A Comparison of the HHI and the Procurement-Based Framework in Merger Review*

Kenny Gong†

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Abstract

The Herfindahl-Hirschman Index (HHI), a measure of market concentration, plays a critical role in the U.S. *Merger Guidelines*. It is used as a threshold metric that marks certain mergers as potentially harmful to consumers. However, the microfoundations for the HHI are grounded in the Cournot oligopoly model, which may not be an appropriate foundation for certain markets, particularly those in which buyers purchase through competitive procurements. Recent developments in Incomplete Information Industrial Organization (IIIO) allow merger analysis to be tailored to such procurement-based markets. While IIIO methods allow one to calculate the probability of an increase in price (PIP) as a result of a horizontal merger, until now no work has been done to compare the HHI approach to merger review with the IIIO approach. In this paper, we find that the IIIO approach is largely consistent with the 2023 *Merger Guidelines* in that we agree that both the post-merger HHI and the change in HHI should be used in merger review, however our results place greater emphasis on the change in HHI in terms of predictive power of the PIP.

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†Duke University, kenny.gong2012@gmail.com.

1 Introduction

On December 18, 2023, the Department of Justice (DOJ) and the Federal Trade Commission (FTC) released the 2023 *Merger Guidelines*. In this version and in prior versions of the guidelines, particularly the 2010 *Horizontal Merger Guidelines*, a key metric used to measure the concentration of a market is the Herfindahl-Hirschman Index (HHI). The DOJ and FTC use the HHI, which is grounded in the Cournot model of oligopoly, to identify mergers that are likely to have anticompetitive effects. In particular, when considering a merger under the 2010 *Horizontal Merger Guidelines*, if the post-merger HHI is greater than 2500 or the change in HHI is greater than 200, then the *Guidelines* identify the merger as presumptively anticompetitive and so likely to be subject to review by the competition authorities. The 2023 revision of the guidelines lowers the standards to 1800 for the post-merger HHI and to 100 for the change in HHI. At the time of writing this paper, it is unclear whether or not the courts will accept and employ these changes into their review. However, the changes indicate that the DOJ and FTC are proposing to tighten control over mergers because decreasing the thresholds will automatically identify more mergers as problematic.

Economists have raised concerns regarding the applicability and usage of HHI-based metrics. Nocke and Whinston (2022) suggest that 1) only the change in HHI is relevant to screening mergers for harmful unilateral effects, and 2) the maximum thresholds established in the 2010 *Horizontal Merger Guidelines* are too lax. While they acknowledge and discuss certain situations that might warrant the use of HHI levels as thresholds, such as when considering coordination or non-price competitor responses, Nocke and Whinston conclude that thresholds for both metrics allow problematic mergers to pass without opposition. Indeed, depending on certain parameters, using the HHI to screen mergers could potentially either allow problematic mergers to occur without review, or block unproblematic mergers. Loertscher and Marx (2019, 2021, 2022a) propose a procurement-based approach to merger review incorporating Incomplete Information Industrial Organization (IIIO) principles. As opposed to traditional oligopoly models, the IIIO framework incorporates bargaining power into merger review, where buyers and sellers negotiate prices, often in the form of a competitive procurement. In the basic many-to-one IIIO setup with symmetric bargaining power, multiple upstream firms sell to a singular downstream firm. The downstream firm holds a procurement, which in the case of efficient contracting (the focus of this paper) can be modeled as a second-price sealed bid procurement.[1] The upstream firm that offers the lowest price wins the procurement and is paid the amount of the second-lowest bid.

[1] Alternatively, if the downstream firm has market power, then contracting can be modeled as an optimal auction (see, e.g., Loertscher and Marx, 2019).

Using this setup, we can derive parameters that allow for useful statements about the market and merger of upstream firms, including the price increase probability (PIP) associated with a merger. The PIP is the probability that the second-lowest cost post-merger is greater than the second-lowest cost pre-merger. Loertscher and Marx (2022b) suggest that a PIP greater than 5% raises concerns about a merger, and can be used analogously to the HHI thresholds. Under certain market conditions, the IIIO approach could be more indicative of problematic mergers than the HHI approach. For example, business-to-business transactions typically resemble IIIO-based procurement markets as opposed to a traditional Cournot oligopoly, in which quantities are determined at the Nash equilibrium of a quantity-setting game among the sellers and the price is the market clearing price under the assumption of a continuum of price-taking consumers. This suggests that, for firms that primarily engage in business-to-business transactions, the procurement-based approach could be more accurate in identifying problematic mergers. In this paper, we compare the HHI approach and the procurement-based approach across various markets, and determine how the procurement-based approach relates to the HHI approach and discuss implications for the use of both methods.

In this paper, we show that, consistent with Nocke and Whinston (2022), the change in HHI is the more powerful predictor of harm compared to the post-merger HHI; however, in contrast to their model, the procurement-based framework suggests that the post-merger HHI also adds some predictive power. We reach this conclusion by examining the relation between the PIP and the post-merger HHI and change in HHI, determining that both HHI metrics are correlated with the PIP, but the effect of the change in HHI is both greater in magnitude and more statistically significant than the post-merger HHI. In addition, we show that the operating margin adds no significant additional information regarding either the PIP or the HHI metrics. This solidifies the operating margin's role in the IIIO approach as purely calibrational. Finally, our results suggest that the addition of the merged firm's market share as a metric in the 2023 *Merger Guidelines* does add not significant additional information compared to the post-merger HHI. Our analysis shows that the effect of the merged firm's market share on the PIP is lesser in magnitude and less statistically significant than the post-merger HHI. Additionally, since both indicators require information on every firm's share in the market, there are no instances in which one indicator may be easier to calculate than the other, which undermines the incremental value of the merged firm's market share in the 2023 *Merger Guidelines*.

Our results lead to a number of policy implications. First, both the change in HHI and the post-merger HHI should continue to be used but with different thresholds and with an emphasis on the change in HHI. The IIIO approach suggest a post-merger HHI of 2600, similar to 2500 outlined in the 2010 *Horizontal Merger Guidelines*, and a change in HHI of 300, which is 100% more than the threshold outlined in the 2010 *Horizontal Merger Guidelines* and 200% more than that in the 2023 *Merger Guidelines*. Second, rather than using both indicators outlined in the 2023 *Merger Guidelines*, it is sufficient to use just the post-merger HHI. The merged firm's market share adds no additional predictive power compared to the post-merger HHI from an IIIO perspective, nor is it ever easier to calculate as both indicators require the same information. Third, the IIIO approach is largely consistent with the merger simulation approach used in Nocke and Whinston (2022). The similar conclusions from both the IIIO approach and the Nocke and Whinston (2022) approach support the use of the PIP in merger review, which has the additional advantage, relative to the merger simulations on which Nocke and Whinston (2022) is based, of ease of calculation, requiring only one additional piece of information compared to the HHI approach, that being the operating margin of one firm.

The remainder of this paper is organized as follows. In Section 2, we examine past literature, focusing particularly on the conceptual foundations of the HHI and its derivation from the Cournot model as well as the conceptual foundations of the IIIO review metrics, and compare the characteristics of markets that better match each model's assumptions. In Section 3, we outline the simulated markets empirical framework used to compare the two models. In Section 4, we discuss the results, and suggest whether the IIIO review metrics support, undermine, or neither support nor undermine the HHI approach. In Section 5, we conclude the paper.

2 Literature review

Since its first inclusion in the 1982 *Horizontal Merger Guidelines*, the HHI has been used for merger review as a screening metric. The Assistant Attorney General at the time, William Baxter, claimed that the HHI's predecessor, the four-firm concentration ratio (CR4), has serious drawbacks, particularly its failure to account for differences in concentration between the largest four firms, as well as that of firms not among the four largest (Calkins, 1983). In the 2010 *Horizontal Merger Guidelines*, the agencies assert that market concentration is a useful indication for certain competitive effects of a merger. The HHI is calculated as the sum of the squared market shares of the firms in an a market:

$$HHI = s_1^2 + s_2^2 + \cdots + s_n^2,$$

where $s_1, s_2, \ldots, s_n$ are the market shares of firms 1 through n in a market. In a monopoly setting, the HHI would consequently be 100^2, or $10,000$. In a perfect competition setting, the HHI would be 0. In the 2010 *Horizontal Merger Guidelines*, the FTC and DOJ use the HHI to define markets as unconcentrated (HHI below 1500), moderately concentrated (between 1500 and 2500), and highly concentrated (above 2500). The FTC and DOJ use these definitions to establish "zones" of concern as shown in Figure 1. It is important to note that the HHI is not used as a definitive screen and is not the absolute authority in merger review. The HHI is used in tandem with a variety of other empirical and theoretical analyses. Should a market be deemed moderately or highly concentrated, then the agencies will adjust the rigor of their subsequent analyses accordingly.

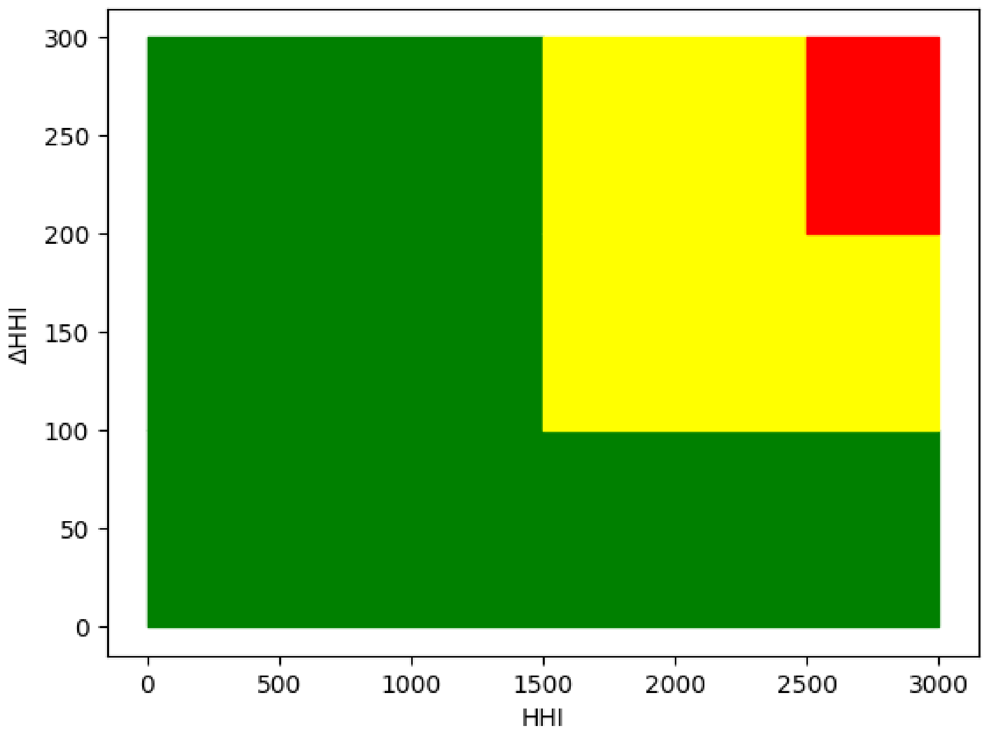


Figure 1: Screening thresholds from the 2010 *Horizontal Merger Guidelines*. Green indicates mergers that are "unlikely to have adverse competitive effects," yellow indicates mergers that "potentially raise significant competitive concerns and often warrant scrutiny," and red indicates mergers that "will be presumed to be likely to enhance market power."

The 2023 *Merger Guidelines* provide two metrics, the "Post-Merger HHI" and the "Merged Firm's Market Share." The "Post-Merger HHI" indicator defines problematic mergers as those with a post-merger HHI greater than 1800 and a change in HHI greater than 100, as shown in Figure 2(a). The "Merged Firm's Market Share" indicator defines problematic mergers as those where the merged firm's market share is greater than 30% and a change in HHI greater than 100, as shown in Figure 2(b). The FTC and DOJ can use either or both these indicators to argue that a merger is likely anticompetitive.

(a) Post-Merger HHI

(b) Merged Firm's Market Share

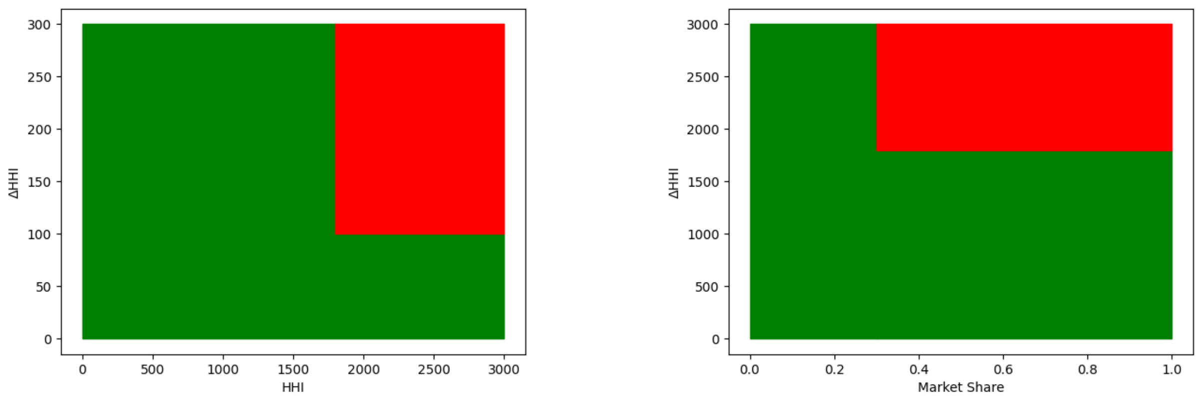


Figure 2: Screening thresholds from the 2023 *Merger Guidelines*. The red indicates mergers that are "presumed to substantially lessen competition or tend to create a monopoly." The green indicates mergers that do not have that presumption.

In comparison to its predecessor, the HHI has several advantages which provided the incentive for its initial introduction in the 1982 *Horizontal Merger Guidelines*. As mentioned before, the HHI captures the shares of all the competitors in a market, while the CR4 only accounts for the top four competitors. In addition, the HHI is responsive to asymmetry in markets. Because the HHI is the sum of the squared market shares, and due to the nature of squared numbers, less symmetric markets with a few large competitors lead to higher HHIs compared to more symmetric markets that have similarly sized competitors. Finally, the HHI is both easy to understand and easy to calculate. Its advantages have led the DOJ and FTC to incorporate the HHI as a key metric for merger review.

2.1 Incomplete information industrial organization

Despite these advantages, in recent years, scholars have begun to challenge the credibility of the HHI usage in merger review. For example, Nocke and Whinston (2022) argue both theoretically and empirically that only the change in the HHI should be used for screening mergers, and the current standards outlined in the 2010 *Horizontal Merger Guidelines* are too lax with respect to the level of synergies required for the merger to remain consumer surplus neutral. In particular, Nocke and Whinston use estimates from Miller and Weinberg

(2017) of the demand systems and marginal costs for major beer brands to simulate every possible merger in Miller and Weinberg's 39 local beer markets. Nocke and Whinston find that the majority of mergers with a post-merger HHI of less than 1500, which is deemed likely unproblematic by the DOJ and FTC, actually led to consumer harm. On the contrary, assuming a three percent synergy gain (or cost reduction) post-merger, any merger with a change in HHI greater than 200 is expected to cause consumer harm regardless of the post-merger HHI. They conclude by suggesting that the 2010 *Horizontal Merger Guidelines* "safe" or unproblematic levels of HHI are too lenient and are letting problematic mergers pass without opposition.

Other economists have similarly used empirical methods in real-world markets to challenge assumptions of the Cournot model and the use of the HHI. More specifically, certain markets with particular behavior are not necessarily captured in the assumptions of the Cournot model. For example, Backus et al. (2020) use data found from over 80 million eBay transactions to showcase that bargaining often leads to a no-deal situation where the two parties are unable to come to agreeable terms for the transaction, known as bargaining breakdown. In addition, Backus et al. found that both buyer and seller behavior change happens gradually over the course of the deal which is inconsistent with many bargaining models. Larsen and Zhang (2022) use a dataset on business-to-business wholesale used-car auctions to similarly display the importance of bargaining power and incomplete information, or the negotiating party's lack of knowledge on other parties' willingness to buy or sell. Larsen and Zhang find that car dealers, the buyers in this transaction, have overall the same bargaining power as manufacturers and other sellers. However, manufacturers, compared to other sellers, have substantially more bargaining power than both other sellers as well as buyers. Similarly, Larsen et al. (2021) examine the effects on auctions of an additional parameter, an intermediary. Larsen and company use the same dataset on business-to-business wholesale used-car auctions and determined that high-performing intermediaries are 22.03% more likely to close a deal compared to low-performing intermediaries. These studies showcase the impact of parameters that are not necessarily captured in the Cournot model.

A recent development in economic literature known as Incomplete Information Industrial Organization (IIIO) has sought to fill these gaps, providing alternatives to the Cournot that account for these assumptions. Loertscher and Marx (2019) use a procurement-based setup in which a buyer designs a competitive procurement in which suppliers compete to sell to the buyer. This approach captures combinations of negotiations and proposals common in procurement settings, and ultimately Loertscher and Marx determine that regardless of buyer power, or the ability for buyers and sellers to negotiate greater take-home shares, a merger between suppliers is harmful to the buyer. Kang and Muir (2022) examine an additional complexity, using the eCommerce site Amazon as their primary example. Kang and Muir study the effect of antitrust policies on a platform that uses the platform to sell to downstream markets and hosts other producers. They use a general mechanism design framework where a platform sells a product to upstream producers, and then competes with the same producers in a downstream market. Kang and Muir find that policies banning a monopolist platform will only harm consumers as platforms that produce positive output in equilibrium always reduce downstream prices. As the platform operates in both upstream activities, its role as a marketplace for other suppliers, and downstream activities, competing with these very same suppliers, a reduction in the platform's production cost results in a substitution towards the platform, ultimately leading to a greater level of production in the downstream market. In the case where the platform is not a monopolist, the other producers then have access to other downstream suppliers or platforms on which to sell, and the platform is incentivized to undermine its supplier competitors. Once again, this framework considers situations that the Cournot model fails to account for. Choné et al. (2021) similarly showcase the importance of bargaining power in vertical mergers. They use a procurement-based framework inspired by Loerstcher and Marx and find that the effects of consumer welfare are heavily dependent on an efficiency gain known as the elimination of double marginalization, where multiple suppliers within different vertical levels of the supply chain markup their prices.

No work has been done to compare the incomplete information approach with the HHI-based approach, so it is unclear whether it provides a more conservative, less conservative, or just a different approach to screening mergers. The rest of the paper makes this comparison, in particular using the PIP as the IIIO counterpart to the HHI. At a high level, the approach of this paper is analogous to that of Nocke and Whinston (2022). They contrast the estimated price increases with the HHI for a large number of hypothetical mergers generated using a merger simulation model applied to the U.S. market for beer. Their estimated price increases are consistent with using a screen based on a change in the HHI of 200, but inconsistent with using a post-merger HHI of 2500 (or other numbers) as a screen. While these results provide evidence against using the post-merger HHI as a screening metric, Nocke and Whinston provide some possible justifications for using the post-merger HHI. For example, their paper examines its usage with respect to unilateral price effects, or the price effects of the activities of the merged entity alone. The post-merger HHI may be used as an indicator for coordinated price effects.

2.2 Conceptual foundations of the HHI

The conceptual foundations of the HHI as a tool for merger review originate from French economist and mathematician Antoine Augustin Cournot. In *The Mathematical Principles of the Theory of Wealth* (Cournot, 1838), Cournot provides the framework for Cournot Competition. Essential to his model is the assumption that $P = f(Q)$, or that the price of a good is a function of the total quantity of that good produced in a market. We can showcase his model using a simple duopoly example, where two firms produce quantities q_1 and q_2 respectively and $Q = q_1 + q_2$. Our example market will have a simple downward-sloping demand curve, $P = a - bQ$, where a and b are positive constants. Thus, the revenue and profit respectively for firm 1 is as follows:

$$Pq_1 = a - b(q_1 + q_2)q_1,$$

$$\Pi_1 = Pq_1 - cq_1 = (a - c - b(q_1 + q_2))q_1,$$

where c is the marginal cost for firm 1. For the sake of simplicity, we assume that marginal cost is a constant, however, we can express marginal cost as a function of quantity produced. The same can be calculated for firm 2. Firms are assumed to maximize their profit with respect to quantity produced, so the condition for profit maximization for firm 1 can be expressed as follows:

$$\frac{\mathrm{d}\Pi_1}{\mathrm{d}q_1} = a - c - 2bq_1 - bq_2 = 0.$$

The same can be calculated for firm 2. Ultimately, we arrive at two equations for q_1^* and q_2^* such that firm 1 and firm 2 simultaneously maximize their individual profits given the quantity choice of their rival:

$$q_1^* = \frac{a - c - bq_2^*}{2b}. \tag{1}$$

$$q_2^* = \frac{a - c - bq_1^*}{2b}. \tag{2}$$

As shown in equations (1) and (2), the profit-maximizing quantity for firm 1 depends on the quantity set by firm 2 and vice versa. We can calculate q_1^* and q_2^* using a system of equations and can calculate the equilibrium price as follows:

$$q_2^* = q_1^* = \frac{a - c}{3b}.$$

$$P^* = a - bQ^* = \frac{a - 2}{3(a - c)} = \frac{a + 2c}{3}.$$

This example can be extended to n firms. The key takeaway is that the quantity produced by each firm is chosen as a "best response" to the quantities of the other firms, and the price is determined by the total quantity produced in the market (Cournot oligopoly). This constitutes the outcome of Cournot competition.

These takeaways provide the motivation for the HHI and its use in merger review. In particular, using Cournot competition, we can rewrite firms' price-cost markup in terms of the HHI (Cournot Oligopoly). To see this, first note that we can express a firm's profit as follows:

$$\Pi_i = P(Q)q_i - c_i(q_i),$$

where Π_i is the profit of firm i, Q is the total quantity in the market, $P(Q)$ is the price set by the market, and $c_i(q_i)$ is the cost of firm i in terms of the quantity produced. As noted above, firms seek to maximize their profit, which can be calculated by taking the derivative of Π_i and setting this to 0:

$$\frac{\mathrm{d}\Pi_i}{\mathrm{d}q_i} = 0 = P(Q) + \frac{\mathrm{d}P(Q)}{\mathrm{d}q_i}q_i - \frac{\mathrm{d}c_i(q_i)}{\mathrm{d}q_i}. \tag{3}$$

In addition, the market share s_i and the elasticity of demand ε can be expressed as follows:

$$s_i = \frac{q_i}{Q} \quad \text{and} \quad \varepsilon = \frac{P(Q)}{\mathrm{d}P(Q)}\frac{\mathrm{d}q_i}{Q(P)}.$$

Using these definitions, we can rewrite equation (3) as follows:

$$\frac{P(Q) - c_i}{P(Q)} = -\frac{\frac{\mathrm{d}P(Q)}{\mathrm{d}q_i}q_i}{P(Q)} = \frac{s_i}{\varepsilon}.$$

Multiplying both sides of the equation by market share and taking the sum across all firms, we can get the share-weighted price-cost markup for the market in terms of the HHI:

$$\sum_{i=1}^{n} s_i \frac{P(Q) - c_i}{P(Q)} = \frac{1}{\varepsilon}\sum_{i=1}^{n} s_i^2 = \frac{HHI}{\varepsilon}. \tag{4}$$

Equation (4) showcases the logic behind using the HHI in merger review. Cournot competition shows that the HHI is positively correlated with share-adjusted price-cost markup. In other words, as a market becomes more concentrated, firms can increase prices further above their cost. Grounded in these conceptual foundations, as well as the agencies' familiarity with the HHI and its ease of calculation and use, the HHI has become an essential metric in merger review.

2.3 Conceptual foundations of the procurement-based framework

While the HHI has the microfoundations based on the Cournot model as described above, some of the underlying assumptions in the Cournot model and subsequently the HHI may not fit certain markets. For example, Cournot competition has no ability to measure buyer power, which is the ability for buyers to use competition among sellers to negotiate better prices. Indeed, as shown in the previous section, many economists observe situations in which the Cournot model and other complete information models cannot capture certain behavior.

Loertscher and Marx (2022b) provide the practical implementation of the incomplete information approach to merger review in procurement markets. They begin by assuming a pre-merger market with $n \geq 2$ suppliers. Each supplier independently draws a cost from a continuously differentiable distribution G_i, its cost distribution, with support $[0, 1]$, where G_i is assumed to have the following parameterized form:

$$G_i(c) = 1 - (1 - c)^{\alpha_i},$$

with each firm's distributional parameter α_i representing the "ability or capacity" of supplier i. Each supplier draws from their respective cost distribution for each procurement. The higher is a firm's α_i, the more likely is the firm to draw a lower cost. We can use the cost distributions of these firms to calculate the markets shares of these firms and vice versa. In particular, the market share of firm i is defined as the probability that the cost of firm i is the lowest, expressed as follows:

$$\sigma_i = \Pr(c_i \text{ is lowest}).$$

Given this, the market share for firm 1 can be written as follows:

$$\sigma_1 = \Pr(c_1 < \min\{c_2, \ldots, c_n\}).$$

This can be written using the cumulative distribution functions of the cost distributions:

$$\sigma_1 = \int_0^1 \int_0^y g_1(c_1) g_{(1:\{2,\ldots,n\})}(y) \, dc_1 \, dy,$$

where $g_1(c_1)$ is the probability density function for firm 1 and $g_{(1:\{2,\ldots,n\})}(y)$ is the density for the lowest cost draw from the set of firms $\{2, \ldots, n\}$. As shown in Loertscher and Marx

(2022b), this double integral evaluates to the following:

$$\sigma_1 = \frac{\alpha_1}{\alpha_1 + \cdots + \alpha_n}. \tag{5}$$

The same process holds for the market shares of the rest of the firms. However, even if we have information on all the market shares in a market, this is not enough to calculate the αs of the firms because with this parametrization alone we do not have enough information to solve the system of equations. We have n variables and only $n-1$ independent equations because market shares are constrained to add up to 1. Thus, additional information is necessary.

For this additional information, we use the operating margin. The operating margin of only one firm is required, as we need only one additional equation to complete the calibration. We delegate this to firm 1, but the system can be solved with the operating margin of any firm. The operating margin of firm 1 is expressed as follows:

$$\omega_1 = \frac{\Pi_1}{Revenue_1},$$

where Π_1 is the profit of firm 1 and $Revenue_1$ is the revenue of firm 1. The procurement-based framework assumes a second-lowest cost auction, where the winner of the auction is paid the second-lowest bid. With this assumption, the operating margin can be rewritten as follows:

$$\omega_1 = \frac{\mathbb{E}[\min\{c_2, \ldots, c_n\} - c_1 \mid c_1 < \min\{c_2, \ldots, c_n\}]\Pr(c_1 < \min\{c_2, \ldots, c_n\})}{\mathbb{E}[\min\{c_2, c_3, \ldots, c_n\} \mid c_1 < \min\{c_2, \ldots, c_n\}]\Pr(c_1 < \min\{c_2, \ldots, c_n\})}.$$

This can once again be written out in terms of the cumulative distribution function of our cost distributions as follows:

$$\omega_1 = \frac{\int_0^1 \int_0^{c_{2nd}} (c_{2nd} - c_1) g_{(1:\{2,\ldots,n\})}(c_{2nd}) g_1(c_1)\, dc_1\, dc_{2nd}}{\int_0^1 \int_0^{c_{2nd}} c_{2nd}\, g_{(1:\{2,\ldots,n\})}(c_{2nd}) g_1(c_1)\, dc_1\, dc_{2nd}},$$

where c_{2nd} is the second-lowest cost, or the lowest cost draw from the set of firms $\{2, \ldots, n\}$. This equation can be rewritten in terms of the firms' α parameters and the sum of all of the α parameters, denoted $A_N \equiv \alpha_1 + \cdots + \alpha_n$. Solving for A_N, we have:

$$A_N = \frac{1}{\frac{1}{\omega_1} - 2 + \frac{\alpha_1}{A_N}}.$$

Because $\sigma_i = \alpha_i / A_N$ by equation (5), we can solve for the distributional parameter of any firm in the market in terms of firm 1's margin and market share by multiplying the above expression by σ_i. Doing so, we conclude that for each firm $i \in \{1, \ldots, n\}$,

$$\alpha_i = \frac{\sigma_i}{\frac{1}{\omega_1} - 2 + \sigma_1}. \tag{6}$$

Using equation (6), along with all the firms' market shares and firm 1's margin, we have enough information to solve for all the α parameters and our calibration is complete. With these parameters in hand, we have the calibrated cost distribution of each firm.

From here, we can use the cost distributions of the firms to model markets and analyze mergers in these markets. Consider a hypothetical procurement market that utilizes a second-lowest price auction. In this market, a single buyer wishes to purchase a single product from $n \geq 2$ suppliers. In a second-lowest price auction, the winner is the supplier that offers the lowest bid, but then is paid the second-lowest bid. Consequently, the price increase probability for a buyer is the probability that the second-lowest cost post-merger is greater than the second-lowest cost pre-merger. Assuming that when two firms merge, the merged entity has a cost equal to the minimum of the costs of the two firms pre-merger, a merger between firm 1 and firm 2 produces a merged entity that draws its cost from the following cost distribution:

$$G_{1,2}(c) \equiv 1 - (1 - G_1(c))(1 - G_2(c)) = 1 - (1 - c)^{\alpha_1 + \alpha_2}.$$

This implies that $\alpha_{1,2} = \alpha_1 + \alpha_2$. After a merger between two firms, the cost of the merged entity is assumed to be the minimum of the cost of the two firms pre-merger. Once again assuming a merger between firm 1 and firm 2, we can express the price increase probability (PIP) as follows:

$$\begin{aligned} PIP &\equiv \Pr(\text{2nd}\{\min\{c_1, c_2\}, c_3, ..., c_n\} > \{c_1, \ldots, c_n\}) \\ &= \Pr(\max\{c_1, c_2\} < \min\{c_3, ..., c_n\}). \end{aligned}$$

This then can be rewritten in terms of our cumulative distribution functions:

$$\begin{aligned} PIP &= \int_0^1 \int_0^1 1_{\max\{c_1,c_2\}<\min\{c_3,\ldots,c_n\}} dG_1(c_1) \ldots dG_n(c_n) \\ &= \int_0^1 \ldots \int_0^y dG_{(2,\{1,2\})}(x) \ldots dG_{(1,\{3,\ldots,n\})}(y), \end{aligned}$$

where $1_{\max\{c_1,c_2\}<\min\{c_3,\ldots,c_n\}}$ is an indicator function that outputs 1 when the condition $\max\{c_1, c_2\} < \min\{c_3, \ldots, c_n\}$ is satisfied and 0 otherwise, $G_{(2,\{1,2\})}(x)$ is the cost distribution of the maximum of c_1 and c_2, and $G_{(1,\{3,\ldots,n\})}$ is the cost distribution of the minimum draw from the set of firms $\{3, \ldots, n\}$. As shown in Loertscher and Marx (2022b), this integral evaluates to the following:

$$PIP = \frac{\alpha_1\alpha_2(A_N + A_{N\backslash\{1,2\}})}{A_{N\backslash\{2\}}A_{N\backslash\{1\}}A_N},$$

where the notation $A_{N\backslash X}$ denotes the sum of the α parameters for the firms in the set $N\backslash X$, i.e., $A_{N\backslash X} \equiv \sum_{i\in N\backslash X} \alpha_i$. In other words, $A_{N\backslash X}$ is the sum of all the α parameters except for the α parameters of the firms in set X. For example, $A_{N\backslash\{1,2\}}$ is the sum of the α parameters for all the firms except firms 1 and 2.

In summary, the calibration for the procurement-based framework requires information on firms' market shares as well as the operating margin of one firm within the market. This provides the sufficient information to complete the parametrization and solve for each firm i's distributional parameter α_i. This then allows us to calculate each firm's cost distribution. Using the cost distributions, we can model certain markets and analyze different mergers. For example, we can use the cost distributions of firms pre-merger and post-merger to evaluate the probability of an increase in price post-merger. The PIP can then be used as a threshold metric, analogous to the HHI, where if a merger causes a PIP that is greater than, say, 5%, the merger is flagged as potentially anticompetitive.

3 Empirical framework

This paper seeks to compare two theoretical frameworks using a simulation method. Specifically, we use simulated markets in which a merger occurs to compare the two models discussed above. The parameters needed for the HHI-based approach are the firms' pre-merger market shares. The parameters needed for the procurement-based framework are the firms' pre-merger market shares as well as the operating margin of one firm. The post-merger market shares are assumed to be the same as the pre-merger market share but with the merged entity having a market share equal to the sum of the pre-merger shares of the two merging firms.

The DOJ and FTC use both the post-merger HHI and the change in HHI to identify prospective anti-competitive mergers. The procurement-based approach uses the probability of an increase in price (PIP) of over 5% to identify prospective anti-competitive mergers.[2]

[2]PIP thresholds other than 5% can be considered, but we focus on 5% to be consistent with thresholds used in the literature.

The HHI-based approach and the procurement-based approach require the market shares of all the firms in the market, and the procurement-based approach additionally requires the operating margin of some firm in the market. This additional information is often available in a merger review context because some commonly used merger simulation approaches currently in use rely on having the operating margin of some firm.

We use a Python script to randomly generate 1000 simulated markets. Each market has n firms where $n \in \{3, \ldots, 10\}$. We start by randomly drawing n from $\{3, \ldots, n\}$ with equal probability for each value. Then, given n, the market shares of the firms are randomly drawn from the $n-1$-dimensional simplex with uniform probability over the simplex. We designate first firm's operating margin to be used in the calculation of the PIP. The first firm's operating margin is constrained by the model to satisfy

$$\omega_1 < \frac{1}{2 - \sigma_1},$$

where ω_1 is the operating margin of firm 1 and σ_1 is the market share of firm 1. We randomly draw the operating margin from the uniform distribution on the interval $[0, \frac{1}{2-\sigma_1}]$. From here, we can calculate the metrics. Using the simulated market approach we can analyze an arbitrarily large number of sample mergers. We use these simulations to examine relationships and patterns between which metrics and factors denote which mergers as potentially anticompetitive.

4 Empirical analysis of simulated markets

In this section, we use the metrics for 1000 simulated mergers to empirically analyze the relation between the procurement-based approach and the HHI approach to merger review. For each possible merger, we calculate the post-merger HHI, change in HHI, and PIP. In Figure 3, we plot the post-merger HHI and change in HHI of all 1000 mergers and indicates the PIP level for each merger. A blue cross indicates mergers with a PIP less than 5%, an orange square indicates mergers with a PIP between 5% and 10%, and a green circle indicates mergers with a PIP greater than 10%. (For clarity, the different PIP levels are depicted separately in the panels of Figure 4.)

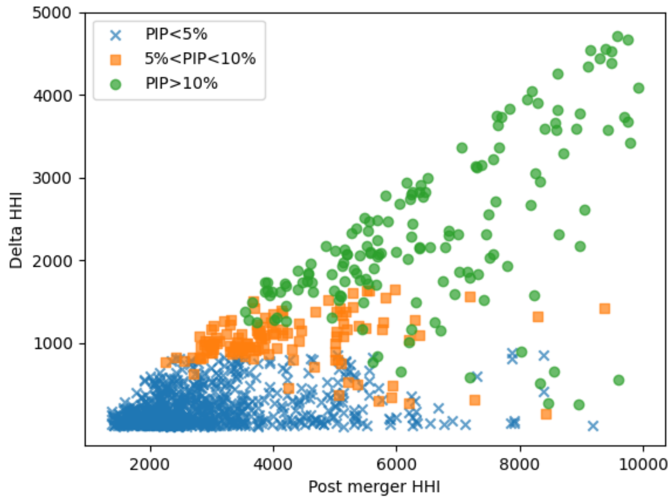
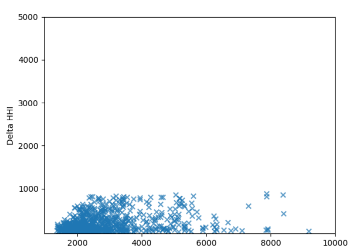
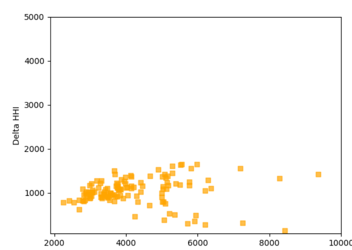


Figure 3: PIP ranges, post-merger HHI, and change in HHI for 1000 simulated mergers. Blue crosses indicate mergers with a PIP less than 5%. Orange squares indicate mergers with a PIP between 5% and 10%. Green circles indicate mergers with a PIP greater than 10%.

(a) PIP < 5%



(b) 5% < PIP < 10%



(c) PIP > 10%

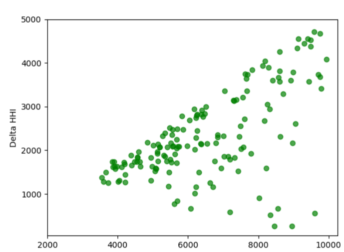


Figure 4: Change in HHI vs. post-merger HHI separated by PIP ranges. Panel (a) shows only mergers with a PIP less than 5%. Panel (b) shows only mergers with a PIP between 5% and 10%. Panel (c) shows only mergers with a PIP greater than 10%.

Similar to the conclusions of Nocke and Whinston (2022), we find that the price increase probability post-merger is only slightly correlated with the post-merger HHI and highly correlated to the change in HHI. In contrast to Nocke and Whinston (2022)'s findings that the post-merger HHI had little predictive value, the procurement-based framework suggests that the post-merger HHI does seem to be able to predict a threshold for the PIP. In particular, a merger with a post-merger HHI of less than approximately 2500 will strictly have a PIP of less than 5%, and a merger with a post-merger HHI of less than approximately 3500 will strictly have a PIP of less than 10%. However, while the post-merger HHI provides a soft lower bound, the change in HHI seems more predictive of the PIP, consistent with Nocke and Whinston.

Indeed, as shown in Table 1, 95% of mergers with a PIP less than 5% have a change in HHI between 3 and 783, 95% of mergers with a PIP between 5% and 10% have a change in HHI between 316 and 1604, and 95% of mergers with a PIP greater than 10% have a change in HHI between 587 and 4527. Also illustrated in Table 1 are the 95% confidence intervals for the post-merger HHI.

Table 1: Summary statistics and confidence intervals (95%).

PIP Range	Num. of Observations	C.I. (ΔHHI)	C.I. (Post-Merger HHI)
$<$5%	721	[3, 783]	[1532, 6255]
5%-10%	124	[316, 1604]	[2692, 7256]
$\geq$10%	155	[587, 4527]	[3836, 9706]

As noted in our visual interpretation of Figure 3, the post-merger HHI does not provide a strong upper bound for the PIP ranges. As shown there, unconcerning mergers with PIP $<$ 5% have a post-merger HHI as high as 9188. With that being said, the lower bound for presumptively anticompetitive mergers seems to agree with the bound established in the 2010 *Horizontal Merger Guidelines*, which establish a post-merger HHI lower bound of 2500. The confidence interval indicates that 95% of mergers with a PIP between 5% and 10% have a post-merger HHI between 2692 and 7256, so the IIIO seems corroborate the DOJ and FTC's use of 2500 as the post-merger HHI threshold for presumtpively harmful mergers in the *2010 Horizontal Merger Guidelines*.

We could use these bounds to suggest new ranges for the *Merger Guidelines* that are empirically backed by IIIO theory. For example, using the bounds on the change in HHI for the three PIP levels, a change in HHI of less than 300 could be a reasonable threshold for safe mergers, as it is unlikely to have a merger with a PIP of greater than 5% when its change in HHI is less than 300. Similarly, the majority of mergers with a PIP greater than 10% have a change in HHI greater than 500, so 500 could act as a lower bound for the region of strongly presumptively harmful mergers. Consequently, mergers with a change in HHI between 5% and 10% could be marked as harmful. These proposed IIIO-backed thresholds suggest that the 2010 *Horizontal Merger Guidelines* are too strict and block safe mergers, which challenges the DOJ and FTC's decision to further tighten these bounds in the 2023 *Merger Guidelines*. These proposed thresholds for merger review are significantly more relaxed than those in both the 2010 *Horizontal Merger Guidelines* and the 2023 *Merger Guidelines* and would ultimately flag fewer mergers as potentially anticompetitive. We illustrate this in Table 2.

Table 2: Thresholds for presumptively harmful mergers from different sources.

	Post-merger HHI	ΔHHI
2010 *Horizontal Merger Guidelines*	2500	200
2023 *Merger Guidelines*	1800	100
Nocke and Whinston (2022)	—	200
Procurement-based framework	2600	300

However, this is not to say that the change in HHI is a perfect predictor for the PIP. For example, one of the simulated mergers has a change in HHI of 269, which would be marked as a presumptively unconcerning merger by our proposed thresholds, but it has a PIP of 19%, so the procurement-based framework classifies it as highly concerning. Therefore, similar to how the DOJ and FTC suggest using the HHI in the current *Merger Guidelines*, the use of the PIP as motivation for HHI-based thresholds for merger review should still be used only as a preliminary screening guideline. It should not be used as the sole decision maker in the merger review process; instead, it should open the door to further investigation.

The simple linear regression in Table 3 is inspired by the corresponding regression as Nocke and Whinston (2022), where they examine the relation between the HHI metrics and the post-merger synergy required to keep the market consumer surplus neutral. In column (1) we instead regress the PIP on the post-merger HHI (labeled "post_merger_hhi"), the change in HHI (labeled "delta_HHI"), and a constant, and in column (2) we run the same regression as in column (1) and include an interaction term between the post-merger HHI and the change in HHI (labeled "post_delta_interact").

The results of Table 3 corroborate my initial visual interpretation of Figure 3. However, as opposed to Nocke and Whinston (2022), the procurement-based framework seems to suggest that the post-merger HHI is relevant in merger review. In column (1), both the post-merger HHI and the change in HHI are statistically significant at the 0.001 level. Our regression results suggest that the post-merger HHI indeed has some predictive power of the PIP and could be a useful metric in measuring the potential harm of a merger. Indeed, in Figure 3, our visual observation that the post-merger HHI provides a lower bound for the PIP ranges

Table 3: Regressions of the PIP on the post-merger HHI and change in HHI.

	(1) PIP	(2) PIP
post_merger_hhi	11.27***	6.952***
	(1.099)	(1.078)
delta_hhi	78.22***	19.61***
	(2.327)	(5.145)
post_delta_interact	—	86.17***
	—	(6.863)
constants	-3.717***	-1.428***
	(0.3384)	(0.364)
R^2	0.788	0.817
N	1000	1000

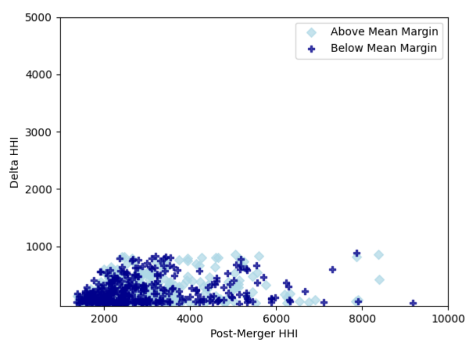
Standard Error in Parentheses
*** $p < 0.001$

likely adds to the significant correlation between the PIP and the post-merger HHI in our regression analysis. Additionally, the existence of a statistically significant interaction term in column (2) indicate that the effect of the post-merger HHI is exacerbated at high levels of change in HHI and vice versa, further corroborating the argument that both should metrics should be considered in merger review.

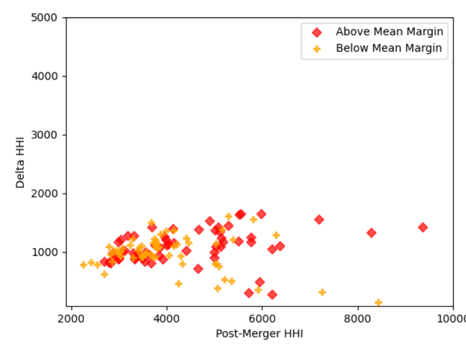
For comparison we provide Nocke and Whinston's analysis of the synergy required to prevent consumer harm as shown in Figure 7 in the Appendix. From a purely visual perspective the synergy levels do not seem to provide any significant bounds or any other type of indicator for the post-merger HHI, which explains why their regression results indicate no significant relation between synergy level and post-merger HHI. However, the results from the procurement-based framework do agree with those of Nocke and Whinston (2022) in that we both agree that the change in HHI is a better predictor for the concern level of a merger compared to the post-merger HHI. In column (1), we see that the relation between the PIP and the change in HHI is both statistically stronger and larger in magnitude than the relation between the PIP and the post-merger HHI. The coefficients are 78.22 and 11.27 for the change in HHI and post-merger HHI respectively, which correspond to t-statistics of 33.61 and 10.25. This result suggests that while both metrics could be useful in merger review, the change in HHI is the better metric for determining the concern level of a merger from a IIIO perspective.

The only additional parameter needed with the IIIO approach to merger review in comparison to the HHI approach is the operating margin of a firm in the market. This raises the question as to what additional information the operating margin might provide about the PIP in comparison to the HHI metrics. In Figure 5, we once again plot the post-merger HHI and the change in HHI separated by PIP ranges, and instead differentiate between mergers above and below the mean operating margin for mergers within that specific PIP range. The diamonds represent mergers with an operating margin above the mean operating margin, and the crosses represent mergers with an operating margin below the mean operating margin. Additionally, the median operating margins for mergers in each PIP range are displayed in Table 4.

(a) PIP $< 5\%$



(b) $5\% <$ PIP $< 10\%$



(c) PIP $> 10\%$

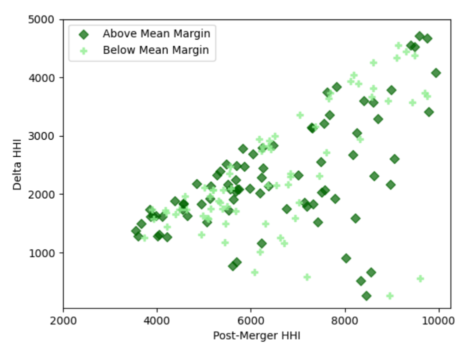


Figure 5: Change in HHI vs. post-merger HHI separated by PIP ranges and mean operating margin. Panel (a) shows only mergers with a PIP less than 5%. Conditional on having a PIP less than 5%, the dark blue crosses indicate mergers with a PIP below the median operating margin of 26.39, and the light blue diamonds indicate mergers with a PIP above the median operating margin. Panel (b) shows only mergers with a PIP between 5% and 10%. Conditional on having a PIP between 5% and 10%, the orange crosses indicate mergers with a PIP below the median operating margin of 30.31, and the red diamonds indicate mergers with a PIP above the median operating margin. Panel (c) shows only mergers with a PIP greater than 10%. Conditional on having a PIP greater than 10%, the light green crosses indicate mergers with a PIP below the median operating margin of 30.43, and the dark green diamonds indicate mergers with a PIP above the median operating margin.

Table 4: Median operating margin for different PIP ranges.

Color	PIP Range	Median Margin
Blue	<5%	26.39
Orange	5%-10%	30.31
Green	>10%	30.43

From a purely visual perspective, the operating margin of the firm in the merger does not seem to have any correlation with either the change in HHI and the post-merger HHI. Additionally, there is little to no correlation between the operating margin and the PIP, with a Pearson Correlation of 0.06, which is not statistically significantly different from 0.[3] This undermines the notion that the margin has any predictive power for the two HHI metrics or the PIP, and solidifies the role of the operating margin as purely for calibration purposes.

With the introduction of the 2023 *Merger Guidelines*, the DOJ and FTC included an additional indicator to be used in merger review. The DOJ and FTC lowered the post-merger HHI and change in HHI thresholds from 2500 to 1800 and 200 to 100 respectively. In addition to these metrics, the DOJ and FTC also stated that if the merged firm's market share is greater than 30% and the the change in HHI is greater than 100, the merger should be flagged as presumptively anticompetitive. Either or both of these indicators can be used as evidence that a merger is presumptively anticompetitive. These two indicators, which the DOJ and FTC call the "Post-Merger HHI" indicator and the "Merged Firm's Market Share" indicator, both utilize the change in HHI.

We have previously explored the "Post-Merger HHI Indicator," examining the effect of the post-merger HHI, change in HHI, and the two metrics in tandem on the PIP. Other than the thresholds provided by the DOJ and FTC, the use of these two metrics in merger review remain the same. Now we will examine the effect of the new indicator, the Merged Firm's Market Share indicator. In Figure 6, we display a similar graph as that shown in Figure 3, plotting the merged firm's market share and the change in HHI for various PIP levels. Once again, a blue cross indicates mergers with a PIP less than 5%, an orange square indicates mergers with a PIP between 5% and 10%, and a green circle indicates mergers with a PIP greater than 10%.

An immediately visually striking aspect of Figure 6 is the seemingly parabolic upper bound for all mergers, which differentiates this graph from Figure 3. This is due to the inherent mathematical relation between the merged firm's market share and the change in HHI. Indeed, if two firm's combine market shares after a merger in a market with n firms,

[3]Using Kanji (1995), the associated t-statistic is 0.63, so we fail to reject the null hypothesis that the correlation is 0.

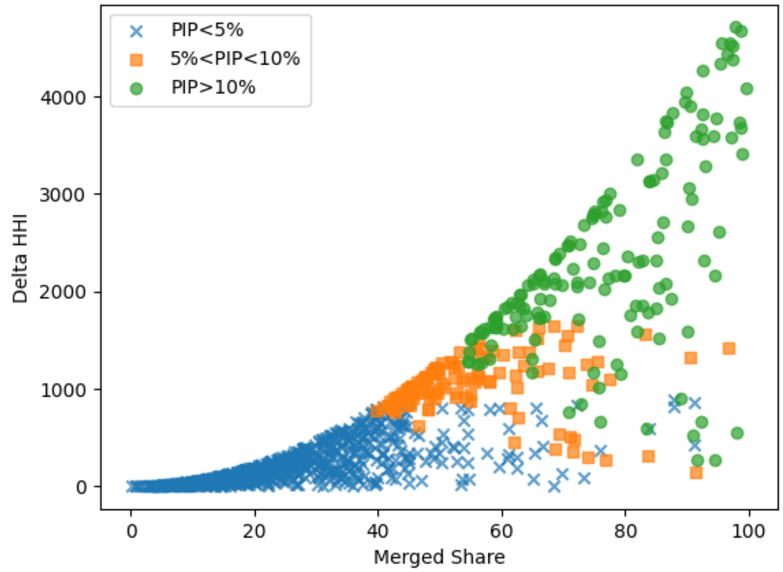


Figure 6: PIP ranges, merged firm's market share, and change in HHI for 1000 simulated mergers. Blue crosses indicate mergers with a PIP less than 5%. Orange squares indicate mergers with a PIP between 5% and 10%. Green circles indicate mergers with a PIP greater than 10%.

then assuming a merger between firm 1 and firm 2 with shares s_1 and s_2 respectively, the change in HHI can be written as follows:

$$\Delta HHI = (s_1 + s_2)^2 + \sum_{i=3}^{n} s_i^2 - (\sum_{i=1}^{n} s_i^2).$$

This is equivalent to the following:

$$\Delta HHI = (s_1 + s_2)^2 - (s_1^2 + s_2^2),$$

which simplifies to the following:

$$\Delta HHI = 2 s_1 s_2. \tag{7}$$

The parabolic shape is explained by the relation between the change in HHI and merged market share shown in equation (7). The change in HHI is upper bounded by the product of the market shares of the two merging firm. This is why we see an upper bound for the change in HHI shaped like a parabola.

More relevant than the shape of the upper bound is the correlation between the PIP and our two independent variables, the change in HHI and merged firm's market share. From both a visual and empirical standpoint, the results provided by the new "Merged Firm's Market Share" indicator introduced in the 2023 *Merger Guidelines* are similar to our results for the "Post-Merger HHI" indicator displayed in Figure 3. Like the post-merger HHI, the merged firm's market share seems to be a weaker predictor for the PIP compared to the change in HHI. There are several instances where a merger may have a large merged firm's market share and still have a small PIP. For example, there are 40 mergers that have a PIP less than 5% and a merged firm's market share greater than 50%. This indicates that the merged firm's market share is an imperfect metric, as relatively large mergers where the merged firm captures the majority of the market are considered safe from the procurement-based framework's 5% PIP threshold. However, similar to the post-merger HHI, the merged firm's market share does seem to provide a strong lower bound for the PIP from a visual perspective. For example, no merger with a PIP greater than 5% has a merged firm's market share less than 39%, and no merger with a PIP greater than 10% has a merged firm's market share less than 54%. In contrast, there exists a merger with a PIP less than 5% with a merged firm's market share as high as 91.2%, and a merger with a PIP less than 10% with a merged firm's market share as high as 97.0%. The 95% confidence intervals for the merged firm's market share by PIP level are shown in Table 5. These results indicate that, similar to the post-merger HHI, the merged firm's market share provides a stronger lower bound and a weaker upper bound.

Table 5: Merged share confidence intervals (95%).

PIP Range	Confidence Interval
<5%	[4.313, 61.18]
5%-10%	[41.94, 83.58]
≥10%	[55.17, 98.51]

We replicate the approach used earlier for the analysis of the post-merger HHI, running the same regression except replacing post-merger HHI with the merged firm's market share to empirically examine the predictive power of the change in HHI, the merged firm's market share, and their interaction on the PIP. These results are shown in Table 6. In column (1) we regress PIP on the merged firm's market share (labeled "merged_share"), change in HHI (labeled "delta_hhi"), and a constant. In column (2) we run the same regression as in column (1) and include an interaction term between the merged firm's market share and the change in HHI (labeled "delta_merged_interact").

Comparing the results in Table 6 and Table 3, we see that the predictive power of the change in HHI largely remains the same. All variables are statistically significant at the 1% level. For the "Merged Firm's Market Share" indicator's regression, the magnitude of the

Table 6: Regressions of the PIP on the merged firm's market share and change in HHI

	(1) PIP	(2) PIP
merged_share	5.903***	8.447***
	(1.058)	(0.9471)
delta_hhi	82.25***	-39.96***
	(2.909)	(7.737)
delta_merged_interact	—	136.6***
	—	(8.157)
constants	-2.054***	-0.7832***
	(0.2968)	(0.2731)
R^2	0.7726	0.8226
N	1000	1000

Standard Error in Parentheses
*** $p < 0.001$

change in HHI's coefficient is 82.25, while for the "Post-Merger HHI" indicator's regression, the magnitude of the coefficient is are 78.22. However, while still statistically significant, the coefficient for the merged firm's market share in Table 6 is less statistically significant and lesser in magnitude compared to its analog the post-merger HHI. The magnitude of the merged firm's market share coefficient is 5.903, compared to post-merger HHI's coefficient of magnitude 11.27.

These results challenge the use of the "Merged Firm's Market Share" in tandem with the "Post-Merger HHI." For both indicators, the change in HHI have comparable effectiveness in terms of predictive power of the PIP. However, the merged firm's market share has less predictive power than the post-merger HHI, from both a magnitude and statistical significance perspective. This undermines the use of both indicators established in the 2023 *Merger Guidelines* and supports the use of only the change in HHI and post-merger HHI used in the 2010 *Horizontal Merger Guidelines.* Additionally, there are no circumstances where one indicator would be more convenient to calculate than the other, as both indicators require the change in HHI which uses the market shares of all the firms in the market, which undermines the potential argument that certain scenarios are more fit for one indicator over the other. However, column (2) of Table 6 displays an interaction term between the change in HHI and the merged firm's market share that is more significant and larger in magnitude than the interaction term between the change in HHI and the post-merger HHI in Table 3. This instead supports the notion that the "Merged Firm's Market Share" indicator should be used, as the effect of the post-merger HHI on PIP is exacerbated by greater merged firm's market share and vice versa.

5 Conclusion

In this paper, we compare the traditional HHI approach to merger review grounded in the Cournot oligopoly model with the procurement-based Incomplete Information Industrial Organization approach. In particular, we examine the relation between the HHI metrics provided in the 2010 *Horizontal Merger Guidelines* and the 2023 *Merger Guidelines* with the price increase probability derived from the procurement-based framework. The 2010 *Horizontal Merger Guidelines* reccomend the use of the post-merger HHI and the change in HHI as thresholds for presumptively anticompetitive mergers, and the 2023 *Merger Guidelines* provide an additional indicator that uses the merged firm's market share and change in HHI.

There are three main takeaways from our empirical analysis. First, our results agree with Nocke and Whinston (2022)'s claim that the change in HHI is the more powerful predictor of harm compared to the post-merger HHI, but disagree with their claim that the post-merger HHI is completely irrelevant. Second, our results indicate that the operating margin adds no significant additional information regarding either the PIP or the HHI metrics, consistent with the operating margin's role in the IIIO approach being purely calibrational. Finally, our results suggest that the 2023 *Merger Guidelines*' addition of the merged firm's market share as an indicator does add not significant additional information compared to the post-merger HHI indicator used in the 2010 *Horizontal Merger Guidelines.* Additionally, since both indicators require information on every firm's share in the market, there are no instances in which one indicator may be easier to calculate than the other, which undermines the use of the merged firm's market share in the 2023 *Merger Guidelines.*

Our results lead to three policy implications. First, both the change in HHI and the post-merger HHI should continue to be used but with different thresholds and with an emphasis on the change in HHI. Second rather than using both indicators outlined in the 2023 *Merger Guidelines*, it is sufficient to use just the post-merger HHI. Third, the IIIO approach is largely consistent with the merger simulation approach used in Nocke and Whinston (2022). The similar conclusions from both the IIIO approach and the Nocke and Whinston (2022) approach support the use of the PIP in merger review.

The PIP is just one of multiple IIIO metrics analogous to traditional approaches for analysis in merger review. Possible future research could explore the IIIO analogs for these approaches and determine whether these analogs support, undermine, or neither support nor undermine the traditional approaches.

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A Appendix: Additional figure

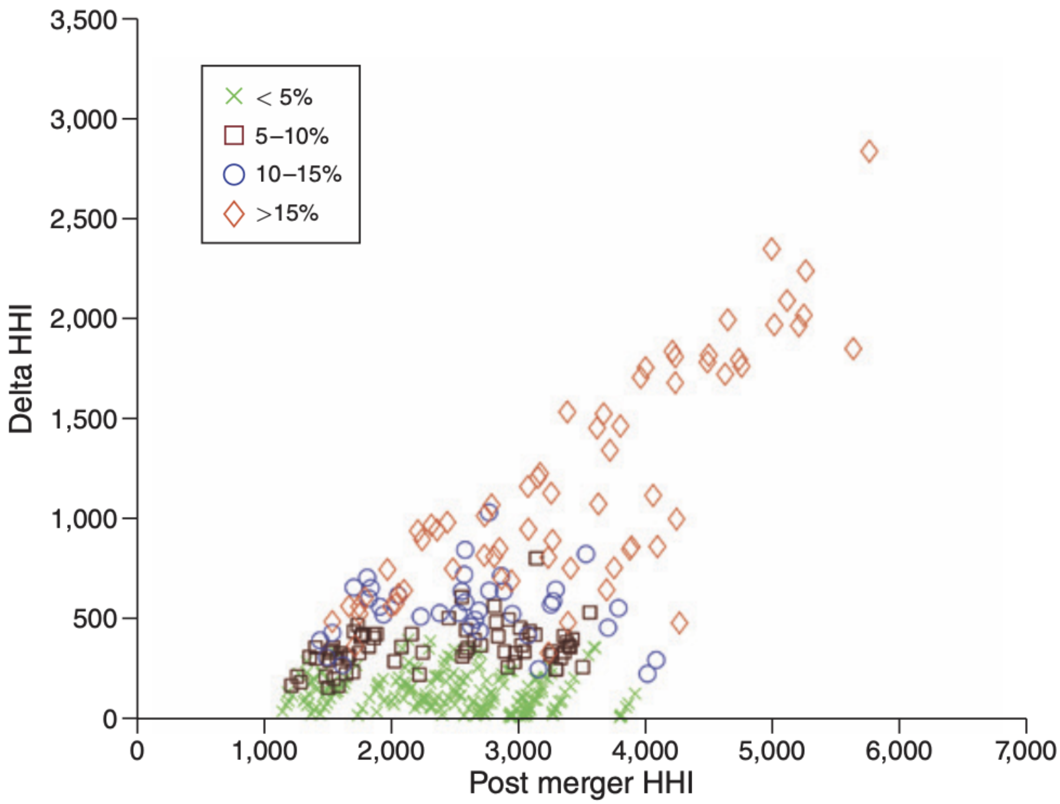

Figure 7: Reproduced from Nocke and Whinston (2022, Figure 5): "Synergy Required to Prevent Consumer Harm, RCNL-1." The figure shows the relationship between the synergy required for a merger to be neutral for consumer surplus and the post-merger HHI and its change, based on Miller and Weinberg (2017)'s RCNL-1 model for the U.S. brewing industry.

B Appendix: Code

Listing 1: Share calculation function. The function takes in an integer parameter *num_firms* and returns an array of size *num_firms* where the ith element in the returned array is the market share of the $i+1$th firm. The market shares of the firms are randomly drawn from the $(num\_firms - 1)$-dimensional simplex with uniform probability over the simplex. The function incorporates the numpy library and its random sampling functionality.

```
def shares(num_firms):
    values = [0.0, 100.0]
    values += list(np.random.uniform(low=0.0, high=100.0,
        size=num_firms - 1))
    values.sort()
    return [values[i+1] - values[i] for i in range(num_firms)]
```

Listing 2: Price increase probability function. The function takes in an array parameter *alphas* where each element represents the alpha for each firm in the market. The function returns a float which is the price increase probability of the simulated market. For the mathematical derivation of the price increase probability see Section 2.

```
def PIP(alphas):
    alphaTotal = 0
    for alpha in alphas:
        alphaTotal += alpha
    alphaOne = alphas[0]
    alphaTwo = alphas[1]
    alphaNoOne = alphaTotal - alphaOne
    alphaNoTwo = alphaTotal - alphaTwo
    alphaNoOneTwo = alphaTotal - alphaOne - alphaTwo
    priceIncProb = (alphaOne * alphaTwo *
                    (alphaTotal + alphaNoOneTwo)) /
                    (alphaNoOne * alphaNoTwo * alphaTotal)
    return priceIncProb
```