The Sub-proportionality of Subjective Probability Weighting in Poker

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Abstract

This study uses Texas Hold'em poker to investigate decision-making under uncertainty and the concept of probability weighting, where individuals may overvalue or undervalue uncertain outcomes. I conduct an experiment to assess Cumulative Prospect Theory's relevance to subjective probabilities in poker by simplifying the game to compare complex and simple gamble evaluations. The research aims to understand how risk preferences and probability estimation without complete information are influenced by individuals' poker experience and framing effects. We find that deviations from what theory predicts in the subjective-probability Poker frame can be explained well by the framing effects made in the decision maker's editing phase. By examining the difference in the predictive power of decision making models in explicit vs subjective probability gambles, the study seeks to improve comprehension of cognitive processes in navigating uncertainty.

Keywords: probability weighting, common ratio effect, prospect theory, experimental economics, decision-making under uncertainty

JEL Classification: C91, D80, D91

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1 Introduction

We make small decisions all the time. In many of these decisions, outcomes are uncertain, payoffs are marginal, preferences are near indifference, and losses are psychologically not internalized. Decision-making under these conditions thus represent a critical area of inquiry within cognitive science and behavioral economics. The complex interplay between risk assessment and strategic decision-making underpins a wide range of human activities, from financial investments to everyday choices. This paper utilizes the structured yet uncertain environment of Texas Hold'em poker as a novel experimental setting to examine these phenomena. Texas Hold'em, a game characterized by incomplete information and strategic interaction among players, provides an apt metaphor for the uncertainty inherent in real-world decision-making scenarios.

The primary objective of this research is two fold. The first objective is to test whether there is more probability weighting when gambles are difficult to evaluate compared to when they are easier to evaluate. The second is to assess how poker provides reasonably good control (via experience) for how difficult the difficult gambles are. By abstracting away the game's traditional mechanics to focus solely on the decision-making process between taking a known reward versus engaging in a gamble, we aim to isolate and analyze participants' ability to estimate probabilities when they are not explicit, and then observe how preferences change as we fix the payoffs and scale the probabilities down proportionally. This methodological simplification allows for a controlled examination of how individuals make judgments in the absence of complete data, reflecting broader psychological and economic theories of decision-making under uncertainty.

This study utilizes a quantitative approach to understanding the minimum payoffs that individuals are willing to accept to forego potential uncertain higher rewards. Through this lens, the experiment seeks to contribute to the ongoing discourse on cumulative prospect theory (CPT), expected utility theory (EUT), and the psychological biases that influence decision-making in uncertain contexts. The incorporation of random elements via card draws from a deck further enriches the experimental design, introducing a stochastic component that mirrors the unpredictable nature of real-world scenarios.

Situating the experiment within the setting of Texas Hold'em provides conditions ripe for testing the properties probability weighting functions, a concept central to CPT. Developed by Daniel Kahneman and Amos Tversky as an extension of Prospect Theory, CPT offers a refined lens through which to view the non-linear ways people distort probabilities when faced with risky choices. Unlike the Expected Utility Theory (EUT), which assumes a linear relationship between actual probabilities and their perceived weight, CPT reveals that individuals often overestimate low probabilities and underestimate high ones, leading to decision-making patterns that diverge from what classical models like EUT would predict. CPT and other prominent non-expected utility theories formalize the probability weighting function π, having four fundamental properties[1]. These properties, together, serve as descriptive explanations for human tendency to arrive at suboptimal outcomes despite the existence of

[1] 1. Subadditivity of small p: $\pi(rp) > r\pi(p)\ \forall r \in (0,1)$, 2. Overweighting of small p: $\pi(p) > p$ for sufficiently small p, 3. Sub Certainty: $\pi(p) + \pi(1-p) < 1\ \forall p \in (0,1)$, 4. Sub Proportionality: $\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}\ \forall p, q, r \in (0,1)$

enough information to suggest otherwise. Kahneman and Tversky (1979) also introduce a value function in their works which is inherently skewed from expected utility as it no longer weights outcomes linearly. In poker, each hand presents a prospect, defined as a set of i disjoint probabilities $P \coloneqq \{p_i : 0 < p_i < 1, \sum_i p_i = 1\}$ and a corresponding set of i possible outcomes X, where $x_i \in X$ denotes the payoff in outcome i. Prospect theory then assigns a value V to the prospect:

$$V(X, P) = \sum_{x_i \in X} \pi(p_i) v(x_i)$$

While V is defined on prospects, v is defined on actual outcomes. In poker, each round presents a large but finite amount of outcomes. Without loss of generality to prospect theory, I simplify the experiment by limiting the number of players to 2 and reducing the player's choice set to a binary choice of their preference between two simple prospects. Due to gambling laws in North Carolina, I cannot take money from subjects in the experiment. The set X must then be restricted to the domain of gains, where $x_i \geq 0$. There are many assumptions about what the behavior $v(x)$ exhibits, but one important assumption CPT makes is that $v(0) = 0$. To further reduce complexity for easier analysis of sub-proportionality, I set $X = \{x_1, x_2\}$, $P = \{p_1, p_2\}$ and $x_2 = 0$ such that $\pi(p_2)v(x_2) = 0$, and V becomes $V(X, P) = \pi(p_1)v(x_1)$. That is, in each prospect, there is a probability p_1 of winning x_1 dollars, and a probability $1 - p_1$ of winning nothing.

This paper sets out to scrutinize the sub-proportionality property assumed in probability weighting functions mentioned above. Sub-proportionality—a condition where the subjective weighting given to probabilities does not align proportionally with their objective magnitude—underscores a fundamental deviation from rational choice models. This paper aims to experimentally test whether sub-proportionality holds in a more realistic context; when gamble probabilities are not explicit and the decision maker has to make an informed estimation of what the probability of success in a gamble truly is. By comparing the decisions made in this setting with equivalent "simple" gambles where probabilities are instead given, I establish the baseline preferences of the subjects upon which I can test the change in descriptive power of CPT.

The contributions of this paper are manifold. By delving into the nuances of sub-proportionality, I endeavor to enhance the present understanding of how individuals deal with incomplete information and make decisions in uncertain environments. Ultimately, by weaving together insights from CPT and a unique experiment, this paper strives to foster a more holistic understanding of human behavior in the face of uncertainty within a familiar context. The investigation into the sub-proportionality of probability weighting functions not only illuminates the intricacies of human decision-making but also contributes to a broader comprehension of behavioral biases in preference formulations between two prospects.

By examining certainty equivalents[2] and an extension of the Allais Paradox[3] to

[2] A *certainty equivalent* (CE) is a sure-thing return that somebody facing a gamble would accept to avoid the gamble. The smaller (larger) the CE is, the more risk-averse (risk-seeking) the person is.

[3] The *Allais Paradox*, sometimes also called the *common ratio effect*, demonstrates that people's choices in risk scenarios can contradict expected utility theory by showing that gambles preferences can shift illogically when payoffs remain fixed and probabilities scale down proportionally.

Texas Hold'em[4], I aim to shed light on whether the assertions made by Allais, Kahneman and Tversky extend to such domains. Furthermore, this study endeavors to contribute to the evolving discourse on behavioral economics, and decision sciences, offering valuable insights into how individuals perceive risk, make choices, and navigate complex decision environments in poker.

2 Literature Review

Recent research has made significant strides in the field of decision theory under uncertainty, particularly in the context of human decision behavior (Etner, 2009; Etner, 2023). This work has expanded the theoretical framework, incorporating psychological approaches and judgmental heuristics (Kahneman & Tversky, 1994). Economic theories have also been applied to this area, with implications for policy analysis (Viscusi, 1991). Decision-theoretic principles have been developed for reasonable care under uncertainty, including the use of maximin and minimax-regret criteria when deliberation is costly (Manski, 2019). The application of decision theory to optimize the value of computation under resource constraints has been explored (Horvitz, 1988), as well as the allocation of benefits under income uncertainty (Naga, 1995).

The principal decision theory model which this paper aims to apply and extend

[4] Texas Hold'em is a poker game in which each player gets two pocket cards (known only to them), while five community cards are dealt face up on the table. Players win if they can make the best 5 cards using their pocket cards and the community cards. Texas Hold'em is by far the most played format of Poker, and the game of chance which I myself am most familiar with. These two factors are the two primary reasons why I selected it as the mode for my experiment, as opposed to any other context where probabilities are unknown yet verifiable

is Advances in prospect theory: Cumulative representation of uncertainty (Kahneman & Tversky 1992). This paper builds upon their earlier work on Prospect Theory, introducing significant refinements to better account for how people make decisions under risk. The theory addresses the limitations of expected utility theory by incorporating psychological insights into the decision-making process, highlighting how individuals evaluate potential losses and gains differently. Key to Cumulative Prospect Theory is the idea that people tend to overvalue small probabilities and undervalue large probabilities, leading to risk-averse behavior in choices involving gains and risk-seeking behavior in choices involving losses, a theory which has come to be known as myopic loss aversion. The cumulative aspect of the theory comes from how people integrate the probability of outcomes, suggesting a more complex weighting function than previously considered. This groundbreaking work has profoundly influenced the fields of economics, psychology, and finance by providing a more accurate description of how people assess risk and make choices, challenging traditional economic models that assume rational behavior. Additionally, the paper elucidates the diminishing sensitivity to changes in magnitude and the reflection effect, whereby risk preferences are influenced by the framing of choices as gains or losses.

The history of Prospect Theory is as studied as it is contentious. The paper *Prospect Theory, a Literature Review* (Edwards 1996) examines how prospect theory has evolved and been criticized over the years. It compares the theory against the Neumann-Morgenstern theory of utility (1944) and maps them out in a conceptual framework for the decision-framing process, segmenting both models into an editing phase, where framing effects impact the way a decision is perceived, and an evaluation phase, where considerations such as the endowment effect, loss aversion, and the information available to the decision maker are taken into account. In the context of poker, this implies that prospect theory predicts that other factors beyond pure expected value, pot odds and equity will alter the way agents perceive the decision in front of them. Edwards also illuminates the compartmentalization of the decision making process, so that efforts can be focused on studying an individual segment of the decision making process without conflating several segments at once. I incorporate this theory into the experimental design by testing one group which has no segmentation of phases, and another in which I ask the subjects for their estimation of the perceived probabilities of success before they make their decision, to make the intermediate decision that gets made subconsciously in the first phase more salient.

The Allais Paradox (Allais 1953) challenges the expected utility theory by demonstrating that people's choices can violate the independence axiom, which suggests that if an option is preferred to another, it should remain preferred when both are altered by a common consequence[5]. It involves a set of choices between lotteries where individuals consistently make decisions that are inconsistent with expected utility maximization, revealing a preference for certainty over probabilistic gains, even when the expected value suggests a different choice. This paradox highlights the complexities of human decision-making, showing that real-life choices often deviate from what would be predicted by purely rational economic models. Borch,

[5] *The common consequence effect* is a family of phenomena under which preferences shift after a common consequence is applied. the *common ratio effect* is one instance of this family of phenomena, where the common consequence is a proportional decrease in probability

1968 suggests that the paradox may be due to the way the questions are posed, and Blavatskyy, 2022 has highlighted the experimental fragility of the Allais Paradox. Kopylov, 2007 and Fan, 2002 have further investigated the paradox, with Kopylov formulating representation results for subjective probabilities and Fan finding that the paradox experimentally vanishes for small-payoff variants in both real and hypothetically incentives. These studies are pertinant to my paper because I am financially constrained to offer only modest payoffs to participants, and Kopylov and Fan both agree that such payoff domains hinder the predictive power of the paradox. I will set out to see whether their conjecture holds within the context of poker. Birnbaum, 1999 and Karmarkar, 1979 propose alternative models that can account for the paradox, while Brady, 1993 and Harman and Gonzalez, 2015 explore the role of information and experience in decision-making, with the latter finding that an overweighting or underweighting of small probabilities depending on experience. Despite these efforts, the paradox remains a complex and multifaceted phenomenon, as evidenced by Munier, 1991 and the discussion of Allais' broader contributions to economic theory therein.

Research on probability weighting functions has revealed a number of key findings. Tversky and Wakker, 1995 formalize axioms[6] which are assumed to hold on all preference structures without restiction. Li and Winter, 2012 and Hantoute et al., 2017 both discuss the subadditivity property of these functions, with Hantoute specifically focusing on Gaussian distributions. Bleichrodt and Pinto, 2000 and G.

[6] These axioms are sixfold. For the curious reader, the definitions can be found in Tversky and Wakker, 1995 Appendix 1: Representation Assumptions. The six axioms are: Weak ordering, strict stochastic dominance, certainty equivalent condition, continuity in probabilities, simply continuity, and tradeoff consistency.

Wu, 1999 provide empirical evidence of subadditivity in probability judgments, with Bleichrodt noting both lower and upper subadditivity. F. Wu et al., 1996 and Al-Nowaihi and Dhami, 2011 further explore the non-linear nature of these functions, with Wu proposing a concave-convex model and Al-Nowaihi emphasizing the importance of probability weighting functions in addressing certain stylized facts. Prelec, 1998 and Hantoute et al., 2017 both provide subdifferential characterizations of continuous probability functions.

There are many studies which have explored the factors influencing decision-making in poker. Laakasuo et al., 2015 found that emotional and social factors, such as anger and being watched, can reduce mathematical accuracy. St. Germain and Tenenbaum, 2011 identified differences in decision-making and thought processes between expert, intermediate, and novice players, with experts outperforming the others outperforming the others when given sufficient time to evaluate, but not otherwise. Rubin and Bellamy, 2012and Ponsen et al., 2009 both focused on the strategies used in poker, with Rubin developing case-based strategies and Ponsen analyzing the evolutionary dynamics of these strategies. Nicholson et al., 2006 and Oliehoek, 2005 both explored the use of AI in poker, with Nicholson developing a Bayesian decision network and Oliehoek proposing a unified approach using game theory. Lastly, Findler and van Leeuwen, 1979 and Palomäki et al., 2013 both considered the role of experience in decision-making, with Findler using poker as a vehicle for studying human decision-making and Palomäki finding that experienced players engage in less self-rumination and more self-reflection. Unlike these studies, my paper seeks to abstract away from the nuances of Texas Hold'em (bluffing, implied odds, position at the table, opponent's range) that can overcomplicate the fundamental decision question of how much a player believes their cards are worth.

A range of studies have explored various aspects of poker through experimental methods. Notkin et al., 1988 and Billings et al., 1998 both discuss the use of poker in parallel programming, with Notkin focusing on the Poker Parallel Programming Environment and Billings et al. on decision-making under uncertainty. Félix and Reis, 2008 and Van Essen and Wooders, 2015 both investigate the role of experience in poker, with Félix developing algorithms for online opponent modeling and Van Essen comparing the behavior of experts and novices. Ponsen et al., 2009 and Seale and Phelan, 2010 both use game theory to analyze poker strategies, with Ponsen focusing on the evolutionary dynamics of strategic behavior and Seale proposing a simplified poker game and observing player behavior. Meyer et al., 2013 challenges the notion of poker as a game of skill, finding that card distribution is a more decisive factor. Utilizing my survey and the differences in simple vs complex gamble evaluations, I seek to determine whose conjecture around the importance of poker experience is correct.

Another means by which I could assess prospect theory as a descriptive model is on the aggregate, using data compiled by observer bots over millions of hands. Such was the methodology employed in Social and Psychological Challenges of Poker (Siler 2010), which sought to observe the strategic payoff structures and demographics across different stake levels. While insightful, this paper and others that use large online poker databases have several limitations. There are several factors which these papers cannot not control when evaluating the decisions of players, such as known advantages and other important factors in poker such as stack size, position at table, the mucked cards held by the opponents, and experience level of players. With my experiment, I aim to control for all of these factors to isolate the variables of interest in the decision making process. These principal variables of interest, in the context of sub-proportionality within a probability weighting function, include only a player's preference between different gambles and their certainty equivalent. From these variables, I can tease out what their probability weighting may look like.

3 Theoretical Framework

The exploration of decision-making under uncertainty is deeply rooted in the seminal works of Daniel Kahneman and Amos Tversky, whose introduction of Prospect Theory in 1979 and its later extension, Cumulative Prospect Theory, have revolutionized our understanding of how individuals evaluate risk and make decisions in uncertain environments. These theories offer a compelling departure from the classical Expected Utility Theory, which had long dominated economic thought regarding decision-making under risk.

Expected Utility Theory (von Neumann, Morgenstern 1944) posits that individuals make decisions by evaluating the expected outcomes of their choices, weighted by their probabilities, and selecting the option that maximizes their utility. This theory assumes rational actors with consistent preferences and a linear perception of risk and reward. However, Kahneman and Tversky's Prospect Theory challenges these assumptions, suggesting that individuals are not perfectly rational utility maximizers but are influenced by biases and heuristics. Prospect Theory (Kahneman, Tversky 1979) introduces the concept of value functions, which are defined over gains and losses rather than final wealth states, and are characterized by diminishing sensitivity and loss aversion. This implies that the pain of losing a certain amount is greater than the pleasure of gaining the same amount, leading to risk-averse behavior in choices involving gains and risk-seeking behavior in choices involving losses. Furthermore, the theory highlights the impact of reference points, where outcomes are perceived as gains or losses relative to this reference rather than absolute outcomes.

Cumulative Prospect Theory (Kahneman, Tversky 1992) extends these insights by incorporating a probability weighting function, which captures the tendency of individuals to overweight small probabilities and underweight large probabilities, deviating from the linear probability weighting assumed in Expected Utility Theory. This refinement allows for a more accurate description of decision-making under risk, accommodating a wider range of observed behaviors, including those involving unlikely but impactful events. In his subsequent work, Prelec (1989) formalizes conditions of diagonal concavity[7] (better known as strict monotonicity), compound invariance [8], and subproportionality[9] that must hold to invoke his formulated (one

[7]From Prelec, 1998, A preference structure is *diagonal concave* if "there is no nondegenerate interval $[s,t]$ such that $\succsim$ is quasiconvex and strictly CE-quasiconcave on" intervals sufficiently near to $[s,t]$

[8]From Prelec, 1998, "$\geq$ exhibits *compound invariance* if for any outcomes $x, y, x', y' \in X$, probabilities $q, p, r, s \in [0,1]$, and compounding integer $N \geq 1$: If $(x,p) \sim (y,q)$ and $(x,r) \sim (y,s)$, then $(x', p^N) \sim (y', q^N) \implies (x', r^N) \sim (y', s^N)$."

[9]A definition of *strict subproportionality* says that a function $\pi(p)$ is subproportional if and only if $\log \pi$ is convex in $\log p$

parameter) weighting function:

$$\pi^{+}(p) = \pi^{-}(p) = e^{-(-\ln p)^{\alpha})}$$

Where $0 < p, \alpha < 1$. For the purposes of this paper, I will be assuming the conditions of compound invariance and strict monotonicity in the preference structure of participants, although further work can be done to construct similar experiments which formally test these conditions in poker scenarios. I will be testing the common ratio effect (CRE) (Allais 1953) to assess the validity of sub-proportionality. If subproportionality holds, I will apply the Prelec weighting function above to the gamble preferences observed and determine a range of plausible values of α. I can alternatively relax the sub-proportionality assumption if it does not hold, and need only assume compound invariance to apply the functional form[10] of the probability weighting function proposed by Tversky and Kahneman, 1992 in their work on CPT:

$$\pi^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{\frac{1}{\gamma}}}$$

Their experiment finds a median γ value of 0.61. In this study, I will assume preference homogeneity (representative preferences) and use the median certainty equivalent observed in the experiment to construct a solution set of possible α and γ values, where α is the parameter of the generally accepted power function representation of the value function in the domain of gains: $v^{+}(x) = x^{\alpha}$. A comparison of the shapes of

[10]This functional form is quite useful because it does not assume strictly monotonic preferences, it only has one parameter that needs to be estimated, and it doesn't assert that $\pi(0.5) = 0.5$

these two weighting functions is included below in figures 1 and 2. Another interesting distinction between the two highlighted in the figure is that the Prelec assumes a constant fixed point (where $\pi(p) = p$, and Kahneman and Tversky allow the fixed point to vary. Further work can be done to assess where these fixed points occur in the content of Texas Hold'em.

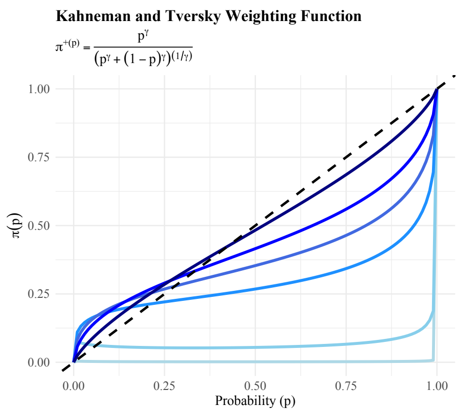

Figure 1: Kahneman & Tversky probability weighting

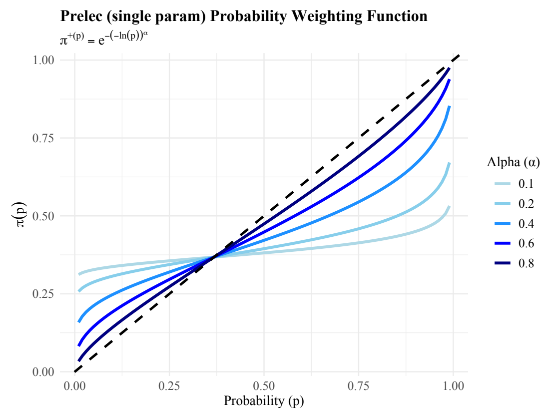

Figure 2: Prelec's probability weighting function

In the setting of Texas Hold'em, these theories are particularly pertinent. The game's structure, with its combination of skill, chance, and strategic interaction among players, mirrors the conditions under which Prospect Theory and Cumulative Prospect Theory provide explanatory power. The decisions made by players, from risk-taking in bets to strategic folds, can be analyzed through the lens of these theories, offering insights into the cognitive processes and biases at play.

In Poker, as well as financial markets, investment decisions, financing decisions, and smaller scale decisions economic agents make on a daily basis, the true probability of outcomes is not given. As such, I want to examine how probability weighting can be adapted for such "difficult" decisions, using probability weighting for "easy" decisions where probabilities are known as a baseline. I control risk preferences and poker experience within subjects to get a better idea both of underlying preferences and how difficult the difficult choices are for the decision maker, and utilize a within-subject framework in order to get more explanatory power out of less sample size.

The experimental design detailed in Section IV was carefully constructed to extend the Allais Paradox to Texas Hold'em. Using a Texas Hold'em solver, I establish Texas Hold'em gambles in which the raw probabilities to win are as near to 100% vs. 98% in decision 1, 10% vs. 9.8% in decision 2, and 1% vs. 0.98% in decision 3 as I can make them. In actuality, the finite nature of the number of outcomes in a Texas Hold'em gamble make it such that it impossible to find gambles with precisely those probabilities, but I got as close as possible, using a Texas Hold'em solver which the curious reader can try out at https://b-inary.github.io/poker/. The true raw probabilities in the Texas Hold'em gambles (derivations in Appendix C) are $P(D1) \in \{0.9808, 1\}$, $P(D2) \in \{0.0985, 0.101\}$ and $P(D3) \in \{0.009796, 0.0106\}$. In each decision, the lower probability corresponds to a payoff of \$3. Thus, the payoff in the 98% gamble is set to \$3. The first decision then asks the participant to come up with a fair certainty equivalent (CE). The certainty equivalent is defined as the minimum certain payoff necessary for the decision maker to prefer receiving the certain payoff to participating in the gamble.

Before making the decision, the participant learns that there is a market dynamics-inspired mechanism in place to promote honesty. A random decimal number between 0 and 3 will be generated to simulate the amount that an imaginary buyer is willing to pay the participant to take their place in the gamble. If that amount is less than the participant's selected certainty equivalent, then there does not exist an imaginary buyer that is willing to pay the participant their certainty equivalent, and so the participant must face the gamble. If the randomly selected number is larger than the certainty equivalent, then there does exist an imaginary buyer willing to pay the certainty equivalent to receive the gamble, and thus the participant receives their certainty equivalent. Although a buyer's willingness to pay is random, the mechanism imposes market discipline on the subject to the point where they should set the price of the gamble closer than they otherwise would to the level where they are indifferent between receiving the price and not.

By definition of certainty equivalent, a decision maker is indifferent between their certainty equivalent and the gamble in question. Thus, for all utility functions $U(.)$, $0.98 \times U(\$3) \sim U(\$CE)$. In the experiment, I then add a small premium ϵ to the certainty equivalent, such that if a participant was indifferent before, it follows that $0.98 \times U(\$3) \prec U(\$CE + \epsilon)$. Expected Utility Theory asserts that if that preference holds, then it still holds for proportionally scaled down probabilities: $\frac{0.98}{10} \times U(\$3) \prec \frac{1.00}{10} \times U(\$CE + \epsilon)$ and $\frac{0.98}{100} \times U(\$3) \prec \frac{1.00}{100} \times U(\$CE + \epsilon)$. CPT and Allais make different predictions. They argue that "for a fixed ratio of probabilities, the ratio of the corresponding decision weights is closer to unity when the probabilities are low than when they are high. This property, called sub-proportionality, imposes considerable constraints on the shape of (the probability weighting function): it holds if and only if (the logarithm of the probability weighting function) is a [strictly] convex function of (the logarithm of the raw probability)" (Kahneman and Tversky 1979, p. 282). Formally, CPT and Allais argues that for a probability weighting function $\pi(p)$, the following behavior should be observed:

$$\frac{\pi(0.98)}{\pi(1)} < \frac{\pi(0.098)}{\pi(0.1)} < \frac{\pi(0.0098)}{\pi(0.01)} < \cdots < 1$$

How does this proposition hold up in poker settings, where probabilities are not explicit? Can these results be generalized to decisions in which probabilities must be inferred? How does the degree of true probability hiddenness, as proxied by a one's experience and familiarity with making these decisions impact the weighting distortions? Can experience even serve as such a proxy? These are the questions I wish to investigate in the following experiment.

4 Experimental Design

4.1 The Survey

Prior to participating in the experiment, the player will complete a Qualtrics survey to illuminate some qualitative behavioral factors. The survey will consist of the following multiple choice questions. Each response is rated based on how much I believe the answer implicates a player's degree of experience or risk tolerance within the context of Texas Hold'em. The contents of the parenthesis after the response indicates how that answer would be weighted when calculating their Poker Experience Scale (PES) and Risk Tolerance Scale (RTS). These weightings, while arbitrary to some degree, allow for a pseudo-ranking of participants based on experience and risk tolerance. Both of these scales become less ambiguous at their endpoints. For example, a PES of 0 implies that the subject has never played poker before, and a maximum PES implies that the subject has played for over 3 years and plays everyday. In the formulation of PES and RTS, the raw scores obtained by the survey responses are normalized to a scale from 0 to 10, with a higher number loosely indicating more poker experience and risk tolerance, respectively.

The first three questions are meant to get an idea of a player's experience playing poker. The last three questions are used to elucidate a player's risk profile as it pertains to poker. After taking the survey, participants will be selected to partake in the Poker experiment, which will enable me to diversify across subjects such that there is representation of all experience levels, as well as risk profiles.

Survey Questions:

1. **How long have you been playing Texas Hold'em?**

    - I've never played (PES+0)
    - Less than 6 months (PES+2)
    - 6-12 months (PES+4)
    - 1-3 years (PES+7)
    - More than 3 years (PES+10)

2. **On average, how often do you play Texas Hold'em?**

    - Daily (PES+20)
    - 2-3 times per week (PES+10)

- Weekly (PES+7)
- Monthly (PES+5)
- Less than Monthly (PES+2)
- Never (PES+0)

3. **Approximately how many hands of Texas Hold'em have you played?**
    - Less than 1,000 hands (PES+1)
    - 1,000-5,000 hands (PES+5)
    - 5,001-10,000 hands (PES+10)
    - More than 10,000 hands (PES+20)
4. **When you believe you have a strong hand, which best describes how do you typically proceed?**
    - I bet aggressively to maximize the pot (RTS+2)
    - I bet cautiously to keep other players in the game (RTS+0)
    - I vary my strategy to keep opponents guessing (RTS+1)
5. **How do you typically react after experiencing a significant loss?**
    - I take a break and analyze my gameplay. (RTS+0)
    - I immediately play another game to try and win back my losses. (RTS+4)
    - I reduce my bet sizes but continue playing as usual. (RTS+1)
    - I do not alter my strategy at all (RTS+2)

6. **How often do you use bluffing as part of your Texas Hold'em strategy?**

   - Frequently, I consider bluffing an essential part of the game. (RTS+4)
   - Occasionally, when the situation seems right. (RTS+2)
   - Rarely or never, I prefer to play based on the strength of my hand. (RTS+0)

Experiment Procedure

Introduction script: "Thank you for your time in participating in my experiment. The purpose of this experiment is to ascertain how we make decisions and weigh choices without having full information available. The medium for this analysis is Texas Hold'em, with a few simplifying modifications. The procedure will be as follows:

You will be presented with a series of 2 scenarios, which you must choose between. There is no calling, no folding, no re-raising or going all in. Rather, your opponent (me) will offer you $X if you can beat me in the scenario. If we tie, there will be no splitting the money; you must make a strictly better hand than me to gain the reward. Every card with a question mark in the scenario is a card that we will randomly draw from the set of all unrevealed cards.

The decisions 1 and 4 will be presented will be slightly different. You will be faced with a gamble, and a sure-thing. For these decisions, consider that you are in possession of the gamble, and there exists a buyer that wants to purchase the gamble from you. The decision asks you to determine the minimum amount you would accept to give up the gamble. This experiment is constructed for you to be as honest in your answer as possible. After you have made all of your choices, we will play the chosen scenarios and you will walk away with whatever money you have won. For clarity, we will begin with an example."

The sample decision below is an example, included to provide clarity and an opportunity for the subject to ask questions before making a series of six decisions that will be recorded.

Example Decision:

Scenario 0A: (Figure 3) The opponent offers \$3 if you can beat them.

Scenario 0B: (Figure 4) You face a guaranteed win scenario but must decide the minimum amount you would accept instead of taking the gamble.

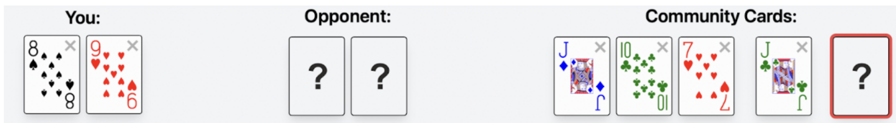

Figure 3: Scenario 0A

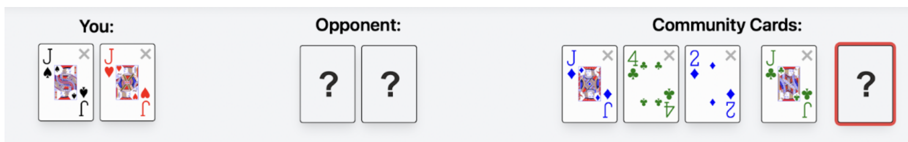

Figure 4: Scenario 0B

Decision 1: Certainty equivalent evaluation

Scenario 1A: (Figure 5) The opponent offers \$3 if you can beat them.

Scenario 1B: (Figure 6) You face a guaranteed win scenario but must decide the minimum amount you would accept instead of taking the gamble.

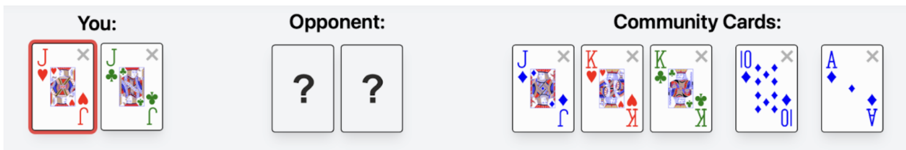

Figure 5: Scenario 1A

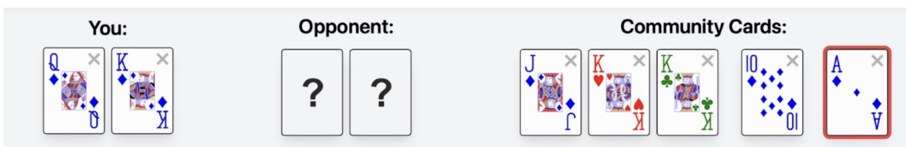

Figure 6: Scenario 1B

Decision 2: Preference between two gambles

Scenario 2A: (Figure 9) The opponent offers \$3 if you can beat them.

Scenario 2B: (Figure 10) The opponent offers you a payoff slightly higher than your certainty equivalent, P_1, if you can beat them.

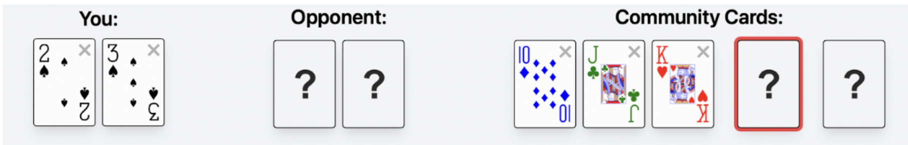

Figure 7: Scenario 2A

Figure 8: Scenario 2B

Decision 3: Preference between two gambles

Scenario 3A: (Figure 9) The opponent offers \$3 if you can make a full house (see hand rank reference sheet) after the first three community cards are revealed.

Scenario 3B: (Figure 10) The opponent offers you a payoff slightly higher than your certainty equivalent, P_1, if you can beat them.

Figure 9: Scenario 3A

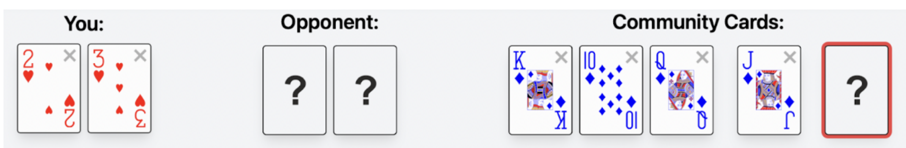

Figure 10: Scenario 3B

1. **Decision 4: Certainty equivalent evaluation**

   Scenario 4A: Win \$3 with a probability of 98%.

   Scenario 4B: Win a certain amount (C) less than \$3 with a probability of 100%. Participants are asked to specify the smallest amount C they would accept to choose Scenario B over Scenario A.

   After specifying an amount, a random decimal number between 0 and 3 is generated to determine whether a random buyer is willing to pay the subject their certainty equivalent for the \$3 gamble.

2. **Decision 5: Preference between two gambles**

   Scenario 5A: Win \$3 with a probability of 9.8%.

   Scenario 5B: Win a payoff slightly higher than your certainty equivalent, P_2, with probability 10%.

3. **Decision 6: Preference between two gambles**

   Scenario 6A: Win \$3 with a probability of 0.98%.

   Scenario 6B: Win a payoff slightly higher than your certainty equivalent, P_2, with probability 1%.

Participants are asked to make their choices based on the given scenarios. Only the poker scenarios are played for money. In the preference between two gambles decisions, the premium is calculated as $P_i = \$(CE + \epsilon)$, where $\epsilon = \frac{3-CE}{10}$.

5 Empirical Data

Table 1: Summary of Survey Variables used in analysis

Label	**Type**	**Description**	**Interpretation**
Respondent_ID	Numeric (Integer)	A unique key for the subject	The first respondent will have ID = 1, the second will have ID = 2, and so on.
PES	Numeric (Integer)	Calculated as the normalized sum of their response scores in questions 1-3.	A score of 0 indicates low experience belief, a score of 10 indicates high experience belief.
RTS	Numeric (Integer)	Calculated as the normalized sum of their response scores in questions 1-3.	A score of 0 indicates low risk tolerance belief, a score of 10 indicates a high risk tolerance belief

Table 2: Summary of Experimental Data Labels

Label	**Type**	**Description**	**Interpretation**
Difficult CE	Numeric (Floating point)	The certainty equivalent chosen by participant in Decision 1 (difficult frame)	quantitatively determines a subject's risk aversion in Texas Hold'em
Easy CE	Numeric (Floating point)	The certainty equivalent chosen by participant in Decision 4 (easy frame)	Closer to EV implies closer to risk neutral preferences in general.
CE difference	Numeric (Integer or Floating Point)	Difference between easy and difficult CE	Captures the effect of hidden probabilities. Larger difference implies more probability distortion.
Decision 2	Categorical (Binary)	A choice variable, 0 (1) if subject chose scenario A (B)	Captures the participant's choice in Decision 2.
Decision 3	Categorical (Binary)	A choice variable, 0 (1) if subject chose scenario A (B)	Captures the participant's choice in Decision 3.

Continued on next page

Table 2 – continued from previous page

Label	**Type**	**Description**	**Interpretation**
Decision 5	Categorical (Binary)	A choice variable, 0 (1) if subject chose scenario A (B)	Captures the participant's choice in Decision 5.
Decision 6	Categorical (Binary)	A choice variable, 0 (1) if subject chose scenario A (B)	Captures the participant's choice in Decision 6.
D2equalsD3?	Categorical (Binary)	A binary variable, with 1 indicating that the participant either the riskier gambles twice or the safer gamble twice.	If 1, then proportionality holds in the difficult frame.
D5equalsD6?	Categorical (Binary)	A binary variable, with 1 indicating that the participant either the riskier gambles twice or the safer gamble twice.	If 1, then proportionality holds in the easy frame.

I initially wished to analyze differences in these variables for subjects with different PES and RTS, as well as the potential mixed effects of PES and RTS. I hypothesized that PES and RTS are correlated to some extent. This is due to the fact that Poker is a game of gambles. Thus, one may expect that there is a certain degree of risk seeking associated with developing poker experience. I later determined that either my RTS was too crude, or that risk preference in general aren't explanatory enough at such low stakes. With respect to the other variables being collected in the experiment, I include a brief overview of my initial hypotheses of how these variables may differ between more and less experienced players prior to running the experiment:

Difficult CE: Experienced players are more likely to deduce a more accurate weighted probability, so I expect *Difficult CE* to be nearer to *Easy CE* for more experienced players.

Easy CE: *Easy CE* is more dependent on the risk preferences of a participant. If a participant is more risk seeking, then their certainty equivalent will be relatively lower to account for the marginal utility that the gamble itself provides.

CE difference: As mentioned above, since I expect more experienced players to be more accurate at determining the probability in the difficult frame, I expect a *CE difference* nearer to 0 for more experienced players.

Decision 2,3,5,6: I expect these binary variables to be independent of experience level. I would expect a more risk averse participant to have a higher frequency of 1's across these decisions as a value of 1 represents a preference towards the less risky prospect. Similarly, I would expect a more risk loving participant to have a higher frequency of 0's.

D2equalsD3?: This is a principle variable of interest, as CPT and EUT do not yet explain whether Allais paradox holds or not for hidden probabilities. If it does largely hold, then a higher proportion 0's will be observed, specifically 0's which come about from a movement from 1 to 0. If EUT is a better descriptive model, then I expect to see a higher frequency of 1's.

D5equalsD6?: CPT and EUT make different assertions about what the value of this binary variable should be. CPT expects a higher frequency of 0's, while EUT asserts that the value should always be 1 for a rational decision maker.

The key variables of interest are *CE difference*, the frequencies of 0's and 1's in *D2* and *D3*, and the degree of concordant and discordant with-in subject pairs (D2,D3) and (D5,D6) in order to analyze trends preference consistency as probabilities scale down proportionally. It will be further useful to decompose the group of discordant pairs into positive and negative discordant subgroups. Sub-proportionality and the CRE both suggest that as probabilities scale down, preferences tend to shift in favor of the riskier, higher-payoff gamble. Thus, theory predicts the frequency of negative discordant pairs (subjects that transition from L (1) in D2 and D5 to H (1) in D3 and D6). For similar reasons, the difference between the frequencies of 0's and 1's in *D2* and *D3* will determine whether or not the CRE holds for decisions when probabilities are non-explicit. Allais predicts a shift in preferences towards the riskier gamble when probabilities scale proportionally down, which translates to a higher proportion of 1's in *D2* than in *D3*. *CE difference* will also serve as a heuristic for the the effect of decision difficulty on probability weighting when probabilities are near certainty. I can then observe the extent to which the heuristic for subject experience, PES, explains the differences between the two certainty equivalents and, by the same token, how well experience can serve as a proxy for gamble difficulty and probability subjectivity.

6 Results

6.1 Experiment Iteration 1

Let's first begin with an overview of the observed survey responses (see Table 3). The survey responses collectively indicate a broad profile of participants, revealing a wide spectrum of engagement with the game, from novices to seasoned veterans. The mode of 10 in *Exp Q1* reveals that a significant proportion (16 of 46) of respondents have been playing for more than 3 years, suggesting a dedicated core of players who likely possess a deeper strategic understanding of the game. This is further corroborated by a subset of players reporting frequent play and a considerable number of hands played, pointing to a high level of commitment and ongoing interaction with the game. Despite this, the overall frequency of play skews towards less regular engagement, with many participants playing monthly or less. This diversity in play frequency indicates that the study attracted both casual players and those who play often. The broad range of engagement levels suggests varied motivations for playing, from casual entertainment to more skill-based interests.

When examining risk preferences and reactions to game situations, the data reveal a general preference for a conservative approach to poker decision making. Participants reflect that they tend to proceed cautiously with strong hands, perhaps indicating general preferences skewed toward risk aversion. This cautiousness extends to reactions following significant losses, where a predominant strategy involves

Table 3: Descriptive statistics

Survey Data: Poker Context					
	Survey Response Weights (Interpretation)				
Survey Questions	**Median**	**Mode**	**Max**	**Min**	**SD**
Exp Q1: Years of Poker Experience	7(1-3yr)	10(>3yr)	10(>3yr)	0(none)	3.47
Exp Q2: Frequency of poker play	5(mth.)	2(<mth.)	20(daily)	0(never)	3.91
Exp Q3: Number of hands played	5(1-5k)	1(<1k)	20(>10k)	1(<1k)	7.32
Risk Q1: Behavior when strong	1(varies)	1(varies)	2(aggr.)	0(cautious)	0.76
Risk Q2: Behavior after big loss	1(bet small)	1(bet small)	4(chase loss)	0(take break)	1.67
Risk Q3: Bluffing tendency	2(occ.)	2(occ.)	4(freq.)	0 (rarely)	1.51
Poker Experience Scale (PES)	3.6	1	10	0.2	2.88
Risk Tolerance Scale (RTS)	4	4	10	0	2.57

Experiment Data: Certainty Equivalents					
	Certainty Equivalent Choices (USD)				
Certainty Equivalents	**Mean**	**Median**	**Max**	**Min**	**SD**
Easy Frame CE	2.90	2.94	3	1.60	0.20
Poker Frame CE	2.65	2.77	2.97	1.8	0.31
Within subjects CE Difference	-0.2519	-0.15	0.4	-1.14	0.317

Experiment Data: Binary Choice Variables ($H = 3$, $L = CE + \epsilon$)	Frequency Statistics (N=46)	
Poker Gamble Choices	**Preferred H**	**Preferred L**
D2: \$H with 9.8% *vs.* \$L with 10%	26(56.52%)	20(43.48%)
D3: \$H with 0.98% *vs.* \$L with 1%	13(28.26%)	33(71.74%)
Simple Gamble Choices	**Preferred H**	**Preferred L**
D5: \$H with 9.8% *vs.* \$L with 10%	15 (30.43%)	32 (69.57%)
D6: \$H with 0.98% *vs.* \$L with 1%	36 (78.26%)	10 (21.74%)

Experiment Data: Concordance				
	Discordant (Not EUT)		Concordant (EUT)	
Variables	**H to L**	**L to H**	**H to H**	**L to L**
Poker Consistency from D2 to D3	20(43.48%)	7(15.22%)	13(28.26%)	6(13.04%)
Simple Consistency from D5 to D6	0(0.00%)	22(47.83%)	10(21.74%)	14(30.42%)

taking a break to analyze game-play or reducing bet sizes, strengthening the case for a skew towards risk aversion, though testing such risk aversion ended up not being explicitly part of the analysis, in part due to the inconsequential payoffs at stake that would necessitate a subject's utility function to exhibit exceedingly strict convexity on all non-degenerate intervals, implying unrealistic risk preferences as payoffs scale larger (Fullenkamp et al. 2003).

The summary of the concordance table suggests that CPT and the Common Ratio Effect serve as better descriptive model for the simple scenarios than the poker gambles. Interestingly, the exerpiment did not observe a single participant switch their preferences from the riskier to safer gamble as probabilities scaled down in the safer frame, but the vast majority (74.07%) of subjects did so in the poker frame. To test the significance of this result, considering the paired nature of our data, I construct a 2x2 contingency table using the discordant and concordant values from the table and conduct McNemar's test, which yields a p-value of 0.021 for pairs (D2,D3) and a p-value of 7.562×10^{-6}. These p-value indicates the probability of observing the test results under the null hypothesis, which, in the context of McNemar's test, is that there is no difference in the discordant proportions (i.e., the changes from one condition to the other are symmetric). As such, there is a statistically significant movement in the proportions of preferences towards risky vs. safe gambles in both the poker and simple frame. The unexpected result that I subsequently set off to explore, unable to be captured by McNemar's non-directional test, yet evident in Table 3 is that the shift in preferences between the two frames moves opposite directions.

The statistical summary of certainty equivalents (CE) from the experiment reveals insightful variations in decision-making under risk across the two framing conditions. The average CE for the Easy Frame condition is significantly higher than for the Poker Frame, with means of \$2.90 and \$2.65, respectively, suggesting that participants generally demanded less premium to forgo a gamble in the former setting. This disparity in CE valuations is further highlighted by a negative mean difference (-0.2519) when comparing CE across frames within subjects, indicating a systematic reduction in the CE under the Poker Frame. Given the data entries are within subject, and thus dependent, I test the comparison of Easy CE and difficult CE distributions using the Wilcoxon Signed-Rank Test and found a P-value of $1.2 * 10^{-6}$ and a test statistic of 875.50, which is to say that there is an overwhelming amount of evidence to reject the null hypothesis that the difference between two observations has a mean signed rank of 0.

Interestingly enough, I measured the correlation between the Easy Frame CE and Difficult Frame CE and found a correlation coefficient of 0.2815, I tested this coefficient using Pearson's product-moment correlation and obtained a 95% confidence interval for the true correlation to be between -0.009 and 0.528 with a p value of 0.058, which suggests a 5.8% of observing the data under the null hypothesis that the two certainty equivalents are uncorrelated. There isn't enough data however to suggest that there is a statistically significant relationship between the two. The total distributions of observed certainty equivalents is visualized in Figure 11. The first immediate takeaway is that the distribution of Easy CE is much tighter than that of difficult CE. While I had initially hoped that Easy CE would be a better control for risk aversion, it was evident very quickly into my experiment that at such low stakes, participants were very risk neutral, and would easily calculate the expected value of a 98% chance of $3. The wider distribution, skewed well below the expected value, of difficult CE may underscore the influence of framing on risk perception and decision-making, where the Poker Frame possibly evokes stronger feelings of risk or uncertainty, leading to more conservative valuations. This behavior aligns with the concept of ambiguity aversion (Ellsberg 1961), where individuals exhibit a preference for known risks over unknowns, potentially explaining subjects consistently selecting a smaller CE in the poker frame than in the simple frame. The variations in standard deviations between the two frames (0.20 for Easy Frame and 0.31 for Poker Frame) also suggest greater variability in participants' risk assessments and decision-making processes under the Poker Frame, further indicating the impact of framing on ambiguity aversion tendencies.

Perhaps the most surprising result is found in the frequency statistics of the binary choice variables in Table 3. I have highlighted in the table an apparent violation of the common ratio effect (Allais 1953), evident in the contrasting preferences between D2 and D3. While the decision outcomes for Poker Decision 2 showed a slight majority (56.52%) favoring the riskier option, when true probabilities scale down by a factor of 10 in Poker Decision 3, I observe a pronounced preference shift in favor of the safer prospects (71.74%). The common ratio effect posits that preferences should shift in the other direction; that a greater proportion of the subjects should prefer the riskier option as probabilities scale proportionally down nearer to zero. Indeed, when true probabilities are known, as in Simple Decision 5 and 6, the common ratio

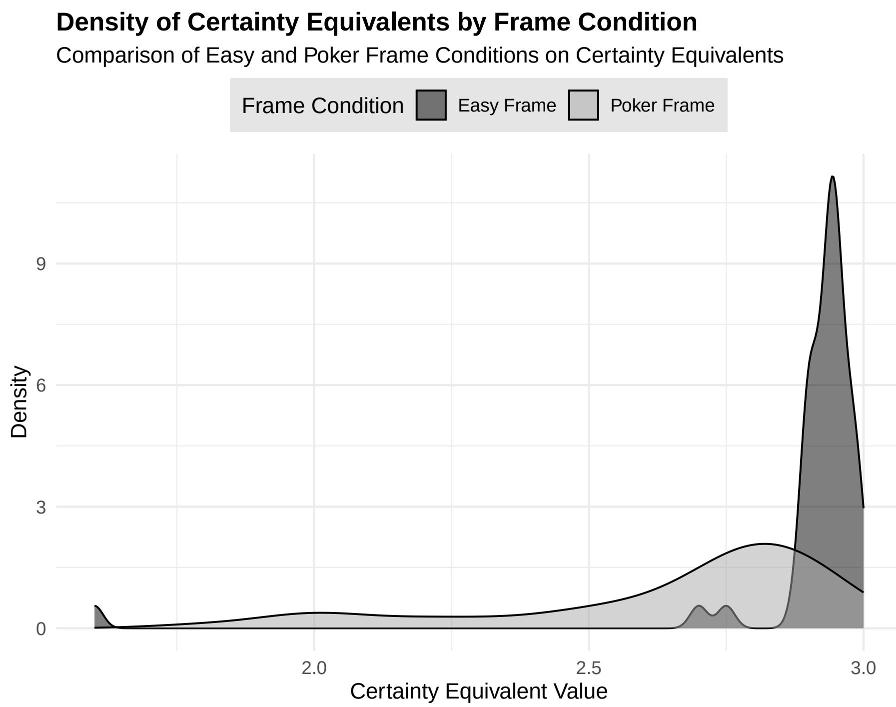

Figure 11: Density Plot of Certainty Equivalents: Easy vs. Poker Frame

effect prevails significantly, with preferences initially favoring the safer gamble by a large majority in D5 (69.57%) when $p_{risky} \approx 0.098$ and $p_{safer} \approx 0.1$, but a larger majority (78.26%) favoring the riskier, higher-payout in D6 when $p_{risky} \approx 0.0098$ and $p_{safer} \approx 0.01$.

6.2 Framing Effects and Perceived Probabilities

So why then does the opposite effect take place in the poker frame? There are several possible judgement heuristics explained by framing effects. The first I will discuss is representativeness (Kahneman & Tversky 1974), which can bias a decision maker to be more pessimistic about sample outcomes that are less representative of the population. Success in the full house scenario (Figure 7) implies a precondition of outcome homogeneity. In the classic "gambler's fallacy", the representativeness heuristic biases the decision maker into believing that in light of the last 8 roulette spins resulting in outcome black, it is likelier than usual that the next outcome will be black, the rationale being that the latter outcome would make the total observed sample more representative of the general population. Poker is a little bit different, because with respect to rank there are only 4 of each type in the set of all outcomes. So now, the occurrence of an outcome type directly reduces the likelihood of simultaneously observing another instance of that type, potentially making the decision maker even more pessimistic. Subjects may view a community card outcome which results in a full house in scenario 3A (Figure 7), such as $(7\clubsuit, 7\blacklozenge, 7\spadesuit)$, to be less likely than outcomes that do not result in a full house, because the former is less representative of the population of all unrevealed cards. This of course is a fallacy, as all community card outcomes are equally likely, but it does provide an explanation for why such a violation may occur. By contrast, they may view a winning outcome in scenario B, for example opponent holding $(8\clubsuit, 4\spadesuit)$ and a revealed community card $(2\clubsuit)$ to be more more representative of the underlying population of all unrevealed cards.

Another judgement heuristic potentially at play is availability (Kahneman & Tversky 1974). When subjects have a certain degree of familiarity with a prospect, they reason about how available a memory of a recent success is as a heuristic for how prospect success frequency. As such, it is possible that the memory of flopping[11] a full

[11] The *flop* refers to the first three community cards in Texas Hold'em. It is common terminology in Texas Hold'em that "flopping" a certain holding means to become endowed with 5 cards that

house, as in 3A, may be less available than the other poker scenarios, as flopping a full house is not captured in the objective function of poker. This may lead to the event's occurrence being relatively less salient than prospects like 3B, where success is framed as having a better hand than your opponent at showdown, more conducive with what success entails in a real poker environment. Thus, an experienced subject would have been conditioned with payoffs each time they experienced a success comparable to success 3B, making them more acutely aware of and better poised to estimate the true frequency of the success they've observed.

The final kind of judgement heuristic mentioned outlined by Kahneman & Tversky (1974) is adjustment and anchoring[12], but this heuristic tells the least compelling story about why 3A is more prone to bias of this type than the other scenarios are.

I realized at this point in my analysis that if I wanted to better understand the framing effect in my data, I would need to alter my experiment to better parse the 2 phases of the subject's decision making and isolate the phase in which framing is believed to take place.

6.3 Experiment Iteration 2

Prospect Theory (Kahneman & Tversky 1979) asserts that when we make decisions, we split the decision into 2 phases: an editing phase and an evaluation phase. Framing effects take hold on the decision maker in the editing phase, when decision makers organize the available information into well packaged "frames" that are easier

jointly form the holding after the first 3 and only first 3 community cards have been revealed.

[12] adjustment and anchoring is a bias by which decision makers will anchor on an initial information endowment, and then adjustment around the anchor until a plausible probability comes to mind

for them to evaluate in the evaluation phase. To separate these phases and better isolate the framing effect, I altered my experiment for a second iteration, this time asking subjects what their perceived probabilities of success were in each scenario of the poker frame before they make their decision. I conducted this updated experiment on 30 subjects, and a summary of the observed perceived probabilities are reported in Table 4.

Table 4: Summary Statistics of Perceived Probabilities in Poker Scenarios

Scenario	Mean	Median	Max	Min	SD
2A	0.0503	0.045	0.12	0.01	0.0299
2B	0.062	0.055	0.15	0.01	0.0349
3A	**0.004**	**0.005**	**0.01**	**5e-05**	**0.0035**
3B	0.012	0.01	0.03	2e-04	0.0078

It is important to note that in the second iteration of the experiment, I did not observe any significant trend shifts in the proportions of the binary variables from the first iteration of the experiment. As evidenced in the table, the average participant believed their probability of success in scenario 3B (0.012) to be three times likelier than their probability of success in scenario 3A (0.004), despite the true probabilities of success in those two scenarios differing by only 0.002. It is also interesting to note that there were subjects who underestimated the probability of success in 3A by a factor of 100. After conducting a power analysis for a one-sample t-test of the observed sample mean against a hypothesized population mean μ, a power level of 0.8 to reject the null with 95% confidence is attained after setting $\mu = 0.006$, and a power level of 1 to reject the null with 95% confidence is attained after setting $\mu = 0.0081$, well below the true probability of 0.0098. That is to say that there is sufficient sample size to suggest that the true average perceived probability in scenario 3A is well below the true probability of 0.0098.

One important consideration to touch on here is that the maxmin expected utility with non-unique priors model[13] (Gilboa & Schmeidler, 1989) posits there exists a set of probabilities which individuals consider possible, but when asked to evaluate a gamble, they pick the worst possible probability from that set. Thus, subjects consistently underestimating the true probability of a poker gamble may be a manifestation of this phenomenon, attributable to ambiguity aversion considerations (Ellsberg 1961) as opposed to something inherent only to poker. Since I did not explicitly ask subjects for a point estimation, which would involve them selecting the mean of their set of believable probabilities, I cannot know whether the believed probability represents the mean, the lower bound, or any other point of this set. The only thing I am able to assume is that their specified probability belief is included in the set.

I test the statistical significance of the difference in percent deviation between believed and true probability (measured as $\frac{p_b - p}{p}$) between scenarios using a piecewise Friedman test, and find significant p-values between all scenario pairs except for one. Interestingly enough, this pair is (Scenario 2A, Scenario 3A), with a p-value of 0.8527. The interpretation is that there is insufficient evidence to suggest that the percentage difference between believed and true probabilities in scenario 3A and 2A come from different underlying populations. Thus, there is insufficient evidence to suggest that the salient full house frame presented in scenario 3A, theorized to

[13] The *maxmin EU with non-unique priors model* proposes that the optimal decision under uncertainty is the one with the least bad worst outcome, in this case meaning the least worst case probability.

be responsible for the paradigm shift in preferences in the poker gambles, affected the subject's accuracy in estimating a probability belief as a percentage of the true probability between the proportional risky gambles.

With the added information of what the average perceived probabilities are in scenarios 2A, 2B, 3A, and 3B, I attempt to control for the framing effect in scenario 3A and determine now how the observed perceived probabilities are weighted by the functional form of the Kahneman and Tversky (K&T) probability weighting function $\pi^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{\frac{1}{\gamma}}}$. Taken together with the generally accepted value function in the domain of gains, $v^{+}(x) = x^{\alpha}$ (Kahneman & Tversky 1992), and assuming relative preference homogeneity among poker players, the observed preferences in the experiment aggregate to a representative preference structure that satisfy both

$$3^{\alpha} * \frac{0.0503^{\gamma}}{(0.0503^{\gamma} + (1-0.0503)^{\gamma})^{\frac{1}{\gamma}}} > (CE+\epsilon)^{\alpha} * \frac{0.062^{\gamma}}{(0.062^{\gamma} + (1-0.062)^{\gamma})^{\frac{1}{\gamma}}}$$

in Decision 2 and

$$3^{\alpha} * \frac{0.004^{\gamma}}{(0.004^{\gamma} + (1-0.004)^{\gamma})^{\frac{1}{\gamma}}} < (CE+\epsilon)^{\alpha} * \frac{0.012^{\gamma}}{(0.012^{\gamma} + (1-0.012)^{\gamma})^{\frac{1}{\gamma}}}$$

in Decision 3. Taking $CE + \epsilon$ to be the observed median $CE + \epsilon = \$2.79$, I attempt to obtain a general solution set of feasible parameters as (γ, α) pairs in Figure 12.

It should be noted that the preference homogeneity assumption is perhaps a heroic one in this case, but it is a cornerstone of general equilibrium theory, and it allows us to make a general conjecture about at least a subset of possible feasible (γ, α) pairs. This figure suggests a γ between approximately 0.25 and 0.5, well below

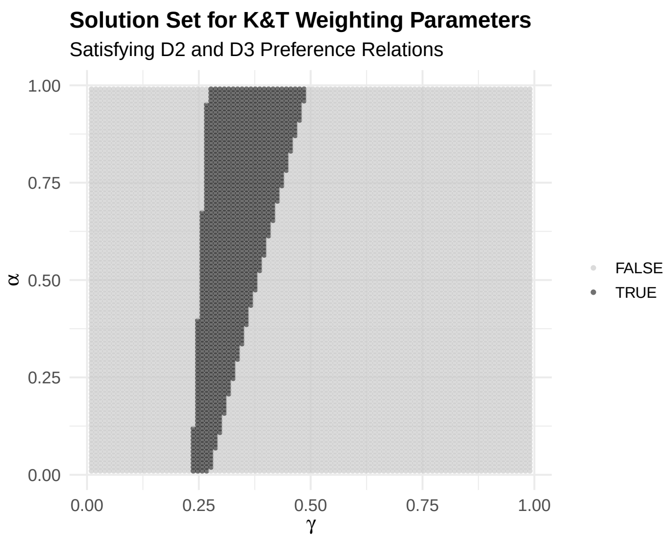

Figure 12: Solution Set of K&T Weighting and Value Function Parameters

the traditionally assumed value of 0.61 (Tversky Kahneman 1992 p. 312). The interpretation of this result is that given the underestimation of the true probability due to the framing of the gambles, participants actually overestimate the likelihood of their perceived probability occurring.

6.4 Quantifying Noise due to Framing

There is a fair degree of noise in my analysis of the framing effect. In a noble yet over-simplistic attempt to quantify the noise of the poker gambles, I define a framing function $F(p, N) = p_b$ that takes inputs hidden true probabilities p and a noise quantifier N, whose magnitude is the extent to which p is hidden due to framing, and maps them to a probability belief p_b. When thinking about how to model this function, there are a few considerations to keep in mind. Firstly, I would expect $R : \{p_b | 0 < p_b < 1, p_b \in \mathbb{R}\}$. Secondly, I would expect $|p_b - p|$ to be increasing in $|N|$. To keep it relatively simple while still satisfying the nonlinear nature of psychological biases and boundedness, I define my Framing function $F(.)$ to be

$$F(p, N) = p_b = logit^{-1}(logit(p) + N)$$

Where $logit(p) = \log \frac{p}{1-p}$ and $logit^{-1}(x) = \frac{1}{1+e^{-x}}$. Taking the logistic transformation of both sides of this equation and rearranging gives $N = logit(p_b) - logit(p)$. In other words, I propose a simple heuristic for noise to be the difference between the log odds of the believed probability and the log odds of the true probability. I Of course, this is certainly a over-simplification of a very complex phenomenon, but for the purpose of my study, it serves as a useful heuristic for the hiddenness of the true probability due in part to the framing effect. In truth, perhaps a more accurate description of what it being captured by this heuristic is the degree of "wrongness" in probability belief due to the framing effect.

6.5 Experience as a proxy for gamble difficulty

Now to return to one of the central questions my paper seeks to answer, which is how Poker Experience implicates the difficulty of a poker gamble? Using the noise heuristic N defined above, I regress N on PES to check if there is any significant relationship between the two variables. Starting with poker scenario 2A, observe the results of the regression in Table 5.

While there is an incredibly low P-value on both the intercept and PES coefficient, the R-square and adj. R-squared suggest only a moderate amount of noise variation explained by PES. Be that as it may, the F-statistic of 19.75 on 1 and 28

Table 5: Linear Regression Results of Noise Heuristic N on PES in Scenario 2A

Statistic	Estimate	Std. Error	t value	P-value
Intercept	−1.895 40	0.241 95	−7.834	0.001
Coefficient (PES)	0.199 01	0.044 78	4.444	0.001

Residual standard error: 0.5921 on 28 degrees of freedom

Multiple R-squared: 0.4136, Adjusted R-squared: 0.3926

F-statistic: 19.75 on 1 and 28 DF, p-value: 0.0001266

DF taken together with the significant P-value indicates evidence sufficient to reject the null hypothesis that the PES coefficient is 0 at the 1% significance level. Thus, the regression model, oversimplified as it may be, reveals statistically significant relationship between the noise heuristic and PES. Also important to note is the fact that the noise is negative. This is the case because in scenario 2A, the believed probability was consistently and significantly below the true probability. To better visualize what is going on, I have included a plot of the linear regression in Figure 13, where one can verify a significant trend of the magnitude of N approaching 0 as PES increases from 0 to 10.

I also would like to test how this model behaves in the full house scenario in decision 3, where I suspect framing to be the culprit behind the subjects' difficulty in estimating the true probability of success. I run the same regression but instead use the N observed within each subject in scenario 3A[14] and report the results in

[14] I also run the corresponding regressions for scenarios 2B and 3B, and find a significant p-value and F-statistic in scenario 2B, but insignificant p-values and F-statistic in 3B

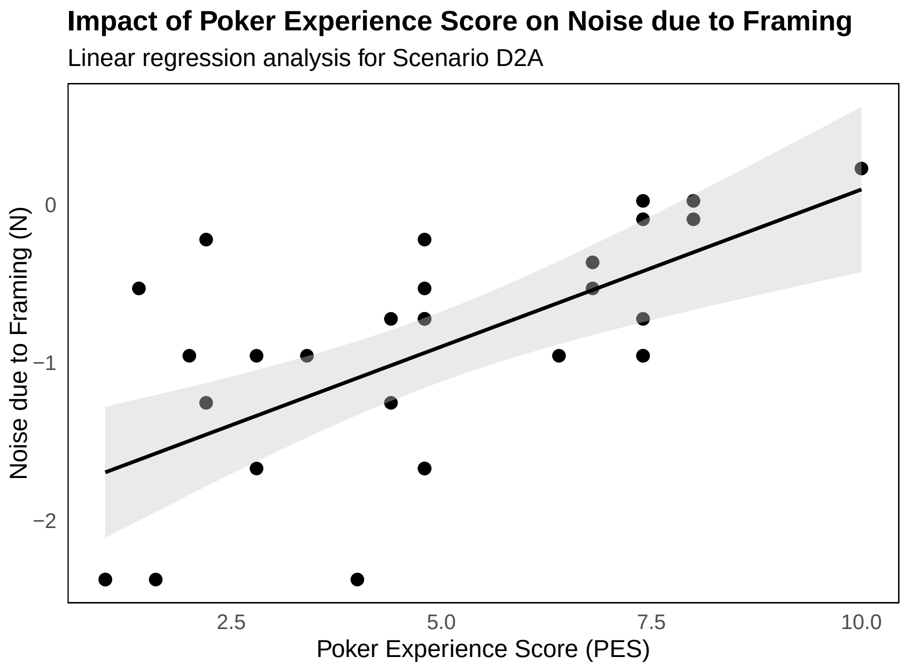

Figure 13: Linear regression of Noise Heuristic N on Poker Experience Scale

Table 6. A quick observation of the table reveals that PES has lost a great deal of predictive and explanatory power on the noise heuristic. The smaller estimate of the coefficient for PES also suggests that an increase in poker experience does not reduce the magnitude of the noise in scenario 3A by as much as it did in 2A. In other words, the noise in scenario 3A is ”stickier” with respect to a change poker experience. To better illustrate what happened, I visualize the scenario 3A regression results in Figure 14.

What is revealed by Figure 14 is that there seem to be 3 outlier points with $N < -4$ that are hindering the goodness of fit of the regression. While it would be nice to simply remove these outliers, doing so would be premature. The mean N in scenario 3A is -1.38 and the standard deviation is 1.427. Taken together with the

Table 6: Regression Analysis Summary for Scenario 3A

Statistic	Estimate	Std. Error	t value	P-value
Intercept	−1.9896	0.5793	−3.435	0.001 87**
Coefficient (PES)	0.1263	0.1072	1.178	0.248 68

Residual standard error: 1.418 on 28 DF
Multiple R-squared: 0.04723, Adjusted R-squared: 0.0132
F-statistic: 1.388 on 1 and 28 DF, p-value: 0.2487

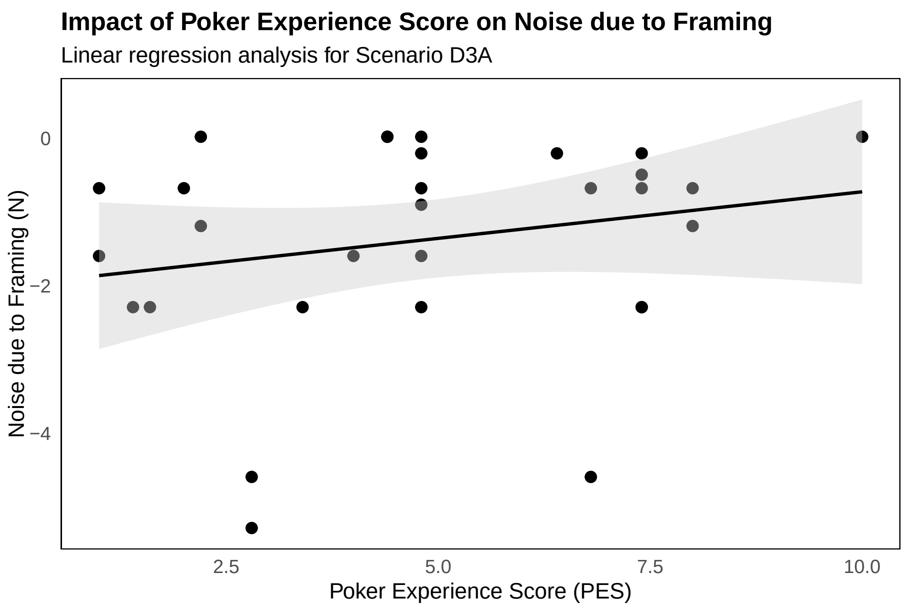

Figure 14: Linear regression of Noise Heuristic N on Poker Experience Scale

noise N observed in the 3 apparent outliers, I obtain 3 z-scores of -2.25, -2.739, -2.253. There may be an argument to be made for removing these outliers if $|z| > 3$, but the observed z-scores taken together with the information that scenario 3A had sufficient framing effects to violate Allais's common ratio effect in the experiment, I would deem these data-points to be true outliers, which to remove would be irresponsible.

The final result which I would like to comment on is the relationship between Poker Experience Scale and the difference between the Easy and Difficult certainty equivalent (CE Difference). Intuitively, I would expect CE Difference to be decreasing in PES. The more experience someone has with Poker, the better I would expect them to be at assessing the true probability of success in D1, and thus, the closer I would expect their certainty equivalent in the poker frame to be to their certainty equivalent with complete information. I light of observing a general trend of concavity in the plot of CE difference vs. PES, I decided to move forward with a logarithmic regression model. I report the findings of the logarithmic regression in Table 7.

Table 7: Logarithmic Regression Results on the Relationship Between PES and diff_CE

Variable	**Estimate**	**Std. Error**	**t value**	**P value**
Intercept	-0.4049	0.0683	-5.93	< 0.001 ***
log(PES)	0.1471	0.0517	2.85	0.0069 **

Residual standard error: 0.2918 on 44 DF
Multiple R-squared: 0.1734
Adjusted R-squared: 0.1546
F-statistic: 9.227 on 1 and 44 DF, p-value: 0.004001

Similar to the regressions on PES in Table 6 and Table 5, I observe very significant p-values for the intercept and coefficient but low R-squared values, indicating a low amount of the variance of CE difference can be explained by PES. That said, the F-statistic of 9.227, well above the 1% critical value of 7.248 on 1 and 44 DF, together with the significant p value, asserts that we can reject the null hypothesis that there is no relationship between PES and CE difference with 99% confidence. Visualizing this regression in Figure 15 provides some contextualizing information.

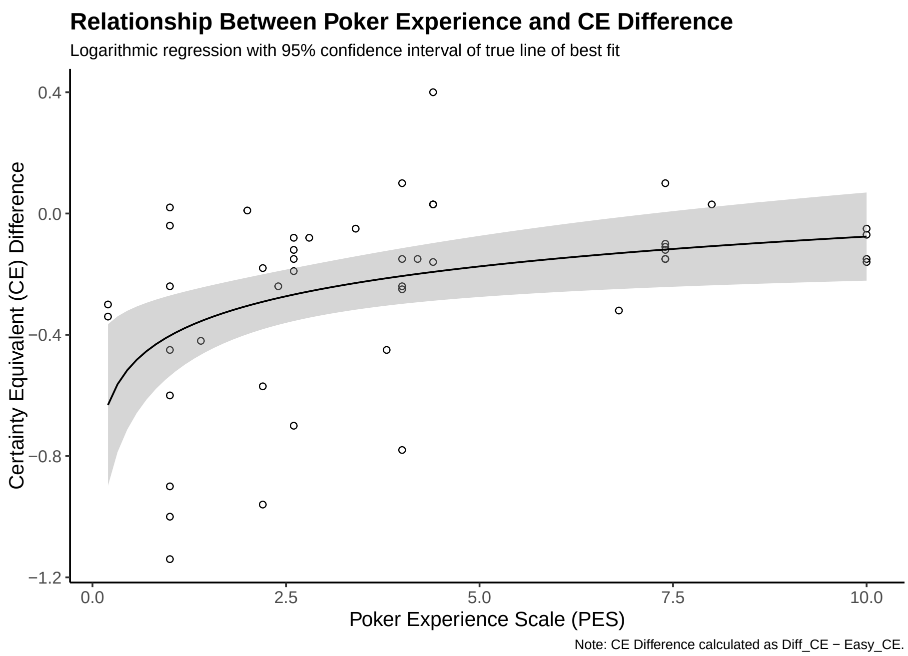

Figure 15: Logarithmic regression of CE Difference on PES

The logarithmic regression suggests positive and diminishing returns to evaluating poker probabilities. Additionally, the graph shows that the variance between subject's CE difference and the CE difference that the logarithmic regression model predicts decreases as PES increases. This is a rather intuitive result, as one might expect that a poker players accuracy in assessing the true probability of a poker gamble improves with time. I would further assume that this model converges to a CE difference of 0 as PES approaches the upper echelons of professional poker players. I did not have access to such players when conducting my experiment, but interesting future work would be to incorporate such players into the sample and assess the true upper bound of CE difference.

7 Conclusion

This work reports on a number of key contributions. First, I find evidence that Allais' Paradox and sub-proportionality hold for small-payoff simple gambles with comparable expected values in the domain of gains, in a more pronounced manner than Fan, 2002 finds in his work. Second, I find that when I express those simple gambles as equivalent poker scenarios, sub-proportionality and the common ratio effect are violated, due to framing effects that cause participants to systematically underestimate true probabilities. Discussion of the underlying psychological factors responsible for this violation remain speculative. They include but are not limited to ambiguity aversion (Ellsberg 1961), and the maxmin expected utility with multiple-priors model (Gilboa & Schmeidler 1989). Third, I find that the more difficult the gamble, proxied by the inexperience of the decision maker, the more ambiguous the gamble becomes. The rest of my analysis admittedly cannot be classified as contributions in a rigorous sense, but are substantive conjecture regarding the violations of sub-proportionality observed. I find positive but diminishing returns to true certainty equivalent assessment from my experience heuristic PES. I construct a simple heuristic for approximating noise quantitatively, perhaps more accurately described as the degree of wrongness in assessing true probability, and show that the magnitude of this heuristic significantly decreases with experience when poker gambles are further from 0. I also use average probability beliefs to conjecture about a solution set of CPT model parameters for the average participant in my experiment.

Despite my attempt at mitigating limitations to these findings, I am not naive to their existence. In conducting this experiment, I was able only to pay participants inconsequential sums of money. There is an existing corpus of work mentioned in the literature review that finds the Allais Paradox to lose descriptive power as gamble payoffs decrease. Secondly, the decisions in my experiment were constructed such that subjects would be near indifference to the gambles. I constructed it this way because at small gain payoffs, even the most risk averse subjects are not risk averse enough to prefer the safer gamble if it's expected value is significantly less than the the risky gamble, so I would expect EUT to dominate, which isn't interesting. The trade off is that near indifference, it can be difficult even for subjects to establish strict preference relations. Another consequence of using small gains which I found empirically to be true was that my risk tolerance heuristic (RTS) provided little explanatory power, simply because subjects are not assessing risk that rigorously at such small payoff levels. Furthermore, I established heuristics, such as N, PES, RTS and probability beliefs which may be viewed as dubious. They are my best attempt at quantifying experimental factors that are qualitative and subjective in nature, but are not founded by any empirical or theoretical consensus. I still argue that at the endpoints, PES is suitable for my needs because I defined it such that PES=0 $\implies$ never having played poker before and PES=10 $\implies$ someone who has played poker for over 3 years and continues to play daily, but near the middle of the scale is where ranking participants by experience becomes more difficult.

In light of these limitations, further work on sub-proportionality in the context of Poker and other games of uncertainty is needed before compelling conclusions can be drawn. Compound invariance in poker gambles needs to be empirically tested, and insights into how my findings respond when the probability scaling factor is varied, when payoffs are increased, and when different frames are applied to hide the true probabilities.

To conclude, I'll re-enforce the motivations of why someone should care about what may seem at first like an unrealistic and oversimplified abstraction of a popular Las Vegas table game. The vast majority of the decisions we make every day are very inconsequential. In most cases, the outcomes of these decisions are only marginally different, payoffs are small, probabilities of desired outcomes are unknown, preferences between options are near indifference, and losses such as time and energy are negligible. My thesis aimed to investigate how well a behavioral economics theory, worthy of a Nobel prize, models these type of decisions, leveraging the setting of Texas Hold'em, where probabilities are subjective yet verifiable. While this paper offers another step in the direction of an improved understanding of decision making under uncertainty in the aforementioned conditions, much still remains to be uncovered.

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Appendix

A IRB Compliance and Information

This experiment was conducted on human subjects, and as such, was subject to review by the Duke Institutional Review Board (IRB). After months of working closely with the campus IRB to ensure a procedure with low/no risk to subjects, I procured approval for my protocol, uniquely identified by IRB Protocol ID 2024-0317. In compliance with the protocol guidelines, the experiment does not report or collect any direct or indirect identifiers. In addition, participants are asked in the survey to acknowledge the participation consent form.

Figure 16: Participation Consent Form

	Key Information
Introduction	Hi. My name is Liam. I'm an undergraduate doing research for my Sr. Honors Thesis.
Purpose/Requirements	I am studying the way we make decisions under incomplete information and uncertainty. In order to participate in this study, participants must be 18 or older and must have at least a cursory familiarity with the rules and hand-rankings of Texas Hold'em
Procedures	If you agree to participate, I will ask you to make 6 total decisions about which gamble you prefer, and then we will play out 3 of the gambles for the prospect of winning real money. Your participation will take about 10 minutes.
Risks	There are no psychological or financial risks to you as a result of this experiment; you can only win money.
Benefits	There are no benefits to you besides the utility you may receive from winning a gamble
Confidentiality	I will provide you with my venmo username so that you may request your winnings from the experiment, and thereafter no identifying information will be collected or used in the analysis of this study. The researcher's Venmo account is set to private to help protect the privacy of the research participant's payout
Voluntariness	It is completely up to you whether to participate. You may skip questions or withdraw at any time for any reason.
Compensation	You will receive the amount you win from the gambles you choose.
Questions	If you have any questions about this research, please ask me now. If you have questions at a later time, you can contact me at liam.clark@duke.edu, or you can also contact my faculty advisor, Philipp Sadowski, Ph.D. at p.sadowski@duke.edu .For questions about your rights, contact the Duke University Institutional Review Board at 919-684-3030 or campusirb@duke.edu. To facilitate a response, let them know that you have participated in a student for protocol #2024-0314

B Data Collection

The study was conducted over the course of several weeks at Duke University. Leveraging email channels of Duke University Student Unions, particularly Economics and Computer Science, students were directed to the Bryan Center Plaza, where I was conducting the experiment on a rolling basis in person. The sample population of Duke students (largely undergraduates studying Economics and Computer Science) self selected to participate in the experiment, which may skew the sample towards those interested in poker and outgoing enough to sit down and engage in an experiment ran by a student they did not know. The BC Plaza is a site on Duke's campus that receives large traffic during the day and has presentation tables available to be rented for 3 hour periods. As such, there were many participants who were just passing by and had their interest peaked.

C Poker Probability Derivations

Decision 3A: Given that you begin with a pair of 2s, there are two scenarios in which you can make a full house after the first three community cards. The first way is if one of the community cards is 2, and other two are of equal rank that is not 2. The second way is if all three community cards are equal rank. Let's count the number of ways this can happen, taking into account that order does not matter. There are $\binom{2}{1}$ 2's left in the deck. For the other 2 community cards, there are $\binom{12}{1}$ other ranks in the deck, and we there are $\binom{4}{2}$ ways we can have 2 among the 4 cards of each rank appear in the community cards. This implies that there are $\binom{2}{1} * \binom{12}{1} * \binom{4}{2}$ ways that the first scenario can occur. For the second scenario, there are $\binom{12}{1}$ other ranks in the deck, and for each there are $\binom{4}{3}$ ways that 3 cards can be selected out of 4 to appear in the community cards. This implies that there are $\binom{12}{1} * \binom{4}{3}$ ways the second scenario can occur. The sum of the ways scenario 1 and scenario 2 can occur is equal to the total number of favorable scenarios. For the total number of possible scenarios, there are $\binom{50}{3}$ ways we can draw 3 of the 50 remaining cards in the deck. Since all scenarios are equally likely, the expression for the raw probability of success in this gamble is:

$$\frac{\binom{2}{1} * \binom{12}{1} * \binom{4}{2} + \binom{12}{1} * \binom{4}{3}}{\binom{50}{3}} = \frac{192}{19600} = 0.009796$$