Clean Growth: Emissions taxes and endogenous technological change

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Abstract

This paper studies the long-run effects of imposing a tax on the emissions from energy usage. We impose an endogenous tax on polluting emissions from energy use in a model where energy firms can do R&D in order to clean up their energy product. The model is an extension of that developed by Peretto (2007), who found that imposing an exogenous tax on energy could cause a long-run increase in welfare, because labor originally employed in energy production would be reallocated toward productivity enhancing R&D in manufacturing. In this paper, the emissions tax once again causes labor to be reallocated away from energy production to final goods production and R&D, but some of that R&D is emissions reducing rather than productivity enhancing. This dampens the positive long-run welfare effect, because it reduces the impact of the tax on manufacturing firms. The model does not include any preference for environmental quality: were such a preference included, the effect on welfare could instead be positive. [1]

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1 Introduction

The subject of emissions taxes is very relevant to the current debate on pollution and global warming. The primary reason is that imposing an emissions tax gives firms an incentive to reduce their use of inputs that pollute. In particular, firms have an incentive to improve their production technologies. Hicks called it the induced innovation hypothesis:

> A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind – directed to economizing the use of a factor which has become relatively expensive [Hicks 1932, pp. 124-125]

This paper studies the effects of imposing such an emissions tax in the form of a per-unit tax on energy, in an economy with endogenous growth, and R&D. It is focused specifically on the long-run effects on welfare and the level of technological innovation in reducing pollution. To investigate these effects, we extend a model of energy taxation developed by Peretto (2007) to include energy cleanliness research.

The model consists of a single household selling a growing supply of labor to a manufacturing sector and an energy sector. The manufacturing sector is in monopolistic competition, with each firm producing a different good (and the household has a preference for variety). Manufacturing firms use energy and labor as inputs to production. Additionally, they hire labor to do productivity enhancing R&D. There are knowledge spillovers in this sector, such that productivity grows endogenously; this captures the sector's improving ability to do research as the overall knowledge level increases.

The energy sector is structured as a duopoly. Energy firms hire labor and import oil in order to produce energy, and additionally can hire labor to do cleanliness R&D. Cleanliness R&D refers in this paper to research undertaken by the energy sector to reduce the pollution tax associated with using its product. It does not increase productivity in the same way as R&D in the manufacturing sector does. The model does not include differentiated energy options, so the government simply taxes at the average pollution level.

Peretto found that imposing an energy tax in his model, without the possibility of cleanliness R&D, could lead to a positive effect on welfare in the long-run. The tax reduces energy demand, which causes a reallocation of labor away from energy toward the manufacturing sector, to be used in production and in productivity enhancing R&D. In the long-run, the increase in productivity offsets the short-run decrease in consumption caused by the higher energy price. Steady-state consumption is therefore increased due to the energy tax.

In the case of an emissions tax, the availability of cleanliness R&D modifies these effects. Upon imposition of the tax, some of the labor that was reallocated to the manufacturing sector is now allocated to cleanliness R&D instead, so that less productivity-enhancing R&D is undertaken. In this paper, we find that cleanliness R&D has a positive long-term growth rate, implying that investment in cleanliness R&D is definitely occurring. In steady-state, then, the positive welfare effect that existed in the energy tax case is dampened. This economy does not, however, include a preference for environmental quality in the household's utility function. Were there such a preference, the dampening effect might no longer occur, if the added welfare from environmental utility offset the dampening effect of R&D substitution.

The rest of the paper is structured as follows. Section 2 discusses some of the relevant literature. Section 3 sets up the model. Section 4 constructs the general equilibrium of this economy. Section 5 analyzes the steady-state outcomes. Section 6 draws conclusions from this analysis and offers suggestions for future research.

2 Literature

The reader completely unfamiliar with endogenous growth theory is encouraged to consult Barro & Xala-i-Martin (2003) or Aghion & Howitt (1998) for an overview of the topic.

This paper is primarily based upon Peretto (2007), which has already been introduced. He presents a model of a small open economy where a single dynastic household supplies labor to energy and manufacturing sectors. Market structure is endogenous, so that entry into the manufacturing sector and R&D to improve total factor productivity (TFP) are both present. He finds that while the introduction of an energy tax does not affect the steady-state growth rate, labor substitution away from an energy sector in which R&D is unavailable (imported oil) to the manufacturing sector where it can increase firm-specific TFP can cause a transient increase in the growth rate of TFP in the economy. While there will be a short-term decrease in welfare due to increased after-tax energy prices leading to increased prices of consumer goods, the price reduction at the end of the transition drives consumption upward into a higher steady-state than the initial case with zero tax.

There has been rapid (and fairly recent) progress on modeling of energy use and conservation in economies with endogenous growth. Smulders and de Nooij (2003) introduced a model of energy conservation in which both the rate and direction of technological progress are endogenous. "The model captures four main stylized facts: total energy use has increased; energy use per hour worked increased slightly; energy efficiency has improved; the value share of energy in GDP has steadily fallen" (abstract). They find that policies that reduce level of energy use do not affect long-run growth, while policies that reduce the growth rate of energy inputs reduce long-run growth. Both types of policy reduce output levels. The model presented here is of the first type - we therefore expect to see long-run growth unchanged by the tax (as is the case in Peretto's model).

3 Model Setup

3.1 The Household

This section is unchanged from Peretto (2007):

The economy consists of a single representative household, which provides a growing supply of labor. It supplies this labor in a competitive market, however supply is inelastic.

$$\dot{L} = L_0 e^{\lambda t} \tag{1}$$

The household maximizes lifetime utility:

$$U(t) = \int_t^\infty (e^{-(\rho-\lambda)(s-t)} \log u(s) ds), \rho > \lambda > 0. \tag{2}$$

It has preferences over a continuum of differentiated consumption goods, and gains utility from consuming as many different types of goods as possible. Its instantaneous preferences are as follows:

$$\log u = \log \int_0^N (\frac{X_i}{L})^{\frac{\epsilon-1}{\epsilon}} di^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1 \tag{3}$$

However the household's consumption decisions are constrained by the asset flow equation:

$$\dot{A} = rA + WL + \tau E + \Pi_E - Y, \tau \geq 0 \tag{4}$$

This equation says that the household can borrow and lend in a competitive financial market. It receives interest on its loans (assets) rA, a wage for labor sold WL, a lump-sum payment of the energy tax τE from the government, and dividends from the energy duopoly Π_E, and Y is total household expenditure (these variables are all presented in more detail shortly).

The reservation rate of return for this household is:

$$r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda \tag{5}$$

so the household saves if assets earn above this rate.

Y is total expenditure:

$$Y = \int_0^N P_i X_i di, \tag{6}$$

where P_i is the price of a differentiated consumption good, and X_i is the amount of the good consumed.

The household maximizes (3) subject to (6) to get the following demand schedule for consumptions goods X_i:

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \tag{7}$$

3.2 Manufacturing sector

The manufacturing sector's setup is also identical to that of Peretto (2007), with the exception of τ which is now an endogenous tax on the pollution caused by using a unit of energy (abatable by using cleaner energy), rather than an exogenous per-unit tax on energy as in the original model.

Manufacturing firms produce differentiated goods using labor and energy as inputs. In addition to producing consumption goods, labor is also used to establish new businesses (for entrants into the market) and to undertake cost-reducing R&D.

Each firm produces a single consumption good:

$$X_i = Z_i^\theta F(L_{X_i} - \phi_i, E_i), 0 < \theta < 1, \phi > 0 \tag{8}$$

X_i is output of good i. L_{X_i} is employment of labor for final goods production, ϕ is a fixed labor cost, E_i is the energy consumed in this process.

Z_i^θ is the firm's Total Factor Productivity or TFP. The firm can do R&D to improve its TFP by increasing its stock of firm-specific knowledge, Z_i.

The total cost of production for a manufacturing firm is:

$$TC_X = W\phi + C_X(W, P_E + \tau)Z_i^{-\theta}X_i \tag{9}$$

where C_X is a unit-cost of production function. θ is a constant representing the elasticity of unit-cost reduction with respect to knowledge. τ represents the per-unit tax on energy emissions. This tax is a function of the cleanliness of the energy being employed, and will be further developed in the subsection on the energy sector.

The firm uses the following R&D technology to generate firm-specific cost-reducing knowledge:

$$\dot{Z} = \alpha K L_{Z_i}, \alpha > 0. \tag{10}$$

αK is the productivity of labor in cost-reducing R&D. α is an exogenous parameter, and K is the stock of public knowledge as follows:

$$K = \int_0^N \frac{1}{N} Z_i d_i \tag{11}$$

Public knowledge accumulates because when a single manufacturing firm undertakes cost-reducing R&D, they also generate non-excludable public knowledge, or spillovers, which enhance the productivity of future research. This is a form of economy-wide learning by doing: as the economy does more cost-reducing R&D, it gets better at it.

3.3 Energy firms - and energy taxes

The energy sector consists of two energy firms operating in a Cournot duopoly. This differs from Peretto (2007), where a competitive market structure is assumed in the energy sector. This duopoly structure was chosen over a single monopolist because a manufacturing sector with inelastic demand for energy would cause the energy producer to price exorbitantly. However, some sort of imperfect market structure is required in order for the energy sector to have an incentive to do R&D, which is funded from monopoly rents. In perfect competition, these rents are non-existent, thus no cleanliness R&D would take place.

The energy duopoly firms hire labor and purchase oil in order to produce energy. Each firm thus produces energy with the technology $E = G(L_E, O)$ where L_E is labor devoted to energy production and O is the oil input. The associated total cost is

$$TC_E = C_E(W, P_O)E \tag{12}$$

The economy imports oil (or some raw material) from an infinitely elastic international supply. The price of oil is therefore constant, and is an exogenous parameter in our model. There exists no domestic supply of oil, so all energy production requires oil purchases from abroad. The unit cost of energy $C_E(W, P_O)$ is a function of constants, so we can consider this to be a fixed parameter in this model, hereby referred to as just C_E.

The new feature introduced to the model here (relative to Peretto (2007)) is that the per-unit energy tax imposed on manufacturing firms is no longer a constant exogenous parameter. Rather it is a function of J, a variable representing the "cleanliness" of energy.

Firms in the energy sector can make quality improvements to their product by reducing the associated per-unit emissions levels. This type of quality improvement is exemplified in "developing gas that burns cleaner." These quality improvements are equivalent (in this model) to decreasing the tax burden on firms purchasing energy (the manufacturing sector).

Manufacturing firms experience the tax as

$$\tau = J^{-\zeta}\tau_0, \tag{13}$$

ζ is an exogenous parameter representing the elasticity of energy tax reduction with respect to knowledge. τ_0 is an exogenous parameter representing the initial value of the per-unit emissions tax. J is the mean cleanliness of the energy product:

$$J = \frac{J_1 + J_2}{2} \tag{14}$$

Energy firms therefore have an incentive to undertake cleanliness R&D because it can increase demand for their product by lowering the unit-cost of energy in the manufacturing firms' production functions.

Each energy firm does its own energy cleanliness R&D as follows

$$\dot{J}_1 = \psi L_{J_1} \tag{15}$$

where J_1 is the amount of cleanliness knowledge developed by the first energy firm, and L_{J_1} is the labor employed to do so. ψ is an exogenous parameter representing the productivity of labor in energy cleanliness R&D.

4 Market equilibrium

This section constructs the market equilibrium for the whole economy. First we characterize the symmetric Nash equilibrium of the manufacturing sector, then the simultaneous equilibrium for the Cournot duopoly in the energy sector. Finally, we impose general equilibrium. The wage rate is the numeraire, i.e. $W \equiv 1$.

4.1 Manufacturing

This section is almost entirely unchanged from Peretto (2007):

The typical manufacturing firm is maximizing the present discounted value of net cash flow:

$$V_i(t) = \int_t^\infty e^{-\int_t^s r(v)dv} \Pi_{X_i}(s)ds \tag{16}$$

V_i is the value of the firm at time t, which is the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technology (10), the demand schedule (7), $Z_i(t) > 0$ (initial knowledge stock is given as non-zero), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given its rivals' innovation paths), and $Z_j(t') \geq 0 fort' \geq t$ (innovation is irreversible). The solution of this problem yields the maximized value of the firm given the time path of the number of firms.

To characterize entry, Peretto follows Etro (2004) and assumes that upon payment of a sunk cost $\beta P_i X_i$, an entrepreneur can start a new firm, with productivity (Z_i) equal to the industry's average. Once in the market, the new firm solves the problem outlined above, so that entry yields value V_i. Free entry equilibrium therefore requires that $V_i = \beta P_i X_i$.

Using the cost function in (9), instantaneous profits are:

$$\Pi_{X_i} = [P_i - C_X(1, P_E + \tau)Z_i^{-\theta}]X_i - \phi - L_{Z_i} \tag{17}$$

The firm's optimization problem in (16) is time-separable, so we can use the Current-Value Hamiltonian to find the optimal strategies:

$$CVH_i = [P_i - C_X(1, P_E + \tau)Z_i^{-\theta}]X_i - \phi - L_{Z_i} + z_i \alpha K L_{z_i} \tag{18}$$

Note that profits are a function of the variable τ. In Peretto's model, τ was a constant. This meant that the firm's utility was a function of scale-free variables and constants. The term "scale-free" is used here to main variables that are firm-specific (i.e. subscripted) or variables that do not scale with the mass of firms. In this extension of the model, however, τ is an inverse function of the economy-wide stock of cleanliness J, which is not scale-free - J grows with aggregate output. (This is shown in section 5).

The first order conditions are:

$$1 = z_i \alpha K \tag{19}$$

This defines the shadow price z_i of cost-reducing R&D (in terms of current-value profit), as an inverse function of the productivity of labor. This is intuitive - higher labor productivity means that the next unit of Z costs less. This condition can be re-phrased as equality between the marginal revenue and marginal cost of knowledge.

$$P_i = C_X(1, P_E + \tau)Z_i^{-\theta} \frac{\epsilon}{\epsilon - 1} \tag{20}$$

This is the firm's pricing strategy. Peretto shows that this pricing strategy, the R&D technology (10), symmetry and aggregation across firms lead to the following conditional factor demands:

$$E = Y \frac{\epsilon - 1}{\epsilon} \frac{S_X^E}{P_E + \tau} \tag{21}$$

$$L_X = Y \frac{\epsilon - 1}{\epsilon} S_X^L + \phi N \tag{22}$$

S_X^E and S_X^L are share of manufacturing costs attributed to energy and labor (these add to 1). These shares are, respectively:

$$S_X^E \equiv \frac{(P_E + \tau)E_i}{C_X(1, P_E + \tau)Z_i^{-\theta} X_i} = \frac{\partial \log C_X(1, P_E + \tau)}{\partial (P_E + \tau)} \tag{23}$$

$$S_X^L \equiv \frac{W L_{X_i}}{C_X(1, P_E + \tau)Z_i^{-\theta} X_i} = \frac{\partial \log C_X(1, P_E + \tau)}{\partial W} \tag{24}$$

Associated to these factor demands are the return to cost reduction and entry respectively:

$$r_Z = \alpha(\frac{Y\theta(\epsilon - 1)}{\epsilon N} - \frac{L_Z}{N}) \tag{25}$$

$$r_N = \frac{1}{\beta}(\frac{1}{\epsilon} - \frac{N}{Y}(\phi + \frac{L_Z}{N})) + \hat{Y} - \hat{N} \tag{26}$$

4.2 Energy sector

We now construct the equilibrium for the energy sector, a Cournot duopoly. Each firm chooses a point on its supply curve simultaneously, taking the other firm's choice as given. Each firm experiences the price of energy as a function of their own energy supply choice and constants.

This is a symmetric Nash equilibrium, so firm 2's choices will be identical. First we invert the energy demand function (21) (to find price as a function of total energy):

$$P_E = \frac{Y^{\frac{\epsilon-1}{\epsilon}} S_X^E}{E} - \tau \tag{27}$$

The manufacturing sector will purchase all the output from both energy producers in equilibrium, so

$$E = E_1 + E_2 \tag{28}$$

Each firm will choose a production point such that the market price of energy delivers optimal profits. Therefore each firm faces an optimization question: how to choose a time series of energy production levels so as to optimize profits?

Instantaneous profits are

$$\Pi_{E_1} = [\frac{Y^{\frac{\epsilon-1}{\epsilon}} S_X^E}{E_1 + E_2} - \tau - C_E]E_1 - L_{J_1}, \tag{29}$$

and each duopoly firm seeks to optimize

$$V_{E_1}(t) = \int_t^\infty e^{-\int_t^s r(v)dv} \Pi_{E_1}(s) ds. \tag{30}$$

We use the current-value Hamiltonian

$$CVH_{E_1} = \Pi_{E_1} + j_1\dot{J}_1 = [\frac{Y^{\frac{\epsilon-1}{\epsilon}}S_X^E}{E_1+E_2} - (\frac{J_1+J_2}{2})^{-\zeta}\tau_0 - C_E]E_1 - L_{J_1} + j_1\psi L_{J_1} \tag{31}$$

where j_1 is the shadow price of a unit of cleanliness knowledge. Here E_1 and L_{J_1} are control variables (chosen by the energy firm at each time step to maximize profit). J_1 is the state variable.

The first order conditions:

$$\frac{dCVH_{E_1}}{dL_{J_1}} = 0 = -1 + j_1\psi \Rightarrow j_1 = \frac{1}{\psi} \tag{32}$$

This tells us that the marginal benefit of investing in cleanliness R&D is the productivity of labor in developing it, and is constant, therefore $\dot{j} = 0$.

$$\frac{dCVH_{E_1}}{dE_1} = 0 = \frac{E_1}{(E_1+E_2)^2}Y\frac{\epsilon-1}{\epsilon}S_X^E - \tau - C_E \tag{33}$$

This gives us the response function for a single energy producer (how much energy firm 1 will produce given firm 2's production choice). These equations are symmetric for firm 1 and 2 so we can solve for a single firm:

$$E_1 = \frac{Y^{\frac{\epsilon-1}{2\epsilon}}S_X^E}{2(\tau+C_E)} \tag{34}$$

From (28) we can obtain the total energy supply:

$$E = \frac{Y^{\frac{\epsilon-1}{2\epsilon}}S_X^E}{(\tau+C_E)} \tag{35}$$

By setting that equal to the energy demand (21), we can obtain the price of energy:

$$P_E = \tau + 2C_E \tag{36}$$

The last first-order condition relates the state variable J to the return on investing in energy cleanliness research:

$$\frac{dCVH_{E_1}}{dJ_1} = r_J j_1 - \dot{j}_1 = \frac{r_J}{\psi} = \frac{\zeta}{2}(\frac{J_1+J_2}{2})^{-\zeta-1}\tau_0 E_1 \tag{37}$$

However we know that in symmetric equilibrium, $J_1 = J_2 = J$, so we can solve for r_J (and simplifying):

$$r_J = \frac{Y}{4(J\tau_0 + J^{\zeta+1}C_E)}\psi\zeta\tau_0\frac{\epsilon-1}{2\epsilon}S_X^E \tag{38}$$

This gives us the return on investing in cleanliness research in terms of J, Y, and constants.

The remainder of this subsection is essentially unchanged from Peretto (2007):

Define the share of oil in energy costs as:

$$S_E^O \equiv \frac{P_O O}{C_E 1, P_O E} = \frac{\partial \log C_E(W, P_O)}{\partial \log P_O} \tag{39}$$

The associated demands for labor and oil are written:

$$L_E = Y\frac{\epsilon-1}{\epsilon}\frac{P_E S_X^E}{P_E+\tau}(1 - S_E^0) \tag{40}$$

$$O = Y\frac{\epsilon-1}{\epsilon}\frac{P_E S_X^E}{P_E+\tau}S_E^0 \tag{41}$$

where S_E^O is share of energy production costs attributed to oil, and $1-S_E^O$ is share of energy production costs attributed to labor (since the shares must add to 1).

Note that P_O, the price of oil, is constant. Therefore $C_E(W, P_O) = C_E(1, P_O)$ is constant. For brevity, it will hereby be referred to as C_E. P_E, the price of energy, however, is not a constant in this model.

4.3 General Equilibrium

We now impose general equilibrium. General equilibrium requires returns equilibration:

$$r = r_J = r_Z = r_A = r_N, \tag{42}$$

subject to the economy-wide budget constraint:

$$L = L_N + L_X + L_Z + L_E + L_O + L_J \tag{43}$$

This budget constraint is in terms of labor (recall that wage $W \equiv 1$ is the numeraire. We can solve the asset flow equation (4) for aggregate labor, substituting to obtain (43). This represents the constraint that labor is supplied inelastically. X represents manufacturing, Z represents manufacturing R&D (increasing total factor production), N represents entry into the manufacturing sector, E represents energy production, O represents oil (we trade labor in exchange for oil), and J represents cleanliness R&D decreasing the tax.

The symmetric Nash equilibria derived earlier gave us:

$$r_A = \rho + \frac{\dot{Y}}{Y} - \lambda \tag{44}$$

$$r_Z = \alpha(\frac{Y\theta(\epsilon - 1)}{\epsilon N} - \frac{L_Z}{N}) \tag{45}$$

$$r_N = \frac{1}{\beta}(\frac{1}{\epsilon} - \frac{N}{Y}(\phi + \frac{L_Z}{N})) + \hat{Y} - \hat{N} \tag{46}$$

We also have r_J from (38)

The set-up and equilibration of the model is now complete.

5 Steady-state

For simplicity, in this paper, we focus purely on the steady state. We take "steady-state" here to mean the state of the economy as $t \to \infty$ where all growth rates are constant. We consider two important variables in steady-state: the growth rate of the aggregate energy cleanliness level ($\hat{J}$), and the level of expenditure per capita.

5.1 Energy cleanliness

In the general equilibrium imposed in the previous section, the returns to saving, entry, cost-reducing R&D, and cleanliness R&D are all equilibrated. Moreover, the returns must all be constant. We will use this fact to investigate steady-state levels of J.

Taking logs and time-derivatives of (38), the equation for r_J, we can obtain an expression for the growth rate of the return to investing in cleanliness R&D. Clearly, in steady-state equilibrium, this must be equal to 0.

$$\frac{\dot{r_J}}{r_J} = 0 = \hat{Y} - \hat{J}\frac{\tau_0 + (\zeta + 1)J^{\zeta}C_E}{\tau_0 + J^{\zeta}C_E} \tag{47}$$

We now use that equality to derive an expression J^*, the steady-state value of J:

$$J^* = \left(\frac{\tau_0(\hat{Y} - \hat{J})}{C_E(\hat{J}(\zeta + 1) - \hat{Y})}\right)^{\frac{1}{\zeta}} \tag{48}$$

This expression shows that there is no constant steady-state value of J. Trying to find one entails setting $\hat{J} = 0$. We can see from (48) that this implies that either $\hat{Y}$ must also be 0, or that $J^* = (-\tau_0)^{\frac{1}{\zeta}}$ which violates the non-negativity constraint on J, or that $\hat{r_J}$ is non-zero, which violates our steady-state assumptions.

Rather, $\hat{J}$ is clearly a function of J, $\hat{Y}$, and constants. This makes sense: the cleanliness tax τ asymptotically approaches 0 as J increases, and therefore so does the marginal value of an added unit of cleanliness knowledge. Population growth is driving steady but positive growth in economy-wide output ($\hat{Y} > 0$), and the demand for cleanliness knowledge is in turn a function increasing in Y. We can solve (47) for $\hat{J}$:

$$\hat{J} = \frac{(\tau_0 + J^\zeta)\hat{Y}}{C_E(\zeta + 1)J^\zeta + \tau_0} = \frac{\tau_0 \hat{Y}}{C_E(\zeta + 1)J^\zeta + \tau_0} + \frac{\hat{Y}}{C_E(\zeta + 1) + \frac{\tau_0}{J^\zeta}} \tag{49}$$

We have from this expression that $\hat{J}$ is positive ($\tau_0, \hat{Y}$, and J are all under non-negativity constraints) for all t. The two terms of the expression on the right can be thought of as the transitional and steady-state components of $\hat{J}$ respectively. The first term is a transitional component in that it is an inverse function of J, which is increasing. Thus in steady state, this transitional term will go to zero. The second term is a steady-state component in that it only depends on the tax level $\frac{\tau_0}{J^\zeta}$, and constants. The tax level also approaches zero over time, for the same reason. Thus in steady-state $\hat{J}$ converges to:

$$\hat{J}^* = \frac{\hat{Y}}{C_E(\zeta + 1)} \tag{50}$$

This is the positive steady-state growth rate for J.

5.2 Per-capita expenditure

We consider the steady-state expenditure per-capita here because it measures the flow of goods (in nominal terms) into the instantaneous utility function

(3), and thus is relevant to drawing conclusions regarding welfare in the presence of the energy tax and emissions-reducing R&D.

We use (4) to define per-capita expenditure as:

$$y^* \equiv \frac{Y}{L} = \frac{1}{1 - \beta(\rho - \lambda) - \tau\frac{E}{Y} - \frac{\Pi_E}{Y}}. \tag{51}$$

The energy supply function (35) is divided through by Y to yield:

$$\frac{E}{Y} = \frac{\frac{\epsilon-1}{\epsilon}S_X^E}{P_E + \tau} = \frac{\frac{\epsilon-1}{2\epsilon}S_X^E}{C_E + \tau} \tag{52}$$

The energy firm's instantaneous profit function in (29) (generalized in symmetric equilibrium to represent a typical energy firm rather than one of two duopolists specifically) is used to obtain:

$$\frac{\Pi_E}{Y} = (P_E - C_E)\frac{E}{Y} - \frac{L_J}{Y} \tag{53}$$

Before the tax is imposed, we have (for $\tau_0 = 0,\ J = 0,\ \hat{J} = 0,\ L_J = 0$):

$$y^*_{pretax} = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{2\epsilon}S_X^E} \tag{54}$$

After the tax is imposed, $\frac{L_J}{Y}$ is no longer zero - we must therefore find a value for this term in steady-state. We use the cleanliness R&D technology (15), the return to investment in cleanliness R&D (38), and the steady-state growth rate of cleanliness R&D (50) to obtain the steady-state ratio between labor in cleanliness R&D and aggregate output:

$$(\frac{L_J}{Y})^* = \frac{\hat{Y}^*}{4\rho}\frac{\epsilon - 1}{2\epsilon}S_X^E(\frac{1}{C_E} - 1) \tag{55}$$

Finally, we substitute into (51) to obtain:

$$y^*_{posttax} = \frac{1}{1 - \beta(\rho - \lambda) + \frac{\epsilon-1}{2\epsilon}S_X^E(\frac{\lambda}{4\rho}(\frac{1}{C_E} - 1) - 1)} \tag{56}$$

We now compare the steady-state expenditure before and after the tax. Some manipulation yields:

$$\frac{4\rho}{\lambda} < \frac{1}{C_E} - 1 \Rightarrow y^*_{posttax} > y^*_{pretax} \tag{57}$$

Thus the net effect of the tax depends on constants. ρ and λ are both positive, so the left-hand size of the inequality must be greater than zero. Therefore, in the case where $C_E(1, P_O) \geq 1$, there will be a decrease in per-capita expenditure due to the tax.

6 Conclusion

This paper investigated the effects of imposing an emissions tax on an economy with endogenous growth and emissions-reducing R&D. They are as follows:

Imposing the tax causes energy demand to decrease. This in turn causes the energy sector's demand for labor in production to decrease. The manufacturing sector substitutes away from energy toward labor in the production of final consumption goods, since the after-tax price of energy relative to labor has increased. The manufacturing sector also employs labor to undertake cost-reducing R&D. This R&D is an engine of endogenous growth, due to knowledge spillovers in the economy. In other words, manufacturing firms get better at doing research as the average stock of knowledge increases. This added productivity can outweigh the effect of the tax to yield an increase in welfare in the long-run.

In addition, however, the energy sector is given the option of improving the cleanliness of the energy it produces by employing labor in emissions-reducing R&D. Energy firms therefore can invest in emissions-reducing R&D, which causes the after-tax price of energy to go down. If they do so, the manufacturing firms experience a lower tax burden, which reduces their incentive to do R&D.

In this paper, it was shown that the steady-state growth rate of cleanliness knowledge is positive. In other words, energy firms do indeed invest in cleanliness R&D, and they continue to do so as the economy grows. The intuition behind this is straightforward: as the economy keeps growing, smaller and smaller decreases to the tax burden on manufacturers are required to maintain equilibrium, even as each additional unit of labor invested into cleanliness R&D yields a smaller decrease in the tax than the last. Therefore the manufacturing firms do indeed experience a lower incentive to do productivity-enhancing R&D (as compared to a model with no cleanliness R&D available).

This paper has been focused on a model where there exists no preference for environmental quality. Indeed, there is indirectly a preference for pollution, as the household receives tax revenues as a lump-sum credit. With the endogenous tax on emissions, tax revenues can be re-interpreted to be the flow of pollution from the economy - and so the more pollution, the higher the household's tax credit.

Introducing a preference for environmental quality would offset this effect and allow the household to capture utility from the fact that the economy undertakes emissions-reducing R&D. While the size of this effect on steady-state welfare cannot be known without further analysis, its direction must be positive, since we are adding a new source of utility. Indeed, a rewarding of extension of this paper would be to analyze the case where the household has a preference for environmental quality. Doing so would make the result presented here more robust, and improve the paper's usefulness in evaluating emissions taxes as a policy alternative.

Another useful (but significantly heftier) undertaking would be to solve for the full transitional dynamics of this extended model, in order to be able to derive aggregate welfare. Obtaining a closed-form solution for the model would also gain us a better understanding of the mechanism driving labor reallocation toward the two different types of R&D. It would also be interesting to compare this model to one like that presented by Peretto (2006), where firms undertake costly abatement to reduce their *own* emissions (as opposed to the current model presented here, where energy firms undertake abatement of downstream firms' emissions). This might allow a comparison of the effectiveness (in terms of welfare maximization) of emissions-reducing R&D in different sectors of the economy.

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