases the price for which the bidder wins the item; thus, the best response is to simply submit a bid equal to one's valuation. This status quo method of thinking about auctions lead to much puzzlement over the phenomenon of sniping being labeled as no more than naïve bidding with no strategically rational grounds. Previous literature has proposed hypotheses as to why sniping has been optimal, but has not yet conclusively proven that it is.

I will begin my discussion on previous literature on the theoretical models that have been proposed to model sniping as a rationally informed move, and then talk about the empirical literature that makes up the remainder of the work done on sniping, and this relevance to establishing sniping as a rationally informed move. This overview will motivate the niche that my thesis intends to fill.

Previous literature has considered the structure of the online auction and how the difference between online auction and Vickrey auction structure might affect optimal behavior. Ockenfels and Roth (2006) is the only work to have theoretically modeled the strategy in an independent private value auction. This paper models a two-player eBay auction as a two-stage auction, but considers that both players are fully informed of each other's valuations, and thus that each player may make decisions based on this information. They find three results with respect to independent value auctions.

Their first conclusion is that there are no dominant strategies in the eBay game. Their reasoning is as follows: if *i* chooses to bid early against an opponent *j* who also bids early, this leads to a price rise, furthermore if we assume the opponent is willing to bid up to an amount greater than *i*'s valuation, and *i* knows this, it is clear that *i* is best off not bidding at all in the early stage, such that *j*'s single early bid remains his final bid. On the other hand, if *i* chooses not to bid at all then *j*'s best response would be to submit an early bid rather than submita late bid, since late bids confer a reduced payoff weighted by the reduced probability that such bids are successfully received.

Ockenfels and Roth's (2006) second conclusion is that sniping may be a best reply to incremental bidding, again when bidders' valuations are common knowledge. A sniper has the trade-off between reducing the amount of incremental bidding that takes place and a bid that is received with non-certain probability. However if a sniper is facing an incremental bidder who he knows has a higher valuation than his own, desisting from incremental bidding himself and simply sniping ensures victory with a positive probability. This makes the sniper strictly better off than he would be had he engaged in simple early bidding, since his opponent has a higher valuation than he does and he would inevitably have been outbid.

Their last conclusion is that there exists a Nash equilibrium in which both players snipe when these players have identical valuations, and again that these valuations are common knowledge. Bidding early will eventually drive the price equal to both players' valuations (and not above since bidding above one's valuation entails a negative payoff – a strategically inferior move), which means that winning the item for that price leads to zero payoff. If both players do not bid early but simply snipe, both have a probability of winning the item for a price that is strictly less than their valuations for it, and so a strictly positive payoff.

This overview makes it clear that the findings of Ockenfels and Roth (2006) lie on tenuous assumptions of complete information with regard to each others' valuations, whose applications do not speak broadly about the type of bidding that is occurring on eBay; bidding against competitors of unknown valuations. Nonetheless their findings provide some theoretical support to the data on sniping. Empirical work makes up the remainder, and the bulk of the work done on establishing sniping as a strategically informed move. The change in payoffs resulting from the fixed time close proposed by Ockenfels and Roth (2004, 2006) begged for comparisons between auctions with hard closes and those with soft closes.

Correspondingly, these papers explored evidence from eBay (hard close) and Amazon (soft close) as well as evidence from controlled experiments that eliminate confounding effects from the differences in goods and users that might otherwise account for differences in such behavior. (Barbaro and Bracht, 2006; Asker et. al, 2004; Ockenfels and Roth, 2006) Barbaro and Bracht's study observed that "only 11 percent of the Amazon auctions were extended past the scheduled deadline), while more than half the eBay auctions received bids in the last ten minutes (and over 10% of the eBay auctions received bids in the last ten seconds)", and Asker et. al (2004) reported greater success in hard close auctions when sniping was practiced, as well as increased sniping rates in experienced players, and that less experienced bidders learnt to snipe auctions. All of these results point away from sniping as naïve behavior, and converge to a consensus that sniping is strategic behavior.

These papers provide the groundwork of theoretical models and evidence as to how sniping can be seen within some manner of an equilibrium context, and have used evidence on experienced bidders and successful bids to illustrate the explanatory power of these theories. However, little attention from both theoretical and empirical work has been paid to the specifics that Ockenfels and Roth (2006) overlooked: games of incomplete information and the existence of a general Bayesian Nash equilibrium under these conditions. Given the previous empirical work that has established sniping as an action that yields success with greater probability, as well as the theoretical work that establishes a tenuous equilibrium or very specific circumstances under which a player might snipe, as proposed by Ockenfels and Roth (2006), I argue that the strategic benefit conferred by sniping is great enough to overcome the unknown parameters of a game of incomplete information when competing players' valuations are unknown, and there exists a general Nash equilibrium for the two-person hard close auction. I argue that the benefit from sniping arises from suppressing inflation of the final price, and correspondingly that sniping is at least as good as bidding early, and that there exists a Nash equilibrium of sniping that pareto-dominates bidding early.

III. Bidding on eBay

Bidding on eBay operates through proxy bidding, in which buyers submit a maximum bid ("proxy bid"), for an item being auctioned to eBay. This value is not made public, but provides the proxy with an upper limit such that automated bidding may take place on the buyer's behalf. Furthermore this submitted bid may or may not be equal to the buyer's true valuation of the item, v. Correspondingly, the seller, in submitting an item for auction, also submits a minimum bid, m, at which the bidding starts and for which he is willing to sell the item. Automated bidding will then occur in increments, s, above the highest bid, if not the bidder's own, up till his reservation price. The value of the current bid, $m + ns$, is known to all bidders, n being the number of incremental bids that have been placed. The highest bidder wins the object for a price incrementally higher than the second highest bid, or his reservation price if this amount exceeds it. Sniping on eBay thus takes place when the bidders change their submitted maximum valuation. Since the proxy takes time to react to such a change, sniped bids are successful with a probability, p, less than one.

IV. Theoretical Model

Here I will develop a theoretical model of sniping. Recall that the puzzle associated with sniping is the reduced success associated with placing a last minute bid. Previous literature (Ockenfels and Roth, 2006) has argued that the motivation behind placing a later bid lies in suppressing incremental bidding. Considering this, the model of sniping I develop needs to consider the trade-off between preventing a bidding war and assuring any given bid is successfully received.

Consider a sealed bid, second price, two-stage auction. To mimic early bidding on eBay during the period in which a submitted bid is received with certainty, bids submitted at the first stage will be received with probability $p = 1$. Stage two bids will only be received with probability $0 < p < 1$, to represent last-minute bidding.

I assume that a single object is for sale. Each bidder privately and independently receives some information – a "signal" – about the value of the object. The bidders' valuations are considered private if each bidder's signal is simply her valuation of the object. Otherwise if each bidder's valuation depends on other bidders' signals as well as her own, that the valuations are considered common.

Therefore consider a standard independent private value (IPV) setup in which n bidders independently and privately draw valuations for an auctioned item from some cumulative distribution function $F(x)$. Let v_i denote the valuation of the ith bidder.

First Stage

First stage bidding begins with a starting bid of m and also with a minimum bid increment of s, both of which are exogenously specified. In the first stage the ith bidder submits a sealed bid containing two numbers, a starting price a_i and exit price $b_i > m$, where $b_i > a_i$. These numbers define an interval over which bidder i will be 'active' in the first stage, that is automated bidding for i will occur if the current bid price falls within these two values. That is, bidder i will submit first stage bid of value b_i if the price of the item rises above a_i. This setup is meant to admit behavior that mimics a continuous 'electronic' auction in which a bidder could plan to begin bidding if the price for an item rises above a_i and quit bidding if the price reaches b_i.

Furthermore, let the seller be the 0^{th} bidder and let $(a_0, b_0) = (0, m)$

Since we assume for all other bidders $i \neq 0$, that $b_i > m$, if there exist any bidders $i \neq 0$, that is, any players who is not the seller himself, then the item will be sold for at least price m.

Let $n(\tau) \equiv \{i \mid b_i \geq \tau > a_i\}$ denote the set of bidders who will be 'active' at price τ, that is, the set includes bidders i ($i = 0, 1, ...$) given their bidding window $b_i > a_i$ includes the current bid τ.

Let $b_{(1)}(\tau) \geq b_{(2)}(\tau) \geq ... \geq b_{(|n(\tau)|)}(\tau) \geq \tau$ denote the ordered exit prices of these active bidders

Consider the function:

$$\text{Let } f(\tau) = \begin{cases} \tau & \text{if } |n(\tau)| \leq 1 \\ \min\{b_{(1)}(\tau), b_{(2)}(\tau) + s\} & \text{if } |n(\tau)| \geq 2 \end{cases}$$

That is, this function takes the value of the current bid for the item if there is only one active bidder. So if there is one, or are no active bidders, this function does not change. Otherwise, if there are two or more active bidders, this function takes the value of the minimum of either the highest exit price or an increment over the second highest price.

The closing price for the first stage of the auction is determined as follows. Consider the sequence of prices τ_k, k = 0, 1, … where $\tau_0 = m$ and $\tau_{k+1} = f(\tau_k)$ for $k \geq 1$. That is, the sequence of bid prices within the first stage starts at *m*, the seller's reservation price, and given the number of players willing to bid within this first stage, the price is the minimum of an increment *s* above the second highest bid $b_2(\tau)$ or the highest bidder's reservation price $b_1(\tau)$. This mimics eBay's bidding setup: if there is competition for the item, and there is more than one active bidder, $|n(\tau)| \geq 2$, recall that the highest bidder wins the object for a price incrementally higher than the second highest bid, or his reservation price if this amount exceeds it, hence $\min\{b_{(1)}(\tau), b_{(2)}(\tau) + s\}$. If there are one or less 'active' bidders at a given price the price is not bid up, hence $f(\tau) = \tau$ for $|n(\tau)| \leq 1$.

Furthermore, note that this sequence is weakly increasing since it only takes values from the finite set: $\{b_0, b_1, \ldots, b_n, b_0 + s, b_1 + s, \ldots, b_n + s\}$

Therefore there must be a finite set of steps, *j*, such that $\tau_k = \rho$ for $k \geq j$.

Let the *first stage closing price* of the auction ρ equal this repeating price.

Examples with $m = \frac{1}{4}, s = \frac{1}{8}, n = 2$

- $(a_i, b_i) = (\frac{1}{4}, \frac{3}{4})$ and $(a_j, b_j) = (\frac{3}{8}, \frac{7}{8})$. Since only the seller is active at the opening bid, the closing price is equal to the opening bid and $\rho = \frac{1}{4}$.
- $(a_i, b_i) = (\frac{1}{8}, \frac{5}{8})$ and $(a_j, b_j) = (\frac{3}{8}, \frac{7}{8})$. Since both seller and bidder *i* are active at the opening bid, $\tau_i = f(\frac{1}{4}) = \min\left\{\frac{5}{8}, \frac{1}{4} + \frac{1}{8}\right\} = \frac{3}{8}$. Since only bidder *i* is active at $\tau_1 = \frac{3}{8}, \tau_2 = f(\frac{3}{8})$ and $\rho = \frac{3}{8}$.
- $(a_i, b_i) = (\frac{1}{8}, \frac{5}{8})$ and $(a_j, b_j) = (\frac{5}{16}, \frac{7}{8})$. Since both seller and bidder *i* are active at the opening bid, $\tau_i = f(\frac{1}{4}) = \min\left\{\frac{5}{8}, \frac{1}{4} + \frac{1}{8}\right\} = \frac{3}{8}$. Since bidder *i* and bidder *j* are active at $\tau_1 = \frac{3}{8}, \tau_2 = f(\frac{3}{8}) = \min\left\{\frac{7}{8}, \frac{5}{8} + \frac{1}{8}\right\} = \frac{3}{4}$. Finally, since only bidder *j* is active at $\tau_2 = \frac{3}{4}, f(\frac{3}{4}) = \frac{3}{4}$ and $\rho = \frac{3}{4}$.

Second Stage

In the second stage, the first stage closing price, ρ is public knowledge and bidders are allowed to submit a second-stage bid, c_i, if they wish, subject to the requirement that

$$c_i \geq b_i, \qquad i = 1,2,...,n$$

Each of these second stage bids will independently be received with probability $p < 1$. Similar to Ockenfels and Roth (2006) I assume this probability is determined exogenously.

Bidder *i'*s final bid is then

$$B_i = \begin{cases} c_i & \text{if } c_i \text{ is received} \\ b_i & \text{otherwise} \end{cases}$$

Let $N(\rho) \equiv \{i \mid B_i \geq \rho\}$ denote the set of bidders for which $B_i \geq \rho$, that is, the set of bidders whose bids exceed the first stage closing price, and thus would be willing to bid in the second stage of the auction.

Let $c_{(1)} \geq c_{(2)} \geq ... \geq c_{(|N(\rho)|)} \geq \rho$ denote the ordered values of these final bids.

The second-stage or final price is then determined as:

$$P = \begin{cases} \rho & \text{if } |N(\rho)| \leq 1 \\ \min\{B_{(1)}, B_{(2)} + s\} & \text{if } |N(\rho)| \geq 2 \end{cases}$$

That is, the final price is equal to the first stage closing price, ρ, if there is only one 'active' bidder or less in the second-stage, or if there are two or more 'active' bidders during the second stage, the final price for the auctioned item is the minimum of an increment above the second highest bid and the highest bidder's reservation price, $\min\{B_{(1)}, B_{(2)} + s\}$.

V. Analysis

Assume that $F(x)$ takes the form of the uniform distribution over [0,1].

Suppose that $m = 0$ and $n = 2$.

Recall the concept of a Nash equilibrium. A Nash equilibrium is a set of strategies such that there is no strictly advantageous deviation any single player can make by playing a non-Nash strategy over that dictated by the equilibrium, given all other players play their Nash equilibrium strategies.

My analysis thus considers and compares relevant payoffs of certain strategy sets and identifies advantageous deviations from one strategy set over another, stating best responses in the conclusion of each comparison. Comparisons of payoffs over several cases will elucidate strategic motivation of submitting a last minute bid.

Note that a strategy that does not hedge against early bidding could not possibly be a dominant (weakly or strictly so) strategy. Consider a strategy that specifies inactivity over the first stage but a single sniped bid of value v_i in the second stage. Against a player who plays such a strategy, it is of strict advantage to bid early, since an early bid would be received with certainty. Furthermore an early bid will not face any other first stage competition, and only compete with the single sniped bid with probability $p < 1$, the probability that competing sniped bid is received. It is also clear that the sniper in this case is best of bidding early instead. Since whether or not he snipes he is competing with the same bid, but sniping reduces the probability he gets to compete at all.

This example illustrates the paradox of sniping; the reduced probability of a sniped bid being registered correspondingly decreases the probability of winning the auction. Therefore a strategy that specifies a single last minute bid without a mechanism to hedge against a competitor who bids early is strictly dominated by a strategy that specifies early bids, and no equilibrium story can be told about such a strategy. Let us consider a strategy that can hedge against early bids, and bid early only if competing early bids are submitted, yet otherwise snipe. I claim that such strategy will yield a Nash equilibrium.

Therefore let *snipe* refer to the strategy in which $a_i = m$ and $b_i = c_i = v_i$

This strategy specifies activity in the first stage of the auction if and only if the price of the item rises above the seller's reservation price, that is, the bidder will be inactive in the early stage of the auction unless any other early bids are submitted that raise the price of the item above m. If competing early bids are submitted and the price for the item goes above m, this would trigger the bidder to subsequently submit her entire valuation in the early stage, but otherwise refrain from bidding early until the second stage of the auction. Thus this strategy says that a player who plays this strategy will not bid early unless other early bids are submitted, otherwise she will snipe the auction at the second stage of the auction, submitting a bid equal to the maximum she would be willing to pay, her valuation for the item.

Conjecture (a): There is a non-null range of values of *p* for which it is a best reply to play *snipe* given one's opponent plays *snipe*.

Relevant payoffs:

The expected payoff for *i* to play *snipe* given that *j* is playing *snipe* is

$$E[\prod_i;(snipe,snipe)] = e = p(1-p)(v_i - m - s) + p^2(v_i^2 - \frac{(v_i - s)^2}{2} - sv_i)$$

Since $m = 0$

$$e = p(1-p)(v_i - s) + p^2 \frac{v_i^2 - s^2}{2} \qquad (1)$$

Not winning the auction results in no loss and no gain therefore each player receives a change in payoff if and only if he wins the auction. That is, I assume the players to be risk neutral. Player *i* can win the auction under two circumstances: *i*'s bid is successfully received and his opponent, *j*'s bid is not successful, or both bids are successful, but *i*'s bid is of a higher value.

The first component of the equation: $p(1-p)(v_i - m - s)$ represents the expected payoff to *i* if *i*'s bid is successful and *j*'s bid is not. This is the probability that that *i*'s bid is successful (the *p* component) and the expected value of *j*'s bid not being successful (the $(1-p)$ component), multiplied by the payoff to winning the auction in this manner: *i*'s valuation for the auction itemed, v_i, minus the price he must pay for it: an incrementally higher price of the minimum bid $m + s$; thus $v_i - m - s$.

The second component of the payoff function: $p^2 \frac{v_i^2 - s^2}{2}$ represents the expected payoff if both player's bids are successfully registered, but *i*'s bid is higher than *j*'s. The probability that both bids are successful is therefore the probability *i*'s bid is successful multiplied by the probability that *j*'s bid is also successful, p^2. Furthermore it is necessary to consider that *i*'s bid must be larger than *j*'s bid, since both players' bids are successfully registered in this case. Since we assume that valuations are randomly drawn from a uniform distribution over [0, 1], this probability is simply v_i.

The payoff associated with winning the prize under this circumstance still represents the valuation *i* holds for the item minus the cost of the item. However since there is a competing bid in this case, unlike the previous one in which any competing bid was unsuccessful, the price of the item is going to be bid up. Recall that the eBay bidding system is such that a winner wins the auction for a price incrementally higher, *s*, than the second highest bid or simply his reservation price if this value exceeds it.

The probability that the price of the item is v_i is the probability that *j*'s valuation is less than *i*'s but an increment over *j*'s valuation is larger than *i*'s valuation alone, that is: $v_j < v_i < v_j + s$, which is equivalent to, over the uniform distribution [0,1]: $\Pr(v_i < v_j + s < v_i + s) = \Pr(v_i - s < v_j < v_i) = s$. Likewise, the probability that the price is equal to an increment over the second highest bid is equivalent to $\Pr(v_j + s < v_i) = \Pr(v_j < v_i - s) = v_i - s$. The exact value of the expected payment in this situation, *ex ante*, is unknown to player *i*, since he does not know the value of v_j, however has expectations of the value of the highest bid, given that it is smaller than his own: $E[v_j \mid v_j < v_i - s]$. Over the uniform interval [0,1] this is $\frac{v_i - s}{2}$.

Thus with probability v_i, i wins the auction and gets payoff v_i, hence v_i^2. Given that i wins, with probability s the price she pays is v_i, whilst with probability $v_i - s$ the price she pays is $\frac{v_i - s}{2}$. Simplifying all these components gives the resulting equation (1).

Consider the outcome if i was to defect from the strategy set (*snipe*, *snipe*) and submit $b_i = v_i - s$ in the first stage. Define *bid early* as the strategy in which a player engages only in early incremental bidding, and is willing to bid up to the maximum amount he would be willing to pay, $b_i = v_i - s$, in the first stage, that is, $a_i < 0$, $b_i = v_i - s$ such that the new strategy set would be (*bid early*, *snipe*). In this case, i's expected payoff would be

$$E[\prod_i;(bidearly, snipe)] = d = (v_i^2 - \frac{(v_i - s)^2}{2} - sv_i)$$

$$d = \frac{v_i^2 - s^2}{2} \qquad (2)$$

Recall that the *snipe* strategy specifies a trigger mechanism such that bidding will not occur in the first stage unless competing bids are submitted. Since we are considering the payoffs when *bid early* plays *snipe*, i's early bidding will trigger j to bid early too. The probability any bid made in the first stage is $p = 1$, so the payoff function above simply illustrates the probabilities and the payoff of winning the auction; the valuation of the good minus the cost of the good. Again in this case the expected price of the good is v_i with probability $\Pr(v_i - s < v_j < v_i) = s$. Likewise, the probability that the price is equal to an increment over the second highest bid is equivalent to

$\Pr(v_j + s < v_i) = \Pr(v_j < v_i - s) = v_i - s$. Again this expected price is $E[v_j \mid v_j < v_i - s]$. Over the uniform interval [0,1] this is $\frac{v_i - s}{2}$.

Simplifying all these components gives the resulting equation (2).

Comparison of payoffs:

Recall that equation (1) represents the expected payoff of playing (*snipe*, *snipe*) while equation (2) represents the expected payoff to playing (*bid early*, *snipe*). Thus the advantage to not defecting and remaining with the strategy (*snipe*, *snipe*) instead of changing to (*bid early*, *snipe*) is (1) – (2);

$$e - d = \frac{1-p}{2}(v_i - s)(2p - (1+p)(v_i + s))$$

This expression is positive when $\frac{1-p}{2}(v_i - s)(2p - (1+p)(v_i + s)) > 0$

$$2p > (1+p)(v_i + s)$$

$$p(2 - (v_i + s)) > (v_i + s)$$

$$\text{i.e. } p > \frac{v_i + s}{2 - (v_i + s)}$$

Therefore for some values of p it is advantageous to play *snipe* over *bid early*.

Conclusion (a)

This result illustrates that for some values of v_i and s sniping is a best reply to *snipe*. Namely it says that for an arbitrarily large s or v_i, sniping is no longer a best reply. Note that if $s = 0$, $\frac{v_i}{2 - v_i} = 1$ when $v_i = 1$ and $\frac{v_i}{2 - v_i} = 0$ when $v_i = 0$. Therefore for some values of v_i and s then, the condition requires that $p > 1$; in such cases *snipe* has no advantage over *bid early*. Thus to formulate an equilibrium story with the strategy *snipe* let us assume $s = 0$.

Working on this assumption, essentially what this inequality tells us is that the probability a sniped bid is successfully received, p, must be large relative to v_i in order for *snipe* to be a best reply to *snipe*. Intuitively this speaks to the effect that a player who values an item greatly will be more hesitant in adopting a strategy that specifies withholding from early bidding in the absence of competing bids; players with high valuations risk higher forgone benefits if their bid is unsuccessful, furthermore whilst bidding early against an opponent who plays *snipe* will trigger a bidding war, however a player with a high valuation is more likely to win in incremental bidding. That is, bidders with high valuations will be more willing to *bid early* than *snipe* when their opponent plays *snipe*. Likewise, bidders with low valuations will be more willing to *snipe* than *bid early* when their opponent plays *snipe*; the risk does not involve forgoing a high payoff, and furthermore bidding early against an opponent who plays *snipe* is going to trigger a bidding war, driving up the item price which most likely a player with a low valuation cannot afford.

Nonetheless, our result illustrates that should the condition $p > \frac{v_i}{2 - v_i}$ hold, and assuming $s = 0$, then there is a non-null range of p for which playing *snipe* is a best reply. That is to say, the strategy *snipe* weakly dominates[3] *bid early* for some values of v_i.

[3] *Weak Dominance* – a strategy is weakly dominant if, a player can do no worse by playing this strategy instead of playing any other possible strategies and does better in at least one case that the weakly dominant strategy is played, over its alternatives

<u>Conjecture (b):</u> The strategy *snipe* does at least as well as *bid early* against an opponent who plays *bid early*.

<u>Relevant payoffs</u>

$$E[\prod_i;(bidearly,bidearly)] = (v_i^2 - \frac{(v_i - s)^2}{2} - sv_i)$$

$$= \frac{v_i^2 - s^2}{2} \qquad (2)$$

Note that this payoff is identical to equation (2), the payoff to *i* when he plays *bid early* against *snipe*.

Again, since the probability any bid made in the first stage is $p = 1$, the payoff function above simply illustrates the probabilities and the payoff of winning the auction; the valuation of the good minus the cost of the good. Again in this case the expected price of the good is v_i with probability $\Pr(v_i - s < v_j < v_i) = s$. Likewise, the probability that the price is equal to an increment over the second highest bid is equivalent to $\Pr(v_j + s < v_i) = \Pr(v_j < v_i - s) = v_i - s$. Again this expected price is $E[v_j \mid v_j < v_i - s]$. Over the uniform interval [0,1] this is $\frac{v_i - s}{2}$.

Simplifying all these components gives the resulting equation (2). The reason why the payoffs are identical is immediately obvious; recall that the *snipe* strategy specifies a trigger mechanism such that bidding will not occur in the first stage unless competing bids are submitted. If early bids are submitted, the game becomes identical to one in which both players *bid early*. Thus when *bid early* plays *snipe* the game is equivalent to (*bid early, bid early*).

By the same reasoning: $E[\prod_i;(snipe,bidearly)] = (v_i^2 - \frac{(v_i - s)^2}{2} - sv_i) = \frac{v_i^2 - s^2}{2}$

<u>Conclusion (b):</u>

Thus, playing *snipe* against *bid early* confers no strict disadvantage to the player who *snipes*, since the trigger mechanism specified by *snipe* ensures that an incremental bidder does not take advantage of the fact that sniped bids are received with uncertain probability. However the very nature of the trigger mechanism that turns the *snipe* strategy into one identical to *bid early* should any competing early bids be submitted, is also the very reason why *snipe* can do no better, or worse, than *bid early* when one's opponent plays *bid early*.

<u>Results:</u>

In fact, our results, conditional on $p > \frac{v_i + s}{2 - (v_i + s)}$, $p > \frac{v_j + s}{2 - (v_j + s)}$, and working on the assumption also that $s = 0$, when illustrated in strategic form[4] take the form of the table below. Recall that this assumption was made since the weak dominance of *snipe* over *bid early* falls apart for some large values of v_i when s is positive.

Player *i*'s payoffs are the row entries, whilst *j*'s payoffs are the column entries. The best

[4] *Strategic form* – the standard representation of games. The two rows correspond to the two possible actions of player *i*, the two columns correspond to the two possible actions of player *j*, and the expressions in each box are the players' payoff functions (as derived previously) to the action profile to which the box corresponds. Player *i*'s payoffs correspond to the bottom left expression, which *j*'s corresponds to the top right expression.

response for each player given his opponent's action is highlighted.

	Snipe	*Bid early*
Snipe	$p(1-p)(v_j - s) + p^2 \frac{v_j^2 - s^2}{2}$ (1) $p(1-p)(v_i - s) + p^2 \frac{v_i^2 - s^2}{2}$ (1)	$\frac{v_j^2 - s^2}{2}$ (2) $\frac{v_i^2 - s^2}{2}$ (2)
Bid early	$\frac{v_i^2 - s^2}{2}$ (2) $\frac{v_i^2 - s^2}{2}$ (2)	$\frac{v_j^2 - s^2}{2}$ (2) $\frac{v_i^2 - s^2}{2}$ (2)

The table illustrates that *snipe* is a weakly dominant strategy in this game and the intersection of best responses, (*snipe*, *snipe*) as well as (*bid early, bid early*) are Nash equilibria. Furthermore note that under the conditions that (*snipe*, *snipe*) is a Nash equilibrium: $p > \frac{v_i + s}{2-(v_i + s)}$, $p > \frac{v_j + s}{2-(v_j + s)}$, $s = 0$, the (*snipe, snipe*) equilibrium is pareto-dominant over the (*bid early, bid early*) equilibrium. That is, all players are strictly better off in the *snipe* equilibrium than the *bid early* equilibrium if the probability that sniped bids are received are sufficiently large.

Thus I conclude (*snipe*^{2}, *snipe*^{2}) is a Nash equilibrium that weakly dominates early bidding. Furthermore, pareto-optimality of such equilibrium motivates the notion that sniping is a collusive strategy that bidders choose to play to suppress incremental bidding and the final sale price of the item.

VI. Conclusion

Standard auction theory dictates that that strategy set in which each player bids his own valuation constitutes a Nash equilibrium. For this reason, the phenomena of sniping in hard close auctions was initially a mystery to many game theorists; late bids were not guaranteed to be received with certainty, furthermore there is no apparent advantage in submitting first stage bids not equal to one's valuation. Previous theoretical modeling established sniping as optimal only under extremely specific conditions not applicable to a standard eBay bidding game. This thesis intended to make an original contribution in proving a general Bayesian Nash equilibrium for a two-player, two-stage auction where players' valuations for the auctioned item were firstly unknown to their competitor, and secondly, fell over a range of values.

Analysis of expected payoffs for different action profiles under the condition of incomplete information of each other's valuations yielded several results. What was immediately obvious was that a strategy that did not hedge against an incremental bidder had no equilibrium story to tell; that placing an early bid equal to one's entire valuation strictly dominated placing a bid of the same value in the last minute stage of the auction. Thus when considering a sniping strategy that incorporated a trigger mechanism that would be activated if competing early bids were placed, I found that such strategy does at least as well as a strategy that involved bidding early only against a competitor who also bids early. Furthermore I found that for a range of sufficiently large probabilities sniped bids are received, this strategy does at least as well as, and sometimes better than bidding early against a player who also plays the snipe strategy.

Together the findings of my model are a strong argument for the advantage of sniping as an alternative strategy to early bidding wars. Furthermore my paper has elucidated the parameters under which sniping is optimal; that the probability that a late bid is successfully received must be sufficiently high, and that players with higher valuations would be more inclined to bid early instead, since high valuations are more likely to win in incremental bidding, and more is at stake with a risky bid. Essentially it illustrates that sniping works as a collusive strategy in which all players attempt to suppress the first stage end price as much as possible and take their chances in the second stage to avoid paying a high price for the item.

The limitations of this paper follow those of theoretical papers; that the accuracy of theoretical models is not guaranteed. Theoretical models are simplifications of real-world phenomena that are constructed to provide explanations of the phenomena. The explanatory power of theoretical models is verifiable only through empirical research. A large limitation of my thesis is indeed that its explanatory power is largely unfounded unless we can ascertain the accuracy of the modeling involved. Indeed comparisons of the results attained from these models to real-world data would be the next step in this line of thinking. Specifically the willingness of different players to snipe under different probabilities that such bids would be successful could provide my results with empirical support. Furthermore controlled studies to compare prices between hard close auctions in which sniping is possible and Vickrey style auctions in which sniping is not possible would also provide support for my argument that the weakly dominant snipe equilibrium has a lower expected price of item and hence is pareto-dominant over the bid early equilibrium.

Other limitations of this paper, separate from empirical concerns, include the fact that this paper considers only a two-player auction game, when eBay auctions are *n* player games. Presumably the result should be similar, that sniping confers some benefit over bidding one's entire valuation in the early stage, yet the parameters under which this strategy is optimal will be different. Extending the method of analysis carried out in this paper for an *n* player auction would be a beneficial future avenue of research. Other interesting extensions might include more realistic distributions of valuations, for example a normal distribution of valuations. My analysis considered a uniform distribution; although I expect similar results for a normal distribution, sniping parameters under a normal distribution might provide more insightful results about sniping.

Lastly, recall that this paper argues that sniping is a strategy played to reduce the final sale price for auctioned items. Sniping only exploits benefit in a hard close auction, since soft close auctions are automatically extended if bids are submitted within the closing 10 minutes of an auction. Considering this, it might be expected that if two identical items were to be auctioned on a hard close online auction (eBay) and a soft close online auction (Amazon), the item on eBay would be expected to sell for less due to snipe bids on eBay suppressing final sale prices. Since eBay profits through a cut of eBay sellers' revenue it would be a profit maximizing strategy to change their auction close to a soft close format to prevent suppressed sale prices.

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Appendix

i. Terms

- *Nash Equilibrium* – A set of strategies such that there is no strictly advantageous deviation any single player can make by playing a non-Nash strategy over that dictated by the equilibrium, given all other players play their Nash equilibrium strategies.
- *Game of incomplete information* – each player is not fully informed of all relevant parameters in a game, for example each player may know their own valuations, but not their opponents'. However each player may have expectations of these unknown parameters
- *Bayesian Nash Equilibrium* – a Nash equilibrium in a game of incomplete information
- *Weak Dominance* – a strategy is weakly dominant if, a player can do no worse by playing this strategy instead of playing any other possible strategies and does better in at least one case that the weakly dominant strategy is played, over its alternatives
- *Private Value Auction* – An auction in which the information each player receives about the auctioned item is simply a valuation for the item independent of every other players' valuation. That is, each player's valuation for the item depends on no one else's valuation.